

N^* Regge Pole and Pion-Nucleon (3,3) Phase Shifts

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The consequence of the hypothesis that the N^* lies on a Regge trajectory for the low-energy pion-nucleon phase shifts in the $J=\frac{3}{2}$, $T=\frac{3}{2}$ state is investigated using the Khuri representation.

THE consequence of the hypothesis of Regge poles on the low-energy behavior of a scattering amplitude has recently been pointed out by Khuri.¹ The Khuri representation has subsequently been used by Khuri and Udgaonkar² to calculate the pion-nucleon phase shifts in the $T=\frac{3}{2}$, $P_{\frac{3}{2}}$ state on the basis of a model in which the scattering process proceeds through a single Regge-pole nucleon intermediate state in the direct channel. The case of neutron-proton scattering proceeding via a Regge-pole deuteron has been discussed elsewhere.³ The purpose of this note is to extend the method of Ref. 2 to calculate the pion-nucleon phase shifts in the $T=\frac{3}{2}$, $P_{\frac{3}{2}}$ state. We consider the scattering process to proceed via a N^* isobar [pion-nucleon (3,3) resonance] intermediate state and treat the isobar as a Regge particle with a variable spin.

Using the Khuri representation, the contribution of the N^* Regge pole to the pion-nucleon $T=\frac{3}{2}$, $P_{\frac{3}{2}}$ partial-wave amplitude can be calculated in a straightforward manner. The result is⁴

$$a_{-}(\frac{3}{2}, W) = -\frac{1}{2} \frac{\beta(W)}{\alpha(W) - \frac{3}{2}} \times [\exp(\alpha - \frac{3}{2})\xi_1 + \exp(\alpha - \frac{3}{2})\xi_2]. \quad (1)$$

In (1), $\alpha(W)$ is the N^* Regge trajectory and $\beta(W)$ the corresponding residuum. W is the total c.m. energy. ξ_1 and ξ_2 are given by

$$\xi_1 = \ln \left\{ 1 + \frac{2}{k^2} + \left[\left(1 + \frac{2}{k^2} \right)^2 - 1 \right]^{\frac{1}{2}} \right\},$$

$$\xi_2 = \ln \left\{ \frac{W^2 - m^2 - 2}{2k^2} - 1 + \left[\left(\frac{W^2 - m^2 - 2}{2k^2} - 1 \right)^2 - 1 \right]^{\frac{1}{2}} \right\}. \quad (2)$$

In (2), k is the c. m. momentum and m the nucleon

mass. The pion mass has been set equal to unity. Following Khuri and Udgaonkar,² we next approximate Eq. (1) based on considerations of the threshold behavior of $\beta(k^2)$. The latter is given by

$$\beta(k^2) \simeq [k^2]^{\alpha_0 - \frac{1}{2}} \quad \text{as } k^2 \rightarrow 0; \quad \alpha_0 = \alpha(W)|_{k^2=0}, \quad (3)$$

while the terms involving the exponential behave as

$$\exp(\alpha - \frac{1}{2})\xi_1 \simeq [k^2]^{\alpha - \frac{1}{2}},$$

$$\exp(\alpha - \frac{1}{2})\xi_2 \simeq [k^2 / (2m - 1)]^{\alpha - \frac{1}{2}}, \quad \text{as } k^2 \rightarrow 0, \quad (4)$$

so that the product $\beta \exp(\alpha - \frac{1}{2})\xi_1$ will be slowly varying and essentially real near threshold. We approximate it by a real constant C for the entire energy range under consideration

$$\beta \exp(\alpha - \frac{1}{2})\xi_1 = C. \quad (5)$$

We next consider the form of the N^* trajectory $\alpha(W)$. The real part of $\alpha(W)$ may be written in the form

$$\text{Re}\alpha(W) = \frac{3}{2} + \epsilon(W - m^*). \quad (6)$$

In (6) m^* denotes the mass of the N^* isobar and ϵ the slope of the N^* trajectory. For the imaginary part of $\alpha(W)$ we assume the form

$$\text{Im}\alpha(W) = C_1 [W - (m + 1)]^{\frac{1}{2}}. \quad (7)$$

The form (7) for $\text{Im}\alpha(W)$ satisfies the requirement that $\alpha(W)$ be purely real below threshold. It is also consistent with the requirement of the correct threshold behavior⁵ of $\text{Im}\alpha(W)$, viz.,

$$\text{Im}\alpha \simeq k^{2\alpha_0 + 1}, \quad k^2 \rightarrow 0. \quad (8)$$

The constant C_1 , occurring in (7), may be expressed in terms of the width Γ of the (3,3) resonance using the relation

$$\Gamma = \frac{1}{m^*} \left[\text{Im}\alpha(W) / \frac{d}{dW^2} \text{Re}\alpha(W) \right]_{W=m^*}. \quad (9)$$

From (6), (7), and (9), we obtain for the trajectory

$$\alpha(W) = \frac{3}{2} + \epsilon(W - m^*) + \frac{1}{2}i\Gamma \epsilon \left[\frac{W - (m + 1)}{m^* - (m + 1)} \right]^{\frac{1}{2}}. \quad (10)$$

The partial-wave amplitude $a_{-}(\frac{3}{2}, W)$ finally becomes,

⁵ A. O. Barut and D. E. Zwanziger, Phys. Rev. **127**, 974 (1962).

