Charge Dependence of Nuclear Forces and the Breakup of Deuterons and Tritons

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The proton spectrum at 4.8° from the reaction $D(n,p)2n$, $E_n=14.4$ MeV, was measured. The high-energy part of that spectrum was analyzed yielding for the neutron-neutron ${}^1S_0$ scattering length the value: $a_{nn}=-21.7\pm1$ F. The correction due to magnetic interaction is 0.8 F and the pure nuclear a_{nn} is therefore $a_{nn}=-22.5\pm1$ F. This experimental value is compared with the calculations of Wong and Noyes, and Lin, and indicates that nuclear forces depart by 2–3% from the charge symmetry. The charged particle spectrum at 4.8° from the reaction $n+\mathrm{T}\to n+\mathrm{T}$, $2n+d$, and $3n+p$, $E_n=14.4$ MeV, was also measured. The effects of the final-state interactions are evident in the spectrum.

1. CHARGE DEPENDENCE OF NUCLEAR FORCES

THE study of the proton-proton and neutron-proton scattering suggests the charge independence[1,2] of nuclear forces. This hypothesis is corroborated by the nuclear-structure data. Particularly, the analysis of the energy levels of mirror nuclei indicates the equality of neutron-neutron and proton-proton forces. The comparison between mirror reactions, such as p-d and n-d or p-He^3 and n-T does not reveal any significant departure from charge symmetry.[3] By now the available data indicate that nuclear forces are to a large extent charge independent. If there is any charge dependence, it should be small. In fact, there are some evidences that nuclear forces are slightly charge dependent. These indications are:

(1) The difference between the energies of the isobaric triplet states in light nuclei cannot be explained in terms of Coulomb forces, the neutron-proton mass difference, and the difference in the electromagnetic interaction of the core with the magnetic moments of the neutron and the proton. When all these effects are taken into account, there remains a residual energy difference[4–6] which is, e.g., for the $A=14$ triad[5,6]:

$$C^{14}-N^{14*}=68\ \mathrm{keV},$$
$$C^{14}-N^{14*}=50\ \mathrm{keV},$$

indicating that n-p interaction should be somewhat stronger than p-p and n-n interaction.

(2) The conserved vector-current theory[7] of the universal Fermi interaction predicts the equality of the coupling constant G_μ for the muon decay and the polar-vector coupling constant G_ν for the beta decay. Recently, G_μ has been determined[8] very accurately. The determination of G_ν requires an accurate measurement of the ft value and a precise knowledge of the nuclear matrix element. The matrix element for the $0^+\to0^+$ transition between the states of the same isobaric spin T is well known. Two $0^+\to0^+$ transitions have been carefully measured: $O^{14}\to N^{14*}$ [9] and $Al^{26}\to Mg^{26*}$.[10] The coupling constant G_ν extracted from these two measurements, after various nonradiative corrections have been applied, differs from G_μ in the case of $O^{14}\to N^{14*}$ by $(1.0\pm0.2)\%$ and in the case of $Al^{26}\to Mg^{26*}$ by $(0.2\pm0.2)\%$. The radiative corrections[11] to G_μ and G_ν are fairly uncertain, but presumably they even increase this discrepancy. The evaluation of G_ν is based on the assumption that the nuclear matrix element is $\sqrt{2}$, i.e., that the wave functions do not contain isobaric spin impurities. The change in the value of the matrix element can be due to Coulomb forces and/or charge-dependent nuclear forces. It has been believed[12] that the Coulomb correction is much too small and that the reconciliation of the $G_\mu-G_\nu$ discrepancy requires the introduction of charge-dependent nuclear forces with the strength relative to the charge-independent forces of $\sim$1–2%.[13] The Coulomb correction has been calculated using pure $1p$ shell-model wave functions. The configurational mixing can, however, produce a strong enhancement of this correction.[14]

The measurement of the β-γ circular polarization

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[2] G. Breit, M. H. Hull, K. E. Lassila, Jr., and K. D. Pyatt, Phys. Rev. Letters **4**, 79 (1960); and Phys. Rev. **120**, 2227 (1960).

[3] See, for example, I. Šlaus, in *Nuclear Interactions*, edited by B. Lalović (Federal Nuclear Energy Commission, Belgrade, 1963), and references therein.

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[7] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1953).

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[12] W. M. MacDonald, Phys. Rev. **110**, 1420 (1958).

[13] R. J. Blin-Stoyle and J. Le Tourneaux, Phys. Rev. **123**, 627 (1961).

