Total Neutron Cross Section of Manganese*

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(Received 20 January 1964)

The total neutron cross section of manganese has been measured from 0.01 eV to about 50 keV and a multilevel analysis of the data from 0.01 eV to 15 keV has been carried out. This analysis shows that the strength function is $\bar{\Gamma}_n^0/D=9.5_{-2.5}^{+6.5}\times10^{-4}$ and that the potential-scattering amplitude for the $J=3$ series of levels is substantially less than 4 F.

INTRODUCTION

THE total neutron cross section of manganese has been the subject of a great deal of investigation over the past fifteen years, largely because of the several strong resonances below 10 keV. The first clear case of the interference between two s-wave scattering resonances was observed in manganese,[1] and studies of the total cross section of manganese were important in establishing[2] that a maximum in the s-wave strength function occurs near $A=55$, the mass of manganese. Krotkov[3] attempted to fit the observed total cross section of manganese with an R-matrix approximation, but was unable to find a set of parameters that would describe the total cross section and give a value of the coherent-scattering amplitude that agreed with the measured value. We have remeasured this well-studied cross section in order to take advantage of advances in experimental technique.

Three main questions could be answered by high-quality results obtained with good resolution: (1) Had some resonances below 20 keV escaped detection? (2) What were the spins of the resonances at 7100 and 8800 eV? (3) Could the total cross section be adequately described by a multilevel analysis? By the time of the second Geneva Conference on the Peaceful Uses of Atomic Energy, held in the summer of 1958, these questions had been answered[4] to a fair degree. A new resonance had been found at 17.8 keV,[5] the spins of the resonances at 7100 and 8800 eV had been determined, and the data for neutron energies between 250 eV and 10 keV had been reasonably well fitted by a simple multilevel approximation.[6] Preliminary attempts to fit the data over a wider range of energy were reported[7] early the following year. The results discussed at that time were derived from an analysis based on the approximations of Refs. 3 and 6, which gave experimentally indistinguishable values of the cross section over the entire range, except for the very bottom of the deep minimum near 1200 eV. For the resonances above 10 keV, parameters that were consistent with the results of the present measurements and those of Marshak and Newson[8] were used with parameters of possible bound levels (adjusted to obtain as good a fit as possible to the data for neutron energies between 0.1 and 15 keV). An entirely satisfactory fit was never obtained with this approach. When the R-matrix analysis of Firk, Lynn, and Moxon[9] (hereafter referred to as FLM) became available, it was adopted as the most useful method of analyzing the problem. Their analysis involved less severe approximations than those in Refs. 3 and 6 and eliminated the need to specify directly the properties of levels outside the range of interest. In addition to this simplication, the relations between the residual R functions used to take account of the effects of distant resonances and such quantities as the strength function and the effective nuclear radius actually allow more information to be extracted from the data in some cases. The present paper describes the results of applying their analysis to the data on manganese.

EXPERIMENTAL DETAILS

The measurements were made by the transmission method with the Argonne fast chopper[4] at the heavy-water research reactor CP-5. The rotor that was used gave a neutron burst whose full width at half-maximum is 1.2 μsec for a speed of 15 000 rpm. The neutrons were detected in boron-loaded liquid scintillators[10] located at distances of 60 or 120 m from the chopper. The best over-all time-of-flight resolution of the system was about 12 nsec/m. All of the data were stored in a 1024-channel time analyzer.[11] For most of the data, both sample and open-beam counts were recorded alternately

* Work performed under the auspices of the U. S. Atomic Energy Commission.

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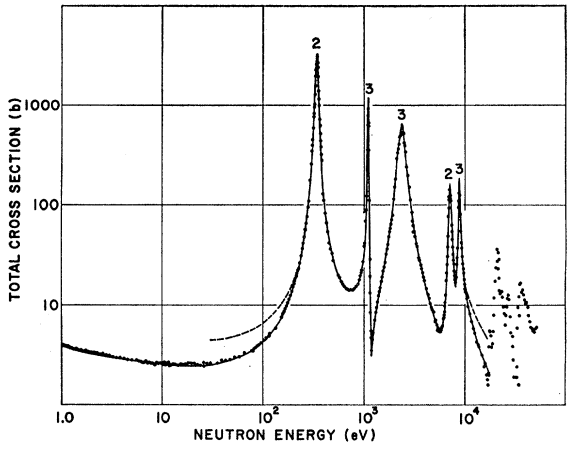

FIG. 1. The observed total neutron cross section of manganese. The solid curve was computed for the values of the parameters listed in Table I and the residual R functions as described in the text. The dashed curve was computed for values of the parameters of the unbound levels listed in Table I and with a constant nuclear radius of 5 F. The statistical errors are, for most of the points, the size of the dots or smaller. For a small number of points these errors are as much as three times as large as the dots.

on a 4-min cycle over periods of 15–20 h. Such a procedure eliminates most of the systematic errors that are likely to produce data of poor quality.

