Angular Correlation Functions for Compound Inelastic Nucleon Scattering

ERIC SHELDON
Laboratorium für Kernphysik, Eidg. Technische Hochschule, Zürich, Switzerland
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To aid the analysis of nucleon-γ angular correlation results for target nuclei having zero and nonzero ground-state spins, the double-differential cross section has been evaluated in explicit form for compound inelastic scattering of spin-$\frac{1}{2}$ particles to first or to second excited states of nuclei having g.s. spin $0+$, $\frac{1}{2}+$, $1+$, $\frac{3}{2}+$, $\frac{3}{2}-$, $\frac{5}{2}+$, $\frac{7}{2}-$. In several instances, provision has been made for the coincident γ radiation to be of mixed multipolarity or for cascades to include an unobserved intermediate γ-decay step preceding the γ transition under observation. Theoretical results are illustrated quantitatively by correlation curves for inelastic neutron scattering at suitable energies around 3 MeV upon appropriate representative target nuclei (Ge^{70}, Zn^{66}, Ni^{64}, Fe^{56}; Si^{29}, P^{31}; P^{32}; S^{33}; Cu^{63}; Zr^{91}; Co^{57}, respectively).

1. INTRODUCTION

ALTHOUGH the formal theory of angular correlation for compound inelastic scattering of spin-$\frac{1}{2}$ nuclear particles is now well established,[1–12] the numerical evaluation of the requisite Racah arithmetic has been carried through[3,11,12] only for one particular nuclear spin transition sequence, namely for the sequence $0+ \rightarrow J_1\pi_1 \rightarrow 2+ \rightarrow 0+$, such as one obtains for scattering to the first level of *e-e* nuclei via compound nucleus (CN) states of spin J_1 and parity π_1. Though this admittedly embraces an extensive class of investigations, the need to extend evaluations to cover other spin sequences is evident. The results presented in the sections which follow aim to satisfy this need, at least in part, and to provide a basis of hand-calculated formulas which may later be used to advantage in checking more general angular correlation and distribution computer programs founded upon the statistical model.

The numerical expressions derived from the basic correlation theory are applicable not only to inelastic nucleon scattering, but also to nuclear reactions of the type $(p,n\gamma)$, $(n,p\gamma)$, $(t,n\gamma)$, $(t,p\gamma)$, $(He^3,n\gamma)(He^3,p\gamma)$, etc., of which, the latter feature the advantage of leading to fairly high excitation of the CN even when the incident energy is low and thus offer conditions conducive to the validity of the continuum assumption. The last-named reaction has, indeed, been subjected to (unpublished) correlation investigations by the Maryland group,[13] whereas proton inelastic scattering to the *second* level of the *e-e* nucleus Ar^{40} at $E_p = 5.6$ MeV has been utilized in p'-γ and γ-γ correlation studies by the Osaka group.[14] The latter group observed isotropy (to within the statistical error of about 4%) in the p'-γ correlation with the p' counter perpendicular to the incident beam and with five settings (between 0° and 120° in the scattering plane) of the γ counter used to register γ rays making the transition from the second to the first level of Ar^{40}. This, together with evidence of pure $E2$ γ multipolarity and the absence of direct γ transition to the $0+$ ground state enabled spin $0+$ to be assigned to the second level at 2.13 MeV of Ar^{40}, a conclusion supported by γ-γ coincidence studies of the cascade radiation. It is symptomatic of the scope of such correlation studies that nucleon-gamma investigations are in general aimed toward elucidation of reaction mechanism,[15] whereas γ-γ (and β-γ) studies aim in the main toward establishing spin-parity assignments and elucidating nuclear structure. The potentialities of the latter field of investigation have been exploited by various groups, and notably by Gove *et al.* at Chalk River, whose most recent results have been presented in the

