Fermi-Liquid Effects in Cyclotron Resonance

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(Received 19 November 1963)

The steady-state properties of transverse circularly polarized waves propagating along a static magnetic field in a uniform plasma (metal) are considered. Using the Landau theory of Fermi liquids, we compute the effect of correlations on the reflection properties of a semi-infinite metallic plasma. The expressions are numerically evaluated and discussed for a range of parameters pertinent to the alkalis.

I. INTRODUCTION

THE properties of electromagnetic waves propagating in a plasma along a magnetic field have been extensively studied.[1–3] In previous calculations the electrons were treated as the current carrying constituent of a noninteracting gas. For real metals the *a priori* neglect of electron-electron interaction effects is unsatisfactory since the mean energy of interaction of the electrons (Coulomb energy) is of the order of their mean kinetic energy. Under such conditions substantial correlation effects in the motion of the electrons may be expected. From a practical point of view, however, it is known that for a wide range of physical parameters, treatments which neglect explicit correlation effects provide an accurate description of the electromagnetic properties of metals and semimetals. We are interested in determining the range of physical parameters, frequency, external magnetic field, carrier density, etc., where the effect of electron-electron correlation may, hopefully, be measured experimentally.

In 1956, Landau[4] constructed a semiphenomenological theory of a system of fermions, such as He_3, interacting via a short-range two-body force. This work was later extended by Silin[5] to a Fermi liquid with long-range Coulomb interactions. We make use of the Fermi-liquid theory to compute the reflection properties of a semi-infinite slab of metal placed in a magnetic field oriented perpendicular to its surface. In this geometry for electromagnetic waves incident normally, it is known[1,3] that the metallic sample will exhibit an absorption edge at approximately the Doppler shifted cyclotron frequency.

$$\omega_c/\omega = 1 + (q/q_0)(V_F/c)\,, \tag{1}$$

where ω_c is the electron cyclotron frequency, q is, crudely speaking, the wave number of the electromagnetic field in the medium, q_0 the wave number in free space, and V_F is the Fermi velocity. The resonance in the electrons motion which exhibits itself as a rapid fluctuation in the surface impedance of the sample is shifted from the value $\omega_c/\omega = 1$. This comes about since the electrons traveling with the phase velocity of the wave see a Doppler shifted frequency. It is these electrons which interact most strongly with the wave. As a result the resonance in the impedance is shifted. Specifically we are interested in possible shifts in the position of, and modifications of the shape of the Doppler shifted electron-cyclotron resonance as the result of electron–electron interactions.

II. CALCULATION

The basic assumption of the Fermi-liquid theory is that it presupposes that as the interaction is turned on, the single-particle states in the neighborhood of the last occupied one remain approximate eigenstates of the interacting system, and that there is a one-to-one correspondence between these states and the single-electron states of the noninteracting Fermi gas. These approximate eigenstates are called quasiparticle states. For slowly varying external disturbances, the transport properties of the Fermi liquid are completely described by the quasiparticle distribution function $f(\mathbf{P},\mathbf{X})$ in momentum and configuration space.[6] (The external disturbance varies slowly enough in space so that the lack of commutivity of $\mathbf{P}$ and $\mathbf{X}$ is unimportant.) In equilibrium and at zero temperature the distribution of quasiparticles $f_0(\mathbf{P},\mathbf{X})$ is

$$\begin{aligned} f_0(\mathbf{P},\mathbf{X}) &= 1\,, \qquad |P| < P_F\,,\\ &= 0\,, \qquad |P| > P_F\,. \end{aligned} \tag{2}$$

For the spherically symmetric electron gas the quasiparticle energy is $E^0(P) = P^2/(2m^*)$, where m^* is an effective mass whose value depends on the dynamics of the interaction between quasiparticles. In nonequilibrium situations, the quasiparticle distribution function satisfies a transport equation similar to the Boltzmann equation. We are interested in the transport equation in the linear approximation, i.e., when the deviation from f_0 is small. If we write $f = f_0 + f_1$, with

[1] P. B. Miller and R. R. Haering, Phys. Rev. **128**, 126 (1962).
[2] P. M. Platzman and S. J. Buchsbaum, Phys. Rev. **128**, 1004 (1962).
[3] P. M. Platzman and S. J. Buchsbaum, Phys. Rev. **132**, 2 (1963).
[4] L. D. Landau, Zh. Eksperim. i Teor. Fiz. **30**, 1058 (1956) [English transl.: Soviet Phys.—JETP **3**, 920 (1956)].
[5] V. P. Silin, Zh. Eksperim. i Teor. Fiz. **33**, 495 (1957) [English transl.: Soviet Phys.—JETP **6**, 387 (1958)].