Six thicknesses of samples were used; the thinnest had 4.02×10^{20} atoms/cm^2, the thickest had 2.70×10^{23} atoms/cm^2. All but the two thickest were made from the same manganese powder, which was shown by chemical analysis to contain no impurities that could cause any gross errors in the measurement. For the measurements of the cross section in the thermal region and for most of the measurements of the cross section where this value is below 10 b, the samples were made from purer manganese obtained in 1963.

EXPERIMENTAL RESULTS

The primary results obtained from the measurements are the transmissions of the various samples of manganese as functions of the energy of the incident neutrons. From these primary results, the total cross sections were obtained under the assumption of perfect resolution, i.e., by the expression $\sigma=-(1/n)\ \ln T$, where T is the measured transmission and n is the sample thickness in atoms per barn. The resulting plots of total cross section versus neutron energy are shown in Figs. 1 and 2. As is customary for the construction of such curves, the points near the peaks were taken from the measurements made with thin samples to minimize the effects of the finite resolution width of the system and the points in the off-resonance regions were chosen from thick-sample data (for which $\Delta T/T$ and hence $\Delta\sigma/\sigma$ is small).

The neutron resonances at 6 and 35 keV in the aluminum of the reactor tank cause the time-of-flight spectra obtained with no sample in the beam (i.e., the open beam) to vary rapidly in the vicinity of these resonances. Because of the finite resolution of the system, it is possible that such variations may not cancel properly. Hence, at these energies the difference between the measured and computed cross section may be larger than the standard statistical errors associated with the data points.

The results shown in Fig. 1 established the existence of the "resonance" at 17.8 keV and made possible the assignment of $J=2$ for the resonance at 7100 eV and $J=3$ for that at 8800 eV. The latter assignments can be made solely on the basis of the peak heights of the resonances, but are confirmed by the detailed multilevel fit in the wings of the resonances. The remainder of the paper deals with the question of a detailed multilevel fit over the entire energy region from 0.01 eV to 15 keV. The region above 15 keV is not included because to do so would require the accurate assessment of large resolution corrections.

The attempts to fit the data were originally restricted to neutron energies between 1 eV and 15 keV. Rather severe difficulties arose in attempts to fit the data between 1 and 250 eV while at the same time maintaining agreement between all other known data and the computed values. Therefore, wherever the total cross section is less than 20 b, it was remeasured from 0.01 to 1300 eV with new pure samples. The data shown in Figs. 1 and 2 include the results of these new measurements. They disagree with previous results[12] over some parts of the range.

ANALYSIS

Since manganese is known to lie close to the peak of the s-wave neutron strength function, it was assumed that all resonances were associated with s-wave interactions.

For the data treated in this paper, there is little difference between the cross sections computed on the basis of the equations derived in Refs. 3, 6, and 9, so the

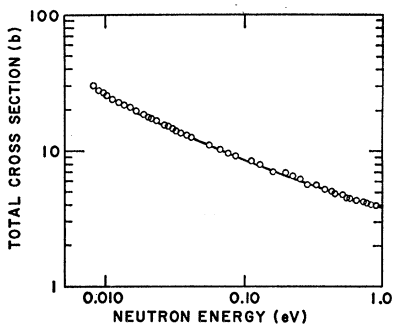

FIG. 2. The observed total cross section of manganese from 0.01 to 1.0 eV. The solid curve was computed for the values of the parameters listed in Table I and the residual R functions as described in the text.

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discussion will be limited to the use of the analysis of FLM. FLM use the reduced R matrix of Thomas[13] to simplify the general Wigner-Eisenbud[14] theory for use in deriving explicit formulas for the various cross sections. This reduction is valid if the partial width for the reaction in each channel other than the entrance channel is much less than the level spacing.

