Form Factors for Magnetic-Dipole Electron Scattering*

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Transition probabilities have been calculated for magnetic-dipole transitions excited by inelastic scattering of electrons of Li^6 and C^{12}. The form factors are found to be quite insensitive to the degree of spin-orbit coupling. The harmonic-oscillator length parameters determined from fitting experimental form factors tend to be larger than those determined from charge scattering.

I. INTRODUCTION

RECENT experiments with electrons scattered inelastically at 180° are providing information about nuclear form factors.[1] An accompanying paper[2] reports investigations on two well-known magnetic-dipole excitations between states with isobaric spins $T=0$ and $T=1$.

The isobaric-vector part of the $M1$ operator is needed to calculate these transition probabilities as functions of the momentum transfer q. The z component in units of nuclear magnetons can be written as

$$\mathfrak{M}_z=(3/4\pi)^{1/2} \times\sum_k\left\{-\tfrac{1}{2}l_z(k)[j_0(qr_k)+j_2(qr_k)]\tau_3(k)+\left(\frac{\mu_n-\mu_p}{2}\right) \times\{\sigma_z(k)j_0(qr_k)-(2\pi)^{1/2}[Y_2\sigma]_z(k)j_2(qr_k)\}\tau_3(k)\right\}. \quad (1)$$

Here j_0 and j_2 are spherical Bessel functions, and $[Y_2\sigma]$ represents the vector coupling of a spherical harmonic with σ, i.e.,

$$[Y_2\sigma]_z(k)=\sum_m (21-mm|10)Y_{2,-m}(\Omega_k)\sigma_m(k). \quad (2)$$

In the limit of small momentum transfer q, $j_0(qr)$ goes to unity and $j_2(qr)$ vanishes. This results in the more familiar form of the $M1$ operator.

II. CALCULATION OF MATRIX ELEMENTS

For nuclei in the $1p$ shell, one can calculate the transition-matrix elements with wave functions representing varying strengths of spin-orbit coupling. The reduced transition probability, which is in effect the square of the matrix element, has the general form

$$B_{M1}(q)=B_{M1}(q=0)[\langle j_0\rangle+\rho\langle j_2\rangle]^2. \quad (3)$$

For these $1p$ nuclei, $\langle j_L\rangle$ is the radial integral

$$\langle j_L\rangle\equiv\int_0^\infty R_{1p}^2(r)j_L(qr)r^2dr. \quad (4)$$

The values for $B_{M1}(0)$ and ρ depend on the relative importance of spin-orbit coupling, but they are independent of the momentum transfer. The square of the bracket in Eq. (3) is then the form factor for the $M1$ transition.

The two transitions which will be compared with experiment are (a) the transition from the $I=0=T$ ground state of C^{12} to the $I=1=T$ excited state at 15.1 MeV, and (b) the transition from the $I=1$, $T=0$ ground state of Li^6 to the $I=0$, $T=1$ state at 3.56 MeV. Table I lists the calculated results for $B_M(I=1\rightarrow I=0,q=0) =|\langle 1|\mathfrak{M}_z|0\rangle|^2$ and for the coefficient ρ as functions of the intermediate-coupling parameter[3] ξ. It is clear that while B_{M1} may be quite sensitive to the degree of coupling, as it is in C^{12}, the coefficient ρ is quite insensitive. In C^{12} the contribution to ρ from the operator $[Y_2\sigma]_z$ in Eq. (1) is about 0.27, changing only by 10% in going from $\xi=4.5$ to the jj limit $\xi=\infty$. Although ρ is fairly small, the term $\rho\langle j_2\rangle$ can become important for values of the momentum transfer such that the integral $\langle j_0\rangle$ vanishes.

III. COMPARISON WITH EXPERIMENT

The experiments[2] on Li^6 and C^{12} were carried out with electrons having initial energy between 40 and 70 MeV. Under these conditions one can extract the desired transition strengths from the cross section by applying the plane-wave Born approximation and neglecting nuclear recoil. The differential cross section[4]

TABLE I. Reduced transition probability (at vanishing momentum transfer q) and coefficient ρ in Eq. (3) as functions of the spin-orbit coupling parameter ξ. Values are given for Li^6 and C^{12}.