[6] The distribution function $f(\mathbf{P},\mathbf{X})$ is a matrix in the spin variables $F(\mathbf{P},\mathbf{X}) = f(\mathbf{P},\mathbf{X}) + \mathbf{m}(\mathbf{P},\mathbf{X})\cdot\boldsymbol{\sigma}$. The function $\mathbf{m}(P,x)$ is pertinent only when quantities which depend on the spin (the susceptibility) are of interest. We will be concerned only with $f(\mathbf{P},\mathbf{X})$.

$f_1 \equiv \delta[E^0(P)-E_F]g$ the linearized transport equation for g becomes

$$\partial g/\partial t+\mathbf{V}\cdot\nabla_{\mathbf{X}}(g+E_1)+(e/c)(\mathbf{V}\times\mathbf{H})\cdot\nabla_{\mathbf{P}}(g+E_1)-e\mathbf{E}\cdot\mathbf{V}=-\nu_c g. \quad (3)$$

The quantity ν_c is a phenomenological short-range collision term which permits the system to relax to equilibrium; $V_\alpha \equiv \partial E^0(P)/\partial P_\alpha = P_\alpha/m^*$ is the velocity of the quasiparticles and

$$E_1=\int S(\mathbf{P},\mathbf{P}')\delta[E_F-E^0(P')]g(\mathbf{P}',\mathbf{X})\times[d^3P'/(2\pi\hbar)^3] \quad (4)$$

is the change in energy of the quasiparticles produced by a change in their distribution function.

The correlation, or interaction function $S(\mathbf{P},\mathbf{P}')$ is the basic quantity characterizing the Fermi liquid. Owing to the presence of the delta function in the distribution function f_0 and in the transport equation, the value of $S(\mathbf{P},\mathbf{P}')$ is only of interest for $|P|=|P'|=P_F$, where P_F is the Fermi momentum. $S(\mathbf{P},\mathbf{P}')$ may then be considered to be a function of the angle between $\mathbf{P}$ and $\mathbf{P}'$. It is conveniently represented by an infinite sequence of Legendre polynomials.

$$S(\hat{P}\cdot\hat{P}')=\textstyle\sum_n S_n P_n^{(0)}(\cos\theta)\,, \quad (5)$$

where $\hat{P}$ is a unit vector in the direction of $\mathbf{P}$. For convenience, we define a set of dimenionsless quantities

$$\delta_0 \equiv S_0 m^* P_F/(2\pi)^2\hbar^3$$
$$\delta_n \equiv [S_n m^* P_F/(2\pi)^2\hbar^3][(n-1)!/(n+1)!]n\geq 1. \quad (6)$$

The correlation function $S(\hat{P}\cdot\hat{P}')$ contains all the information pertaining to explicit dynamical many-body effects. The limit $S\rightarrow 0$ is the free electron or Hartree limit. In this limit the transport equation, Eq. (3) becomes the usual Vlasov equation and the only "correlation" effect which is included is the self-consistent electromagnetic field.

