(The constants of proportionality and the phase angle ϕ depend on the chosen C_n's.) Thus, the plasma-type excitations consist of oscillations in charge density, while the free-particle excitations consist of mass-density oscillations.

A similar interpretation arises from calculations of the transition probabilities between the ground state and states with one of the two types of pseudoparticles. Again it is found that

$$\begin{aligned}&\langle 0|\rho_{\text{mass}}|k_1\rangle=0,\\&\langle 0|\rho_{\text{charge}}|k_1\rangle\sim e^{i(kr-\omega_k t)},\\&\langle 0|\rho_{\text{mass}}|k_2\rangle\sim e^{i(kr-\omega_k t)},\\&\langle 0|\rho_{\text{charge}}|k_2\rangle=0,\end{aligned}\tag{65}$$

where $|k_i\rangle$ denotes a state with one pseudoparticle of type i with momentum k and $\langle 0|$, of course, corresponds to the state with no pseudoparticles present. Thus, excitations of the first type could be induced by an electromagnetic field and excitations of the second type by a gravitational field.

In order that the Boguliubov approximations be applicable the number of particles in excited states must be a small fraction of the total number of particles or

$$\langle N-N_0/N_0\rangle=\langle n-n_0/n_0\rangle\ll 1\tag{66}$$

for both species of bosons.
In both cases this reduces to

$$\frac{1}{\pi^2 n_0}\int_0^\infty B^2k^2dk\ll 1,\tag{67}$$

where B is given by Eq. (48).

This integral corresponds to that of Foldy[5] if his n_0 is replaced by $2n_0$. Thus,

$$\langle n-n_0/n_0\rangle=2^{5/3}Qr_{s0},\tag{68}$$

where

$$\begin{aligned}&Q=1.905,\\&r_{s0}=(3/4\pi)^{1/3}[mq^2/\hbar^2(2n_0)^{1/3}],\end{aligned}\tag{69}$$

and again the approximations are valid at high densities.

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Verdet Constant of the "Active Medium" in a Laser Cavity

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It is shown that the frequency of the intensity modulation observed when the output of a gas laser in a homogeneous axial magnetic field is veiwed through a polarizer is simply related to the Verdet constant of the "active medium." This method is used to determine the Verdet constant of "active" neon at 0.633 μ. A value of 5.9×10^{-7} rad/cm-Oe is obtained. Theoretical expressions for the Verdet constant of a dilute monatomic gas at a frequency close to the center of a Doppler-broadened line are derived for the three allowed transitions, $\Delta J=0$ and $\Delta J=\pm1$. The results of the experiment and the theory are used to estimate the threshold values of the absorption coefficient and the population inversion density in the present case.

I. INTRODUCTION

MORE than a century ago Michael Faraday discovered the effect which today bears his name. He observed that when plane-polarized light is passed through matter which has been placed in a homogeneous longitudinal magnetic field, the plane of polarization of the emergent light is rotated through some angle θ with respect to the incident beam. The amount of rotation per unit field strength per unit optical path length is commonly referred to as the Verdet constant V.

Any optical activity exhibited by a medium, be it this magnetically induced type or its natural counterpart, is indicative of a nonzero value of the quantity $n_r(\nu)-n_l(\nu)$, where $n_r(\nu)$ and $n_l(\nu)$ are the indices of refraction of the medium for right and left circularly polarized light of frequency ν, respectively. In terms of θ, the optical path length d, and the vacuum speed of light c, we may write for $n_r(\nu)-n_l(\nu)$

$$n_r(\nu)-n_l(\nu)=(c/\pi\nu)(\theta/d),\tag{1}$$

and in terms of V and the magnitude of the longitudinal

magnetic field H

$$n_r(\nu)-n_l(\nu)=(c/\pi\nu)VH. \quad (2)$$

The large positive absorption coefficient usually encountered at frequencies in the vicinity of an atomic or molecular transition complicates the direct measurement of the Verdet constant in such a region. The absorption coefficients associated with the many laser transitions are, of course, negative. The various "active media" might, therefore, appear to be ideal materials from which one can determine the Verdet constant associated with a large number of line centers by direct measurement. Indeed, for each transition, one has available an intense source of monochromatic linearly polarized radiation of precisely the frequency required.

