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Production of Strange Particles by 2.77-BeV/c π^+ Mesons on Hydrogen*

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The Brookhaven 20-in. liquid-hydrogen bubble chamber was exposed to a beam of 2.77-BeV/c separated π^+ mesons. In a detailed study involving strange-particle production the well-known K^* (mass 900 MeV) and Y_1^* (mass 1385 MeV) were observed. The former was produced in the final state $\Sigma^+\pi^+K^0$, and the latter in the states $\pi^+\pi^+\Lambda^0K^0$ and $\pi^+K^+\Lambda^0$. It was necessary to include both K and K^* exchanges in order to explain the K^* decay angular distributions. The decay angular distribution of the Y_1^* gave evidence for a strong Y_1^* polarization as well as spin $\geq 3/2$. Furthermore, an examination of the reaction $\pi^+pK^0K^0$ showed that the (π^+p) emerged as $N^*_{3/2,\,3/2}$ coupled with a (K^0K^0) enhancement in the mass region from 1000 to 1040 MeV.

1. INTRODUCTION

A LARGE quantity of data is available concerning the production of strange particles by π^-+p interactions over a wide range of pion momentum.[1] π^++p reactions, on the other hand, have been studied primarily for the study of pion and pion-nucleon resonances,[2] little information being available concerning the production of strange particles, and strange-particle resonances in these reactions.[3] The present work supplements the information on those latter features of π^++p interactions.

2. EXPERIMENTAL SETUP

A 2.77-BeV/c positive π-meson beam was obtained from the Brookhaven (AGS) by using a two-stage separated beam.[4] The beam momentum was determined both by measuring straight through beam tracks, and by varying the initial beam momentum in the kinematic fits of events of known character, and observing their χ^2 variation. The momentum resolution was $\pm 1.5\%$.

A total of about 67 000 pictures was taken in the Brookhaven 20-in. liquid-hydrogen bubble chamber. The pictures were scanned twice for events containing charged and neutral decays. From the results of these two independent scans, the scanning efficiency was determined to be 98%. There was no appreciable difference in the scanning efficiency among the various event topologies.

The scanned events were measured in the standard manner on digitized measuring projectors. The events were then geometrically reconstructed by the BNL-TRED program and kinematically analyzed by the BNL version of KICK on an IBM-7090 computer.

A particular kinematic hypothesis was accepted if: (a) it had a χ^2 probability greater than 1%, and (b) the ionization densities of the tracks involved were consistent with those predicted by this hypothesis. Unless there was a factor of 10 or greater in the χ^2 probabilities among ambiguous kinematic fits for a given event, all hypotheses meeting the above criteria were accepted. The ambiguous events constituted about 30% of the total, excluding the $\Lambda^0-\Sigma^0$ ambiguity, which was close to 80%. For a study of the various effective-mass distributions, however, only unambiguous events were used.

3. CROSS SECTIONS

Many of the events had multiple fits as mentioned in the previous section. This was primarily due to the difficulty of measuring short tracks in the events with charged decays. For the purpose of cross-section calculations, the ambiguous events were apportioned according to the ratios of the unambiguous events. A check was made on the consistency of such a ratio by taking a ratio based on the χ^2 probabilities, where the fit with the better χ^2 was taken as the only fit. These ratios were in good agreement with those based on unambiguous events for all hypotheses.

A further correction was necessary for events which fit as $\Sigma^+\pi^+K^0$ with an unseen K^0. This was due to large errors in the measurement of the direction and energy of the Σ^+, resulting in a large error for the associated missing mass. Thus, an event which was actually $\Sigma^+\pi^+K^0\pi^0$ could also fit as the hypothesis $\Sigma^+\pi^+K^0$. It is not possible to kinematically fit the first reaction, since it involves two unseen particles. However, an estimate of the number of unfitted $\Sigma^+\pi^+K^0\pi^0$ was made from the number of events which fit as $\Sigma^+\pi^+K^0\pi^0$ with a visible K^0. Furthermore, an ideogram was made of the square of missing mass from the events which fit as $\Sigma^+\pi^++$ unseen K^0. This ideogram had a peak at the K^0 mass, and a shoulder on the high-mass side of the peak starting at

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[1] See for example, Ref. 1 of L. Bertanza, B. B. Culwick, K. W. Lai, I. S. Mittra, N. P. Samios *et al.*, Phys. Rev. **130**, 786 (1963).