The correlation function is as important to the many electron problem as the shape of the Fermi surface is in the one-electron problem. Unfortunately, the function $S(\hat{P}\cdot\hat{P}')$ (for the region of metallic densities), cannot be computed from first principles.[7] Hopefully then a transport experiment of the kind to be considered here might enable one to measure the characteristics of the scattering function $S(\hat{P}\cdot\hat{P}')$. Ideally one would like to perform an experiment in which a specific moment of the scattering function is measured. It was first pointed out by Landau[4] that the zeroth and first moments of the scattering function were simply related to two elementary properties of the quasiparticle gas. The clothed mass m^* of the quasiparticles is related to the first moment S_1 by

$$m^*/m=1+\tfrac{4}{3}\delta_1 \quad (7)$$

and the velocity of ordinary sound v_S is related to the zeroth moment S_0 by

$$v_S=(\delta_0+1)^{1/2}[P_F/(3mm^*)^{1/2}]. \quad (8)$$

A cyclotron resonance experiment in the extreme anomalous limit, in the so-called Azbel'-Kaner geometry,[8] measures only the effective mass.[9] On the other hand, the cyclotron resonance experiment in the so-called Galt geometry[10] involves, as we shall see, all of the moments. In the Galt geometry a static magnetic field perpendicular to the surface of the sample is utilized and the reflection (or absorption) of circularly polarized electromagnetic waves propagating along the magnetic field is measured as a function of the applied field. The solution, neglecting correlations, of the coupled Maxwell-Boltzmann equations in the presence of a single boundary (assuming specular reflection of the carriers at the surface) may be reduced to the solution of an equivalent infinite medium problem.[3] The final expression for the surface impedance Z is

$$\frac{Z}{Z_0}=(2i/\pi)\int_0^\infty \frac{dq}{[q^2-\epsilon(q,\omega)]}, \quad (9)$$

where Z_0 is the impedance of free space. The quantity $\epsilon(q,\omega)$ is the finite wave number and frequency-dependent dielectric constant for the infinite medium, $[\epsilon(q,\omega)\equiv 1+\sigma^+(q,\omega)/i\omega]$.

The quantity E_1 is Eq. (4) is related to a first-order change in the distribution function by the relationship

$$E_1=\int S(\mathbf{P},\mathbf{P}',\mathbf{X},\mathbf{X}')\delta[E_F-E^0(P')]g(\mathbf{P}',\mathbf{X}')\times\frac{d^3P'd^3X'}{(2\pi)^3\hbar^3}. \quad (10)$$

In writing Eq. (4) we have assumed that $S(\mathbf{P},\mathbf{P}',\mathbf{X},\mathbf{X}')=S(\mathbf{P},\mathbf{P}')\delta(\mathbf{X}-\mathbf{X}')$. With this assumption the spatial part of the problem is identical with that in the noninteracting case. For an interacting gas of quasiparticles in real materials correlations are not local in space but do in fact extend over distances of the order of a Debye length ($\lambda_D \simeq V_F/\omega_P$). For metallic densities this is of the order of the interparticle spacing. The Debye length is small compared with distances over which the field varies appreciably, typically distances of the order of the high-frequency classical skin depth ($\lambda_S \simeq c/\omega_P$), so that to a good approximation we can neglect nonlocal effects. With the locality assumption on the function

[7] C. Herring has recently computed the scattering function S as a power series in $r_s(r_s\equiv n^{1/3}a_H$ where n is the electron density and a_H is the Bohr radius) for metals $r_s\approx 2$–6 (private communication).

[8] M. Ya. Azbel, E. A. Kaner, Zh. Eksperim. i Teor. Fiz. **30**, 811 (1956) [English transl.: Soviet Phys.—JETP **3**, 772 (1956)].

[9] This statement has been proved by J. M. Luttinger (private communication).

[10] J. K. Galt, W. A. Yager, F. R. Merritt, B. B. Cetlin, and A. D. Brailsford, Phys. Rev. **114**, 1396 (1959).

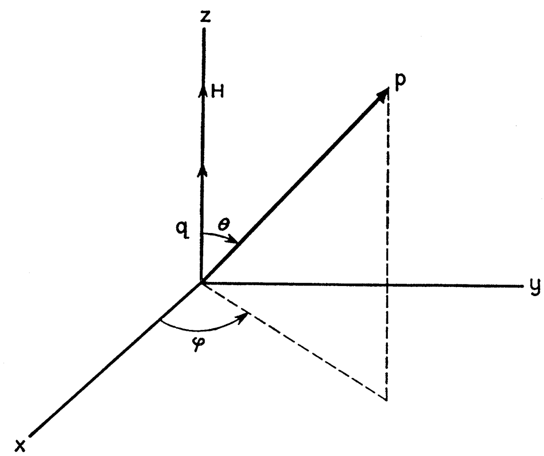

FIG. 1. Polar coordinate system used for solution of the transport equation, Eq. (3). H is the magnetic field, q is the propagation vector, and P is the quasiparticle momentum.