Fork and Patel describe such an experiment for the $5d[\frac{3}{2}]_1{}^0-6p[\frac{3}{2}]_1$ (Racah notation) transition in xenon which occurs at a wavelength of 2.026 μ.[1] Assuming a gain of 4.5 dB per m, they report a calculated Faraday rotation of 0.39 deg/Oe-m. It is seen that for small fields, for which nonlinear effects can be neglected, the magnitude of the rotation is small. The magnitude will be smaller still for most of the other gas laser transitions since the rotation is proportional to the absorption coefficient, and all but a few of the laser transitions are of low gain, unlike the xenon transition studied. Here again, for most of the laser transitions, the direct measurement of the Verdet constant is difficult.

Statz, Paanenen, and Koster have reported a modulation of the intensity of the 1.153-μ output of a helium-neon gas laser which had been placed in a magnetic field.[2] The observed modulation was only evident if the beam were passed through a polarizer, and from the reported data it can be shown that for small field strengths, the modulation frequency is proportional to the field strength.

It is the purpose of this paper (1) to show how the Verdet constant of the "active medium" in any gas laser at the frequency of the laser transition can be determined from an experiment such as that described by Statz *et al.*; (2) to give the results of a measurement of the Verdet constant of "active" neon in a laser cavity at a frequency of 0.633 μ; and (3) to derive an expression for the quantity n_r-n_l at an atomic line center for the three allowed transitions, $\Delta J=\pm 1$ and $\Delta J=0$. The indirect method employed here for determining V at a line center will be seen to be applicable with equal facility to all of the gas laser transitions, irrespective of the magnitude of the gain.

II. THE VERDET CONSTANT FROM EXPERIMENT

For the oscillation frequency ν of one of the modes in a Fabry-Perot cavity we can write

$$\nu=(mc/2L)\{1-(L_a/L)[n(\nu)-1]\}, \quad (3)$$

where m is an integer, L is the distance between the mirrors, L_a is the length of the active medium, and $n(\nu)$ is the index of refraction of the active medium at the frequency ν. We have assumed that the index of refraction of anything other than the active medium has a negligible effect on ν. Further, we can neglect the contribution to $n(\nu)$ from all lines except that which comes from the laser transition.

We now place the laser cavity in an external longitudinal magnetic field. Under these conditions at every frequency it is generally true that $n_r\neq n_l$. Thus, the given cavity mode will be split into two modes of slightly different frequencies: ν_r and ν_l. When the light in each mode is made plane-polarized, the frequency difference $\Delta\nu$ ($\equiv\nu_l-\nu_r$) is observed as a modulation of the intensity of the resultant radiation.

We see from Eq. (3) that

$$\Delta\nu=\nu_0(L_a/L)[n_r(\nu_r)-n_l(\nu_l)], \quad (4)$$

where ν_0, the frequency of the line center, was taken equal to $mc/2L$. Since the variation of the index of refraction over the interval $\Delta\nu$ is extremely small compared to unity, to good approximation, $n_r(\nu_r)-n_l(\nu_l)=n_r(\nu)-n_l(\nu)$. Thus, we obtain

$$\Delta\nu=\nu_0(L_a/L)[n_r(\nu)-n_l(\nu)] \quad (5)$$

or, for the Verdet constant in terms of $\Delta\nu$,

$$V=(\pi/cH)(L/L_a)\Delta\nu. \quad (6)$$

The experimental arrangement is described as follows. An all quartz, helium-neon laser having internal mirrors defining a hemiconcentric cavity 48 cm in length and operating at a wavelength of 0.633 μ was oriented coaxially within an electromagnet. The magnet, a five-element design,[3] is an improvement over the Helmholz coil pair and provided an extremely uniform axial field of 0.850 Oe/A over a large central volume. The entire assembly of magnet and laser, and the immediate surroundings, were free of magnetic materials. The laser radiation was passed through a Nicol prism to a photomultiplier detector, the output of which was read simultaneously by a digital frequency meter and an oscilloscope.