[1] L. C. Biedenharn and M. E. Rose, Rev. Mod. Phys. **25**, 729 (1953) and earlier references therein.
[2] M. E. Rose, Oak Ridge National Laboratory Report ORNL-1555, 1953 (unpublished).
[3] G. R. Satchler, Phys. Rev. **94**, 1304 (1954); and **104**, 1198 (1956).
[4] H. Frauenfelder, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), p. 531.
[5] A. A. Kraus, Jr., J. P. Schiffer, F. W. Prosser, Jr., and L. C. Biedenharn, Phys. Rev. **104**, 1667 (1956).
[6] S. Devons and L. J. B. Goldfarb, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 42, p. 362.
[7] M. E. Rose, Oak Ridge National Laboratory Report ORNL-2516, 1958 (unpublished).
[8] L. J. B. Goldfarb, in *Nuclear Reactions*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Company, Amsterdam, 1959), Vol. I, p. 159.
[9] L. C. Biedenharn, in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, p. 732.
[10] A. E. Litherland and A. J. Ferguson, Can. J. Phys. **39**, 788 (1961).
[11] G. R. Satchler and E. Sheldon, in *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach Publishers Inc., New York, 1963), p. 832.
[12] E. Sheldon, Rev. Mod. Phys. **35**, 795 (1963), and references therein.
[13] Private communication by W. F. Hornyak and C. A. Ludemann, to whom the author desires to express his appreciation of the opportunity to peruse and discuss correlation and distribution results for the reaction $C^{12}(He^3,p_{(2)}\gamma)N^{14}$ at $E_{He}=2.25$ and 2.45 MeV prior to publication. A preliminary report by C. A. Ludemann, H. D. Holmgren, and W. F. Hornyak had been submitted as a short contribution to the Topical Conference on Compound Nuclear States, Gatlinburg, 1963.
[14] T. Wakatsuki, Y. Hirao, and I. Miura, Nucl. Phys. **39**, 335 (1962).
[15] The above Ar^{40} $p'-\gamma$ measurements constitute an exception in that isotropy of the correlation with respect to the γ-emission angle is a *model-independent* consequence for γ decay proceeding from a $0+$ state, as is well known.

paper of Broude and Gove.[16] These authors, making use of the extensive correlation parameter tabulation of Ferguson and Rutledge,[17] evaluated γ-γ correlation functions (for which the tabulation is especially suited) for several spin sequences[18] of the type $0+ \rightarrow J_1\pi_1 \twoheadrightarrow J_2\pi_2 \rightarrow 2+ \rightarrow 0+$ which represent an advance upon the single transition sequence mentioned earlier, but which cannot directly be taken over for nucleon-gamma correlations. Although calculation of the latter is somewhat facilitated by employing these and other parameters,[1,17,19,20] it is nevertheless appreciably more complicated in that the procedure involves additional nuclear barrier penetrabilities and "particle parameters,"[1,6,9] for which reason it was deemed commensurately straightforward to use the more basic Racah functions, as tabulated numerically in various reports,[21–30] and to employ the modified Ferguson-Rutledge parametrization in occasional spot checks only. The final expressions have in each instance been checked by comparison with identical calculations carried out independently,[31] also to some extent by checks of internal consistency, and in part by integration and comparison with appropriate distribution expressions cited by Van Patter[32] in a privately circulated manuscript.

Apart from underlying assumptions and simplifications in the basic correlation theory and the basic reaction theory discussed in Refs. 1, 3, 12 and by Feshbach,[33] respectively, the hand calculations have been reduced to manageable complexity by arbitrarily assuming further that spin-orbit interaction need not be considered and that the influence of higher partial waves than those with $l=2$ is negligible. Though detailed investigations[12] have shown these simplifying assumptions to be justified at relatively low energies in the case of the familiar $0+ \rightarrow J_1\pi_1 \rightarrow 2+ \rightarrow 0+$ sequence, they may in some other instances be too restrictive, but it is precisely in such cases that hand calculation would become unfeasibly complicated if they were to be relaxed. Provision has, however, been made for intermediate unobserved cascade transitions and for mixed γ multipolarity. After a summary of the basic theory in Sec. 2, explicit correlation functions are presented in absolute form for target nuclei having $0+$ ground states in Sec. 3 and for those with nonzero ground-state spin in Sec. 4. Of the innumerable combinations of spin sequences which could have been taken, only those were selected from a comprehensive compilation[34] of nuclear energy levels and spin assignments which would be suitable for experimental analysis involving stable and fairly abundant target nuclei in the range $29 \leqslant A \lesssim 100$. In each instance the theoretical results are presented both in an intermediate form in terms of Legendre polynomials and hyperpolynomials valid for all azimuths and immediately reducible to angular *distributions* of particles or γ radiation, and in a final form valid when the radiations are coplanar (azimuth $\varphi=0°$) in function of the particle-emission angle θ_1 and the γ-emission angle θ_2 referred to the incident direction in the center-of-mass system. This final form is easy to code for computation over the entire angular range, with numerical coefficients and transmission coefficients constituting the entire input. To illustrate the expressions quantitatively, such a program has been compiled for the Zürich ERMETH computer, and correlation curves evaluated for inelastic neutron scattering at suitable energies around 3 MeV upon appropriate representative target nuclei. These are shown for scattering to either the first (Sec. 3A) or the second (Sec. 3B) level of the target nuclei Ge^{70}, Zn^{66}, Ni^{64}, and Fe^{56} having a ground-state spin $0+$, and for scattering to the corresponding levels of the nuclei Si^{29} or P^{31}, having ground state spin $\frac{1}{2}+$ or P^{32}, S^{33}, Cu^{63}, Zr^{91}, Co^{57}, respectively having ground state spins $1+$, $\frac{3}{2}+$, $\frac{3}{2}-$, $\frac{5}{2}+$, $\frac{7}{2}-$ (Secs. 4A, 4B, 4C).