[14] H. A. Weidenmüller, Phys. Rev. **128**, 841 (1962).

asymmetry parameter has revealed[15] several cases where the Fermi matrix element M_F for $\Delta J=0$, $\Delta T=\pm 1$ is different from zero. Since M_F in such cases should be zero if the conserved vector-current theory is valid, and if there are no isobaric spin impurities in the wave functions, these experiments can be interpreted as the evidence in favor of the charge dependence of nuclear forces.[16]

Until now it has not been possible to find a charge-dependent potential that would quantitatively explain[6] all data which indicate the charge dependence of nuclear forces.

All these evidences are not very conclusive. They are derived from the study of the atomic nuclei and, therefore, can yield information only about the effective nuclear forces in nuclear matter for which we do not know whether they are equal or not to the forces between two free nucleons. Also, there are a number of uncertain corrections about which very little is known.

While *n-p* scattering directly gives a_{np}, *p-p* scattering gives a scattering length which includes both nuclear and Coulomb effects. The extraction of a_{pp} depends upon the assumed shape of the nucleon-nucleon potential. The values of the $^1S_0$ scattering lengths are[17]:

$$a_{np}=-23.678\pm 0.028 \text{ F},$$
$$a_{pp}\sim -17 \text{ F}.$$

The magnetic interaction also contributes to the experimental scattering lengths. This effect was at first calculated by Schwinger,[18] who assumed a point magnetic moment. The magnetic correction depends upon the shape of the nucleon-nucleon potential and Schwinger found that the charge independence can be restored only for potentials singular at the origin, e.g., the Yukawa potential. This calculation suffers from two unrealistic assumptions. First, the nucleon-nucleon potential contains a large repulsive core. Salpeter took this into account and found that the agreement between a_{np} and a_{pp} is lost.[19] Second, the electron-nucleon scattering data[20] show that the charge and current distribution has a finite extension. Riazuddin repeated Schwinger's calculation taking instead of the point magnetic moment the experimentally established values and found that the agreement between a_{np} and a_{pp} is destroyed even for the Yukawa potential without the hard core.[21]

Nucleon-nucleon interaction is due to the exchange of mesons. The masses of charged and neutral pions are different and this can produce a difference between *n-p* and *p-p* forces. This effect was studied by Sugie[22] and Riazuddin,[21] who found that the correction is in the right direction to remove the $a_{pp}-a_{np}$ discrepancy, but is larger in magnitude than is required.

The nucleon-pion interaction is characterized by the coupling constants: $G_{\pm p}$, $G_{\pm n}$, G_{0p}, and G_{0n}, where the first subscript denotes the pion and the second the nucleon. Riazuddin claims that the $a_{pp}-a_{np}$ difference can be removed if one takes the charged pion-nucleon coupling constant to be $\sim 1.5\%$ smaller than the neutral pion-nucleon coupling constant. The value of the nucleon-pion coupling constant derived from the nucleon-nucleon scattering data is[2]

$$G^2=13.5\pm 0.9.$$

The uncertainty in G^2 is several times larger than the small difference between $G_{\pm np}{}^2$ and $G_{0np}{}^2$ required by Riazuddin to fit the $a_{pp}-a_{np}$ difference.

Lin[23] has recently calculated nuclear potentials of Brueckner and Watson[24] up to the fourth order in the perturbation calculation taking into account the different masses of charged and neutral pions. These potentials are then used to calculate the *n-p* and *p-p* scattering lengths and the energy of the first excited $T=1$ state in Li^6. Three coupling constants $G_{\pm np}$, G_{0p}, and G_{0n}, together with the hard core radius r_0, have been treated as free parameters to be determined by fitting the experimental data. Table I gives the relative magnitudes of the coupling constants and the hard-core radii obtained in this way.

The two upper rows in Table I belong to the choice for the nuclear radius used to evaluate Li^6, $R=1.45A^{1/3}$ F, while the two lower rows are for $R=1.2A^{1/3}$ F. The last column lists the results of the calculation of the a_{nn}.

An interesting and important feature of Table I is that a very small variation in coupling constants yields a complete agreement with the experimental data.

Wong and Noyes[25] have investigated the electrostatic corrections to the singlet-state scattering amplitudes within the framework of the dispersion relations. The calculation is based on the use of nonrelativistic wave

TABLE I. The hard-core radii and the relative magnitudes of the coupling constants used in Lin's calculation.

r_0 in F	$G_{\pm np}{}^{\text{relative}}$	$G_{0p}{}^{\text{relative}}$	$G_{0n}{}^{\text{relative}}$	a_{nn} in F
0.3	1	0.997	0.999	−19.478
0.4	1	0.993	1.007	−20.048
0.3	1	0.980	1.014	−22.182
0.4	1	0.993	1.0008	−20.028

[15] S. D. Bloom, L. G. Mann, and J. A. Mishel, Phys. Rev. **125**, 2021 (1962).
[16] R. J. Blin-Stoyle and Lj. Novaković (to be published), and (private communication, 1963).
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[18] J. Schwinger, Phys. Rev. **78**, 135 (1950).
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[22] A. Sugie, Progr. Theoret. Phys. (Kyoto) **11**, 333 (1954).
[23] D. L. Lin (private communication), and Bull. Am. Phys. Soc. **7**, 348 (1962).
[24] K. A. Brueckner and K. M. Watson, Phys. Rev. **92**, 1023 (1953).
[25] D. Y. Wong and H. P. Noyes, Phys. Rev. **126**, 1866 (1962).

functions and since it contains only static effects, it is uncertain for relativistic and radiation corrections which are of the order of $(v/c)^2$ and $e^2/\hbar c$, respectively.

