S-Wave $\Lambda-\Lambda$ Interaction for Odd $\Sigma-\Lambda$ Parity*†

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The $^1S_0$ $\Lambda-\Lambda$ zero-energy scattering length is calculated by solving the Schrödinger equation for the coupled $\Lambda-\Lambda$ and $\Sigma-\Sigma$ channels, using pionic exchange potentials (with hard core of radius x_0) derived for the case of odd $\Lambda-\Sigma$ parity. Using the value of x_0 and the coupling constants $f_{\Sigma\Sigma}$ and $g_{\Lambda\Sigma}$ which fit the low-energy Λ-N scattering data, the $\Lambda-\Lambda$ interaction is found to be so strongly attractive as to bind in the $^1S_0$ $\Lambda-\Lambda$ state. Since the $\Lambda-\Lambda$ interaction deduced phenomenologically from the analysis of $\Lambda-\Lambda$ hypernuclei is not strong enough to bind, we conclude that the assumption of odd $\Lambda-\Sigma$ parity leads to contradictory results in the calculation of the low-energy Λ-N and $\Lambda-\Lambda$ scattering.

1. INTRODUCTION

RECENTLY Dalitz and Rajasekaran[1] have used variational procedures to calculate the binding energy $B_{\Lambda\Lambda}$ in the experimentally observed[2] system $_{\Lambda\Lambda}Be^{10}$ as a function of the $\Lambda-\Lambda$ potential strength. With the experimentally observed value of $B_{\Lambda\Lambda}$, the result given by these authors for the volume integral $v_{\Lambda\Lambda}$ of the $^1S_0$ $\Lambda-\Lambda$ potential is 322 ± 26 MeV F^3 (assuming an intrinsic range of 1.482 F for the $\Lambda-\Lambda$ potential[3]).

The $\Lambda-\Lambda$ force is expected to arise principally from the exchange of two pions. In addition, if one assumes a neutral vector boson coupled universally to all the baryons, there may be a repulsive core of radius $x_0\approx0.35\,\mu^{-1}$ dominating the inner region (μ=average pion mass=138.1 MeV/c^2). The closed $\Sigma-\Sigma$ channel, coupled to the $\Lambda-\Lambda$ channel by pionic exchange, may also contribute significantly. For even $\Lambda-\Sigma$ parity, for which there is good experimental evidence,[4] de Swart[5] has given a discussion of the S-wave $\Lambda-\Lambda$ interaction. For odd $\Lambda-\Sigma$ parity, the expressions for the $\Lambda-\Lambda$ potential $V_{\Lambda\Lambda}$ (without the $\Sigma-\Sigma$ channel) have been given by Deloff[6]; however, if there is a hard core in the $\Lambda-\Lambda$ force, and in addition also effects coming from the closed $\Sigma-\Sigma$ channel, it is not evident what the net $\Lambda-\Lambda$ interaction will be. The purpose of the present work is to calculate the zero-energy $\Lambda-\Lambda$ scattering length $a_{\Lambda\Lambda}$ for odd $\Lambda-\Sigma$ parity, including the effect of the closed $\Sigma-\Sigma$ channel and a hard core in the potentials. Here, the influence of the closed $\Xi-N$ channel will be neglected. For even $\Sigma-\Lambda$ parity, a calculation[7] has shown its influence to be rather small; for odd parity, the coupling to the $\Xi-N$ channel is again through K and K^* exchange, and the effect is again expected to be small.

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[1] R. H. Dalitz and G. Rajasekaran, Nucl. Phys. (to be published).
[2] M. Danysz, K. Garbowska, J. Pniewski, F. Pniewski, J. Zakrewski, E. R. Fletcher, J. Lemonne, P. Renard, J. Sacton, W. F. Foner, D. O'Sullivan, F. P. Shah, A. Thompson, P. Allen, M. Heeran, A. Montwill, J. E. Allen, M. J. Beniston, D. H. Davis, D. A. Garbutt, V. A. Bull, R. C. Kumar, and P. V. March, Phys. Rev. Letters **11**, 29 (1963).
[3] This corresponds to a two-pion exchange potential.
[4] H. Courant, H. Filthuth, P. Franzini, R. G. Glasser, A. Minguzzi-Ranzi, A. Segar, W. Willis, R. A. Burnstein, T. B. Day, B. Kehoe, A. J. Herz, M. Lakitt, B. Sechi-Zorn, N. Seeman, and G. A. Snow, Phys. Rev. Letters **10**, 409 (1963).
[5] J. J. de Swart, Phys. Letters **5**, 58 (1963).
[6] A. Deloff, Phys. Letters **5**, 147 (1963).