$S(\mathbf{P},\mathbf{P}',\mathbf{X},\mathbf{X}')$, the expression for the impedance within the framework of the Fermi-liquid theory is still given by Eq. (9). However, the dielectric constant $\epsilon(q,\omega)$ will now include the effects of correlations.

This dielectric constant is obtained by linearizing the transport equation, Eq. (2), with an rf field varying as $\exp(i\mathbf{q}\cdot\mathbf{X}-\omega t)$ and solving for the induced current. The current is given by

$$I^\alpha(q)=\frac{ne}{m}\int\frac{d^3P}{(2\pi)^3\hbar^3}P^\alpha g(\mathbf{P},q)\delta(E_P{}^0-E_F) \qquad (11)$$

or, equivalently, using Eq. (4)

$$I^\alpha(q)=\frac{ne}{m^*}\int\frac{d^3P}{(2\pi)^3\hbar^3}P^\alpha(g+E_1)\delta(E_P{}^0-E_F)\,. \qquad (12)$$

We choose our coordinate system (see Fig. 1) so that the static magnetic field H and the propagation vector $\mathbf{q}$ point along the z axis. The angles θ and Φ are polar angles specifying the direction of $\mathbf{P}$. For a right-handed circularly polarized wave $(\mathbf{E}=E_x\hat{U}x+iE_y\hat{U}_y)$ the transport equation for $g(\mathbf{P},q)$ is,

$$\begin{aligned}-i\tilde{\omega}g(\mathbf{P},q)&+\frac{iqP}{m^*}\cos\theta g(P,q)+\omega_c{}^*\frac{\partial g}{\partial\Phi}\\&+\left[\frac{iqP}{m^*}\cos\theta+\omega_c{}^*\frac{\partial}{\partial\Phi}\right]\\&\times\int d^3P'/(2\pi\hbar)^3S(\mathbf{P},\mathbf{P}')\delta(E_F-E_{P'}{}^0)g(\mathbf{P}',q)\\&\times eEe^{i\Phi}\frac{P}{m^*}\sin\theta\,, \qquad (13)\end{aligned}$$

where $\omega_c{}^*=|eH/m^*c|$ and $\tilde{\omega}=\omega+i\nu_c$.

If we set $g(\mathbf{P},q)\equiv g(\theta,\Phi)=g(\theta)e^{i\Phi}$, then it is easily shown that

$$g(\theta)=\frac{ieEV_F{}^*\sin\theta}{\xi}+\frac{(qV_F{}^*\cos\theta+\omega_c{}^*)}{\xi}\Lambda(\theta)\,, \qquad (14)$$

where $V_F{}^*=P_F/m^*$ and

$$\xi=(\tilde{\omega}-\omega_c{}^*-qV_F{}^*\cos\theta)\,. \qquad (15)$$

The quantity $\Lambda(\theta)$ is

$$\Lambda(\theta)=\sum_{n=0}^{\infty}\delta_nP_n{}^{(1)}(\cos\theta)\times\int_\pi^0 g(\theta')P_n{}^{(1)}(\cos\theta')d(\cos\theta')\,. \qquad (16)$$

Equation (14) is an infinite set of coupled linear equations for the quantities $\Omega_n=\int_\pi^0 P_n{}^{(1)}(\cos\theta)g(\theta)d(\cos\theta)$. If we terminate the spherical harmonic expansion for $S(\hat{P}\cdot\hat{P}')$ after a finite number of terms, then Eq. (16) may be solved self-consistently for the quantity $g(\theta)$. Knowing $g(\theta)$, we may substitute into Eq. (11) and evaluate the current.

