Form Factors for Strong $M1$ Transitions in Light Nuclei

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Strong peaks corresponding to magnetic-dipole excitation of the nucleus were observed in the energy spectrum of electrons scattered at 180° from targets of Li^6, C, Mg, and Si. The momentum transfer dependence of the excitation of these peaks was investigated by using primary electrons of 40, 55, and 70 MeV. From the measured cross sections and the Born approximation theory the inelastic form factors can be obtained. The radiative transition probability from the excited state to the ground state gives the first point on the curve of form factor as a function of momentum transfer. In the case of C^{12} a shell-model theory of the magnetic-dipole transition gives a form-factor curve in fair agreement with the experimental results.

STRONG $M1$ transitions in light nuclei can be investigated favorably in inelastic electron scattering at very large angles (~180°). The reason for this can be seen in the general expression for inelastic electron scattering[1,2] (in Born approximation)

$$\frac{d\sigma}{d\Omega}(J_f\leftarrow J_i)=\frac{k_2}{k_1}\frac{8\pi\alpha^2}{\Delta^4}\Bigg[V_L(\theta)\sum_{J=0}^{\infty}\frac{1}{2J_i+1}\times\langle J_f||M_J(q)||J_i\rangle^2+V_T(\theta)\sum_{J=1}^{\infty}\frac{1}{2J_i+1}\times(\langle J_f||T_J^{\rm el}(q)||J_i\rangle^2+\langle J_f||T_J^{\rm mag}(q)||J_i\rangle^2)\Bigg]. \quad (1)$$

In Eq. (1), k_1 and k_2 are the initial and final electron wave numbers, $\mathbf{q}^2=(\mathbf{k}_2-\mathbf{k}_1)^2$ and $\Delta^2=\mathbf{q}^2-(k_2-k_1)^2$ are the 3- and 4-momentum transfers. θ is the electron scattering angle,

$$V_L(\theta)=(\Delta^4/q^4)2k_1k_2\cos^2(\theta/2)\,, \quad (2a)$$

$$V_T(\theta)=(2k_1k_2/q^2)\sin^2(\theta/2)\times[(k_1+k_2)^2-2k_1k_2\cos^2(\theta/2)], \quad (2b)$$

and $M_J(q)$, $T_J^{\rm el}(q)$, $T_J^{\rm mag}(q)$ are the longitudinal, transverse electric, and transverse magnetic operators. The nuclear ground state is denoted by the subscript i and the excited state by the subscript f. The operators $T_J^{\rm el}(q)$ and $T_J^{\rm mag}(q)$ are exactly the same ones that describe emission and absorption of real photons but in this case the relation between the momentum transfer and energy of the photon is

$$\hbar c|\mathbf{q}_{\rm ph}|=\Delta E=E_{fi}. \quad (3)$$

At 180° the longitudinal term goes to zero and only the two transverse terms remain; the investigation of electric transitions (the $E1$ giant resonance) induced by the operator $T_J^{\rm el}(q)$ is given in another publication.[3]

Magnetic-dipole transitions are induced by the operator

$$T_{JM}^{\rm mag}(q)=\int d\mathbf{x}[\boldsymbol{\mu}_N(\mathbf{x})\cdot(\nabla\times j_J(qx)\mathfrak{Y}_{JJ1}^M(\Omega_x))+\mathbf{j}_N(\mathbf{x})\cdot j_J(qx)\mathfrak{Y}_{JJ1}^M(\Omega_x)], \quad (4)$$

where $e\mathbf{j}_N(\mathbf{x})$ and $e\boldsymbol{\mu}_N(\mathbf{x})$ are the nuclear current and magnetization density operators, e is the proton charge, $j_J(qx)$ is the spherical Bessel function of order J, and $\mathfrak{Y}_{JJ1}^M(\Omega_x)$ are the vector spherical harmonics.[4]

In previous experiments done at Stanford[5,6] electrons of 41 MeV from the Mark II accelerator were used to excite strong $M1$ transitions in Li^6, C^{12}, Ne^{20}, Mg^{24}, Si^{28}, and S^{32}. Calculations by Kurath[7] revealed there is a tendency for the transition strength to be concentrated in a few levels (as in the $E1$ giant resonance). This feature was confirmed by experiment.[6]

In this paper we present results of our recent measurements on the $M1$ transitions in Li^6, C^{12}, Mg^{24}, and Si^{28} (where they are quite outstanding) at different momentum transfers. The Mark II Stanford Linear Accelerator, modified to give energies up to 75 MeV, was used in these experiments; the experimental setup was the same as described previously.[3]

Figure 1 shows typical results obtained for the 15.1-MeV level in C^{12} at energies of 41.5, 54, and 70 MeV for the incident electrons. From this data and using formula (1) evaluated at 180° one obtains the square of the reduced matrix element for this state $\langle 1^+||$

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TABLE I. Data on $M1$ transitions.