2. UNDERLYING THEORY

A. Basic Expressions for the Double-Differential Cross Section in the Absence of Unobserved Intermediate Radiations

The derivation of the correlation function for inelastic nucleon scattering to the first excited state of target nuclei on the basis of a pure CN mechanism has been

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[16] C. Broude and H. E. Gove, Ann. Phys. (N. Y.) **23**, 71 (1963).

[17] A. J. Ferguson and A. R. Rutledge, Chalk River Report CRP-615, AECL-420, 1957, revised 1962 (unpublished).

[18] The symbol $\twoheadrightarrow$ throughout this paper betokens an unobserved intermediate transition.

[19] M. Ferentz and N. Rosenzweig, Argonne National Laboratory Report ANL-5324, 1955 (unpublished).

[20] G. R. Satchler, Proc. Phys. Soc. (London) **A66**, 1081 (1953).

[21] A. Simon, *Numerical Table of the Clebsch-Gordan Coefficients*, Oak Ridge National Laboratory Report ORNL-1718, 1954 (unpublished).

[22] B. E. Chi, *A Table of Clebsch-Gordan Coefficients* (Rensselaer Polytechnic Institute, Troy, New York, 1962).

[23] B. J. Sears and M. G. Radtke, *Algebraic Tables of Clebsch-Gordan Coefficients*, Chalk River Report TPI-75, 1954 (unpublished).

[24] M. Rotenberg, R. Bivins, N. Metropolis, and J. K. Wooten, Jr., *The 3-j and 6-j Symbols* (Technology Press, Cambridge, Massachusetts, 1959).

[25] A. Simon, J. H. Vander Sluis, and L. C. Biedenharn, *Tables of the Racah Coefficients*, Oak Ridge National Laboratory Report ORNL-1679, Special, 1954 (unpublished).

[26] K. Alder, Helv. Phys. Acta **25**, 235 (1952).

[27] K. Smith and J. W. Stevenson, *A Table of Wigner 9-j Coefficients*, Argonne National Laboratory Report ANL-5776, 1957 (unpublished).

[28] K. Smith, *Supplement to a Table of Wigner 9-j Coefficients*, Argonne National Laboratory Report ANL-5860, Parts I and II, 1958 (unpublished).

[29] J. M. Kennedy, B. J. Sears and W. T. Sharp, *Tables of X-Coefficients*, Chalk River Report CRT-569, 1954 (unpublished).

[30] W. T. Sharp, J. M. Kennedy, B. J. Sears and M. G. Hoyle, *Tables of Coefficients for Angular Distribution Analysis*, Chalk River Report CRT-556, revised 1960 (unpublished).

[31] By Mr. J. Costandi at Zürich, to whom the author is indebted for his painstaking and laborious work.

[32] D. M. Van Patter, *Angular Distributions of* $(n,n'\gamma)$ *and* $(p,p'\gamma)$ *Radiations—Satchler's Theory*, revised, Bartol Research Foundation, 1961 (unpublished).

[33] H. Feshbach, in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, Chapters V A and VI D, pp. 625 and 1034.