For the purposes of this paper we will include the first three moments of $S(\hat{P}\cdot\hat{P}')$ S_0, S_1, and S_2. We have no real justification for terminating this series after so few terms; however, there are at least some crude arguments which suggest that a Legendre expansion for $S(\hat{P}\cdot\hat{P}')$ converges rapidly. Landau has shown that the function $S(\hat{P}\cdot\hat{P}')$ is proportional to the negative of the forward scattering amplitude of two quasiparticles. A typical type of exchange scattering diagram which enters into a microscopic calculation of $S(\hat{P}\cdot\hat{P}')$ is shown in Fig. 2. The dashed line is a bare Coulomb line and the solid lines represent electron (hole) propagators. This type of scattering diagram contributes a term to $S(\hat{P}\cdot\hat{P}')$ of the form

$$S_{\text{exch.}}(\hat{P}\cdot\hat{P}')\sim\frac{-e^2}{|\mathbf{P}'-\mathbf{P}|^2\epsilon_L(|\mathbf{P}'-\mathbf{P}|,0)}\,, \qquad (17)$$

where $\epsilon_L(q,0)$ is the zero-frequency longitudinal dielectric constant. If ϵ_L is computed by summing the

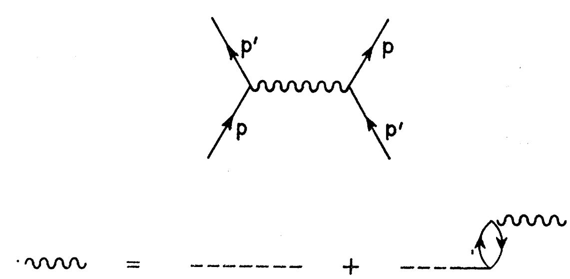

FIG. 2. A typical type of exchange scattering diagram which contributes to $S(\hat{P}\cdot\hat{P}')$. Solid lines are electron propagators and dashed lines represent Coulomb propagators.

simple set of bubble diagrams shown in Fig. 2 (RPA approximation), we find

$$\epsilon_L(q,0)=1+(q_D^2/q^2)F(q)\,, \tag{18}$$

where $F(q)\approx 1$ for $q\ll q_F$ and $F(q)\approx\frac{1}{2}$ for $q=2q_F$. Using this expression for the dielectric constant, we find

$$S(0)/S(\pi)=\tfrac{1}{2}[1+8P_F^2/q_D^2]\,. \tag{19}$$

For potassium $S(0)/S(\pi)\approx 0.7$ so that on the basis of this crude computation the scattering is roughly spherical.

If we truncate Eq. (5) after three terms, solve Eq. (16) and substitute into Eq. (11), we find for the scalar conductivity

$$\sigma^+=\tfrac{3}{4}\omega_P^2 iV^*\left[1+4\delta_1/3-\tilde{\omega}\delta_1 V^* -\frac{(1+4\delta_1/3)}{[1+(V^*/\tilde{\omega}W^{*2})(1/(9\delta_2)+4/15-\omega\gamma W^*)]}\right]^{-1}, \tag{20}$$

where $\gamma=(\tilde{\omega}-\omega_c^*)/qV_F^*$, and $\omega_P=(4\pi ne^2/m)^{1/2}$. The functions V^* and W^* are defined by

$$V^*=\int_\pi^0 \frac{\sin^2\theta d(\cos\theta)}{(\tilde{\omega}-\omega_c^*-qV_F^*\cos\theta)}, \tag{21}$$

$$W^*=\int_\pi^0 \frac{\sin^2\theta\cos\theta d(\cos\theta)}{(\tilde{\omega}-\omega_c^*-qV_F^*\cos\theta)}. \tag{22}$$

The conductivity, although it is reasonably complicated, approaches some rather simple and physically meaningful limiting values. In the limit $q\rightarrow 0$, σ_+ approaches

$$(\sigma_+)_{q\rightarrow 0}=i\omega_P^2/(\tilde{\omega}-\omega_c)\,. \tag{23}$$

There are no effects due to correlations in this limit. This is a consequence of the fact that total momentum is conserved in electron-electron collisions. The induced-current (for a zero wave vector) electric field is not affected by the electron-electron interactions. In the limit $\omega/\omega_c\rightarrow 0$, the effect of correlations to lowest order in ν_c/ω_c vanishes. This is true not only for our truncated scattering function, but for a general scattering function as well. If we examine the transport Eq. (3) and the second expression for the current Eq. (12), then neglecting the relaxation term and the time derivative term we see that $(g+E_1)$ satisfies the usual transport equation, without correlations. The energy delta function in Eq. (12), is just sufficient to produce a factor m^* canceling the $1/m^*$ in front of the integral leading to the uncorrelated conductivity.