[2] See for example, C. Alff, D. Berley, D. Colley, N. Gelfand, U. Nauenberg *et al.*, Phys. Rev. Letters **9**, 322, 325 (1962); A. Abolins, R. L. Lander, W. A. W. Melhop, N. h. Xuong, and P. M. Yager, *ibid.* **11**, 381 (1963).

[3] C. Baltay, H. Courant, W. J. Fickinger, E. C. Fowler, H. L. Kraybill *et al.*, Rev. Mod. Phys. **33**, 374 (1961); H. W. J. Foelsche and H. L. Kraybill, Bull. Am. Phys. Soc. **8**, 342 (1963); D. Berley and N. Gelfand, Post-deadline paper, Washington Meeting of the American Physical Society, 1963 (unpublished).

[4] C. Baltay, J. Sandweiss, J. Sanford, H. Brown, M. Webster, and S. Yamamoto, Nucl. Instr. Methods **20**, 37 (1963).

TABLE I. Cross section for production of various final states by 2.77-BeV/c π^+ on protons.

Final state	σ in mb
$K^+\Sigma^+$	0.105 ± 0.010
$K^+\Sigma^+\pi^0$	0.120 ± 0.010
$\pi^+\Sigma^+K^0$	$0.09\ \pm0.010$
$\pi^+\Sigma^+K^0\pi^0$	0.037 ± 0.006
pK^+K^0	0.059 ± 0.013
$pK^+K^0\pi^0$	0.022 ± 0.008
$\pi^+K^+\Lambda^0/\Sigma^0$ [a]	0.163 ± 0.018
$\pi^+K^+\Lambda^0/\Sigma^0\pi^0$	0.084 ± 0.013
$\pi^+\pi^+\Lambda^0/\Sigma^0K^0$	0.085 ± 0.012
$p\pi^+K_1{}^0K_2{}^0$	0.013 ± 0.005
$p\pi^+K_1{}^0K_1{}^0$	0.005 ± 0.003
$\pi^+K^+K^0n$	0.019 ± 0.007
$\pi^+\pi^+K^+\Sigma^-$	0.013 ± 0.004
$\pi^+\pi^-K^+\Sigma^+$	0.007 ± 0.003

[a] Λ^0/Σ^0 stands for Λ^0 or Σ^0.

about 0.4 BeV^2, which corresponds to the mass of $K^0+\pi^0$. An estimate was made of the $\Sigma^+\pi^+K^0\pi^0$ contamination from this ideogram. Both estimates were in good agreement.

Events with a visible Λ^0 decay were almost always ambiguous between Σ^0 and Λ^0 hypotheses. Since it was usually impossible to distinguish between them in this momentum region, no apportionment was made.

The pion-beam flux was estimated by using the biased sample method.[5] The muon contamination was estimated to be less than 5%. The proton contamination was negligible because of the two stages of mass separation. Only one K^+ decay in flight was observed in the whole exposure, thus demonstrating the K^+ contamination to be also negligible.

Table I lists the cross sections for the observed channels. No correction was made for the muon contamination, and the quoted errors are statistical.

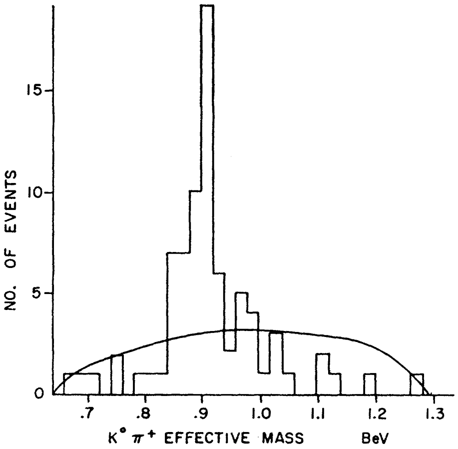

FIG. 1. The $K^0\pi^+$ effective mass from $\pi^++p\rightarrow\Sigma^++\pi^++K^0$. The solid curve is phase-space normalized to the total number of events.