ξ	$B_{M1}(1\rightarrow 0,q=0)$	ρ
	Li^6	
0	5.29	0
1.5	5.53	0.022
3.0	5.59	0.028
	C^{12}	
4.5	0.79	0.256
6.0	1.37	0.220
∞	3.75	0.160

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[4] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 (1956), especially p. 476. The relationship between B_{M1} and the quantity used in Ref. 2 is

$$(\hbar cq)^2B_{M1}(1\rightarrow 0,q)=5.28\times 10^6\times\langle 1\|T_1^{\mathrm{mag}}(q)\|0\rangle^2.$$

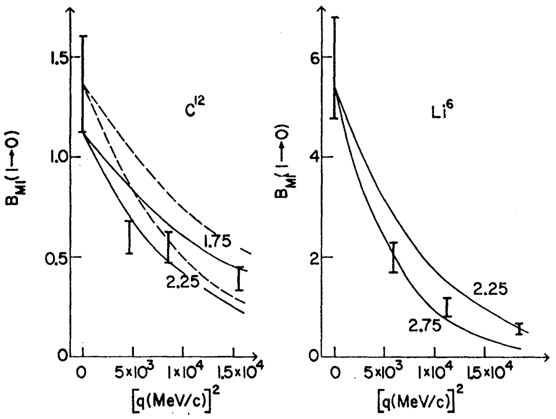

FIG. 1. The reduced transition probability, $B_{M1}(1\rightarrow 0)=|\langle 1|\mathfrak{M}_z|0\rangle|^2$ for Li^6 and C^{12} as a function of q^2, the square of the momentum transfer. The points are experimental; the curves are calculated for different values of the harmonic-oscillator parameter $a=(\hbar/m\omega)^{1/2}$, which is given in units of fermis.

at 180° can be written as

$$\left(\frac{d\sigma}{d\Omega}\right)_{M1}(180°)=(1.643\times 10^{-32}\ \mathrm{cm}^2)\times(1+\epsilon)^{-2}B_{M1}(I_i\rightarrow I_f,q)\,, \quad (5)$$

where $\epsilon=(E^*/\hbar cq)$, E^* being the nuclear excitation energy. The experimental values of $B_{M1}(1\rightarrow 0,q)$ are plotted in Fig. 1 as functions of q^2, the square of the momentum transfer.

In addition to the data from electron scattering, values for $B_{M1}(q\rightarrow 0)$ (the long-wavelength limit) have been obtained[5,6] from nuclear resonance-fluorescence experiments. These points are also plotted in Fig. 1. The values of $B_{M1}(q\rightarrow 0)$ can be compared with the calculated values in Table I to find the values of the spin-orbit coupling parameter which are compatible with experiment. It is of interest to point out that this is the strongest evidence against treating the C^{12} ground state as a filled $1p_{3/2}$ level. That picture would give a value of $B_{M1}(1\rightarrow 0,q=0)$ which is two or three times the experimental value.

Theoretically the dependence on momentum transfer is contained in the radial integrals of Eq. (3), defined in Eq. (4). These integrals are evaluated with a harmonic-oscillator radial function

$$R_{1p}=N_{1p}r\exp[-\tfrac{1}{2}(r/a)^2]. \quad (6)$$

The present accuracy of the experiments does not warrant the refinement of including the effects of a finite extension for the nucleon moments, or center-of-mass corrections[7] to the shell-model functions. The radial integrals[8] are simply

$$\langle j_0\rangle=[1-\tfrac{1}{6}(qa)^2]\exp[-\tfrac{1}{4}(qa)^2],$$
$$\langle j_2\rangle=+\tfrac{1}{6}(qa)^2\exp[-\tfrac{1}{4}(qa)^2]. \quad (7)$$

The experimental data are fitted by selecting values of $B_{M1}(0)$ and ρ from the range indicated in Table I and varying the oscillator parameter a. At present the experimental data lie in a range of momentum transfer for which the integral $\langle j_0\rangle$ is much larger than $\rho\langle j_2\rangle$. These contributions do not become comparable for momenta below $q\approx 200$ MeV/c.

In Li^6, the calculation is quite insensitive to the intermediate-coupling parameter ξ; the photon point $(q\rightarrow 0)$ is fitted well. Two curves are given in Fig. 1. For the upper curve, the oscillator parameter is $a=2.25$ F; the lower corresponds to $a=2.75$ F. These two values span the region of rough agreement with the observations.