2. THE POTENTIALS

Starting with the charge-independent Hamiltonian density

$$H_{\rm int}=g_{\Lambda\Sigma}(4\pi)^{1/2}[\Lambda^\dagger\mathbf{\Sigma}+\mathbf{\Sigma}^\dagger\Lambda]\cdot\boldsymbol{\pi} +(f_{\Sigma\Sigma}/\mu)(4\pi)^{1/2}[\mathbf{\Sigma}^\dagger\times\sigma^j\mathbf{\Sigma}\cdot\nabla_j\boldsymbol{\pi}], \quad (1)$$

and deriving the potentials with the Brueckner-Watson[8] procedure, we obtain the following results:

$$V_{\Lambda\Lambda}=3g_{\Lambda\Sigma}{}^4({}^XX^{(4)}+{}^{II}X^{(4)}), \quad (2)$$

$$V_{\Lambda\Sigma}=-\sqrt{3}g_{\Lambda\Sigma}{}^2\,X^{(2)}+2\sqrt{3}f_{\Sigma\Sigma}{}^2g_{\Lambda\Sigma}{}^2({}^{II}Y^{(4)}-{}^XY^{(4)}), \quad (3)$$

$$V_{\Sigma\Sigma}=-2f_{\Sigma\Sigma}{}^2V^{(2)}+2f_{\Sigma\Sigma}{}^{(4)}({}^XV^{(4)}+2{}^{II}V^{(4)}) +g_{\Lambda\Sigma}{}^4({}^XX^{(4)}+3{}^{II}X^{(4)})-4f_{\Sigma\Sigma}{}^2g_{\Lambda\Sigma}{}^{2X}U^{(4)}. \quad (4)$$

U, V, X, and Y refer to the contributions of the various graphs to the potential. ${}^XU^{(4)}$ is the contribution to $V_{\Sigma\Sigma}$ of those fourth-order crossed graphs which have one Λ particle in the intermediate state. Fourth-order uncrossed graphs with one Λ particle in the intermediate state do not contribute to a transition from an $I=0$ $\Sigma-\Sigma$ state to an $I=0$ $\Sigma-\Sigma$ state. This is a consequence of the requirement of isotopic spin conservation, as can be seen by the following argument. The isotopic spin factors are the same for all uncrossed graphs which differ only by time ordering. So if the isospin factor is zero for one time ordering, it is zero for all time orderings. But for those graphs whose time ordering is such that the intermediate state consists of one Σ particle and one Λ particle, the isotopic spin factor must be

[7] J. N. Pappademos, Phys. Rev. **134**, B1132 (1964), following article.
[8] K. A. Brueckner and K. M. Watson, Phys. Rev. **92**, 1023 (1953).

zero, since isotopic spin is not conserved in the intermediate state for such graphs.

$V^{(2)}$, ${}^XV^{(4)}$, and ${}^{II}V^{(4)}$ represent the contributions to V of the second-order graphs, and the fourth-order crossed and uncrossed graphs with two Σ particles in intermediate states. ${}^XX^{(4)}$ and ${}^{II}X^{(4)}$ are the contributions of the fourth-order crossed and uncrossed graphs to $V_{\Lambda\Lambda}$. $X^{(2)}$ is the contribution of the second-order graph to $V_{\Lambda\Sigma}$, while ${}^{II}Y^{(4)}$ and ${}^XY^{(4)}$ are the contributions of the fourth-order uncrossed and crossed graphs to $V_{\Lambda\Sigma}$. They are given by the expressions

$$ {}^XY^{(4)}=\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,{}^XY_\sigma{}^{(4)}(x)+S_{12}\,{}^XY_T{}^{(4)}(x), \tag{5}$$

$$ {}^{II}Y^{(4)}=\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,{}^{II}Y_\sigma{}^{(4)}(x)+S_{12}\,{}^{II}Y_T{}^{(4)}(x), \tag{6}$$

where x is measured in Yukawas (1 yukawa $=1\,\mu^{-1}$ $=1.4289$ F).

Although the channel mass difference $\Delta=153.8$ MeV/c^2 is large, it turns out that the mass difference corrections[9] to the range and strength of the second-order transition potential vanish identically for the case of two equal mass particles in each channel.