[34] K. Way, N. B. Gove, C. L. McGinnis, and R. Nakasima, in *Energy Levels of Nuclei* (Springer-Verlag, Berlin, 1961), Group I, Vol. 1 of Landolt-Börnstein, *Nuclear Physics and Technology*, New Series.

presented in detail by Satchler,[3] whose approach is followed in the treatment below. Apart from correction of several errors,[11,12,35] the theory has been modified only to embrace spin-orbit coupling,[12] but for simplicity this latter development has not been incorporated into the present paper. In the following, transition sequences of the form

$$J_0\pi_0(j_1=l_1\pm\tfrac{1}{2})J_1\pi_1(j_2=l_2\pm\tfrac{1}{2})J_2\pi_2(L_2,L_2') \rightarrow J_3\pi_3[(L_3,L_3')J_4\pi_4]$$

are considered, and the usual statistical assumption as to the absence of interference between the various possible levels $J_1\pi_1$ of the CN is made. The transition designated by square brackets enters in the case of nucleon scattering to the second level of a target nucleus, which decays by an (unobserved) γ transition followed by an (observed) γ transition in cascade to the ground state. This will be treated as a special case later; it is more straightforward first to consider the sequence $J_0\rightarrow J_1\rightarrow J_2\rightarrow J_3$ which applies to the case of inelastic scattering either to the first excited state followed by γ decay or to the second level followed by γ decay to the first level or direct to the ground state. The γ radiation may be of mixed multipolarity L_2, L_2', where $L_2'=L_2+1$ and the mixing ratio is given by

$$\Delta_2^2=(J_3\|L_2'\|J_2)^2/(J_3\|L_2\|J_2)^2. \tag{1}$$

The double-differential cross section can then be written absolutely as

$$\frac{d^2\sigma}{d\Omega_1 d\Omega_2}=\frac{\lambda\!\!\!^{-2}}{32\pi}\left(\frac{\hat{J}_2}{\hat{J}_0}\right)^2\sum NCWMX\tau S_{\mu\nu\lambda}, \tag{2}$$

using the notation

$$\hat{k}\equiv(2k+1)^{1/2}, \tag{3}$$

and summing over the momenta J_1, j_1, j_2 and the (positive even) transition parameters μ, ν, λ, restricted in the range of possible values by triangle relations which must be obeyed by the following triads,

$$(J_0j_1J_1),\ (J_1j_2J_2),\ (J_2L_2J_3),\ (J_2L_2'J_3),\ (j_1j_1\mu),\ (J_1J_1\mu),\ (j_2j_2\nu),\ (J_2J_2\lambda),\ (L_2L_2\lambda),\ [(L_2L_2'\lambda)],\ (L_2'L_2'\lambda),\ (\mu\nu\lambda),$$

wherein the triad in square brackets refers to nonzero λ.

The separate terms in Eq. (2) are, respectively,

$$N=(-)^{J_0-J_1-J_2+J_3+j_2}(\hat{J}_1)^4(\hat{j}_1)^2(\hat{j}_2)^2, \tag{4}$$

$$C=\langle\mu 0|j_1j_1\tfrac{1}{2}-\tfrac{1}{2}\rangle\langle\nu 0|j_2j_2\tfrac{1}{2}-\tfrac{1}{2}\rangle, \tag{5}$$

$$W=W(J_1J_1j_1j_1;\mu J_0), \tag{6}$$

$$\begin{aligned}M=M_\lambda^{(2)}\equiv&[1+\Delta_2^2]^{-1}[(\hat{L}_2)^2\langle\lambda 0|L_2L_21-1\rangle\\&\times W(J_2J_2L_2L_2;\lambda J_3)\\&+2\Delta_2\hat{L}_2\hat{L}_2'\langle\lambda 0|L_2L_2'1-1\rangle W(J_2J_2L_2L_2';\lambda J_3)\\&+\Delta_2^2(\hat{L}_2')^2\langle\lambda 0|L_2'L_2'1-1\rangle W(J_2J_2L_2'L_2';\lambda J_3)],\end{aligned} \tag{7}$$

$$X=X(J_1J_1\mu;j_2j_2\nu;J_2J_2\lambda), \tag{8}$$

[35] F. D. Seward, Phys. Rev. **114**, 514 (1959).

for the spin-dependent "geometrical" factors, wherein C represents a product of Clebsch-Gordan coefficients, W a Racah coefficient, and X a Fano X-coefficient (Wigner 9-j symbol). In practice, it is convenient to express results in function of terms $M_0^{(2)}$, $M_2^{(2)}$, $M_4^{(2)}\cdots$ corresponding to increasing values of λ up to the highest value permitted by the above triangle relations. For $\lambda=0$, Eq. (7) yields

$$M_0^{(2)}=(-)^{J_2-J_3-1}/\hat{J}_2, \tag{9}$$

a value independent of the multipolarity mixing ratio Δ_2, as one would expect physically. The $M_\lambda^{(2)}$ for $\lambda=2, 4, \cdots$, as given by Eq. (7) take on the form of simple numerical functions of Δ_2 only for any given spin sequence, whence expressing the correlation function in terms of these $M_\lambda^{(2)}$ permits it to be evaluated readily for any scattering sequence in which the mixing ratio Δ_2 is known. It is obvious that for pure multipolarity, with $\Delta_2=0$, very considerable simplification becomes possible. Equation (7) then reduces to

