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Magnetic Properties of Nearly Free Electrons.* Nonoscillatory Magnetic Susceptibility

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The partition function is derived for a gas of electrons in the presence of a weak periodic potential and uniform magnetic field. From it the steady terms in the free energy and the zero-field–zero-temperature total electronic magnetic susceptibility are calculated. The results are applied to the alkali metals. The non-additivity of the paramagnetic and diamagnetic susceptibilities and the inadequacy of the effective mass approximation are discussed.

I. INTRODUCTION

THE purpose of this note is to derive expressions for the nonoscillatory free energy and magnetic susceptibility of a degenerate, nearly free electron gas. That is, we consider a gas of noninteracting electrons in the presence of a weak potential, due to a lattice of positive charges, and a uniform magnetic field. The free energy of the system is evaluated to second order in the temperature and field parameters with both paramagnetic and diamagnetic terms in the Hamiltonian taken into account; however, only nonoscillatory behavior is considered. The effect of the lattice on the de Haas–van Alphen oscillations will be the subject of a future report.

The calculation was made with two ends in view. First, considering the success of the pseudopotential concept,[1–3] it appears that the nearly free electron approximation has computational value as well as being merely instructive. Second, we wish to go beyond the limitations of the effective mass approximation for describing lattice effects in the context of magnetic properties. A calculation is performed in Sec. III using available pseudopotential parameters for the alkali metals Li, Na, and K. There exists no completely satisfactory calculation of the role of correlation in metallic properties for the density range into which these metals fall and, since this topic is outside the scope of the present study, we shall only point out that corrections can be estimated by interpolation methods.[4] Quantitative accord with experiment thus can not be expected, but it is proposed that for metals with small energy gaps the effect of the lattice is qualitatively accounted for. The criterion suggested is that the ratio of the energy gaps to the Fermi level be much less than unity.

II. CALCULATION

We consider N noninteracting electrons moving in a weak periodic potential $V(r)$, which occupies a very large volume Ω, in the presence of a magnetic field represented by the vector potential $\mathbf{A}=(-H_0y,0,0)$. An electron will therefore be described by the Hamiltonian

$$\mathcal{H}=(1/2m)(\mathbf{P}-(e/c)\mathbf{A})^2+2\mu_0H_0S_Z+V(\mathbf{r}) \equiv\mathcal{H}_0+V(\mathbf{r}). \quad (1)$$

As is well known,[5] the eigenstates of $\mathcal{H}_0$ are (apart from normalization)

$$|k_xnk_z,\sigma\rangle=e^{-\frac{1}{2}(\eta-\eta_0)^2}H_n(y-y_0)e^{i(k_xx+k_zz)}\chi_\sigma, \quad (2a)$$

$$E_\sigma(n,k_z)=2\mu_0H_0(n+1/2)+\hbar^2k_z^2/2m+\mu_0H_0\sigma, \quad (2b)$$

where $\mu_0=$ Bohr magneton; $\chi_\sigma=$ spin state, $\sigma=\pm1$; $\eta=(eH/\hbar c)^{1/2}y$, $\eta_0=-(\hbar c/eH)^{1/2}k_x$; $H_n=n$th Hermite polynomial.

The thermodynamic properties of the system can be studied conveniently by means of the partition function

$$Z(\gamma)=\mathrm{Tr}\{\exp[-\gamma(\mathcal{H}_0+V)]\}, \quad \gamma=1/kT. \quad (3)$$

Using the Schwinger trace formula[6] (the variable of integration has been changed to $u=\gamma s_1$)

$$Z(\gamma)\cong\mathrm{Tr}\{e^{-\gamma\mathcal{H}_0}\}-\gamma\,\mathrm{Tr}\{V(\mathbf{r})e^{-\gamma\mathcal{H}_0}\} +\tfrac{1}{2}\gamma^2\,\mathrm{Tr}\left\{\int_0^1 du V(\mathbf{r})e^{-\gamma\mathcal{H}_0(1-u)}V(\mathbf{r})e^{-\gamma\mathcal{H}_0u}\right\} \equiv Z_0(\gamma)-Z_1(\gamma)+Z_2(\gamma). \quad (4)$$