The OPE effect is treated exactly. The calculation of the discontinuity across the OP cut for the Yukawa and Coulomb fields has been done in the first Born approximation, which turns out to be the exact procedure for the Coulomb modification. The difference between the masses of charged and neutral pions has been included in the calculation. The effect of more-than-one-pion exchange contributions has been represented[26] by a single pole. The coupling constants, the position, and the strength of the multipion pole are to be determined from the comparison with the experimental data. In principle, the charge dependence can be contained in either the difference between the coupling constants, or in the difference between the multipion-exchange singularities for p-p, n-n, and n-p systems.

Under the assumptions that $G_{0p}=G_{0n}$ and that the position of the multipion pole is the same for n-p and p-p system, Wong and Noyes adjusted the remaining free parameters to fit the low-energy p-p and n-p data. The result is that if $G_{0p}^2=G_{0n}^2=G_{\pm p}G_{\pm n}$, then the strengths of the multipion-exchange poles are also equal within $\sim 3\%$, which is the inherent inaccuracy of the calculation. This demonstrates that n-p and p-p low-energy ${}^1S_0$ scattering data are consistent with the charge independence hypothesis.

Following the same procedure the n-n ${}^1S_0$ scattering length can be predicted. If one assumes the exact charge symmetry

$$G_{0p}^2=G_{0n}^2=13.5\pm 0.9\,,$$

and the multipion pole strengths

$$\Gamma_{nn}=\Gamma_{pp}\,,$$

one obtains

$$a_{nn}=-27\pm 1.4\ \mathrm{F}\,.$$

If the multipion pole strength Γ departs by as little as 5% from the charge symmetry, a_{nn} could lie, for $G_{0n}^2=G_{0p}^2=13.5\pm 0.9$, anywhere between -18 and -53 F. If the value of the coupling constants is not 13.5 but between 10 and 18, the limits of a_{nn} (with the 5% departure of Γ from charge symmetry) are still broader, comprising all values between -15 and -65 F.

All this shows that a_{nn} is the most sensitive quantitative test for the possible departure for charge independence. A precise determination of a_{nn} is, therefore, very desirable.

2. NEUTRON–NEUTRON SCATTERING LENGTH

In the past, several attempts have been made to determine a_{nn}. Since it is impossible to produce a neutron flux dense enough to allow scattering of two such fluxes on each other and to measure the n-n scattering directly, one is lead to study systems involving more than two particles in order to deduce the a_{nn}. Low energy n-n data can be extracted by investigating processes which result in several particles in the final state. Some of these particles, particularly if their relative energy is low, may strongly interact in the final state.[27]

The final-state interactions influence the spectral shape and the angular correlation. The measurement of spectra and angular correlations of such processes can determine the n-n ${}^1S_0$ scattering length. The cleanest experiments in the sense that there is only one strong final-state interaction, are

$$\pi^-+d\rightarrow 2n+\gamma\,,$$

and

$$\mu^-+d\rightarrow 2n+\nu\,.$$

Phillips and Crowe[28] measured the γ spectrum from the first reaction. It is considerably more convenient to measure neutrons, as suggested by McVoy,[29] and is now so planned by the group at UCLA.[30]

The reactions between very light nuclei even at moderate energies often result in several particles in the final state. The reactions T$(d,\mathrm{He}^3)2n$,[31] T$(t,\alpha)2n$,[32] and D$(n,p)2n$ [33–36] have been studied, and in all cases the strong final-state interaction of two neutrons has been observed. In the spectrum at 4° from the reaction D$(n,p)2n$ a particularly pronounced peak due to the neutron-neutron interaction has been established.[33] This peak is near the maximum proton energy and corresponds to the case where two neutrons have small relative energy. The proton spectrum shows another peak at 5.6 MeV associated with the neutron-proton final-state interaction. Ilakovac *et al.*[37,38] performed the theoretical analysis of the proton spectrum at 4° using the Born approximation and taking into account the final-state interaction exactly. The theory can explain the shape of the entire spectrum, but does not predict the absolute value of the cross section correctly.