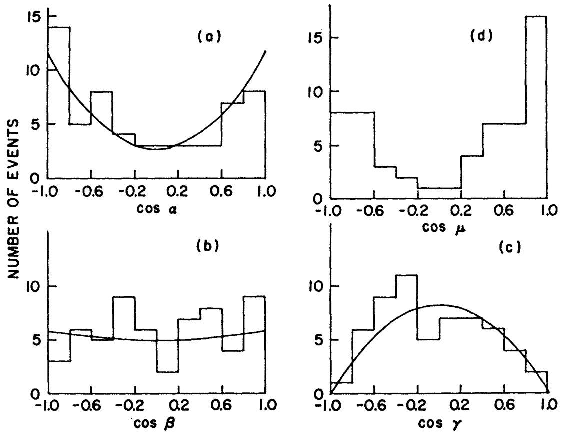

FIG. 2. (a) K^* decay relative to the normal $\hat{n}$ of the Σ^+K^* production plane. The solid curve is a least-squares fit to $(2.79\pm0.32)+(8.87\pm0.98)\cos^2\alpha$ with a χ^2 of 5.4 for 8 deg of freedom (b) K^* decay relative to $\hat{I}$, the direction of the incident π^+ in the K^* rest frame. The solid curve is a least-squares fit to $(4.95\pm0.86)+(0.96\pm1.98)\cos^2\beta$ with a χ^2 of 11.8 for 8 deg of freedom. (c) K^* decay relative to $\hat{I}\times\hat{n}$. The solid curve is a least-squares fit to $(8.15\pm0.84)-(8.18\pm1.32)\cos^2\gamma$ with a χ^2 of 5.6 for 8 deg of freedom. (d) K^*-production angular distribution in the π^++p center-of-mass system.

4. PRODUCTION OF RESONANCES

The following reactions were studied in detail for the production of strange-particle resonances.

$$\pi^++p\rightarrow\Sigma^++\pi^++K^0\,, \quad (1)$$

$$\rightarrow\Sigma^++K^++\pi^0\,, \quad (2)$$

$$\rightarrow\pi^++\pi^++K^0+\Lambda^0\,, \quad (3)$$

$$\rightarrow\pi^++K^++\Lambda^0\,, \quad (4)$$

$$\rightarrow\pi^++p+K^0+\bar{K}^0. \quad (5)$$

A. Reactions (1) and (2).

Figure 1 shows a histogram of the $K^0\pi^+$ effective mass from reaction (1). It is clear that the K^* of mass 900 MeV is strongly produced in this reaction.

It is of interest to compare the reaction (a) $\pi^++p\rightarrow\Sigma^++K^{*+}$ with the reaction (b) $\pi^-+p\rightarrow\Sigma^0+K^{*0}$, since both reactions can be analyzed in terms of a mixture of K and K^* exchange. If the same mechanism is responsible for the production of the K^* in these two reactions, the K^* decay angular distributions, and the K^* production angular distribution from them should be similar. Smith *et al.*[6] made a detailed study of the reaction (2) at π^- momenta of 2.17 and 2.25 BeV/c. In order to study the K^* decay characteristics they defined a set of coordinate axes in the rest frame of the K^*;

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[5] F. S. Crawford, Jr., Rev. Sci. Instr. **30**, 1096 (1959).

[6] G. A. Smith, J. Schwartz, D. H. Miller, G. Kalbfleisch, R. W. Huff, O. I. Dahl, and G. Alexander, Phys. Rev. Letters **10**, 138 (1963).

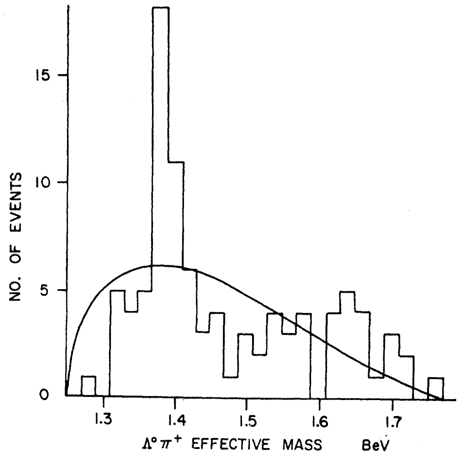

FIG. 3. The $\Lambda^0\pi^+$ effective mass from $\pi^++p\rightarrow\pi^++\pi^++\Lambda^0+K^0$. The solid curve is phase space, normalized to the total number of events.