The Pauli principle limits the S-wave $\Lambda-\Lambda$ interaction to the ${}^1S_0$ state. Hence, none of the tensor force terms in the above potentials will appear in the potential matrix of our problem. Orbital angular momentum as well as spin are good quantum numbers, and the ${}^1S_0$ $\Lambda-\Lambda$ state is connected only to the ${}^1S_0$ $\Sigma-\Sigma$ state.

The momentum space integrals and radial dependences of all of the above potentials are listed in Appendices A and B, with the exception of $V^{(2)}$, ${}^XV^{(4)}$, and ${}^{II}V^{(4)}$, which are given in Ref. 10.

3. RESULTS

In units of $\hbar=c=1$, the Schrödinger equation takes the form

$$\begin{aligned} -(1/M_\Lambda)u_\Lambda''+V_{\Lambda\Lambda}u_\Lambda+V_{\Sigma\Lambda}u_\Sigma&=Eu_\Lambda,\\ -(1/M_\Sigma)u_\Sigma''+V_{\Lambda\Sigma}u_\Lambda+(V_{\Sigma\Sigma}+\Delta)u_\Sigma&=Eu_\Sigma, \end{aligned} \tag{7}$$

where $\Delta=2(M_\Sigma-M_\Lambda)$, and u_Λ and u_Σ are the radial wave functions in the ΛΛ and ΣΣ channels. The numerical values used were (in MeV/c^2) $M_\Lambda=1115.36$ and $M_\Sigma=1192.3$. The latter figure is an average over the members of the charge multiplet. Solving this equation numerically, the zero-energy scattering length $a_{\Lambda\Lambda}$ was obtained for various trial values of coupling constants. Some features of the numerical solution are discussed in Appendix C. The core radius used was $0.35\,\mu^{-1}$. For this core, the values of $g_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$ which lead to a fit to the observed $\Lambda-N$ scattering length were found by de Swart and Iddings[10] to be in the neighborhood of $g_{\Lambda\Sigma}=0.763$, $f_{\Sigma\Sigma}=-0.150$. Trial values of $g_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$ close to the above values were chosen; the results are given in Table I. Since $g_{\Lambda\Sigma}$ enters to the fourth power in the diagonal potential $V_{\Lambda\Lambda}$, while $f_{\Sigma\Sigma}$ does not even enter into $V_{\Lambda\Lambda}$ at all, one would expect the dependence on $g_{\Lambda\Sigma}$ to be quite strong, with a much weaker dependence on $f_{\Sigma\Sigma}$; this is found to be the case. For $0.8<g_{\Lambda\Sigma}<0.72$, and $-0.2<f_{\Sigma\Sigma}<0$, the zero-energy scattering length varies between the limits $1.17\,\mu^{-1}$ and $2.00\,\mu^{-1}$. A positive scattering length can correspond to either a repulsive interaction, or an attraction strong enough to give rise to one or more bound states. One would not expect positive values of $a_{\Lambda\Lambda}$ in this range to correspond to a repulsive $\Lambda-\Lambda$ force, since the diagonal potential $V_{\Lambda\Lambda}$ consists of a hard core of radius $X_0=0.35$ μ^{-1} plus an attractive tail.[11] It follows that if the force were repulsive, $a_{\Lambda\Lambda}$ would be positive but less than $0.35\,\mu^{-1}$. The effect of the closed $\Sigma-\Sigma$ channel is to enhance the attraction.[12]

TABLE I. ${}^1S_0$ zero-energy scattering length $a_{\Lambda\Lambda}$ in Yukawas (1 yukawa = 1.4289 F). $x_0=0.35\,\mu^{-1}$.

$g_{\Lambda\Sigma}$	$f_{\Sigma\Sigma}=$	−0.20	−0.15	−0.10	−0.05	0.00
0.80		1.173	1.216	1.238	1.248	1.251
0.76		1.461	1.512	1.538	1.550	1.554
0.72		1.864	1.939	1.979	1.998	2.003

Thus the values of $a_{\Lambda\Lambda}$ found for the above ranges of $g_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$ suggest that this potential corresponds to the existence of one or more bound states. In order to ascertain how many bound states there are, and also to corroborate the conclusion that the $\Lambda-\Lambda$ force is indicated to be attractive rather than repulsive, the value of $a_{\Lambda\Lambda}$ was calculated for values of $g_{\Lambda\Sigma}$ varying from 0.763 down to 0.15. The value of $f_{\Sigma\Sigma}$ was held constant at -0.15, and $x_0=0.35\,\mu^{-1}$. The results are shown in Table II. For very small $g_{\Lambda\Sigma}$, where the attractive tail is practically negligible, $a_{\Lambda\Lambda}$ approaches the hard-sphere value $0.35\,\mu^{-1}$. As $g_{\Lambda\Sigma}$ is increased, corresponding to more and more attraction, $a_{\Lambda\Lambda}$ at first

TABLE II. Variation of ${}^1S_0$ $\Lambda-\Lambda$ zero-energy scattering length $a_{\Lambda\Lambda}$ with $g_{\Lambda\Sigma}\cdot f_{\Sigma\Sigma}=-0.15$. $x_0=0.35\,\mu^{-1}$.