It has been demonstrated[37] that the shape of the proton spectrum around the maximum energy is quite sensitive to the value of a_{nn}. In spite of the inadequacy of this naive theory it has been used to extract a_{nn}. The

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[37] K. Ilakovac, L. G. Kuo, M. Petravić, and I. Šlaus, Phys. Rev. **124**, 1923 (1961).

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analysis which relies entirely on the fit of the shape of the spectrum gave

$$a_{nn}=-22\pm 2 \text{ F}. \quad (1)$$

The error quoted is purely statistical and does not contain any estimate of the accuracy of the theory. The determination of a_{nn} depends on the knowledge of the resolution of the experimental setup. It also depends on the number of experimental points in the spectrum taken into account. The error associated with these uncertainties has not been included in (1). In the experiment of Ilakovac *et al.*[33] the sensitivity of the proton spectrum to the neutron-neutron scattering length has not been fully exploited due to an insufficient number of points determining the shape of the peak (the analysis has been based on 6 points), and to the large uncertainty in the rather poor energy resolution. Also, the experiment has been done with a 5×20 channel analyzer which allowed taking simultaneously only 5 points. It was, therefore, thought to be worthwhile performing a more accurate measurement of the proton spectrum from the reaction D$(n,p)2n$.[39]

3. EXPERIMENT

The 200-keV Cockcroft-Walton accelerator of the Institute "Ruđer Bošković"[40] provided a yield of 1.5×10^9 n/sec. The neutrons were monitored by counting the associated α particles.

The deuterium target was 3.27 mg/cm^2 thick and 4.2 cm^2 in area heavy paraffin, uniformly deposited on a gold backing.

The charged particles were detected by a counter telescope similar to the one described by Kuo, Petravić, and Turko,[41] consisting of three proportional gas counters followed by a scintillation CsI(Tl) counter. The scintillation counter was used as the energy counter. Its resolution for ~14-MeV protons was ~4%. The proportional counter closest to the E counter was a dE/dx counter. The width at the half-maximum of the distribution of the energy losses for ~12.8-MeV deuterons in the dE/dx counter was ~24%. The first proportional counter rejected the pulses produced by particles originating in the front wall of the telescope. The second counter served for collimation and considerably reduced the background.

Each of the four pulses from the telescope, after passing the amplification stage, were lead to the coincidence-anticoincidence gating unit,[41] from which two pulses were derived: one proportional to the energy loss of the particle in the dE/dx counter, the other one proportional to the residual energy of that particle in the scintillation counter. These pulses were analyzed by a 100×100 channel analyzer.[42] The amplitude-to-time converter transforms the two pulses into numbers read by two independent E1T scalers. A Kienzle printer read the two numbers and printed them on the tape. Because of the relatively long time the printer takes to print and reset (about 0.4 sec), the neutron monitors as well as the analyzer input were gated until the coming of the reset pulse. The numbers from the tape were sorted into tables of ΔE versus E by the sorting and tabulating IBM-421 electronic machine. The resulting three-dimensional graphs show proton, deuteron and triton ΔE versus E spectra simultaneously. Thus, the 100×100 channel analyzer allowed to take the complete energy spectra of protons, deuterons, and tritons at the same time.

In general, it was easy to resolve protons from deuterons. However, in the energy region of 12–14 MeV, where the intense group of elastically scattered deuterons occurs, careful calibrations with the recoil protons and deuterons, as well as the calculations of the theoretical Landau distributions[43] were made in order to unambiguously determine the number of breakup protons. The energy and energy-loss calibrations were performed using a polythene and heavy-paraffin target partially covered with aluminum absorbers of different thicknesses. It is believed that the energy calibration is accurate to ±0.06 MeV. A typical distribution of energy losses in the dE/dx counter is shown in Fig. 1.

A simultaneous measurement of proton and deuteron spectra affords a continuous energy calibration through the intensive peak of elastically scattered deuterons. It

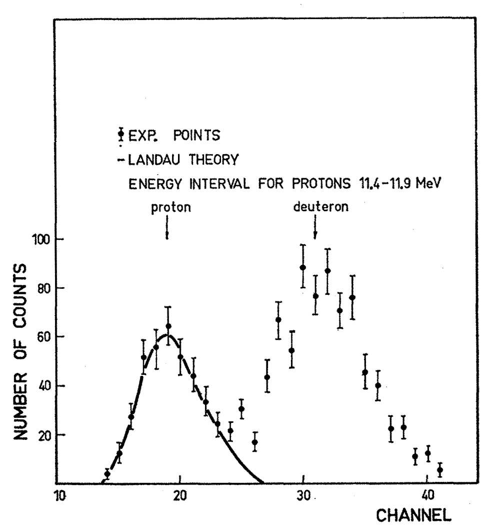

FIG. 1. Spectra of proton and deuteron energy losses in the dE/dx counter. The curve is calculated from the Landau-Symon theory. See Ref. 43.