namely, the production normal $\hat{n}$, the direction $\hat{I}$ of the incident pion in the K^* rest frame, and the direction defined by $\hat{I}\times\hat{n}$. The quantities $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are then the direction cosines of the decay pion from the K^* with respect to $\hat{n}$, $\hat{I}$ and $\hat{I}\times\hat{n}$. Since the spin and parity of the K^* is 1^-,[7] the conservation of angular momentum and parity enables one to predict that the angular distribution of $\cos\beta$ will be of the form $\cos^2\beta$, if a K meson is exchanged, and of the form $\sin^2\beta$, if a K^* is the exchanged particle. In general the decay amplitude for a mixture of K and K^* exchange will be:

$$\Psi=AY_1^1+BY_1^{-1}+CY_1^0, \qquad (1)$$

where A and B are K^* exchange amplitudes, and C the K exchange amplitude. A, B and C are normalized to unity; i.e.,

$$|A|^2+|B|^2+|C|^2=1.$$

From the observed $\cos^2\beta$ angular distribution, Smith *et al.* deduced that the ratio of K to K^* exchange was 0.45 ± 0.10, and using this ratio they were able to predict the form of $\cos\gamma$ and $\cos\alpha$ angular distributions, which were in agreement with their experimental observation.

Figures 2(a), 2(b), and 2(c) show our decay angular distributions for events with $M_K{}^*$ lying between 860 and 940 MeV. They were fitted to a form $A+B\cos^2\theta$ by the least-squares method. These angular distributions are quite similar to those obtained by Smith *et al.* If we assume the amplitudes A, B, and C in Eq. (1) to be real and constant, the form of the $\cos\beta$ distribution will be

$$(1-C^2)+(3C^2-1)\cos^2\beta.$$

The least-squares fit to the observed $\cos\beta$ distribution is $(4.95\pm0.86)+(0.98\pm1.98)\cos^2\beta$, which is consistent with the $\cos^2\beta$ term equal to zero. This leads to a value of $\frac{1}{3}$ for C^2, and K to K^* exchange ratio of 0.5, which is in agreement with the result of Smith *et al.*

Figure 2(d) is the K^* production angular distribution in the reaction center of mass, which deviates substantially from that obtained by Smith *et al.* Without better statistics, however, it will be difficult to determine the significance of this discrepancy.

No evidence was found for a $\Sigma^+\pi^+$ resonance. This is in agreement with the results of Kalbfleisch *et al.*[8]

An effective mass plot of $K^+\pi^0$ from the reaction (2) shows evidence for the production of the K^* with its mass centered at 910 MeV. The high mass is probably due to the contamination of this channel by $\Sigma^+K^+\pi^0\pi^0$ events.

B. Reaction (3).

The reaction (3) is almost always indistinguishable from $\pi^++\pi^++\Sigma^0+K^0$. This, however, does not present a serious problem to the study of $\Lambda^0\pi^+$ resonances in this reaction because: (a) the branching ratio of the well-known Y_1^* at 1385 MeV into $\Sigma\pi$ is less than 6% of the total rate,[9] and (b) no other strong $T=1$ $\Sigma^0\pi^+$ resonance has been observed. Figure 3 shows a histogram of the $\Lambda^0\pi^+$ effective mass. The well-known Y_1^* appears as a peak at a mass of 1385 MeV. In order to check for the effects of the Σ^0 events, the $\Sigma^0\pi^+$ effective mass was also plotted assuming these events to be $\pi^+\pi^+\Sigma^0K^0$. This plot showed no enhancement at 1385 MeV, and a peak was observed at 1450 MeV, which corresponds to a shift in the effective mass due to the $\Sigma^0-\Lambda^0$ mass difference.