The modulation frequency $\Delta\nu$ was obtained at threshold from the frequency meter at a number of field strengths. Figure 1 shows the plot of $\Delta\nu$ versus H. It is seen that, as expected for small H, a straight line passing through the origin is obtained. The slope of the line is 2.80 (kc/sec)/Oe, which, with $L/L_a=2.0$, implies a Verdet constant of 5.9×10^{-7} rad/cm-Oe.

III. THE VERDET CONSTANT FROM THEORY

In this section we consider the derivation of the quantity $n_r(\nu)-n_l(\nu)$ for a dilute monatomic gas in a homogeneous magnetic field when ν is close to ν_0. The three cases are distinguished $J_A\rightarrow J_B=J_A$, $J_A\rightarrow J_B$

[1] R. L. Fork and C. K. N. Patel, Phys. Rev. **129**, 2577 (1963).
[2] H. Statz, R. A. Paananen, and G. F. Koster, J. Appl. Phys. **33**, 2319 (1962); R. Paananen, C. L. Tang, and H. Statz, Proc. Inst. Elec. Electron. Engrs. **51**, 63 (1963).
[3] S. M. Rubens, Rev. Sci. Instr. **16**, 243 (1945).

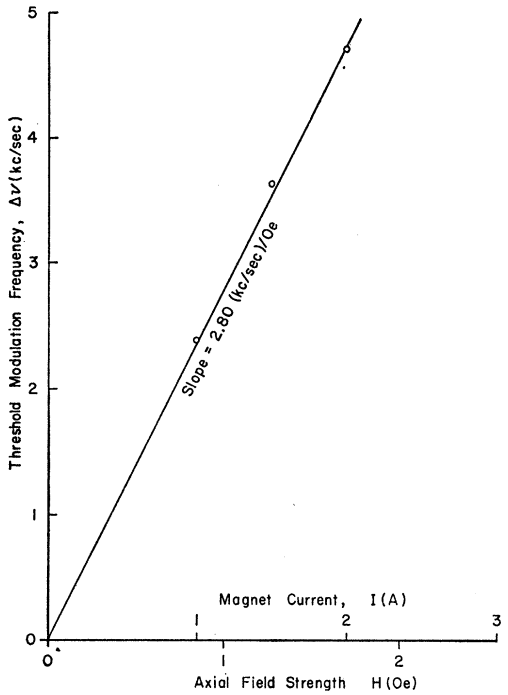

FIG. 1. The frequency at threshold $\Delta\nu$ as a function of the electromagnet current I.

$=J_A-1$, and $J_A\rightarrow J_B=J_A+1$, where J_A and J_B are the total electronic angular-momentum quantum numbers for the lower and upper levels for the transition, respectively. It will be assumed that the line in question is completely Doppler-broadened with a full width at half-maximum of $\Delta\nu_D$, and that the $2J+1$ states associated with each of the two levels have equal populations, with N_B being the total population density of the upper level and N_A that of the lower.