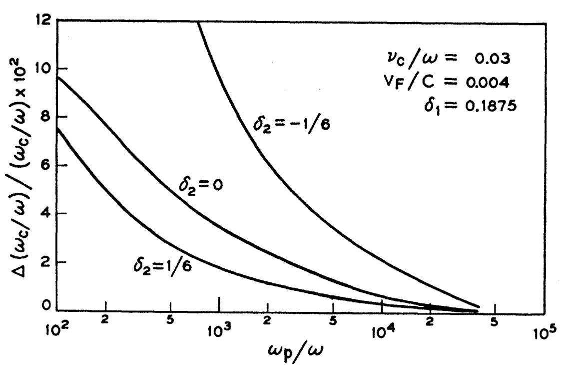

FIG. 3. A plot as a function of ω_p/ω of the relative shift in the position of the Doppler shifted absorption edge due to correlation effects.

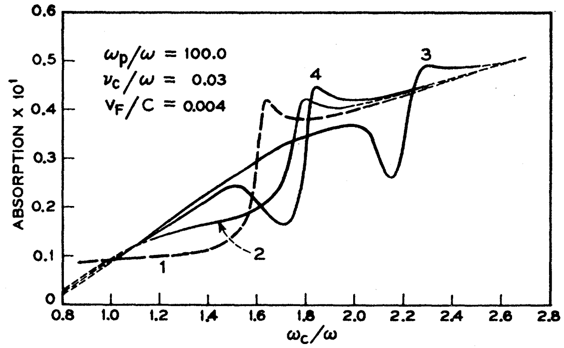

FIG. 4. A plot of the Doppler shifted absorption edge as a function of magnetic field. The plasma frequency, collision frequency and Fermi velocity are fixed. The four curves show the effect of correlation on the edge. The curve labeled (1) has a $\delta_1=0$ and a $\delta_2=0$, i.e., uncorrelated. Curve (2) has $\delta_1=0.1875$, $\delta_2=0$. Curve (3) has $\delta_1=0.1875$, $\delta_2=-0.166$, and curve (4) has $\delta_1=0.1875$, $\delta_2=-0.05$.

III. EVALUATION AND DISCUSSION OF THE SURFACE IMPEDANCE

Typically, the absorption coefficient of a semi-infinite metallic plasma exhibits a rather sharp resonance at the Doppler shifted resonance frequency.[11] If we substitute for q in Eq. (1), the value which is obtained from a simple analysis which leaves out nonlocal effects, i.e.,

$$q^2/q_0^2=-\omega_p^2/[\omega(\omega-\omega_c)]\,. \tag{24}$$

Then we may rewrite the approximate Doppler shifted resonance condition Eq. (1) as

$$\omega_c/\omega=1+[(V_F/c)(\omega_p/\omega)]^{2/3}\,. \tag{25}$$

At low frequencies, where $(\omega_p/\omega V_F/c)\gg 1$ and $\omega_c/\omega\gg 1$, the effects of correlations are not observable. At higher frequencies, where $\omega_p/\omega V_F/c\sim\omega_c/\omega\sim 1$, correlations are important.

Since the simple resonance condition, Eq. (24), at these "high frequencies" is now a function of the "mass" of the particles, in this case the bare mass, and since correlations do play a role, it seems natural to ask what mass comes into play. There are, of course, two masses in the problem; the so-called bare mass and the clothed mass [see Eq. (7)]. In Fig. 3 we have plotted the

[11] J. Kirsch and P. B. Miller, Phys. Rev. Letters 9, 421 (1963).