There was no indication of K^* production in the

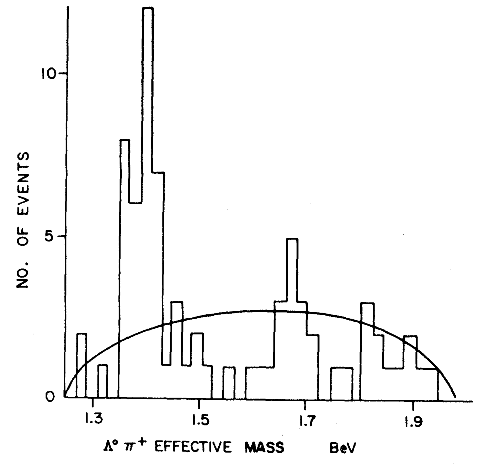

FIG. 4. The $\Lambda^0\pi^+$ effective mass from $\pi^++p\rightarrow\pi^++K^++\Lambda^0$. The solid curve is phase space normalized to the total number of events.

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[7] R. Armenteros, L. Montanet, D. R. O. Morrison, S. Nilsson, A. Shapira *et al.*, in *Proceedings of the International Conference on High Energy Nuclear Physics, Geneva, 1962* (CERN Geneva, Switzerland, 1962), p. 295; W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee and T. O'Halloran, Phys. Rev. Letters **9**, 330 (1962).

[8] G. R. Kalbfleisch, G. Alexander, O. I. Dahl, D. H. Miller, A. Rittenberg, and G. A. Smith, Phys. Letters **4**, 225 (1963).

[9] P. Bastien, M. Ferro-Luzzi, and A. H. Rosenfeld, Phys. Rev. Letters **6**, 702 (1961).

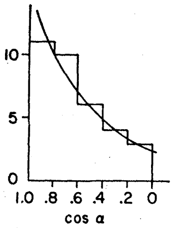

FIG. 5. Folded Y_1^*-decay distribution relative to $\hat{n}$, the normal to K^+Y^*-production plane. The solid curve is a least-squares fit to $(3.07\pm0.55)+(11.04\pm1.72)\cos^2\alpha$ with χ^2 of 0.36 for 3 deg of freedom.

π^+K^0 effective-mass plot. In particular no double resonance such as Y^*+K^* was seen.

C. Reaction (4).

Figure 4 is a histogram of the $\Lambda^0\pi^+$ effective mass from the reaction (4). The 1385-MeV Y_1^* is evident. A plot, as in the previous section, of the $\Sigma^0\pi^+$ effective mass showed that the 1385-MeV peak appeared shifted by the $\Sigma^0-\Lambda^0$ mass difference.

The reaction $\pi^++p\rightarrow Y_1^*+K^+$ is a two-body reaction which is free from $K\pi$ resonances since no $T=\frac{3}{2}$ K^* has been reported. Hence, it could serve as a useful tool in investigating the properties of the 1385-MeV Y_1^*. Figure 5 shows the up-down decay distribution of the π^+ from the Y_1^* in the Y_1^* mass region from 1330 to 1430 MeV. The up-down angle is defined as the angle between the decaying π meson from the Y_1^* in the Y_1^* rest frame and the normal to the $K^+-Y_1^*$ production plane. The distribution was folded to improve the statistics, but the unfolded distribution was symmetric about 0^0 within statistics. The folded distribution was fitted to a curve of the form $A+B\cos^2\alpha$ by the least-squares method. The values of A and B were 3.07 ± 0.55 and 11.04 ± 1.72, respectively. Barring a D-wave interference from nonresonant states,[10] such a distribution indicates a strong Y_1^* polarization, and a Y_1^* spin of $\frac{3}{2}$ or greater. This confirms the results of previous experiments.[11] Stodolsky and Sakurai[12] have pointed out that such a distribution is expected from a spin $\frac{3}{2}$ Y^*, if its production goes via K^* exchange, analogous to the case of the production of $N^*_{3/2,3/2}$ via ρ exchange in π^++p interactions. In either case, under such an assumption, the Adair distribution[13] is not expected to be of the form $1+3\cos^2\theta$, but rather $1-\frac{3}{5}\cos^2\theta$, for a spin $\frac{3}{2}$ isobar or Y^*. We were unable to check the Adair distribution for this case because of the small number of events.

Stodolsky and Sakurai also suggested that a Yang-Treiman angular distribution[14] would have the form $1+2\sin^2\phi$ for this reaction, if the Y^* spin is $\frac{3}{2}$. A plot of the Yang-Treiman angle ϕ at the proton vertex showed an anisotropic distribution with more events in the region around 90° than in the regions near 0° and 180°. This is indicative of an exchange of a particle with a nonzero spin. Because of poor statistics any fit to the $\cos\phi$ distribution would not be significant.