¹ N. N. Khuri, Phys. Rev. **130**, 429 (1963). Khuri's formula was independently obtained by C. E. Jones, University of California Lawrence Radiation Laboratory Report UCRL-10700 (unpublished).

² N. N. Khuri and B. M. Udgaonkar, Phys. Rev. Letters **10**, 172 (1963).

³ S. K. Bose and M. DerSarkissian, Nuovo Cimento (to be published).

⁴ We are following the notation of V. Singh [Phys. Rev. **129**, 1889 (1963)]. The odd J -parity amplitude $a_{-}^0(J, W)$ interpolates to $a_{-}(\frac{3}{2}, W)$. We have assumed that the former amplitude has no other singularity in the complex J plane except for a pole corresponding to N^* .

from (1) and (10),

$$a_{-}(\frac{3}{2}, W) = -\frac{C}{2\epsilon} \left\{ \frac{\exp(-\xi_1) + \exp(-\xi_2) \exp[(\xi_2 - \xi_1)(\alpha(W) - \frac{1}{2})]}{W - m^* + \frac{1}{2}i\Gamma \left(\frac{W - (m+1)}{m^* - (m+1)} \right)^{\frac{1}{2}}} \right\}. \quad (11)$$

If we now specify the values of the parameters m^* and Γ from experiment, i.e., $m^* = 8.91$, $\Gamma = 0.72$, and estimate ϵ from the observed location of the next higher resonance, viz., the pion-nucleon resonance with $J = \frac{7}{2}^+$ at⁶ 1920 MeV (ϵ turns out to be 0.41), the trajectory $\alpha(W)$ then is completely fixed. The (3,3) phase shift which is related to the partial-wave amplitude through the

relation

$$a_{-}(\frac{3}{2}, W) = (e^{i\delta_{33}}/k) \sin \delta_{33} \quad (12)$$

can now be calculated from (11). It is not necessary to know the value of the over-all multiplicative constant C appearing in (11) as the expression for phase shift $\delta_{33} = \tan^{-1}[\text{Im}a_{-}(W)/\text{Re}a_{-}(W)]$ is independent of the latter. The situation here is quite different from that in Ref. 2, where it was essential to determine the constant C (which was done by an extrapolation procedure). The difference between our case and that of Ref. 2 arises because, in the latter, the imaginary part of the (nucleon) trajectory was explicitly neglected. However, once the phase shifts are determined the amplitude $a_{-}(\frac{3}{2}, W)$ is completely fixed and the constant C can then be determined by comparing the amplitude $a_{-}(\frac{3}{2}, W)$ calculated at a fixed energy using (12), with that given by (11). This can, in particular, be done at the resonance energy $W = m^*$. The value of C , thus determined, is $C \simeq 0.3$.

The (3,3) phase shifts calculated from (10) are plotted in Fig. 1. As can be seen from the figure, our result for the energy dependence of δ_{33} is in good agreement with the experimental data.

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Note added in proof. A closely analogous approach to the present problem has been made by DerSarkissian [M. DerSarkissian. Louisiana State University, 1963 (unpublished)].

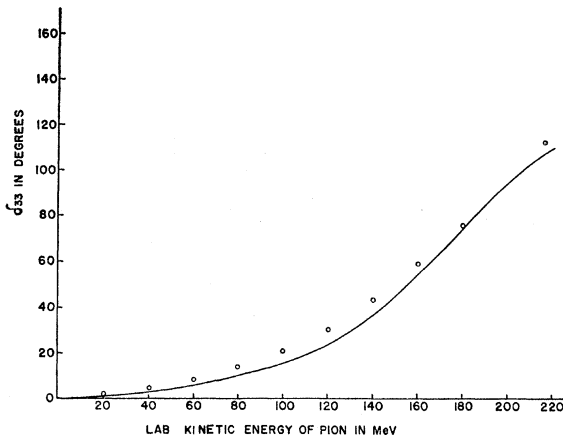


FIG. 1. Plot of the δ_{33} phase shifts in degrees as a function of the kinetic energy in MeV of the incident pion in the laboratory. Experimental points are denoted by circles.

⁶ We have assumed the spin of πN resonance at 1920 MeV to be $\frac{7}{2}^+$. We understand that this point is not yet completely settled, although the presence of a large $\cos^2\theta$ term in the pion angular distribution strongly favors this assignment. This assignment of spin $\frac{7}{2}^+$ has also been suggested by Glashow and Rosenfeld [S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 192 (1963)].