[39] I. Šlaus, in *Progress in Fast Neutron Physics*, edited by G. C. Phillips, J. B. Marion, and J. R. Risser (University of Chicago Press, Chicago, 1963), p. 61.

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[41] L. G. Kuo, M. Petravić, and B. Turko, Nucl. Instr. Methods **10**, 53 (1961).

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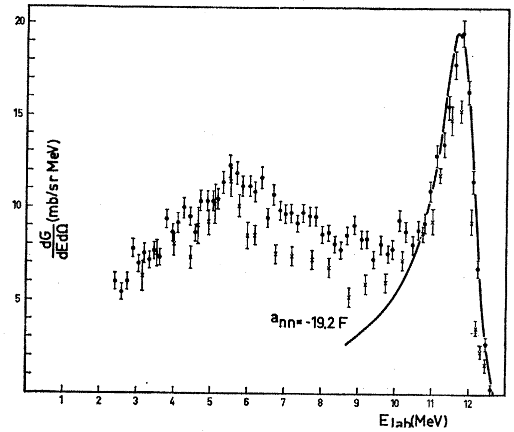

FIG. 2. Energy spectrum of protons from deuteron breakup at 14.4 MeV at 4.8° in the laboratory system. Crosses: measurement by Ilakovac *et al.*, Ref. 37. Dots: present data. The curve represents the theoretical calculated spectrum for $a_{nn}=-19.2$ F after smearing and normalization.

offers a way of correcting possible shifts in the electronic system. Unless properly corrected, such shifts would result in the smearing of the particle spectra.

In the previous experiment[33] the resolution of the counting system was established by measuring protons elastically scattered from the polythene target. The peaks of the elastically-scattered deuterons and protons from the hydrogen contamination in the deuterium target open the possibility of continuous monitoring of the energy resolution of the counting system. Since a noticeable fraction of the over-all resolution was due to the electronic instability, it turned out that such a procedure was essential to determine the resolution correctly. Using this procedure it was found that the resolution was $(5.8\pm0.5)\%$.

The extreme scattering angles accepted by the scintillation counter when the telescope is placed at 0° are $+13°$, 0°, $-13°$. The angle corresponding to the maximum of the window function of the counter is 4.8°. Its width at the half-maximum is $\pm3°$.

Figure 2 shows the spectrum of the breakup protons at 4.8°. The errors shown are statistical errors. The data are corrected for the variation of the energy intervals arising from different energy losses at different energies. The correction is also made for neutrons degraded in energy and/or incident in other directions. The presence of such neutrons is evident from the spectra of elastically scattered protons and deuterons from polythene and heavy-paraffin targets, respectively. Since the angular distribution of neutron-deuteron elastic scattering is anisotropic and strongly peaked backwards,[3] the deuteron spectrum measures mostly neutrons incident in the forward direction. Neutrons degraded in energy and incident obliquely distort the proton spectrum by neutron-proton elastic scattering on hydrogen contamination in the deuterated paraffin target and also by producing breakup protons. The hydrogen contamination is evaluated by measuring the elastically scattered protons and deuterons from the heavy-paraffin target. It is found to be 5%. The effect of the elastic scattering on hydrogen contamination is corrected by subtracting from the proton heavy-paraffiin spectrum the proton polythene spectrum multiplied by the ratio of the amount of hydrogen contamination in the deuterated paraffin target and hydrogen in the polythene target. This correction never exceeded 4% and it was generally 1–2%. The error introduced through this correction is 1%. The number of breakup protons produced by degraded and obliquely incident neutrons is difficult to estimate since the energy and angular variation of the proton spectra from the reaction D$(n,p)2n$ are not known. It is experimentally established that the angular distributions are peaked forward[34,38] and that the total cross section for the deuteron breakup increases almost linearly with the energy[44] up to $E_n\approx14$ MeV. This angular and energy dependence of the cross section is well reproduced by the theory of Frank and Gammel.[45] The corrections are calculated under the assumptions that the spectra are flat and that the energy and angular dependence of the breakup cross section is given by the theory of Frank and Gammel. These corrections are largest in the low-energy part of the spectrum, where they amount to $\sim1.4\%$. It is believed that the error brought up through these corrections is not larger than 1.4%.

The absolute value of the cross section was determined in two ways, one of them being the normalization of proton spectra to the neutron-proton elastic scattering[46] using the polythene target of the known size and thickness, the other one the normalization to the elastically scattered deuterons[34,47] from the same heavy-paraffin target. Both procedures turned out to give the same result much within the experimental error. The over-all accuracy in the measurement of the absolute cross section was 12%, while the two determinations differed by only 2.5%.

The present data were compared with those of Ref. 33 in Fig. 2. The excellent agreement in the shapes of the two spectra is obvious. However, the absolute cross sections do not agree, the present value being $\sim25\%$ larger.