Since the Zeeman term commutes with the remainder of the Hamiltonian, the spin part of the trace is performed very simply to give

$$Z(\gamma)=2\cosh(\mu_0H_0\gamma)[z_0(\gamma)-z_1(\gamma)+z_2(\gamma)]. \quad (5)$$

$z_0(\gamma)$ is known from the work of Sondheimer and

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[*] This study was supported in part by U. S. Air Force Office of Scientific Research Grant No. AF-AFOSR-260-63.

[1] J. C. Phillips and L. Kleinman, Phys. Rev. **116**, 287 (1959).

[2] M. H. Cohen and V. Heine, Phys. Rev. **122**, 1821 (1961).

[3] J. M. Ziman, Phil. Mag. **6**, 1013 (1961).

[4] S. D. Silverstein, Phys. Rev. **130**, 1703 (1963).

[5] L. Landau and E. Lifshitz, *Quantum Mechanics*, (Addison-Wesley Publishing Company, Reading, Massachusetts, 1958), Sec. 125.

[6] A. Saenz and R. O'Rourke, Rev. Mod. Phys. **27**, 381 (1955). There is a misprint in Eq. (1.2.8) which should read:

$$\mathrm{Tr}\{Q(s)\}=\mathrm{Tr}\{e^{-sa}\}-s\,\mathrm{Tr}\{be^{-sa}\}+s\sum_1(-1)^n/n \times\mathrm{Tr}\left\{be^{-sa}\int_0^s ds_1\cdots\int_0^{s_{n-2}}ds_{n-1}b(s_1)\cdots b(s_{n-1})\right\}.$$

Wilson[7]:

$$z_0(\gamma)=\Omega\alpha\gamma^{-3/2}(\mu_0H_0\gamma)\,\operatorname{csch}(\mu_0H_0\gamma)\,,\qquad \alpha=(m/2\pi\hbar^2)^{3/2}. \tag{6}$$

$z_1(\gamma)$ can be calculated directly if $V(\mathbf{r})$ is represented as a Fourier series

$$V(\mathbf{r})=\textstyle\sum_{\mathbf{K}} V_{\mathbf{K}}e^{-i\mathbf{K}\cdot\mathbf{r}}, \tag{7}$$

where the **K's** are reciprocal lattice vectors. Representing the trace in the form

$$\mathrm{Tr}\{B\}=\frac{\Omega^{2/3}}{(2\pi)^2}\int dk_x\int dk_z\sum_{n=0}^{\infty}\langle k_xnk_z|B|k_xnk_z\rangle, \tag{8}$$

and using the generating function for products of Hermite polynomials[8] we obtain

$$z_1(\gamma)=\gamma V_0z_0(\gamma)\,, \tag{9}$$

where V_0 is the average crystal potential. For simplicity it will be assumed that V has a center of symmetry.

The evaluation of $z_2(\gamma)$ is straightforward but tedious. We begin with the expression

$$\begin{aligned}z_2(\gamma)=\tfrac{1}{2}\gamma^2\frac{\Omega^{4/3}}{(2\pi)^4}\int_0^1du\int dk_xdk_x'\int dk_zdk_z'\\ \times\sum_{n,n'}|\langle k_xnk_z|V(r)|k_x'n'k_z'\rangle|^2\\ \times\exp\{-\gamma E(n',k_z')(1-u)+E(n,k_z)u\}.\end{aligned} \tag{10}$$

First the spatial integrals over **r** and **r′** in the matrix elements can be done, except for the y and y' integrals, making use of (8). The sums over n and n' are evaluated next using the generating function[8] and the y and y' integrals then performed in terms of the variables $y\pm y'$. At this point the remaining integrals, except for the u integration, are done without difficulty leading to the result

$$\begin{aligned}z_2(\gamma)=\tfrac{1}{2}\gamma^2z_0(\gamma)\sum_K V_K^2\\ \times\int_0^1\exp\left\{-\frac{\hbar c}{eH}\mathbf{K}_\perp^2\mu-\frac{\gamma\hbar^2}{2m}\mathbf{K}_{\parallel}^2u(1-u)\right\}du.\end{aligned} \tag{11}$$