We write for the contribution α of a single naturally broadened line to the polarizability at a frequency ν near the frequency of the line center ν_0

$$\alpha=\frac{4(\nu-\nu_0)S(A,B)}{3Nh(\Delta\nu_n)^2(2J_B+1)}\times\{N_B-[(2J_B+1)/(2J_A+1)]N_A\}\times\{1+[2(\nu-\nu_0)/\Delta\nu_n]^2\}^{-1},\quad(7)$$

where $\Delta\nu_n$ is the linewidth, $S(A,B)$ is the strength of the line, and N is the total number density of atoms of the species under consideration. In the limit of complete Doppler broadening, this expression becomes[4]

$$\alpha=\frac{4(\ln 2)^{1/2}S(A,B)F(\omega)}{3Nh\Delta\nu_D(2J_B+1)}\times\{N_B-(2J_B+1)/(2J_A+1)N_A\},\quad(8)$$

[4] Equation (8) is equivalent to the expression given for α in the Ref. 1.

where

$$F(\omega)=\exp\{-\omega^2\}\int_0^{\omega}\exp\{y^2\}dy$$

and

$$\omega=[2(\ln 2)^{1/2}/\Delta\nu_D](\nu-\nu_0).$$

In the presence of a homogeneous axial magnetic field H, the substates shift their energy in such a way as to make a line appear at each frequency $\nu_0+(eH/4\pi m_e c)\times[(g_B-g_A)M_A\pm g_B]$, where e and m_e are the electronic charge and mass, g_B and g_A are the "g" factors of the upper and lower levels, and M_A is the magnetic quantum number of one of the $2J_A+1$ substates of the lower level. Here and in what follows the upper sign refers to the transition $M_A\rightarrow M_B=M_A+1$ ($\Delta M=+1$) and the lower sign to the transition $M_A\rightarrow M_B=M_A-1$ ($\Delta M=-1$). Under these conditions the polarizabilities $\alpha_\pm$ associated with the $\Delta M=+1$ and the $\Delta M=-1$ transitions are different. Taking into account the relative intensities of the Zeeman components,[5] we write

$$\alpha_\pm=-[3c\pi^{-5/2}k(\nu_0)/8N\nu_0]R_\pm,\quad(9)$$

where

$$R_\pm=[J_A(J_A+1)(2J_A+1)]^{-1}\times\sum_{M_A=-J_A}^{J_A}(J_A\pm M_A+1)(J_A\mp M_A)\times F[\omega_\pm(M_A)],\quad(10a)$$

for the case $\Delta J=0$;

$$R_\pm=[J_A(2J_A-1)(2J_A+1)]^{-1}\times\sum_{M_A=-J_A}^{J_A}(J_A\mp M_A-1)(J_A\mp M_A)\times F[\omega_\pm(M_A)],\quad(10b)$$

for the case $\Delta J=-1$; and

$$R_\pm=[(J_A+1)(2J_A+3)(2J_A+1)]^{-1}\times\sum_{M_A=-J_A}^{J_A}(J_A\pm M_A+2)(J_A\pm M_A+1)\times F[\omega_\pm(M_A)]\quad(10c)$$

for the case $\Delta J=+1$; the symbol $\omega_\pm(M_A)$ is defined as $\omega-[2(\ln 2)^{1/2}/\Delta\nu_D](eH/4\pi m_e c)[(g_B-g_A)M_A\pm g_B]$. The absorption coefficient at the line center $k(\nu_0)$ has been introduced at this point for later convenience. It is related to the strength $S(A,B)$ as follows[6,7]:

$$k(\nu_0)=-\frac{2(\ln 2/\pi)^{1/2}8\pi^3\nu_0 S(A,B)}{3hc\Delta\nu_D(2J_B+1)}\times\{N_B-[(2J_B+1)/(2J_A+1)]N_A\}.\quad(11)$$

[5] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, Cambridge, England, 1957), Chap. 16, p. 387.

[6] E. U. Condon and G. H. Shortley in Ref. 5, Chap. 4, p. 98.

[7] A. C. G. Mitchell and M. W. Zemansky, *Resonance Radiation and Excited Atoms* (Cambridge University Press, Cambridge, England, 1934), Chap. III.