$g_{\Lambda\Sigma}$	$a_{\Lambda\Lambda}$, μ^{-1}
0.763	1.487
0.650	5.026
0.625	29.4
0.600	−5.38
0.575	−2.010
0.550	−1.026
0.450	0.0108
0.350	0.248
0.250	0.325
0.150	0.347

[9] The reason for not including the mass difference corrections in the fourth-order potentials is discussed by de Swart and Iddings in Ref. 10.

[10] J. J. de Swart and C. K. Iddings, Phys. Rev. **128**, 2810 (1962).

[11] Both the crossed and uncrossed diagram contributions to $V_{\Lambda\Lambda}$ are attractive.

[12] This can be understood by recalling that the energy correction caused by second-order transitions to virtual states lying higher in energy is always negative.

TABLE III. Variation of zero-energy $^1S_0$ $\Lambda-\Lambda$ scattering length $a_{\Lambda\Lambda}$ with hard-core radius, for $g_{\Lambda\Sigma}=0.763$, $f_{\Sigma\Sigma}=-0.15$.

x_0, μ^{-1}	$a_{\Lambda\Lambda}$, μ^{-1}
0.35	1.49
0.36	1.57
0.40	1.98
0.44	2.56
0.48	3.64
0.52	6.73
0.56	2296.
0.60	−5.28
0.64	−2.18
0.68	−1.12

decreases to zero, then goes to very large negative values changing finally to positive values as the attraction becomes sufficient for binding. No more sign changes occur before $a_{\Lambda\Lambda}$ reaches the value $1.487\,\mu^{-1}$ (at $g_{\Lambda\Sigma}=0.763$) and we conclude that values of $g_{\Lambda\Sigma}$ in this neighborhood give rise to only *one* bound state.

The $^1S_0$ zero-energy $\Lambda-\Lambda$ scattering length has been estimated by Dalitz and Rajasekaran[1] (using the data from the $_{\Lambda\Lambda}Be^{10}$ event observed by Danysz *et al.*[2]) to be $a_{\Lambda\Lambda}=-1.76\pm0.33$ F $(=-1.23\,\mu^{-1})$.[13] This corresponds to an attractive force, though not strong enough to give binding.

Thus, the available experimental information on the low-energy $\Lambda-\Lambda$ interaction is in contradiction with the results of a calculation which is based on the assumption of odd $\Sigma-\Lambda$ parity and which uses the same combination of hard-core radius, $f_{\Sigma\Sigma}$ and $g_{\Lambda\Sigma}$ which fit the low-energy $\Lambda-N$ scattering data. If one did not require the cores in the $\Lambda-N$ and $\Lambda-\Lambda$ interactions to be equal in radius, then a simultaneous fit to the $\Lambda-\Lambda$ and $\Lambda-N$ data could be obtained with the *same* values of $g_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$. Dalitz[14] has advanced an argument favoring a somewhat larger core (≈ 0.48 F) in the $\Lambda-\Lambda$ system than in the $^1S_0$ $\Lambda-N$ system. However, a calculation of $a_{\Lambda\Lambda}$ for various core radii (and $f_{\Sigma\Sigma}=-0.150$, $g_{\Lambda\Sigma}=0.763$), the results of which are presented in Table III, shows that the hard-core radius in the $\Lambda-\Lambda$ system must be increased to the neighborhood of $x_0=0.68\,\mu^{-1}$ (0.94 F) in order to fit the value $a_{\Lambda\Lambda}=-1.23\,\mu^{-1}$. We conclude that unless the hard core in the $\Lambda-\Lambda$ system is very much larger than the hard core in the $^1S_0$ $\Lambda-N$ system, in fact unreasonably large, the assumption of odd parity leads to contradictory results in calculation of the low-energy $^1S_0$ $\Lambda-N$ and $^1S_0$ $\Lambda-\Lambda$ interactions. This result is in agreement with the experimental evidence[4] on the relative $\Lambda-\Sigma$ parity.