Two measurements of the reaction D$(n,p)2n$ at the nominal angle of 0° and at 14 MeV were recently performed by Bonnel and Lévy,[35] and by Veljukov and Prokofjev.[36] Both spectra show a pronounced peak at the maximum proton energy and their shape is in good agreement with our work. The absolute cross section

[44] V. J. Ashby, H. C. Catron, L. L. Newkirk, and C. J. Taylor, Phys. Rev. **111**, 616 (1958); H. C. Catron, M. D. Goldberg, R. W. Hill, J. M. LeBlanc, J. P. Steering, C. J. Taylor, and M. A. Williamson, Phys. Rev. **123**, 218 (1961).
[45] R. M. Frank and J. L. Gammel, Phys. Rev. **98**, 1204 (1955).
[46] T. Nakamura, J. Phys. Soc. Japan **15**, 1359 (1960); H. L. Poss, E. O. Salant, G. A. Snow, and L. C. L. Yuan, Phys. Rev. **87**, 11 (1952).
[47] J. C. Allred, A. H. Armstrong, and L. Rosen, Phys. Rev. **91**, 90 (1953).

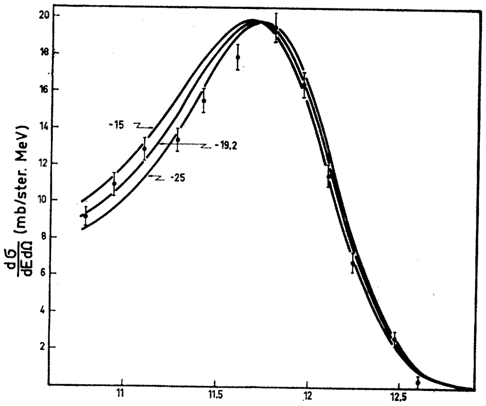

FIG. 3. The experimental data for D$(n,p)2n$ at 14.4 MeV and at 4.8° in the laboratory system compared with three calculated spectra, smeared for the energy resolution of 5.8% and separately normalized.

in Ref. 35 is ~20% larger than the present value. Veljukov and Prokofjev do not explicitly give the cross section, but an estimate made comparing their breakup protons with the elastically scattered protons from the known hydrogen contamination in their target yields a value of ~20 mb/sr in the peak, in fair agreement with the present value.

4. ANALYSIS

The cross section for the breakup of the deuteron has been calculated using the Born approximation and taking into account the final-state interaction of two neutrons. The procedure is the same as described in Ref. 37 and its validity is restricted to a region of the proton spectrum around the maximum energy.

Figure 3 shows the high-energy part of the proton spectrum at 4.8° together with the theoretical curves calculated for $a_{nn}=-15$ F, -19.2 F, and -25 F. The theoretical curves are smeared for the energy resolution of 5.8%. They are also normalized to the experimental points since the theory did not predict the absolute value correctly. It can be seen that there is a distinct difference between the shapes of the three calculated spectra, and that varying a_{nn} a good fit to the data can be obtained in the region around the peak.

The result of the analysis depends upon the number of experimental points taken into account. If the points at lower proton energies are included, the neutron-proton final-state interaction becomes more significant and the theory is no longer valid. As an empirical criterion one can use the stability in the value of a_{nn} when the number of experimental points is varied. Twelve experimental points in the energy region between 10.78 and 12.59 MeV were considered. The value of a_{nn} as a function of the number of experimental points N taken into the analysis shows a flat behavior around $N=9$. Taking for N either more than 10 or less than 8 points would give 1–2 F lower value for a_{nn}. The analysis was based on 9 experimental points in the interval from 11.2 to 12.59 MeV.

The experimental determination of the energy may not quite coincide with the theoretical calculations. In that case an adjustment of the energy scale should be done in such a way as to bring the experimental data in the best agreement with the theoretical curves. It was found that the best fit to the data is obtained by transforming the energy scale linearly so that the shift at 11.79 MeV amounts to +20 keV. Since the required energy shift is well within the experimental accuracy of the energy measurements, it could be safely performed.

The value of a_{nn} deduced from the analysis depends upon the resolution of the counting system. The change of the resolution from 4.9% to 6.1% implies a change in the value of a_{nn} from 20 to 22.5 F if an equally good fit is required.