shift in frequency of the Doppler edge from its uncorrelated value as a function of (ω_p/ω). For fixed values of $(\nu_c/\omega)=0.03$ and $V_F/c=0.004$ there is a shift away from what might be called a bare-mass resonance toward higher values of the magnetic field, i.e., towards a clothed mass resonance. We have taken $\delta_1=\frac{3}{16}$ so that the effective mass equals 1.25 bare electron masses (the observed effective mass in potassium).[12] The three curves in Fig. 3 are for three values of δ_2 ($\delta_2=0$, $\delta_2=\frac{1}{6}$, $\delta_2=-\frac{1}{6}$). A positive δ_2 tends to decrease the magnitude of the computed shift in frequency whereas a negative δ_2 increases the shift. The important thing to note here is that the resonance occurs at *neither* the frequency determined by the bare mass nor by the clothed mass. *The shift is a dynamical function of the frequency.*

In Fig. 4 we have plotted the actual absorption line for fixed values of the plasma frequency $\omega_p/\omega=100$, Fermi velocity, collision frequency, and δ_1. The effect of a negative δ_2 on the line shape is rather striking. It produces an over-all smearing of the line relative to the uncorrelated line. The magnitude of the steeply rising portion of the absorption curve is severely reduced. In addition, a negative δ_2 causes the absorption edge to exhibit a rather prominent minimum. In some cases (i.e., for the case $\delta_2=-\frac{1}{6}$ or $-\frac{1}{10}$) the minimum or hole in the absorption edge effectively causes a splitting of the edge into two peaks.

Physically, we can understand the origin of this decrease in absorption. Suppose for the purpose of this rather crude argument that the electric field inside the metal could be characterized by a single wave number q, as it can be in the local or classical theory. The edge in the absorption is due to a sharp increase in the conductivity when $\omega_c/\omega\approx(1+qV_F/\omega)$. The electrons at the Fermi velocity traveling in the direction of the wave see a static dc field spiral out around the lines of force and pick up energy from the field. Suppose we now ask ourselves if it is possible to introduce a mechanism which, in this region of increasing conductivity, would tend to decrease the induced current or conductivity, thus producing an associated dip in the absorption edge. Electron interactions can produce just this effect. As the quasiparticles (the current carriers in a Fermi liquid theory) are dragged through the surrounding, now slightly incompressible fluid, they create a backflow by pushing other quasiparticles out of their way. This backflow carries current. At a definite wave number q and frequency ω, it is possible for the backward flowing current to exactly cancel the forward flowing current so that the conductivity goes to zero. For the circularly polarized mode we simply require that

$$\int g(\theta)\,\sin\theta d(\cos\theta)\sim\Omega_1=0. \tag{26}$$

This implies that

$$1+\frac{V^*}{\tilde{\omega}W^{*2}}\left(\frac{1}{9\delta_2}+\frac{4}{15}-\tilde{\omega}\gamma W^*\right)=0. \tag{27}$$

For a finite δ_2 (second moment) it is possible to find at least one solution to Eq. (27) (to zeroth order in ν_c/ω). The dips in the conductivity, which show up in this simple model are characteristic of the interacting Fermion system. Had we retained more terms in the multipole expansion of the scattering function, it is likely that there would exist multiple solutions of Eq. (27). In the actual boundary value problem, q is not well defined and the zero in the conductivity appears as a minimum in the absorption curve. If the minimum happens to fall in the high field or classical region of the curve, then the magnitude of the effect is reduced since in this limit $q\rightarrow 0$ and there are no effects due to correlations.

The observation of correlation effects in the alkali metals, unfortunately, is extremely difficult since the magnetic fields required in order to keep the Dopper shifted edge in the neighborhood of the cyclotron frequency are enormous. However, experiments could be performed in semimetals, semiconductors, or doped insulators.

One should look for a material, preferably one with a simple band structure, which has an $r_s>1$. In addition, $\omega_p/\omega v_F/c\sim\omega_c/\omega\sim1$ and $\omega/\nu_c\gg1$. The frequency range in which one can work will primarily be limited by the magnetic fields which are available.

ACKNOWLEDGMENTS

The authors would like to thank Dr. S. J. Buchsbaum and Dr. P. A. Wolff for a number of helpful discussions. We would also like to thank Dr. Buchsbaum for a critical reading of the manuscript.

[12] C. C. Grimes, Bull. Am. Phys. Soc. (to be published).