D. Reaction (5).

The cross section for the reaction (5) is very small as seen in Table I, and only eleven unambiguous events were observed. Even in this small sample of events, however, there are some characteristic features which we wish to point out. These characteristics are best illustrated in Fig. 6 which is a scatter diagram of π^+p and K^0K^0 effective masses. Of eleven events seven are contained in a region bounded by 1000 MeV $\leq M_{K^0K^0}$ $\leq$1040 MeV, and 1180 MeV $\leq M_{\pi p}\leq$1280 MeV, whereas phase space predicts 1.5 events in this $M_{K^0K^0}$ region. The K^0K^0 enhancement corresponds to the recently discovered ϕ meson.[15] The second corresponds to the mass of $N^*_{3/2,3/2}$. We must now examine the nature of the K^0K^0 pairs we observed. Of seven events inside the boundary, one had two V's associated with it, thus making it a $K_1^0K_1^0$ pair. Of the remaining events, which all had one visible K^0, one should be a $K_1^0K_1^0$ pair in which one of the K^0's decays via the neutral mode. The remaining events should be $K_1^0K_2^0$ pairs. Of four events outside the boundary one was a $K_1^0K_1^0$ pair with two visible K^0's.

It may be premature to speculate on the basis of such meager statistics, but we wish to point out that if the ϕ meson was produced in this reaction, it was produced predominantly in association with $N^*_{3/2,3/2}$ in a two-body reaction $N^*+\phi$. Given a large enough number of events such a reaction may be analyzed in terms of ρ exchange, for example, and the properties of the ϕ meson may then be studied from its decay characteristics. If there is any enhancement in the $K_1^0K_1^0$ pairs, this might correspond to entirely different mechanisms such as the S-wave scattering enhancement observed by Erwin *et al.*[16] and Alexander *et al.*[17]

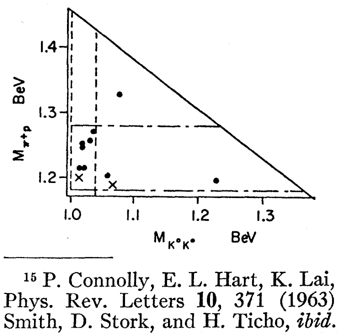

FIG. 6. Scatter diagram of M_{π^+p} and $M_{K^0\,K^0}$ from $\pi^++p\rightarrow\pi^++p+K^0+\bar{K}^0$. The dots are events with one visible K^0, and the crosses are events with two visible K^0's.

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[10] R. K. Adair, Rev. Mod. Phys. **33**, 406 (1961).

[11] L. Bertanza, V. Brisson, P. L. Connolly, E. L. Hart, I. S. Mittra *et al.*, Phys. Rev. Letters **10**, 176 (1963); J. B. Shafer, J. J. Murray, and D. O. Huwe, *ibid.* **10**, 179 (1963); R. P. Ely, S. Y. Fung, G. Gidal, Y. L. Pan, W. M. Powell, and H. S. White, *ibid.* **7**, 461 (1961); D. Colley, N. Gelfand, U. Nauenberg, J. Steinberger, S. Wolf *et al.*, Phys. Rev. **128**, 1930 (1962).

[12] L. Stodolsky and J. J. Sakurai, Phys. Rev. Letters **11**, 90 (1963).

[13] R. K. Adair, Phys. Rev. **100**, 1540 (1955).

[14] S. B. Treiman and C. N. Yang, Phys. Rev. Letters **8**, 140 (1962).

[15] P. Connolly, E. L. Hart, K. Lai, G. London, G. Moneti *et al.*, Phys. Rev. Letters **10**, 371 (1963); P. Schlein, W. Slater, L. Smith, D. Stork, and H. Ticho, *ibid.* **10**, 368 (1963).

[16] A. R. Erwin, G. A. Hoyer, R. H. March, W. D. Walker, and T. P. Wangler, Phys. Rev. Letters **9**, 34 (1962).

[17] G. Alexander, O. I. Dahl, L. Jacobs, G. R. Kalbfleisch, D. H. Miller *et al.*, Phys. Rev. Letters **9**, 460 (1962).