$$M^{(\text{pure }2)}=(\hat{L}_2)^2\langle\lambda 0|L_2L_21-1\rangle W(J_2J_2L_2L_2;\lambda J_3), \tag{10}$$

which can be evaluated explicitly (in practice, the three constituent terms are respectively incorporated within N, C, and W) rather than subjected to the above subdivision into $M_0^{(2)}$, $M_2^{(2)}$, $M_4^{(2)}$, $\cdots$.

The energy dependence of the correlation is contained within the term

$$\tau\equiv T_{l_1}(E_1)\cdot T_{l_2}(E_2)/{\textstyle\sum'}_{jlE}\,T_l(E), \tag{11}$$

where the T_l are transmission coefficients for incident energy E_1 and outgoing energy E_2 of the particle in the center-of-mass (c.m.) system, and the summation in the denominator extends over all permissible channels by which the compound system can decay (a summation hitherto confined to the elastic scattering channel to a 0+ ground state and the inelastic scattering channel to a 2+ first excited state: the "two-channel" approximation, for which the restricted sum is characterized by $\sum'$). The T_l thus vary for different nuclei and different optical potentials chosen to describe the scattering process.

The angular dependence upon θ_1, the scattering angle, and θ_2, the γ-emission angle in the c.m. system, referred to the incident direction taken as the z axis (the y axis being along $\mathbf{k}_0\times\mathbf{k}_1$, where $\mathbf{k}_0$ and $\mathbf{k}_1$ denote the propagation vectors of incident and emergent particle waves), as also upon the azimuth φ, is contained within the Legendre "hyperpolynomial"

$$S_{\mu\nu\lambda}=4\pi(\hat{\mu}/\hat{\lambda})\sum_m(-)^m\langle\lambda m|\mu\nu 0m\rangle \times Y_\nu^{-m}(\theta_1,0)Y_\lambda^m(\theta_2,\varphi), \tag{12}$$

where m is a summation index running over negative and positive integer values up to the lesser of ν, λ. This hyperpolynomial, as introduced and developed by Rose,[7,36] is identical with Seward's Θ_{abc} (Ref. 35) and is

[36] M. E. Rose, J. Math. Phys. **37**, 215 (1958–1959).

closely related to the Λ function of Biedenharn and Rose[1] or the $X_{KM}{}^{N}$ function of Ferguson and Rutledge.[17] Properties of this or analogous functions have also been discussed in a number of publications[9,12,37–39] and explicit values with $\mu, \nu, \lambda \leqslant 4$ have been cited by Sheldon[12] together with a tabulation of numerical parameters which enable $S_{\mu\nu\lambda}$ to be evaluated for $\mu, \nu \leqslant 18, \lambda \leqslant 4$.

The correlation expression (2) for the special case of *e-e* target nuclei can be reduced somewhat, in that $J_0=0$, whence $j_1=J_1$ and

$$\frac{d^2\sigma}{d\Omega_1 d\Omega_2}=\left(\frac{\lambda\!\!\!^{-2}}{32\pi}\right)(\hat{J}_2)^2\sum N'C'MX\tau S_{\mu\nu\lambda} \tag{13}$$

with

$$N'\equiv(-)^{J_1-J_2+J_3+j_2}(\hat{J}_1)^4(\hat{j}_2)^2, \tag{14}$$

$$C'\equiv\langle\mu 0|J_1J_1\tfrac{1}{2}-\tfrac{1}{2}\rangle\langle\nu 0|j_2j_2\tfrac{1}{2}-\tfrac{1}{2}\rangle, \tag{15}$$

and the remaining terms as defined in Eqs. (7)–(12), the summation being extended over J_1, j_2, μ, ν, λ. If additionally $J_3=0$, as is the case for scattering to the *first* level of *e-e* nuclei, then $L_2=L_2'=J_2$ and $\Delta_2=0$, whence

$$\frac{d^2\sigma}{d\Omega_1 d\Omega_2}=\left(\frac{\lambda\!\!\!^{-2}}{32\pi}\right)(\hat{J}_2)^2\sum N''C''X\tau S_{\mu\nu\lambda} \tag{16}$$

with

$$N''\equiv(-)^{J_1+J_2+j_2}(\hat{J}_1)^4(\hat{j}_2)^2, \tag{17}$$

$$C''\equiv\langle\mu 0|J_1J_1\tfrac{1}{2}-\tfrac{1}{2}\rangle\langle\nu 0|j_2j_2\tfrac{1}{2}-\tfrac{1}{2}\rangle\langle\lambda 0|J_2J_2 1-1\rangle, \tag{18}$$

and X, τ, and $S_{\mu\nu\lambda}$ unchanged.

