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Comparison of Moderate Energy Proton-Proton Models. II*

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The published phases of Scotti and Wong, and of the Yale YLAM and YRB1 representations, are compared to the 647 pieces of 10–320-MeV scattering data used in a previous comparison. A ten-parameter comparison representation for the phase shifts is found to fit the data about as well as most models, and considerably better than the Scotti-Wong phases.

I. INTRODUCTION

In a previous communication,[1] the predictions of six recently proposed two-nucleon models were compared to 647 pieces of 10–316-MeV proton-proton scattering data. Equivalent results for several more models are reported here. In addition, a simple comparison representation is set up, in which the least-squares error sum χ^2 is easily minimized with respect to the representation parameters. The values of χ^2 for several numbers of search parameters in this comparison representation are also shown.

II. THE COMPARISON REPRESENTATION

A general characteristic of the two-nucleon models is that the phase shifts for all states of orbital angular momentum $L>1$ smoothly become one-pion-exchange (OPE) as the energy is decreased. In a comparison representation, one also desires that the adjustable parameters have roughly the same types of effects as the model parameters.

The comparison representation (hereafter CR) used was an energy-dependent phase-shift representation. The above desired conditions were at least partly met by representing the phases by OPE values multiplied by polynomials in the scattering energy. For states with $L>1$, the leading (zero power of energy) coefficients were fixed at unity, assuring OPE behavior at low energy. For $L=0$ and 1, the representations were rather arbitrarily taken as singlet even-parity OPE phases multiplied by polynomials. In addition, the $^1S_0$ contained added Coulomb-corrected scattering length and effective range contributions. Details are shown in Table I.

The scattering-length and effective-range values for $^1S_0$ were set by fitting the 10–50-MeV Hamada-Johnston[2] model $^1S_0$ phase shifts. The polynomial coefficients were initially set equal to values which produced phases close to the Hamada-Johnston values. The coefficients were then adjusted so as to obtain a least-squares fit to the 647 data. The fitting method reported by Lietzke[3] was used, resulting in definite minimization of χ^2. The standard deviations on the search parameters (polynomial coefficients) were obtained as by-products from the second-derivative matrix used in the minimization.

Examination of the parameters and their standard deviations, resulting from the above procedure, disclosed several parameters which were not significantly

TABLE I. Least-squares sum χ^2 for various models and 647 proton-proton scattering data in the energy range 10–316 MeV. CR(N) denotes the comparison representation, described in the text, with N being the number of searched-upon parameters. Some results previously obtained are shown for comparison.

Model	χ^2	χ^2/χ^2[CR(21)]
CR (21)	1657	1.00
YLAM	2189	1.32
CR (10)	2564	1.55
YRB1	2753	1.66
Scotti-Wong	6612	4.00
Yale[a]	2477	1.49
Hamada-Johnston[a]	3061	1.85
Saylor-Bryan-Marshak[a]	4454	2.69
Brueckner-Gammel-Thaler[a]	37678	22.8

[a] See Ref. 1.

TABLE II. Polynomial coefficients for CR (10) and CR (21). x=lab energy/100. $a_0=-7.773$ F. $r_0=2.767$ F.

Phase shift (nuclear bar)	Polynomial OPE phase	Polynomial Power of x	Coefficients CR (10)	Coefficients CR (21)
$^1S_0$	$^1I_6$	0	-65.5	-101.4
		1	-2.47	32.2
		2		-7.18
$^1D_2$	$^1D_2$	1	1.15	1.057
$^1G_4$	$^1G_4$	2		0.1616
$^3P_0$	$^3P_0$	0	1.24	0.8742
	$^1G_4$	0	-93.1	-47.54
		1		-9.313
$^3P_1$	$^3P_1$	0	0.697	0.7421
		1		-0.07042
		2		0.02645
$^3P_2$	$^3P_2$	0	5.11	6.406
		1		-1.880
		2	-0.241	0.2706
$\bar{\epsilon}_2$	$\bar{\epsilon}_2$	1	-0.239	-0.3262
		2		0.04084
$^3F_2$	$^3F_2$	1		-0.5439
		2	-0.170	0.1075
$^3F_3$	$^3F_3$	1		-0.1380
$^3F_4$	$^3F_4$	2		0.2889
$\bar{\epsilon}_4$	$\bar{\epsilon}_4$	2		-0.05525

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different from zero. Such coefficients were eliminated and others tried. Only those significantly different from zero were finally kept.

III. RESULTS

The values of the goodness of fit parameter χ^2 is shown in Table I for the Scotti-Wong[4] model published phases, and for the Yale[5] energy-dependent phase analyses YLAM and YRB1. The values for two comparison representations are also listed: The corresponding polynomial coefficients are shown in Table II.

Examination of the contributions to χ^2 from the individual data points revealed that the single datum (10.2°) at 98 MeV contributed 421 to χ^2 for CR(21). The five lowest-angle cross section points at 98 MeV (including the 10.2° point) contributed a total of 548. It is to be strongly recommended that these data and their associated experimental standard deviations be re-examined.

It would appear that the Scotti-Wong model, as represented by the published phases, is rather poor if judged as a phenomenological model against CR(10). Quite different criteria should be applied, of course, if the Scotti-Wong model is judged theoretically.

The computations reported here were carried out in the Computation Center of the Pennsylvania State University and the Atomic Energy Commission Computation Center at New York University.

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Negative Pion Capture From Rest on Complex Nuclei

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The capture of negative pions from rest by light nuclei, primarily O^{16}, is investigated by means of a shell-model calculation. The conventional pion-nucleon ps–(ps) interaction is used to calculate the probability of one-nucleon and two-nucleon ejection from the nucleus. Due to the effects of distorted waves, one-nucleon ejection is not found to be suppressed as has been previously supposed but is comparable to the two-nucleon mode. It is also found that back-to-back ejection of a nucleon pair is dominant over parallel ejection, and that the capture probability as a function of the angle between an ejected pair will show structure.

I. INTRODUCTION

THOUGH the capture of the π^- meson from rest by deuterons was used 13 years ago[1,2] to determine that the pion was a pseudoscalar, little experimental or theoretical work since has developed on the capture from rest by more complex nuclei. More recently, interest has developed experimentally[3–5] on such processes and for the first time experiments are being done using counters[5] rather than emulsion or cloud chamber techniques. Experimental data are sparse and for the most part, with a few exceptions,[6,7] theoretical calculations of the various modes of capture[8] are nonexistent.

In the early theoretical work of Brueckner, Serber, and Watson[2,9] calculations were made of π^- capture on complex nuclei by means of extrapolating the deuterium capture in an obvious way by saying that

$$(1/Z)\sigma[\pi^-+A \rightarrow \text{star}]=\Gamma\sigma[\pi^-+D \rightarrow 2n], \quad (1)$$

where the left-hand side of this expression contains the cross section σ for absorption on a nucleus of number A and the right-hand side contains a factor Γ which allows in a vague way for the effect of the remaining $(A-2)$ nucleons on the capturing pair. Since no analytic expression for the pion-nucleon interaction existed at the time of their work, it was necessary to calculate $\sigma[\pi^-+A \rightarrow \text{star}]$ by means of a phenomenological R-matrix approach with a partial closure approximation.

This calculation suffered the additional disadvantage

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