The reduced R function (for angular momentum J) is

$$R_J(E)=\sum_\lambda \frac{\gamma_{\lambda Jn}^2}{E_{\lambda J}-E-\frac{1}{2}i\Gamma_{\lambda Jr}}, \quad (1)$$

where the sum is over levels λ of angular momentum J with eigenvalues $E_{\lambda J}$ and reduced neutron widths $\gamma_{\lambda Jn}^2$. The width $\Gamma_{\lambda Jr}$ is the total reaction width of the level, i.e., it is the total width $\Gamma_{\lambda J}$ minus the neutron width $\Gamma_{\lambda Jn}$. For s-wave neutrons, the neutron width is related to the reduced neutron width by

$$\Gamma_{\lambda Jn}=2ka\gamma_{\lambda Jn}^2,$$

where k is the neutron wave number and a is the arbitrary radius chosen to separate the entrance channel from the internal nuclear region.

The collision function $U_J(E)$, the amplitude of the outgoing wave in the entrance channel resulting from scattering of a unit-flux plane wave, is (for s waves)

$$U_J(E)=e^{-2ika}\frac{1+ikaR_J(E)}{1-ikaR_J(E)}.$$

From this the total cross section is

$$\sigma_t(E)=\frac{2\pi}{k^2}\sum_{J=|I-\frac{1}{2}|}^{|I+\frac{1}{2}|} g_J[1-\mathrm{Re}U_J(E)],$$

where I is the angular-momentum quantum number of the ground state of the target nucleus and $\frac{1}{2}[I\pm 1/(2I+1)]$ is the spin statistical factor. The elastic-scattering cross section is

$$\sigma_n(E)=\frac{\pi}{k^2}\sum_{J=|I-\frac{1}{2}|}^{|I+\frac{1}{2}|} g_J|1-U_J(E)|^2$$

and the coherent elastic-scattering cross section is

$$\sigma_{\mathrm{coh}}(E)=\frac{\pi}{k^2}\Big|\sum_{J=|I-\frac{1}{2}|}^{|I+\frac{1}{2}|} g_J[1-U_J(E)]\Big|^2.$$

Because the parameters are known for only a few resonances (those in the region of interest) whereas the function R_J of Eq. (1) is defined over all resonances of the same J, $R_J(E)$ is taken by FLM to be the sum of a local component R_J^{local} and a residual component $R_J^{\mathrm{res}}(E)$ based on resonances outside the region of interest. The local component is calculated as in Eq. (1) and the residual part $R_J^{\mathrm{res}}(E)$ is expressed in terms of a power series as

$$R_J^{\mathrm{res}}(E)=R_J^{\infty}+\sum_\mu \frac{\gamma_{\mu Jn}^2}{E_{\mu J}-E_{1/2}}+(E-E_{1/2})\sum_\mu \frac{\gamma_{\mu Jn}^2}{(E_{\mu J}-E_{1/2})^2}+(E-E_{1/2})^2\sum_\mu \frac{\gamma_{\mu Jn}^2}{(E_{\mu J}-E_{1/2})^2}+\cdots+\frac{i}{2}\sum_\mu \frac{\gamma_{\mu Jn}^2\Gamma_{\mu Jn}}{(E_{\mu J}-E_{1/2})^2}+\cdots,$$

where $E_{1/2}$ is the energy at the center of the region of interest and μ signifies levels outside of this region. FLM abbreviate this expansion as

$$R_J^{\mathrm{res}}(E)=A_J+B_J(E-E_{1/2})+C_J(E-E_{1/2})^2+D_J(E-E_{1/2})^3+\cdots+iK_J+\cdots.$$

The procedure followed to obtain values of $E_{\lambda J}$, $\gamma_{\lambda Jn}^2$, $\Gamma_{\lambda Jr}$, and the constants A_J, B_J, etc., was much the same as that followed by FLM. First attempts to fit the data used the cross section above 20 b—i.e., in the peaks of the resonances—since these regions are relatively insensitive to the choice of the nuclear radius and to the effects of neighboring resonances. In Fig. 1 the data are compared with a curve (the dotted curve) computed on the basis of the parameters of the five prominent resonances only and a nuclear radius of 5 F. The curve almost completely satisfies the data over the range from 200 eV to 10 keV. However, above 10 keV and below 200 eV the agreement is unacceptable. In addition, it is impossible to satisfy the data that pertain to the thermal region.