$\mathbf{K}_\perp$ and $\mathbf{K}_\parallel$ are the components of **K**, perpendicular and parallel to the magnetic field, and

$$\mu^{-1}=\coth(\mu_0H_0\gamma u)+\coth[\mu_0H_0\gamma(1-u)]. \tag{12}$$

Combining (5)–(11) we have

$$\begin{aligned}Z(\gamma)=Z_0(\gamma)\{1-\gamma V_0+\tfrac{1}{2}\gamma^2V_0^2\\ +\tfrac{1}{2}\gamma^2{\textstyle\sum}'G(\mathbf{K},\gamma)V_K^2\},\end{aligned} \tag{13}$$

where $G(\mathbf{K},\gamma)$ is the integral in (11) and

$$Z_0(\gamma)=2\Omega\alpha\gamma^{-3/2}(\mu_0H_0\gamma)\coth(\mu_0H\gamma) \tag{14}$$

is the partition function for free electrons. For convenience, the term $K=0$ has been separated from the sum and this is denoted by the prime.

For comparison the partition function has also been calculated for the case where the diamagnetic terms in (1) are omitted. This can be carried out easily using plane-wave states; the result is

$$\begin{aligned}Z'(\gamma)=Z_0'(\gamma)\Big\{1-\gamma V_0+\tfrac{1}{2}\gamma^2V_0^2\\ +\gamma^{5/2}\left(\frac{1}{\alpha}\right){\textstyle\sum}'V_K^2\int\frac{d^3k}{(2\pi)^3}\frac{e^{-\gamma\epsilon(k)}}{\epsilon(k-K)-\epsilon(k)}\Big\},\end{aligned} \tag{13'}$$

where

$$Z_0'(\gamma)=2\Omega\alpha\gamma^{-3/2}\cosh(\mu_0H_0\gamma)\,,\qquad \epsilon(k)=\hbar^2k^2/2m.$$

It follows from these expressions that to second order in the potential the paramagnetic and diamagnetic effects of the field are not simply additive.

The free energy and other thermodynamic properties can now be calculated. Since we consider here only the steady susceptibility, we may simply disregard the poles of (13) lying off the real axis in the inverse temperature plane.[9] It is convenient at this point to separate $Z(\gamma)$ into field-dependent and independent parts retaining terms of no higher than second order in the field. Introducing the parameter $\beta=\mu_0H_0$ we find $\mu\cong\beta\gamma S[1-\frac{1}{3}(\beta\gamma)^2S]$ where $S=u(1-u)$ and $G(\mathbf{K},\gamma)=\frac{1}{2}I_0[\frac{1}{4}\gamma\epsilon(\mathbf{K})]+\beta^2\gamma^3(\epsilon(K_\perp)/96)I_2[\frac{1}{4}\gamma\epsilon(\mathbf{K})]$, where

$$I_n(x)=\int_0^1\frac{y^ne^{-xy}}{(1-y)^{1/2}}. \tag{15}$$

Thus,

$$Z(\gamma)=Z^{(0)}(\gamma)+\beta^2Z^{(1)}(\gamma)\,,$$

where

$$\begin{aligned}Z^{(0)}(\gamma)=2\Omega\alpha\gamma^{-3/2}\{1-\gamma V_0+\tfrac{1}{2}\gamma^2V_0^2\\ +\tfrac{1}{4}\gamma^2{\textstyle\sum}'V_K^2I_0[\tfrac{1}{4}\gamma\epsilon(K)]\}\,,\end{aligned} \tag{16a}$$

$$\begin{aligned}Z^{(1)}(\gamma)=2\Omega\alpha\gamma^{-3/2}\{\tfrac{1}{3}\gamma^2-\tfrac{1}{3}\gamma^3V_0\\ +\tfrac{1}{6}\gamma^4V_0^2+\tfrac{1}{12}\gamma^4{\textstyle\sum}'V_K^2I_0[\tfrac{1}{4}\gamma\epsilon(K)]\}\\ +(1/192)\gamma^5{\textstyle\sum}'V_K^2\epsilon(\mathbf{K}_\perp)I_2[\tfrac{1}{4}\gamma\epsilon(K)].\end{aligned} \tag{16b}$$