By varying both a_{nn} and the normalization factor, the best fit to the experimental data was obtained for

$$a_{nn}=-21.7\pm1\text{ F}. \tag{2}$$

The error is only statistical. The uncertainty in the resolution, which is $(5.8\pm0.5)\%$, together with the uncertainty brought by the variation in the number of experimental points taken into analysis may argue for the increase of the total error to ±2 F. The value (2) is in excellent agreement with the value previously obtained by Ilakovac *et al.*[37]

5. BREAKUP OF THE TRITON

The interaction between the neutron and the triton can result in the elastic scattering

$$n+\mathrm{T}\rightarrow n+\mathrm{T},$$

and in the breakup processes

$$n+\mathrm{T}\rightarrow 2n+d,$$
$$n+\mathrm{T}\rightarrow 3n+p.$$

The Q values for the latter processes are -6.26 and -8.49 MeV, respectively.

The neutron-triton elastic scattering has been studied[48] at several energies up to $E_n=14$ MeV. There are no data on breakup processes.[3]

The reaction T$(n,d)2n$ is analogous to the deuteron breakup D$(n,p)2n$. The measurement of the deuteron spectrum at 0° should reveal indications of the neutron-neutron and neutron-deuteron final-state interactions. The analysis of the higher energy peak (corresponding to the neutron-neutron final-state interaction) could give a_{nn}. The analysis of the lower energy peak (corresponding to the neutron-deuteron final-state interaction) could give information about the doublet and the quartet nucleon-deuteron scattering lengths.

[48] J. H. Coon, C. K. Bockelman, and H. H. Barschall, Phys. Rev. **81**, 33 (1951); J. D. Seagrave, L. Cranberg, and J. E. Simmons, *ibid.* **119**, 1981 (1960).

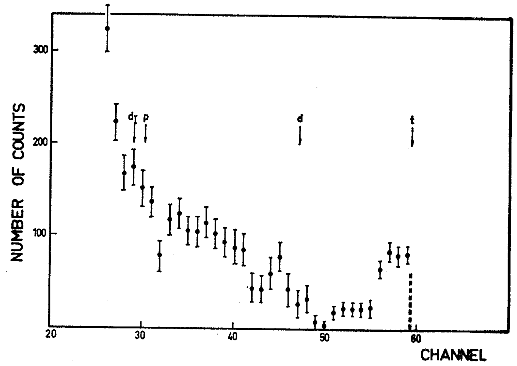

FIG. 4. Energy spectrum of charged particles from the $T+n$ reaction at 14.4 MeV and at 4.8° in the laboratory system, as measured by a 100×100 channel analyzer. The arrows indicate the positions of elastically scattered tritons, deuterons moving forwards in the case when two neutrons are moving backwards, deuterons moving forwards together with a neutron, the other neutron moving backwards, and protons moving forwards when three neutrons are moving backwards.

Even more interesting is the study[3] of the breakup processes

$$n+\mathrm{T}\to 3n+p\,,$$

and the charge symmetric reaction

$$p+\mathrm{He}^3\to 3p+n\,.$$

The energy spectrum of the dissimilar particle could reveal peaks corresponding to the strong final-state interactions. The peak at the maximum proton energy in the first process would correspond to the final-state interaction of three neutrons. The analysis of this peak could show whether nuclear forces contain three-body forces.[49] However, the study of the process $\mathrm{He}^3(p,n)3p$ is experimentally considerably more suitable for that purpose.

The Ti-T target supplied by the Isotope Division of ORNL was bombarded with 14.4-MeV neutrons. A 0.5-mg/cm^2 layer of titanium was deposited on a platinum backing. The amount of tritium was $\lesssim 1$ Ci. The charged particles produced by the neutron bombardment were detected by the same counter telescope. To protect the counter from the tritium beta activity, the target was covered with 0.8-mg/cm^2-thick Mylar foil. The background was measured with an equivalent Ti on Pt target.

The spectrum of charged particles at 4.8° is shown in Fig. 4. The results are preliminary[50] but even at this stage interesting conclusions could be drawn.

The peak around the channel 60 belongs to elastically scattered tritons. The arrows marked d and d_I indicate the deuteron energy corresponding kinematically to the case when two neutrons are recoiling backwards with small relative energy, and to the case when the deuteron and one of the neutrons are moving forward with small relative energy, respectively. The arrow marked p indicates the proton energy which corresponds to the case when three neutrons are recoiling backwards with small relative energy.

The effects of the neutron-neutron and neutron-deuteron final-state interactions are evident in the spectrum. Also, the peak associated with the final-state interaction of 3 neutrons seems to be present. The measurement reveals that at 0° the breakup cross section is almost of the order of magnitude of the elastic cross section.

6. DISCUSSION

The theoretical analysis used to extract a_{nn} is subject to strong criticism. The use of the Born approximation at 14 MeV is not justified. The neutron-proton final-state interaction was neglected. If it is included it would modify the value obtained for a_{nn}. In spite of the inadequacy of the theory, a remarkably good reproduction of the shape of the entire proton spectrum as well as a similar success enjoyed by the same theoretical analysis to analogous problems, raise some confidence in the value for a_{nn} quoted in Sec. 3.