It is of interest to note that the $T=1$ $K^+\bar{K}^0$ effective masses from the final states $K^+p\bar{K}^0$, $K^+\pi^+\bar{K}^0n$, and $K^+p\bar{K}^0\pi^+$ showed no enhancement.

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Dynamical Model of the 1535-MeV Ξ* Hyperon*

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An attempt is made to understand the dynamical origin of the 1535-MeV Ξ* hyperon. We study the problem of scattering in the $J=\frac{3}{2}^+$-state of the $\bar{K}\Lambda$-channel by the N/D method. The influence of all other channels are ignored. The dynamical singularities of the partial-wave amplitude are assumed to arise mainly from the near by cut (due to the nucleon exchange in the crossed channel) and the far left-hand cut $(-\infty<S\leqslant 0)$. The contribution of the former is evaluated explicitly in terms of the ΛNK coupling constant and that of the latter, by the method of Balázs, through the introduction of the effective range pole terms. The analysis is found to be quite insensitive to the choice of the ΛNK coupling constant. The D function is found to have the desired behavior for the occurrence of a resonance or bound state in the energy region of interest. The main force for the existence of the resonance or bound state seems to arise from the singularities associated with the far left-hand cut. The best set of self-consistent solutions for the position and residue indicates the presence of a bound state in the $J=\frac{3}{2}^+$ state of $\bar{K}\Lambda$ system at $S_R\approx 109\,m_\pi^2$ with residue $\mathrm{K}_{\Xi^*}\approx 10$.

I. INTRODUCTION

THE discovery of the Ξ* hyperon[1] with $S=-2$ and $I=\frac{1}{2}$ at 1535 MeV fits beautifully with the scheme of SU(3) symmetry.[2] In order that it may be identified as a member of the tenfold representation,[3] to which the (3,3) πN resonance belongs, its spin and parity should be $\frac{3}{2}^+$, which also seems to be true from the recent UCLA experiment.[4] The present paper is an attempt to account dynamically for the existence of Ξ*. In the present paper, we will confine our attention to the $J=\frac{3}{2}^+$ state only and examine whether in this state one should expect a resonance or bound state (depending upon which channel one is considering) at a mass around 1535 MeV with $I=\frac{1}{2}$, $S=-2$, and $Y=-1$. The criterion used to determine whether or not such a resonance or bound state is expected and if so with what mass and residue, is the vanashing of the D function together with approximate self-consistency between the input and output values of the position and residue of the resonance or bound state in question. It is the same criterion used recently in the dynamical explanations of the ρ meson,[5,6] the (3,3) N^* etc.[7,8]

In each one of the above problems, including the present one, some of the main difficulties from the point of view of practical calculations are:

(1) Inadequate knowledge of the far left-hand cut contribution to the N function.

(2) Inadequacy of the knowledge of the ratio of the total to the elastic partial-wave cross section (the so called R_l function) at higher energies, which through the unitarity condition is material for the evaluation of the D function.

(3) Presence of many channels of strongly interacting particles.

As regards the first difficulty, Balázs[6] introduced a trick by which one can approximately replace the far left-hand cut contribution to the N function, by a few

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[1] G. M. Pjeirou, *et al.*, Phys. Rev. Letters **9**, 114 (1962). L. Bertanza, *et al.*, Phys. Rev. Letters **9**, 180 (1962).

[2] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962). Y. Neeman, Nucl. Phys. **26**, 222 (1961).

[3] M. Gell-Mann, *Proceedings of the International Conference on High Energy Nuclear Physics, Geneva, 1962* (CERN Scientific-Information Service, Geneva, Switzerland, 1962), p. 805. S. L. Glashow and J. J. Sakurai, Nuovo Cimento **26**, 622 (1962).

[4] P. E. Schlein *et al.*, Phys. Rev. Letters **11**, 167 (1963).

[5] F. Zachariasen, Phys. Rev. Letters **7**, 112 (1961). F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962).

[6] L. A. P. Balázs, Phys. Rev. **126**, 1220 (1962).

[7] V. Singh and B. M. Udgaonkar, Phys. Rev. **130**, 1117 (1963).

[8] E. Abers and C. Zemach, Phys. Rev. **131**, 2305 (1963); J. S. Ball and D. Y. Wong (to be published).