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[13] The sign convention used in Ref. 1 is opposite to that used here. We use the convention according to which a hard sphere of radius d has zero-energy scattering length equal to $+d$.

[14] R. H. Dalitz, Phys. Letters 5, 53 (1963).

APPENDIX A: LIST OF MOMENTUM SPACE INTEGRALS

$$^{X}X^{(4)}=\frac{-\mu}{8\pi^4}\int\int d_3k d_3k'\, e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}}\frac{\omega^2+\omega'^2+\omega\omega'}{\omega^3\omega'^3(\omega+\omega')}, \tag{A1}$$

$$^{II}X^{(4)}=\frac{-\mu}{8\pi^4}\int\int d_3k d_3k'\, e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}}\frac{1}{\omega^2\omega'^2(\omega+\omega')}, \tag{A2}$$

$$X^{(2)}=\frac{-\mu}{2\pi^2}\int d_3k\frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{\omega^2}, \tag{A3}$$

$$^{II}Y^{(4)}=\frac{-\mu}{8\pi^4}\int\int d_3k d_3k'\frac{\boldsymbol{\sigma}_a\cdot\mathbf{k}'\boldsymbol{\sigma}_b\cdot\mathbf{k}'\, e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}}}{\omega^2\omega'^2(\omega+\omega')}, \tag{A4}$$

$$^{X}Y^{(4)}=\frac{-\mu}{8\pi^4}\int\int d_3k d_3k'\boldsymbol{\sigma}_b\cdot\mathbf{k}\boldsymbol{\sigma}_a\cdot\mathbf{k}' \times e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}}\frac{\omega^2+\omega'^2+\omega\omega'}{\omega^3\omega'^3(\omega+\omega')}, \tag{A5}$$

$$^{X}U^{(4)}=\frac{\mu}{8\pi^4}\int\int d_3k d_3k'\boldsymbol{\sigma}_b\cdot\mathbf{k}\boldsymbol{\sigma}_b\cdot\mathbf{k}' \times e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}}\frac{\omega^2+\omega'^2+\omega\omega'}{\omega^3\omega'^3(\omega+\omega')}. \tag{A6}$$

The momentum space integrals for $V^{(2)}$, $^{X}V^{(4)}$, and $^{II}V^{(4)}$ may be found in Ref. 10.

APPENDIX B: RADIAL FUNCTIONS DESCRIBING THE POTENTIALS

$$X^{(2)}=\mu(-2/x)e^{-x}, \tag{B1}$$

$$^{X}Y_\sigma^{(4)}=(1/3\pi)[(3/x^2)K_1(2x)+(2/x)K_0(2x)], \tag{B2}$$

$$^{X}Y_T^{(4)}=(1/3\pi)[(3/x^2)K_1(2x)+(2/x)K_0(2x)], \tag{B3}$$

$$^{II}Y_\sigma^{(4)}=(1/3\pi)[(e^{-x}/x^2)(2xK_0(x)-K_1(x)) -(1/x^2)(2xK_0(2x)-K_1(2x))], \tag{B4}$$

$$^{II}Y_T^{(4)}=(1/3\pi)[e^{-x}K_0(x)(2/x+3/x^2+3/x^3) +(2e^{-x}/x^2)K_1(x)-(5/x^2)K_1(2x) -(2/x+3/x^3)K_0(2x)], \tag{B5}$$

$$^{X}X^{(4)}=-\mu(2/\pi x)K_0(2x), \tag{B6}$$

$$^{II}X^{(4)}=-\mu(2/\pi x)[e^{-x}K_0(x)-K_0(2x)], \tag{B7}$$

$$^{X}U^{(4)}=-\mu(1/\pi)[(3/x^2)K_1(2x)+(2/x)K_0(2x)], \tag{B8}$$

in which the functions $K_n(x)$ are given by

$$K_n(x)=\frac{\Gamma(n+\frac{1}{2})}{\pi^{1/2}}\left(\frac{2}{x}\right)^n\int\frac{\cos kx}{(k^2+1)^{n+1/2}}dk. \quad \text{(B9)}$$

The radial functions for $V^{(2)}$, ${}^{X}V^{(4)}$, and ${}^{II}V^{(4)}$ are to be found in Ref. 10.