A limited analysis of this type leads to good determinations of the energies $E_{\lambda J}$ and good estimates of most of the $\gamma_{\lambda Jn}^2$, but gives almost no information about the total radiation width Γ_γ, which for manganese is equal to the total reaction width $\Gamma_{\lambda Jr}$. However, either of two pieces of information can be used to estimate Γ_γ. These are the resonance-capture integral and the thermal-capture cross section. We have chosen to use the resonance-capture integral for this purpose, since it is less adversely affected by the uncertainty caused by a lack of detailed knowledge of the bound levels. Recent measurements[15] appear to agree that a value of 8.0 ± 0.5 b is the best value for the resonance-absorption integral without the $1/v$ contribution. From this value, the resonance parameters of Table I, and a correction for the contribution[16] of the resonances above 10 keV, the

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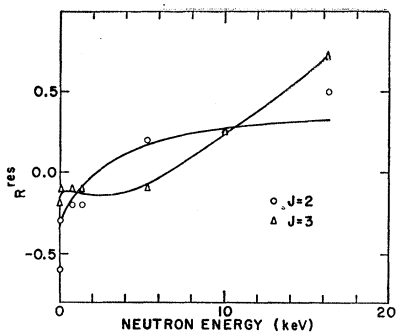

FIG. 3. The residual R functions R_J^{res} for each spin as a function of neutron energy. The curves represent the variation in R_J^{res} with energy, as calculated by the power-series expansion for R_J^{res} with the coefficients given in the text and the bound levels defined by the parameters listed in Table I.

average radiation width $\bar{\Gamma}_\gamma$ is found to be 0.45 ± 0.04 eV. It is assumed, of course, that Γ_γ does not vary from resonance to resonance or with J.

As the final step in the fitting procedure, it is required that the coefficients in the expansion of the residual R function be chosen so that the total cross section below 20 b can be reproduced and that the following thermal data are satisfied. (a) The magnitude of the coherent-scattering cross section is $\sigma_{coh}(th)=1.7\pm0.1$ b and its amplitude is negative.[12] (b) The radiative capture cross section[12] is $\sigma_\gamma(th)=13.3\pm0.2$ b. (c) For polarized manganese nuclei capturing polarized thermal neutrons, the ratio of capture with parallel alignment to that with antiparallel is 0.96 for the particular set of polarizations used in the experiment of Bernstein *et al.*[17] (d) The gamma-ray spectrum observed by Kennett, Bollinger, and Carpenter[18] for the capture of thermal neutrons is very different from the spectrum associated with the capture of neutrons at the resonance at 337 eV, although the contribution of this resonance to the thermal-capture cross section is quite dominant. It is implied, of course, that the $E_{\lambda Jn}$ and $\gamma_{\lambda Jn}^2$ will also be adjusted slightly to improve the quality of the over-all fit as measured by the value of χ^2 for a particular set of parameters.

As was described by FLM, it was found to be expedient to study each sensitive region of the cross section separately to find the best values of the R_J^{res} for each. These values were then plotted as functions of the neutron energy, and the coefficients of the expansion for each R_J^{res} were obtained through a least-squares fit of the polynomial to these values. In cases in which the variation of R_J^{res} was very rapid (an indication of a level just outside the region of interest), a level with parameters that would produce the rapid change in R_J^{res} was postulated and the polynomial was fitted to the remainder. The variations of R_J^{res} for each spin are shown in Fig. 3. The points represent "best" choices of each $R_J^{res}(E)$ for each of the regions discussed above; the curves represent the calculated variation of $R_J^{res}(E)$ with energy. This calculation employed the bound levels listed in Table I and the expansion coefficients used were those defined by

$$R_2^{res}=0.425,$$
$$R_3^{res}=0.203+0.764\times10^{-4}(E-E_{1/2})+0.204\times10^{-8}\times(E-E_{1/2})^2-0.282\times10^{-12}(E-E_{1/2})^3. \quad (2)$$

The final computed total cross section was based on the expansions of Eq. (2) and the parameters listed in Table I. It is shown as the solid lines in Figs. 1 and 2.

TABLE I. Best values of the parameters for the resonances in manganese up to 10 keV and the parameters for the postulated bound levels required in order to satisfy the measured cross section in the positive-energy region.

E_0 (eV)	J	γ_n^2 (eV)	Γ_n^0 (eV)
335.5± 0.4	2	556± 3	1.22
1098 ± 2.3	3	199±10	0.44
2355 ± 7	3	3780±10	8.31
7110 ±35	2	2295±10	5.04
8740 ±45	3	1802±10	3.96
−2830	2	2300	5.05
−78	3	7.3	0.016

DISCUSSION

It is clear from Figs. 1 and 2 that the parameters of Table I and the coefficients of Eq. (2) give as accurate a description of the physical situation as is warranted by the data. Some further refinement might be made, but it is not clear that any new physical content would be added. In addition to describing the total cross section accurately, the final parameters yield (a) the correct sign for the coherent-scattering amplitude, (b) values of the thermal-capture cross section and coherent-scattering cross section that are in agreement with the measured values, (c) agreement with the results of Bernstein *et al.*[17] on the capture of polarized neutrons and, it should be recalled, (d) agreement with the measurements of the resonance-capture integral.