	q MeV/c	Cross section[a]	Reference	$\lvert\langle J_f\Vert T_1^{\mathrm{mag}}(q)\Vert J_i\rangle\rvert^2$ (10^{-3})	$(1/q^2)\lvert\langle J_f\Vert T_1^{\mathrm{mag}}(q)\Vert J_i\rangle\rvert^2$ $[10^{-6}(\mathrm{MeV}/c)^{-2}]$
			15.1-MeV level in C^{12}		
Photons	15.1	(2.05 ± 0.27)	8	0.044	(0.19 ± 0.025)
Electrons	68	(2.0 ± 0.3)	this work	0.55	(0.12 ± 0.018)
Electrons (160°)	68	$(2.6\pm_{0.40}^{0.52})$	5	0.69	$(0.15\pm_{0.022}^{0.030})$
Electrons	93	(2.0 ± 0.3)	this work	0.87	(0.10 ± 0.015)
Electrons	125	(1.5 ± 0.25)	this work	1.10	(0.07 ± 0.012)
			3.56-MeV level in Li^6		
Photons	3.56	$(0.92\pm_{0.15}^{0.20})$	b	0.014	$(1.10\pm_{0.18}^{0.24})$
Electrons	76	(3.0 ± 0.45)	this work	2.0	(0.38 ± 0.057)
Electrons	106	(1.55 ± 0.23)	this work	2.1	(0.19 ± 0.028)
Electrons	136	(0.90 ± 0.14)	this work	2.0	(0.11 ± 0.017)
			11.6-MeV level in Si^{28}		
Photons	11.6	(9.8 ± 2.6)	c	0.016	(1.2 ± 0.32)
Electrons	71.5	(3.5 ± 1.4)	d	0.92	(0.18 ± 0.07)
Electrons	88.4	(3.0 ± 0.75)	this work	1.2	(0.15 ± 0.038)
Electrons	129.4	(1.8 ± 0.45)	tlis work	1.5	(0.09 ± 0.022)
			11-MeV level in Mg^{24}		
Photons	11	(13.1 ± 3.8)	c	0.02	(1.7 ± 0.50)
Electrons	83	(4.4 ± 0.88)	6	1.5	(0.22 ± 0.044)
Electrons	97	(3.5 ± 0.70)	this work	1.6	(0.1 ± 0.034)
Electrons	129	(1.8 ± 0.36)	this work	1.3	(0.081 ± 0.016)

[a] In units of 10^{-27} MeV-cm² for photons and 10^{-32} cm²/sr for electrons.
[b] L. Cohen and R. A. Tobin, Nucl. Phys. **14**, 243 (1960).
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$T_1^{\mathrm{mag}}(q)\Vert 0^+\rangle^2$ at momentum transfers of 68, 93, and 125 MeV/c. Resonance-fluorescence experiments[8] permit an evaluation of the same matrix element at $q=15.1$ MeV/c. The expression to be used in this case was shown by Lewis and Walecka[9] to be

$$\langle J_f\Vert T_1^{\mathrm{mag}}(q_{ph})\Vert J_i\rangle^2=\frac{E_{fi}\int\sigma_{abs}(E)dE}{(2\pi)^3\alpha(\hbar c)^2}(2J_i+1)\,. \quad (5)$$

For a discrete state the integral of the absorption cross section is related by the principle of detailed balance to the partial width $\Gamma_{\gamma 0}$ for radiation to the ground state by the equation

$$\Gamma_{\gamma 0}=\frac{E_{fi}^2}{(\pi\hbar c)^2}\frac{(2J_i+1)}{(2J_f+1)}\int\sigma_{abs}(E)dE\,. \quad (6)$$

In Table I we list the square of the reduced matrix elements $\langle J_f\Vert T_1^{\mathrm{mag}}(q)\Vert J_i\rangle^2$ for the different momentum transfers and several elements.

In column 6 of Table I we give the quantity $(1/q^2)\lvert\langle J_f\Vert T_1^{\mathrm{mag}}(q)\Vert J_i\rangle\rvert^2$ which is proportional to the reduced transition probability $B_{M1}(f\rightarrow i,q)$ used by Alder *et al.*[2] and Kurath.[10]

In the case of C^{12} one can obtain a simple theoretical formula for this matrix element by assuming that the 15.1-MeV level is excited as a single-particle transition from the $(1p_{3/2})$ to the $(1p_{1/2})$ shell using j-j coupling.[11]

The resulting expression is

$$\langle(1p_{1/2})(1p_{3/2})^{-1},J^\pi=1^+,T=1,M_T=0\Vert$$
$$\times\Vert T_1^{\mathrm{mag}}(q)\Vert J=0,T=0\rangle^2=\frac{1}{9\pi}\left(\frac{\hbar q}{Mc}\right)^2$$
$$\times\{[(\mu_p-\mu_n)-\tfrac{1}{2}]F_0(q)+[\tfrac{1}{4}(\mu_p-\mu_n)-\tfrac{1}{2}]F_2(q)\}^2\,, \quad (7)$$

where M is the proton mass, μ_p and μ_n are the proton and neutron magnetic moments in units of nuclear