B. Basic Expressions in Presence of an Unobserved Intermediate γ Transition

If unobserved radiations (irrespective of their nature) feature in intermediate transitions, the correlation becomes modified[20] by one or more (normalized) Racah factors of the form

$$U_K(L_rJ_rJ_{r+1})=(-)^{J_r+J_{r+1}-L_r}\hat{J}_r\cdot\hat{J}_{r+1}\times W(J_rJ_rJ_{r+1}J_{r+1};KL_r), \tag{19}$$

where the index r labels the unobserved transition and an incoherent weighted sum over L_r and L_r' has to be taken in the case of mixed multipoles. Each of the cases considered in Secs. 3B(*ii*), (*iv*), (*vi*), (*ix*) and 4B involves an unobserved γ transition from J_2 to J_3 of pure multipolarity L_2, so that $\Delta_2=0$. The succeeding γ transition from J_3 to J_4 is observed in coincidence with the emergent particles, and may be of mixed multipolarity L_3, L_3', with mixing ratio defined by

$$\Delta_3{}^2=(J_4\|L_3'\|J_3)^2/(J_4\|L_3\|J_3)^2. \tag{20}$$

In this case, the multiplicative Racah factor (19) takes on the form

$$U_K\rightarrow U_\lambda(L_2J_2J_3)=(-)^{J_2+J_3-L_2}\hat{J}_2\cdot\hat{J}_3\cdot W(J_2J_2J_3J_3;\lambda L_2), \tag{21}$$

and the correlation expression (2) has to be modified somewhat to take account of the change in the γ-decay sequence from $J_2\rightarrow J_3$ to $J_2 \twoheadrightarrow J_3\rightarrow J_4$. The step $J_2\twoheadrightarrow J_3$ is accounted for by introduction of the U term (21) and the step $J_3\rightarrow J_4$ by redefinition of the M term to

$$\begin{aligned}M=M_\lambda{}^{(3)}\equiv&[1+\Delta_3{}^2]^{-1}\\ &\times[(\hat{L}_3)^2\langle\lambda 0|L_3L_3 1-1\rangle W(J_3J_3L_3L_3;\lambda J_4)\\ &+2\Delta_3\hat{L}_3\cdot\hat{L}_3'\langle\lambda 0|L_3L_3'1-1\rangle W(J_3J_3L_3L_3';\lambda J_4)\\ &+\Delta_3{}^2(\hat{L}_3')^2\langle\lambda 0|L_3'L_3'1-1\rangle\\ &\times W(J_3J_3L_3'L_3';\lambda J_4)].\end{aligned} \tag{22}$$

The latter is again, for convenience, expanded in increasing permitted values of λ in a manner analogous to that of the previous section, $M_0{}^{(3)}$ again being a pure number, and $M_2{}^{(3)}$, $M_4{}^{(3)}$, $\cdots$ being Δ_3-dependent. This expansion is again redundant when $\Delta_3=0$, for then Eq. (22) reduces to

$$M^{(\text{pure }3)}=(\hat{L}_3)^2\langle\lambda 0|L_3L_3 1-1\rangle W(J_3J_3L_3L_3;\lambda J_4), \tag{23}$$

a product of terms which can be absorbed within the resulting N, C, and W in the requisite modified Eq. (2). The remaining modification consists in "eradicating" from (2) those terms which came from the original (observed) transition $J_2\rightarrow J_3$. The final result is similar in form to (2),

$$\frac{d^2\sigma}{d\Omega_1 d\Omega_2}=\frac{\lambda\!\!\!^{-2}}{32\pi}\left(\frac{\hat{J}_2}{\hat{J}_0}\right)^2\sum N'''C\,W''M_\lambda{}^{(3)}\tau S_{\mu\nu\lambda}, \tag{24}$$

but with M replaced by either (22) or (23) and

$$N'''\equiv(-)^{J_0-J_1+J_2+J_4+j_2-L_2}(\hat{J}_1)^4(\hat{J}_3)^2(\hat{j}_1)^2(\hat{j}_2)^2 \tag{25}$$