APPENDIX C: NUMERICAL SOLUTION OF THE SCHRÖDINGER EQUATION

The procedure followed in integrating the wave equation has been discussed previously.[15] In the present problem, where there is only one orbital angular momentum channel, the two independent solutions of the wave equation are combined into a 2×2 wave function matrix ψ. At the outer edge of the hard core the elements of ψ are set equal to zero, while the slopes of the 11 and 22 elements are arbitrarily set equal to unity. Values of ψ are computed successively, by use of the Noumanoff approximation to the differential equation, at intervals of 0.01 μ^{-1} out to $x=1.0\ \mu^{-1}$, then at intervals of 0.02 μ^{-1} out to $x=5.0\ \mu^{-1}$. At this point the linear combination of the two independent solutions for ψ is determined to match an outer wave function consisting of a sine wave in the open channel and a damped exponential in the closed channel. In this case it was found that the cumulative errors in the numerical solution for ψ were so large as to cause the matching conditions to yield completely erratic results; the difficulty was cleared up by carrying out the calculation at 16 significant figure accuracy instead of the 8 significant figure accuracy customarily used in IBM-7094 calculations.

In order to understand qualitatively the reason for this difficulty,[16] a model two-channel problem was set up in which all of the potentials consisted of a hard repulsive core of common radius, followed by attractive rectangular wells of depths V_L, V_S, and V_{LS} for the $\Lambda-\Lambda$, $\Sigma-\Sigma$ and transition potentials, respectively. The radius x_1 was used for all of the wells. For the ${}^1S_0$ $\Lambda-\Lambda$ state, the Schrödinger equation for the region within the well reads (for zero energy)

$$u_\Lambda''+M_\Lambda V_L u_\Lambda+M_\Lambda V_{LS}u_\Sigma=0\,,$$
$$u_\Sigma''+M_\Sigma V_{LS}u_\Sigma+[(V_S-\Delta)M_\Sigma]u_\Sigma=0\,, \quad \text{(C1)}$$

where u_Λ and u_Σ are the wave functions in the $\Lambda-\Lambda$ and $\Sigma-\Sigma$ channels, M_Λ and M_Σ are the Λ and Σ masses, and Δ is twice the $\Lambda-\Sigma$ mass difference. On assuming solutions of the form

$$u_\Lambda=A\cos(nx+\epsilon)\,,$$
$$u_\Sigma=B\cos(nx+\epsilon)\,.$$

We find that the eigenvalues of n are given by

$$n^2=\tfrac{1}{2}\{M_\Lambda V_L+(V_S-\Delta)M_\Sigma\pm[(M_\Lambda V_L-(V_S-\Delta)M_\Sigma)^2+4M_\Lambda M_\Sigma V_{LS}^2]^{1/2}\}. \quad \text{(C3)}$$

In case $V_S>\Delta$, we see that solutions with n real exist, provided the condition

$$V_{LS}^2<V_L(V_S-\Delta) \quad \text{(C4)}$$

is met. In case $V_S<\Delta$, we find that n is real if the positive sign in (C3) is chosen, imaginary if the minus sign is used. Thus, the wave function within the well is a linear combination of oscillatory and exponential parts. In the actual $\Lambda\Lambda-\Sigma\Sigma$ problem with odd-parity potentials, two characteristic features stand out: (1) The transition potential $V_{\Lambda\Sigma}$ tends to be very strong as well as long ranged by comparison with $V_{\Lambda\Lambda}$ and $V_{\Sigma\Sigma}$, (2) the channel mass difference Δ is large, about 134 MeV/c^2. In making the comparison with our model problem, we should take Δ and V_{LS} large by comparison with V_S and V_L. If, in Eq. (C3), we neglect V_S and V_L, we see that the characteristic length λ of the exponential part of the wave function is given by

$$\lambda\approx\left\{\frac{M_\Sigma\Delta}{2}+\left[\left(\frac{M_\Sigma\Delta}{2}\right)^2+M_\Lambda M_\Sigma V^2{}_{LS}\right]^{1/2}\right\}^{-1/2}. \quad \text{(C5)}$$

From this we see that the wave function in the actual problem will have a very rapid exponential increase with r. The corresponding exponential increase in the rounding-off errors makes it extremely difficult to accurately match this solution to the exponentially decreasing solution which we want in the exterior region.

[15] J. J. de Swart and C. Dullemond, Ann. Phys. (N. Y.) **16**, 263 (1961).

[16] We have found that good results were obtained with only 8 significant figure accuracy in other cases, e.g., the case in which the closed channel is the $\Xi-N$ channel and even $\Lambda-\Sigma$ parity potentials used.