The quantity $\Phi=F-n\zeta$, $(n=N/\Omega)$, will now be calculated using Eq. (14) of Ref. 9. Asymptotic expansions in powers of the degeneracy parameter $(\gamma\zeta)$ can be obtained for the contributions of the first three terms of (16a) and (16b) to obtain Φ. For the remaining terms the procedure described in Ref. 9 would be permitted only if $\gamma\zeta\gg\frac{1}{4}\gamma\epsilon(K)$; since this is not the case for the alkali metals, the relevant parameter for these terms is $a=4\zeta/\epsilon(K)$.

The calculation as outlined gives

$$\Phi=\Phi_0^{(0)}+\gamma^{-2}\Phi_1^{(0)}+\beta^2\Phi_0^{(1)}+(\beta/\gamma)^2\Phi_1^{(1)}\,, \tag{17}$$

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[7] E. H. Sondheimer and A. H. Wilson, Proc. Roy. Soc. (London) **A210**, 173 (1951).

[8] Eq. (A4) of Ref. 7.

[9] M. L. Glasser, J. Math. Phys. (to be published).

where

$$\Phi_0^{(0)}=-(2/\sqrt{\pi})\alpha\{(8/15)\zeta^{5/2}-\tfrac{4}{3}V_0\zeta^{3/2}+V_0^2\zeta^{1/2}+\tfrac{1}{4}\textstyle\sum' V_K^2[\epsilon(K)]^{1/2}f(a)\}\,, \tag{18a}$$

$$\Phi_1^{(0)}=-\tfrac{1}{3}\pi^2(2\alpha/\sqrt{\pi})\{\zeta^{1/2}-\tfrac{1}{2}V_0\zeta^{-1/2}-\tfrac{1}{8}V_0^2\zeta^{-3/2}+\textstyle\sum' V_K^2[\epsilon(K)]^{-3/2}g(a)\}\,, \tag{18b}$$

$$f(a)=\sqrt{a}+(a-1)\tanh^{-1}\sqrt{a}\,,$$

$$g(a)=a^{-1/2}(1-a)^{-1}; \tag{19}$$

$$\Phi_0^{(1)}=-(2\alpha/\sqrt{\pi})\{\tfrac{2}{3}\zeta^{1/2}-\tfrac{1}{3}V_0\zeta^{-1/2}-\tfrac{1}{12}V_0^2\zeta^{-3/2}+\tfrac{2}{3}\textstyle\sum' V_k^2[\epsilon(K)]^{-3/2}g(a)+\tfrac{1}{6}\sum' V_K^2(\epsilon(\mathbf{K}_\perp)/[\epsilon(K)]^{5/2})h(a)\}\,, \tag{20a}$$

$$\Phi_1^{(1)}=-\tfrac{1}{3}\pi^2(2\alpha/\sqrt{\pi})\{-\tfrac{1}{12}\zeta^{-3/2}-\tfrac{1}{8}V_0\zeta^{-5/2}-\tfrac{5}{32}V_0^2\zeta^{-7/2}+(16/3)\textstyle\sum' V_K^2[\epsilon(K)]^{-7/2}g''(a)+\tfrac{4}{3}\sum' V_K^2(\epsilon(\mathbf{K}_\perp)/[\epsilon(K)]^{9/2})h''(a)\}\,, \tag{20b}$$

and

$$h(a)=\tfrac{3}{2}\tanh^{-1}\sqrt{a}+\frac{a^{1/2}(5-3a)}{2(1-a)^2}\,. \tag{21}$$