A word should be said about the agreement between the results of the present analysis and the results of Bernstein *et al.*[17] The latter measured the cross section for capture of polarized neutrons by polarized manganese nuclei as a function of the relative orientations of incident and bombarded particles. The parallel/antiparallel ratio of the capture rates for the particular polarizations of sample and neutron beam used in their experiments was reported to be 0.96 ± 0.003, after correction for incomplete neutron flipping. In the present analysis, the quantity related to their result is the relative cross section for capture in each spin state at thermal neutron energies. To make the present

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[18] T. J. Kennett, L. M. Bollinger, and R. T. Carpenter, Phys. Rev. Letters 1, 77 (1958).

analysis consistent with their result, it was necessary to introduce a rather weak bound s-wave level at -78 eV with a reduced neutron width of 0.016 eV and $J=3$. Although this represents a rather narrow width, there is a 5% probability of finding such a resonance in a population governed by a Porter-Thomas[19] distribution with the average width given by the average of the three positive-energy levels of the same spin. Alternatively, the results could be explained in terms of a bound p-wave level.

The results of Kennett, Bollinger, and Carpenter[18] can be attributed to the bound level that had to be included for $J=2$ and to the existence, established by Coté and Bollinger,[20] of interference in radiative transitions associated with capture in neutron resonances. Lane and Lynn[21] had suggested a possible explanation in terms of direct capture, but they were unaware of the evidence for a strong level near the binding energy.

Let use now consider some of the physical quantities that can be derived from the above results. The strength function computed on the basis of the five dominant resonances is $\gamma^2/D=0.41_{-0.19}^{+0.85}$ and the average level spacing per spin state is $D=4200_{-1250}^{+3100}$ eV. An estimate of the size of the potential scattering length for $J=3$ can be obtained from a consideration of the expansion coefficients for $R_3^{\rm res}$. We estimate that the levels between 20 and 40 keV contribute about 0.15×10^{-4} to B_3. In addition, the levels above 40 keV contribute about 0.1×10^{-4}. This leaves about 0.5×10^{-4} that needs to be accounted for by the negative-energy levels, which appear to be somewhat larger than the positive levels. Hence, $R_3^{\infty}>A_3$ because of the imperfect cancellation of the $\gamma_n^2/(E-E_{1\,2})$ terms from positive and negative levels. This implies that the potential scattering length $a'=a(1-R^{\infty})$ is considerably less than 4 F; a was chosen to be 5 F for all of the calculations. Because the bound level postulated at $E_0=-2800$ eV dominates the variation of $R_2^{\rm res}$, no estimates of the sizes of the coefficients A_2, B_2, etc., can be derived, and therefore the potential-scattering amplitude for the $J=2$ states cannot be determined from the present experiments. FLM found that for vanadium the two amplitudes were measurably different.

The linear term in the expansion of $R_J^{\rm res}$ can be used to obtain another measure of γ^2/D. It was stated by FLM that, under the assumption of equally spaced levels,

$$B_J=\frac{2\gamma^2}{D_J^2}\left(\frac{\pi^2}{6}-\sum_{\lambda=1}^{m}\frac{1}{\lambda^2}\right),$$

where m is the number of level spacings in half of the measured range. The number was taken to be 2 for the present case. The small bound level postulated for the $J=3$ set of levels does not introduce any appreciable uncertainties in the expansion coefficients for $R_3^{\rm res}(E)$; therefore the variation of this quantity may be analyzed by the procedure followed by FLM. Combining the linear term B_3 with a value of the level spacing D leads to $\gamma^2/D=0.4_{-0.12}^{+0.3}$. This value, when combined with the result based on the five resolved levels, leads to a final value $\gamma^2/D=0.4_{-0.1}^{+0.2}$ so that $\bar{\Gamma}_n^0/D=9.5_{-2.5}^{+6.5}\times10^{-4}$.

ACKNOWLEDGMENTS

The authors are very grateful to J. E. Lynn for many helpful discussions while he was a visiting scientist at Argonne National Laboratory and are grateful for the use of his R-matrix code for the IBM-704 with which the final analysis was done. The authors are also grateful to Miss J. P. Marion for assistance with the computations. One of us (R.E.C.) also is happy to acknowledge the assistance given by J. E. Monahan in the early stages of the analysis.

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