$$=(-)^{2J_2-J_3+J_4-L_2}(\hat{J}_3)^2\cdot N; \tag{26}$$

$$W''\equiv W(J_1J_1j_1j_1;\mu J_0)W(J_2J_2J_3J_3;\lambda L_2). \tag{27}$$

As before, the summation extends over momenta J_1, j_1, j_2 and (positive even) parameters μ, ν, λ, where now the range of permitted λ values may differ from that for a $J_2\rightarrow J_3$ transition in consequence of the triangle relations for the additional triads $(J_3J_3\lambda)$, $(L_3L_3\lambda)$, etc. The range of μ and ν is, of course, unchanged. The terms C, τ, and $S_{\mu\nu\lambda}$ have been defined in Eqs. (5), (11), and (12). Throughout this paper, the range of summation has been curbed by restricting the orbital angular momenta of incident and emergent particles to l_1, $l_2\leqslant 2$. In deciding whether to treat any given γ transition as having pure or mixed multipolarity, it has been assumed that an E_L, M_{L+1} mixture is essentially improbable when compared with the likelihood of an M_L, E_{L+1} mixture, and that the multipolarity is dictated by the rule $L=\Delta J$, $1+\Delta J$, except in the case of $2+\rightarrow 2+$ γ transitions

[37] A. J. MacFarlane, Nucl. Phys. 38, 504 (1962).
[38] E. Sheldon, Phys. Letters 2, 178 (1962).
[39] D. Brink and G. R. Satchler, *Angular Momentum* (Oxford University Press, Oxford, 1962).

which involve an $M1+E2$ mixture for which Δ_2 can assume large values ($2\hbar\omega \rightarrow 1\hbar\omega$ transitions in vibrational nuclei).

At this stage, attention may be drawn to a resulting by-product of the above calculations taking a correlation which involves a $J_2 \rightarrow J_3$ step over into one which involves a $J_2 \twoheadrightarrow J_3 \rightarrow J_4$ cascade. It follows from the requisite Racah algebra that *no* modification of the final correlation expression is necessary when $J_4=0$ and $L_2=L_2'=L_3=L_3'(=L)$; the range of summation is also unaltered in this special case.[40] The γ-transition parameter in Satchler's notation[3] for the step $J_2 \rightarrow J_3$ with $\Delta_2=0$ is

$$A_\lambda(L_2J_2J_3)=[(-)^{-J_2+J_3-1}\hat{J}_2(\hat{L}_2)^2][\langle\lambda 0|L_2L_2 1-1\rangle] \times[W(J_2J_2L_2L_2;\lambda J_3)], \quad (28)$$

whereas that for the cascade $J_2 \twoheadrightarrow J_3 \rightarrow J_4$ with $\Delta_2=\Delta_3=0$ is

$$U_\lambda(L_2J_2J_3)A_\lambda(L_3J_3J_4) =[(-)^{J_2+J_4-L_2-1}\hat{J}_2(\hat{J}_3)^2(\hat{L}_3)^2][\langle\lambda 0|L_3L_3 1-1\rangle] \times[W(J_2J_2J_3J_3;\lambda L_2)W(J_3J_3L_3L_3;\lambda J_4)]; \quad (29)$$

both these expressions reduce to the same value, namely

$$(-)^{J_2+L-1}\hat{J}_2(\hat{L})^2\langle\lambda 0|LL1-1\rangle W(J_2J_2LL;\lambda L),$$

under the above conditions. Another, rather trivially obvious instance of equality occurs when λ is restricted to the value $\lambda=0$, for then $U_0=A_0=1$ by definition (irrespective of multipole mixing). Such a situation occurs when J_2 and/or J_3 have the value 0 or $\frac{1}{2}$. The emission of γ radiation is then isotropic, so that the correlation loses its θ_2 and φ dependence and reduces essentially to an inelastic scattering *distribution* [in which case $d^2\sigma/d\Omega_1 d\Omega_2=(4\pi)^{-1}d\sigma/d\Omega_1$].