In order to use these expressions, the chemical potential must be known to second order in the various parameters. It can be calculated by solving the equation $(\partial\Phi/\partial\zeta)_\Omega+n=0$. Except in rare cases, the temperature and field dependences of ζ are small enough to be neglected. Hence we need retain only $\Phi_0^{(0)}$ in this equation and solve for ζ to second order in the lattice potential. This calculation is easily carried out and yields

$$\zeta=\zeta_0[1+(V_0/\zeta_0)-\tfrac{1}{2}\zeta_0^{3/2}\textstyle\sum' V_K^2[\epsilon(K)]^{-1/2}f'(a_0)]\,, \tag{22}$$

where $\zeta_0=\frac{1}{4}(9\pi n^2)^{1/3}(2\pi\hbar^2/m)$ and $a_0=4\zeta_0/\epsilon(K)$.

The magnetic susceptibility can now be calculated. Putting (17) into the formula $\chi=-(1/H)(\partial F/\partial H)|_\Omega$ we find $\chi=-2\mu_0^2[\Phi_0^{(1)}+(kT)^2\Phi_1^{(1)}]$. To second order, the temperature-independent susceptibility is

$$\chi_0=\chi_0^{\text{free}}[1-(1/8\zeta_0^2)\textstyle\sum' V_K^2F(a_0)]\,, \tag{23}$$

where

$$F(a_0)=(a_0)^{1/2}\tanh^{-1}(a_0)^{1/2}-\frac{a_0}{1-a_0}-\frac{1}{4}\frac{\epsilon(\mathbf{K}_\perp)}{\epsilon(K)}a_0^{3/2}h(a_0)\,, \tag{24}$$

and $\chi_0^{\text{free}}=\mu_0^2/\zeta_0$ is the susceptibility for free electrons. A similar formula can be obtained for the temperature-dependent term.

An equation, resembling (23), for the paramagnetic susceptibility was published recently by Abe.[10] His formula is derived along the lines of this calculation in the Appendix.

III. DISCUSSION

In this section the results of the preceding analysis are applied to the alkalis Li and Na, these being the only metals for which the spin susceptibility has been measured. We shall cast our analysis for these metals in the form of a one-parameter model in terms of the lowest nonzero Fourier coefficient of the crystal potential. This is determined by the energy gap at the zone surface and estimates from band calculations[3,10] are listed in Table I. An estimate for sodium based on Callaway's[11] pseudopotential showed that the sum in (23) is given almost entirely by the contributions fron the reciprocal lattice vectors of shortest length. The results obtained by summing over only these vectors and using the Fourier coefficients listed in Table I are shown in Table II. $F(a_0)$ is a universal function for a given lattice; for the bcc case $F(a_0)=-6.33$ for the vectors of type $2\pi/a$ $(1, -1, 0)$ and -4.29 for those of type $2\pi/a$ $(1,0,1)$ where a is the lattice constant. The calculation for Na carried out using Callaway's pseudopotential gave a result much larger than that listed in Table II. This is consistent with the observation that the energy gap (at the point N) found by Callaway is an order of magnitude larger than that found in most other calculations.

TABLE I. Pseudopotential and free electron parameters for the alkali metals.

	ζ_0 (Ryd)	V_{K_0} (Ryd)	$\chi_0^{\text{free a}}$	$\chi^{*\text{a}}$
Li	0.355	0.112	0.54	0.25
Na	0.235	0.010	0.43	0.20
K	0.155	−0.018	0.35	0.18

[a] (×10⁶ cgs vol. units).

Estimates of the correction to the susceptibility due to correlation have been made on the basis of Silverstein's[4] work and are listed under χ^* in Table I. The entries in Table II denoted by a star were obtained by including these adjustments. The starred values of the paramagnetic susceptibility are in close agreement with Abe's[10] values, obtained by a Sampson-Seitz procedure. Considering that the sum in (23) has been underestimated and that correlation has been accounted for in a very crude fashion, agreement with experiment for Li and Na appears to be satisfactory.