We now expand $\alpha_\pm$ in powers of the magnetic field strength as far as the linear term,

$$\alpha_\pm=\alpha_\pm{}^0+\alpha_\pm{}^1 H \tag{12}$$

which allows the summations indicated in Eqs. (10) to be readily performed. The results are

$$\alpha_\pm{}^0=-ck(\nu_0)F(\omega)/4\pi^{5/2}N\nu_0 \tag{13}$$

which is equivalent to Eq. (8) and

$$\alpha_\pm{}^1=\pm\frac{(\ln 2/\pi)^{1/2}ek(\nu_0)[1-2\omega F(\omega)]}{16\pi^3 N\nu_0 m_e\Delta\nu_D}(g_A+g_B), \tag{14a}$$

for $\Delta J=0$,

$$\alpha_\pm{}^1=\pm\frac{(\ln 2/\pi)^{1/2}ek(\nu_0)[1-2\omega F(\omega)]}{16\pi^3 N\nu_0 m_e\Delta\nu_D}\times[(g_A+g_B)+J_A(g_A-g_B)], \tag{14b}$$

for $\Delta J=-1$, and

$$\alpha_\pm{}^1=\pm\frac{(\ln 2/\pi)^{1/2}ek(\nu_0)[1-2\omega F(\omega)]}{16\pi^3 N\nu_0 m_e\Delta\nu_D}\times[(g_A+g_B)+J_B(g_B-g_A)], \tag{14c}$$

$$\alpha_\pm{}^1=\pm\frac{(\ln 2/\pi)^{1/2}ek(\nu_0)[1-2\omega F(\omega)]}{16\pi^3 N\nu_0 m_e\Delta\nu_D}\times[(g_A+g_B)+J_B(g_B-g_A)], \tag{14d}$$

for $\Delta J=+1$.

For light propagating in the direction of the magnetic field,

$$n_r(\nu)-n_l(\nu)=2\pi N(\alpha_-{}^1-\alpha_+{}^1)H. \tag{15}$$

Therefore, from Eqs. (2) and (11) we see that

$$V=(4\pi^2\nu_0 N/c)\alpha_-{}^1. \tag{16}$$

IV. DISCUSSION

Since the measurements were made at the threshold of oscillation in applying Eqs. (14), we can set the factor $1-2\omega F(\omega)$ equal to unity. In addition, at threshold we need not consider the effects of gain saturation; we further know that $k(\nu_0)$ has the value $-\mathfrak{L}/L_a$, where $\mathfrak{L}$ is the fractional single-pass cavity loss.

The 0.633-μ laser transition in neon involves the $5s'[\frac{1}{2}]_1{}^0$ and $3p'[\frac{3}{2}]_2$ levels for which $g_B=1.295$, $g_A=1.301$, $J_B=1$, and $J_A=2$.[8] Assuming a Doppler width of 1.5×10^9 cps, we calculate from the observed V and Eq. (16) a value of $\mathfrak{L}$ of 0.012. The measured mirror transmissivities account for one-third of this value. Since L_a is 24.0 cm, it is also evident that the threshold value of $k(\nu_0)$ is -5.0×10^{-4} cm^{-1}.

The magnitude of the population inversion density $N_B-[(2J_B+1)/(2J_A+1)]N_A$ at threshold can be estimated from the equation

$$N_B-[(2J_B+1)/(2J_A+1)]N_A=-\frac{4\pi k(\nu_0)\Delta\nu_D\nu_0{}^2}{\mathbf{A}(B,A)(\ln 2/\pi)^{1/2}c^2}. \tag{17}$$

Taking a value of 1×10^7 sec^{-1} for $\mathbf{A}(B,A)$, the Einstein coefficient of spontaneous emission, we obtain

$$N_B-[(2J_B+1)/(2J_A+1)]N_A=5\times10^8 \text{ atoms/cc}.$$

[8] C. E. Moore, *Atomic Energy Levels*, National Bureau of Standards Circular No. 467 (U. S. Government Printing Office, Washington, D. C., 1949).