If one accepts $a_{nn}=-21.7\pm 1$ F, what are the implications for the charge independence of nuclear forces?

The experimental value $a_{nn}=-21.7\pm 1$ F measures the effective strengths of nuclear and magnetic interactions. The correction due to magnetic interaction was calculated[51] following the procedure outlined by Riazuddin[21] and the pure nuclear value for the neutron-neutron $^1S_0$ scattering lengths is found to be

$$a_{nn}=-22.5\pm 1 \text{ F}.$$

This value can be compared with the values obtained by Lin[23] and listed in the last column of Table I. A satisfactory agreement can be obtained if one allows $\sim 2\%$ difference in the coupling constants. According to Lin, such a small difference in the coupling constants incorporated in a treatment which takes into account the mass difference between charged and neutral pions can explain the nucleon-nucleon $^1S_0$ scattering lengths and the $T=1$ isobaric levels in the $A=6$ triad. Nuclear forces seem, therefore, to be about 2–3% charge dependent.

Wong and Noyes predicted[25] for a_{nn}, assuming the exact charge symmetry,

$$a_{nn}=-27.0\pm 1.4 \text{ F}.$$

A noticeable difference between our value and this value indicates that nuclear forces may be charge dependent. If one assumes that the coupling constants are all equal and that their value is 13.5±0.9, and if one further assumes that the position of the multipion pole remains

[49] A. I. Baz (private communication, 1962).

[50] The experiment on these processes is now undertaken in the Institute "Ruđer Bošković" by V. Ajdačić, M. Cerineo, B. Lalović, G. Paić, I. Šlaus, and P. Tomaš.

[51] D. Rendić, B. Eman, and E. Coffou (private communication, 1963).

constant for neutron-neutron, neutron-proton, and proton-proton systems, the value for $a_{np}=-22.5$ F implies that the multipion pole strength Γ_{nn} departs from the exact charge symmetry by $\sim$2–2.5%.

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Deuteron Stripping Studies in the Light Isotopes of Nickel*

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Measurements of protons from (d,p) reactions on Ni^{58} and Ni^{60} were made with the Aldermaston tandem Van de Graaff and multigap spectrograph. A large number of states of the final nuclei (well over a hundred in each case) were observed and assigned to single-particle states. For the states in the $28<N\leq 50$ shell, the results for both energies and degree of filling are compared with pairing theory; the agreement is good. A sufficiently large fraction of the $l=2$ states are observed to locate the $d_{5/2}$ and $d_{3/2}$ single-particle states at 6.0 and 9.3 MeV, respectively, in Ni^{59}, and at 5.0 and 8.4 MeV, respectively, in Ni^{61}. A relation between neutron-reduced width Γ_n^0 (from neutron experiments) and the stripping spectroscopic factor S is derived and checked experimentally with two levels observed in both experiments; the agreement is satisfactory. Plots of neutron strength function versus energy are obtained containing both neutron and stripping data, and subjected to the requirements of $\Sigma S=1$ and width$=2W$ (where W is the depth of the imaginary potential in optical model). The results give the location of the $3s_{1/2}$ states as 7.3 MeV in Ni^{59} and 6.0 MeV in Ni^{61}. The distribution of states belonging to each single-particle state is found to have approximately the expected width $2W$ except for the $g_{9/2}$ state in Ni^{61} which is concentrated in a single nuclear level. It is shown that the latter behavior is expected since there are no other positive-parity states expected even nearly within a distance W of the single-particle state. In Ni^{59}, the situation is similar except that states are expected and found at a distance $\approx 1.5W$, and these are mixed in weakly.

I. INTRODUCTION

The reactions $Ni^{58}(d,p)Ni^{59}$ and $Ni^{60}(d,p)Ni^{61}$ have been studied by the present authors[1] and by other investigators.[2–5] In our previous study,[1] we located the single-particle states of Ni^{59} and Ni^{61}; these states were taken as the centers of gravity of the corresponding nuclear levels. It is clear that all the principal nuclear levels of a given shell-model state must be observed and identified to give an accurate center of gravity. In Ref. 1 we gave evidence that not all the $3s_{1/2}$ levels were observed, because they occur in an energy region where the increasing level density made it difficult and finally impossible to resolve individual levels. The situation with the $2d_{5/2}$ and $2d_{3/2}$ states was even less satisfactory, for the same reason.

In this paper we present the results of a recent and more thorough investigation of the (d,p) reactions on Ni^{58} and Ni^{60}, with resolution better by a factor of two than that of Ref. 1. The high resolution spectra were obtained with the Aldermaston tandem Van de Graaff accelerator. The data yielded new information principally in the energy region of s and d levels. Revised centers of gravity of the $s_{1/2}$, $d_{5/2}$, and $d_{3/2}$ states, and also of the $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ states are given on the

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