In C^{12}, $B_{M1}(0)$ is a very sensitive function of the intermediate-coupling parameter, so two pairs of curves are drawn in Fig. 1. In each pair the upper curve corresponds to the value $a=1.75$ F for the oscillator parameter; the lower curve corresponds to $a=2.25$ F. The broken curves have $B_{M1}(0)=1.37$, the central value for the experimental result at the photon point. A better over-all fit is found by going to $B_{M1}(0)=1.13$, the experimental lower limit for the photon point. This value is used for the solid curves for C^{12} in Fig. 1.

IV. DISCUSSION

The values determined for the oscillator parameter by the comparisons of Fig. 1 are those for $1p$ nucleons. These nuclei have also been investigated by elastic electron scattering, and curves fitted to the form factors were obtained with harmonic-oscillator functions[9,10] among others.

Fits to C^{12} data, made on the assumption that the same oscillator parameter applies to both $1s$ and $1p$ protons, led to a best value of $a=1.64$ F. Corrections for finite proton size and for the shell-model center of mass reduced this by about 4%, but since such corrections are not included in the present paper, it is better to compare $a=1.64$ F with our values. If one requires the curve for B_{M1} to go through the photon point, then the $M1$ data indicate that $a\approx 2$ F. This is considerably higher than the value from the analysis of elastic scattering. Of course, the lower value, $a=1.64$ F, can fit the other points if one gives up trying to fit the photon point. This leads to $B_{M1}(0)\approx 0.8$.

The elastic electron scattering from Li^6 requires[11,12]

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different oscillator parameters for 1*s* and 1*p* protons. The most likely parameter for 1*p* protons is $a \approx 2.2$ F. The Li^6 curves of Fig. 1 indicate that a slightly bigger value of the oscillator parameter is desirable, although the curve for $a = 2.2$ F would lie fairly close to the experimental points.

The comparison of experiment and theory exhibited in Fig. 1 shows that one is on the borderline of understanding the results with a simple theory. On the experimental side it would be desirable to have some points at higher and lower momentum transfers. It would also be desirable to improve the values for the photon point, although these are difficult experiments. However, a lower value for the C^{12} photon point would remove much of the disagreement.

The interpretation of the 180° cross-section measurements at lower energy would be more complicated in that distortion effects[13] on the electron waves can be important. For the points included in the present experiments, they are not likely to have much effect.

Ultimately, more accurate experiment and calculation may show that for these *M*1 form factors the simple harmonic-oscillator radial functions are just not adequate. The use of *M*1 transitions provides a more sensitive test than elastic charge scattering since only outer shell nucleons are involved.

ACKNOWLEDGMENT

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Independent Fission Yield of Sb^{127}†

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The fractional independent yield of Sb^{127} has been found to be 0.057 ± 0.010 from the thermal-neutron fission of U^{235}. This value leads to a calculated value of 49.5 ± 0.1 for the "most probable charge" Z_P for fission products with $A = 127$ and implies that there is no pronounced effect of the 50-proton shell on the Z_P function. It was also found that $40.4 \pm 2.4\%$ of fission-product Sb^{127} is formed by beta decay of 4.4-min Sn^{127} and that $53.9 \pm 2.1\%$ is formed by beta decay of 2.15-h Sn^{127}.

INTRODUCTION

It has been suggested[1] that the "most probable charge" Z_P for fission products in the mass region below $A \cong 130$ stays close to and just above 50. This suggestion is based primarily on the independent yield of I^{128}. Wahl and Nethaway[2] have shown that the fractional independent yield of $_{50}Sn^{121}$ is small and, therefore, that Z_P for $A = 121$ is < 50. It seemed desirable to determine the independent yields of other fission products in the mass region between $A = 121$ and $A = 130$ in order to learn more about the behavior of the Z_P function. Therefore, experiments were undertaken to determine the independent yield of Sb^{127}. A description of these experiments and of the results obtained is given below.

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EXPERIMENTAL

Irradiations

Irradiations were made at the Oak Ridge Research Reactor in the pneumatic tube facility where the thermal-neutron flux density was $\sim 6 \times 10^{13}$ neutrons cm^{-2} sec^{-1}. A cadmium ratio for U^{235} fission of ~ 35 was determined by comparing the amount of Sb^{127} produced by fission of unshielded uranium samples with the amount produced by fission of samples shielded by a cadmium absorber 0.40 in. thick. One-milliliter solutions containing 100 micrograms of uranium (93% U^{235}) in ~ 0.1 M HNO_3 were irradiated in high-density polyethylene capsules for periods of 20 sec.