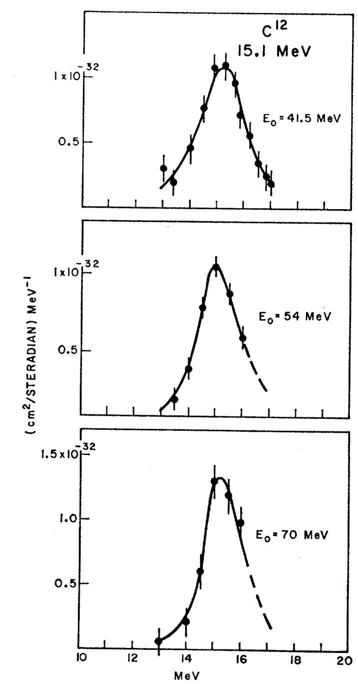

FIG. 1. Results obtained for the excitation of the C^{12} 15.1-MeV level in inelastic electron scattering at 180° for different incident electron energies.

[8] H. Schmid and W. Scholz, Z. Physik **175**, 430 (1963).
[9] F. H. Lewis, Jr. and J. D. Walecka, Phys. Rev. **133**, B849 (1964).
[10] D. Kurath, Phys. Rev. **134**, B1025 (1964), following paper.
[11] Particle-hole calculations carried out by Vinh Mau and Brown using j-j coupling for the 1^+, $T=1$ states in C^{12} show that the 15.1-MeV level is almost a pure $(1p_{1/2})(1p_{3/2})^{-1}$ single-particle state in this coupling scheme. [See N. Vinh Mau and G. E. Brown, Nucl. Phys. **29**, 89 (1962).]

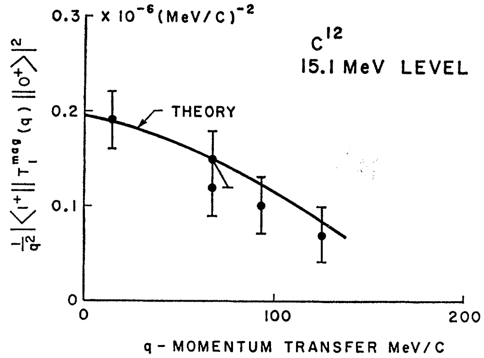

FIG. 2. The experimental values of the quantity $(1/q^2) \times\langle 1^+||T_1^{\rm mag}(q)||0^+\rangle^2$ as a function of the momentum transfer. The theoretical curve was normalized as explained in the text.

magnetons, which we take to be the magnetic moments of free nucleons. The form factors $F_0(q)$ and $F_2(q)$ are defined by

$$F_l(q)=\int_0^\infty |R_{1p}(r)|^2 j_l(qr)r^2dr\,, \tag{8}$$

and $R_{1p}(r)$ is the radial wave function for a particle in the $1p$ shell. Calculation of these form factors using harmonic oscillator wave functions gives

$$F_0(q)=e^{-\frac{1}{4}x^2}[1-\tfrac{1}{6}x^2]\,, \tag{9}$$

$$F_2(q)=e^{-\frac{1}{4}x^2}[\tfrac{1}{6}x^2]\,, \tag{10}$$

where $x=qb$, b=oscillator length parameter. By choosing this parameter to fit Coulomb energy differences in mirror nuclei[12] we obtain $b\cong 1.6$ F.

[12] B. C. Carlson and I. Talmi, Phys. Rev. 96, 436 (1954).

It is clear from expression (7) that if we plot $(1/q^2)\langle 1^+||T_1^{\rm mag}(q)||0^+\rangle^2$ as a function of q the resulting curve varies with q only through $F_0(q)$ and $F_2(q)$ which are derived from the radial wave functions. We made this plot for the case of C^{12} and the resulting curve is about four times higher than the experimental points but approximately parallel to them. This suggests that the q dependence contained in the radial wave functions is approximately correct, but that the type of coupling assumed in calculating the 15.1-MeV transition is not adequate. Figure 2 illustrates this result where the experimental values of $(1/q^2)\langle 1^+| \times|T_1^{\rm mag}(q)||0^+\rangle^2$ are compared with the theoretical prediction divided by a normalization factor of 3.8. The normalized curve is in good agreement with the electron scattering points, but we have plotted only the most recent[8] of the photon results, which has the lowest quoted error, but also is lower in magnitude than any of the other photon scattering results. The parameters of the 15.1-MeV level as determined by earlier photon scattering experiments are also given in Ref. 8. Calculations using intermediate coupling have been done by Kurath,[10] and the results indicate that the absolute value of $(1/q^2)\langle 1^+||T_1^{\rm mag}(q)||0^+\rangle^2$ depends rather sensitively on the type of coupling. Kurath also gives results calculated using a larger value of the oscillator parameter. In this case the form factor falls more rapidly with q, and the resulting curve gives a better fit to the experimental data if all of the photon scattering measurements are included.