Also listed in Table II are values for the "diamagnetic susceptibility" obtained by subtracting χ_p (A5) from

TABLE II. Experimental and calculated susceptibility (×10⁶ cgs vol. units).

	Li	Na	K
χ_0^{a}	1.20*	0.64*	0.56*
	0.95	0.44	0.38
$\chi_0^{\text{b,c}}$	1.94±0.05	0.88±0.03	
χ_p^{d}	1.33*	0.84*	0.73*
	1.08	0.66	0.55
χ_p^{b}	2.08±0.10	0.95±0.10	
$\chi_d^{(\text{calc})}$	−0.13	−0.22	−0.17
χ_d^{b}	−0.14±0.15	−0.07±0.11	

[a] From Eq. (23).
[b] R. T. Schumacher and C. P. Slichter, Phys. Rev. **101**, 58 (1956).
[c] The ionic diamagnetism has been subtracted.
[d] From Eq. (A5).

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[10] R. Abe, Progr. Theoret. Phys. (Kyoto) **29**, 23 (1963).

[11] J. Callaway, Phys. Rev. **112**, 322 (1958).

χ_0 (23). Because of the nonadditivity of diamagnetic and paramagnetic effects, discussed below in more detail, these values correspond only roughly to the Landau-Peierls diamagnetism. There is some evidence[12] that the diamagnetic susceptibility is only slightly affected by correlation; thus, much of the correlation contribution is subtracted off by this procedure. The close agreement between theory and experiment here is strong evidence in favor of a pseudopotential theory of these metals.

In the course of this calculation it was discovered that the same basic method has been applied by Samoilovich and Rabinovich[13] to study the diamagnetism of nearly free electrons; where comparable, their results agree with this calculation. However, due to the nonadditivity of the susceptibilities, the result of adding their expression to Abe's differs from (23); indeed

$$\Delta=[\chi_d(\mathrm{SR})+\chi_p(\mathrm{Abe})-\chi_0(23)]/\chi_0^{\mathrm{free}}$$
$$=\frac{1}{16\zeta_0^2}\sum{}' V_K^2\frac{\epsilon(\mathbf{K}_\perp)}{\epsilon(K)}[a_0(1-a_0)^{-2}-\tfrac{1}{2}a_0^{3/2}h(a_0)].$$

For lithium, $\Delta=0.62$, while for sodium, $\Delta=0.012$. Therefore care must be taken in separating the paramagnetic and diamagnetic susceptibilities for comparison with experiment. Since the diamagnetic susceptibility is not directly measured, it is believed that the estimates in Table II are more significant than those of Samoilovich and Rabinovich. The existence of this nonadditivity has been discussed in a general way by Blount.[14] This feature would appear to be a source of considerable error in previous calculations of diamagnetic susceptibilities.

In conclusion, a comparison can be made with the effective-mass approximation within the scope of the nearly free electron model. The susceptibility is given in terms of the effective mass ratio by[7]

$$\chi_0/\chi_0^{\mathrm{free}}=\tfrac{3}{2}(m^*/m)^{3/2}-\tfrac{1}{2}(m/m^*)^{1/2}. \qquad (25)$$

In the nearly free electron approximation m^*/m, averaged over the Fermi surface, is

$$m^*/m=1+(1/4\zeta_0^2)\sum{}' V_K^2(a_0)^{1/2}\tanh^{-1}(a_0)^{1/2}. \qquad (26)$$

Combining (25) and (26) gives

$$\chi_0/\chi_0^{\mathrm{free}}=1+(5/8\zeta_0^2)\sum{}' V_K^2(a_0)^{1/2}\tanh^{-1}(a_0)^{1/2},$$

which differs significantly from (23).

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APPENDIX

Using (13′) and proceeding as in the body of the text we find

$$\Phi'=\Phi_0'+\Phi_1'+\Phi_2'+\Phi_2'', \qquad (\mathrm{A1})$$

where

$$\Phi_0'=-\pi/\gamma(m/2\pi\hbar^2\gamma)^{3/2}[B_5\{\gamma(\zeta+\mu_0H)\}+B_5\{\gamma(\zeta-\mu_0H)\}],$$
$$\Phi_1'=\pi V_0(m/2\pi\hbar^2\gamma)^{3/2}[B_3\{\gamma(\zeta+\mu_0H)\}+B_3\{\gamma(\zeta-\mu_0H)\}],$$
$$\Phi_2'=-\pi\gamma/2V_0^2(m/2\pi\hbar^2\gamma)^{3/2}[B_1\{\gamma(\zeta+\mu_0H)\}+B_1\{\gamma(\zeta-\mu_0H)\}],$$

and

$$B_\sigma(\lambda)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}s^{-\sigma/2}\csc\pi s e^{\lambda s}ds$$

is an integral discussed in another paper.[9] It is convenient to handle Φ_2'', corresponding to the last term in (13′), somewhat differently. We first calculate $z''(E)$, the inverse Laplace transform of the corresponding term in $Z'(\gamma)/\gamma^2$. Then we obtain

$$\Phi_2''=\sum{}' V_K^2\int_0^\infty\frac{\partial f_0}{\partial E}\{g(K,E+\mu_0H)+g(K,E-\mu_0H)\}dE,$$

where f_0 is the Fermi function and

$$g(K,\lambda)=(2\pi)^{-3}\int_{\epsilon(k)<\lambda}d^3k[\epsilon(\mathbf{k}-\mathbf{K})-\epsilon(\mathbf{k})]^{-1}.$$

In the zero-temperature limit (A1) becomes

$$\begin{aligned}-\Phi(T=0)=&[1/\Gamma(\tfrac{7}{2})](m/2\pi\hbar^2)^{3/2}\\&\times\{[(\zeta+\mu_0H)^{5/2}+(\zeta-\mu_0H)^{5/2}]\\&-\tfrac{5}{2}V_0[(\zeta_0+\mu_0H)^{3/2}+(\zeta-\mu_0H)^{3/2}]\\&+(15/8)V_0^2[(\zeta+\mu_0H)^{1/2}+(\zeta-\mu_0H)^{1/2}]\}\\&+\sum{}' V_K^2\{g(K,\zeta+\mu_0H)+g(K,\zeta-\mu_0H)\}. \qquad (\mathrm{A2})\end{aligned}$$

This leads to

$$\chi_p=\frac{4}{\sqrt{\pi}}\mu_0^2\left(\frac{m}{2\pi\hbar^2}\right)^{3/2}\zeta^{1/2}\left[1-V_0/\zeta-\tfrac{1}{8}(V_0/\zeta)^2+\frac{2\pi}{\zeta}\left(\frac{m}{2\pi\hbar^2}\right)\sum{}'\frac{V_k^2}{K^2-4k_0^2}\right], \qquad (\mathrm{A3})$$

where k_0 is the free electron Fermi momentum. The chemical potential is obtained as before. Inserting (22) into (A3) leads to Abe's[10] expression

$$\chi_p/\chi_p^{\mathrm{free}}=1+(1/8\zeta_0^2)\times\sum\frac{V_K^2}{u^2-1}\left\{1+\frac{1-u^2}{2u}\ln\left|\frac{u+1}{u-1}\right|\right\}, \qquad (\mathrm{A4})$$

where $\chi_p^{\mathrm{free}}=3\mu_0^2n/2\zeta_0$, and $u=K/2k_0=1/\sqrt{a_0}$.

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[12] H. Kanazawa and N. Matsudaira, Progr. Theoret. Phys. (Kyoto) 23, 433 (1960).

[13] A. Samoilovich and E. Rabinovich, Fiz. Tverd. Tela 5, 778 (1963) [English transl.: Soviet Phys.—Solid State 5, 567 (1963)].

[14] E. I. Blount, Phys. Rev. 126, 1636 (1962).