C. Reduction of a Double-Differential to a Differential Cross Section

As justification for expressing correlation results in an intermediate form involving Legendre polynomials and hyperpolynomials, it was stated earlier that not only do such expressions have the merit of being applicable to *any* value of φ, but that they are amenable to straightforward reduction to the angular distribution of emitted particles and of γ radiation. It follows from the relations[12]

$$\frac{d\sigma}{d\Omega_1}=4\pi\left.\frac{d^2\sigma}{d\Omega_1 d\Omega_2}\right|_{\lambda=0} \quad\text{and}\quad \frac{d\sigma}{d\Omega_2}=4\pi\left.\frac{d^2\sigma}{d\Omega_1 d\Omega_2}\right|_{\nu=0}, \quad (30)$$

that (i) the scattering distribution $d\sigma/d\Omega_1$ is obtained by multiplying correlations of the above form by 4π after setting $P_\nu(y)=P_\nu(w)=S_{\mu\nu\lambda}=0$ for $\mu=\nu\neq 0$ and $\lambda=0$; (ii) the γ distribution $d\sigma/d\Omega_2$ is obtained by multiplying correlations of the above form by 4π after setting $P_\lambda(x)=P_\lambda(w)=S_{\mu\nu\lambda}=0$ for $\mu=\lambda\neq 0$ and $\nu=0$. Herein for convenience the abbreviated notation

$$x\equiv\cos\theta_1,\quad y\equiv\cos\theta_2, \quad w\equiv\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2\cos\varphi \quad (31)$$

has been employed, where w stands for the cosine of the angle between the emitted particles and the γ radiation measured in coincidence. Another abbreviation to be used later may be introduced at this stage, namely

$$z\equiv\cos\theta_1\cos\theta_2\sin\theta_1\sin\theta_2\equiv xy[(1-x^2)(1-y^2)]^{1/2}. \quad (32)$$

From (31) and (32),

$$w=xy+(z/xy)\cos\varphi. \quad (33)$$

The total compound inelastic cross section is, of course,

$$\sigma=\tfrac{1}{2}\pi\lambda\!\!\!^{-2}\textstyle\sum_{J_1}(\hat{J}_1/\hat{J}_0)^2\tau. \quad (34)$$

D. Numerical Computation of Double-Differential and Differential CN Cross Sections

Although it is quite feasible to evaluate correlations numerically when expressed in Legendre polynomial and hyperpolynomial form, the computation is very much simpler and faster when the correlation is first reduced further by hand calculation to a form such as used in the present paper for the $\varphi=0°$ plane in terms of the entities x, y, z defined in Eqs. (31) and (32), e.g.,

$$d^2\sigma/d\Omega_1 d\Omega_2 =(E_1)^{-1}[x^4y^4(a_{440})+x^4y^2(a_{420})+x^2y^4(a_{240})+x^4(a_{400}) +y^4(a_{040})+x^2y^2(a_{220})+x^2(a_{200})+y^2(a_{020})+(a_{000}) +x^2y^2z(a_{221})+x^2z(a_{201})+y^2z(a_{021})+z(a_{001})], \quad (35)$$

where the a_{pqr} are an abbreviation for respective weighted sums of τ terms,

$$a_{pqr}=\textstyle\sum_i a_{pqr}{}^{(i)}\tau^{(i)}, \quad (36)$$

with p, q, r denoting the powers in the corresponding term $x^py^qz^r$ of the series. With E_1 in MeV, the expression (35)—which acquires additional terms when μ, ν, $\lambda>4$—can readily be coded to yield the double-differential cross section in mb sr^{-2} at predetermined intervals of emission angles θ_1, θ_2. The same program can also be employed for numerical calculation of the differential cross sections in mb sr^{-1}; in the expression for $d\sigma/d\Omega_1$ all coefficients except a_{400}, a_{200}, and a_{000} vanish, whereas in that for $d\sigma/d\Omega_2$, all except a_{040}, a_{020} and a_{000} vanish. The program can also evaluate the total cross section σ as given from Eq. (34); in the form corresponding to Eq. (35), only the a_{000} are nonzero. Clearly the respective nonvanishing coefficients take on unique values in each instance different from their correlation counterparts.

[40] Considerations of the angular correlation of radiations with parallel angular momenta by U. Fano, Nuovo Cimento **5**, 1358 (1957) and earlier references therein have some bearing upon the result discussed here and also cited by A. E. Litherland and A. J. Ferguson (Ref. 10). This condition is also implicit in the remark which D. M. Van Patter attributes (Ref. 32) to M. E. Rose concerning identity of the distributions of cascade γ radiations when $J_4=0$ and $J_3=L_2=L_2'=L_3=L_3'$.

The label (i) has been introduced in (36) to take account of the diversity of τ terms $[\tau^{(i)}]$ each weighted by a coefficient $a_{pqr}^{(i)}$ which differs for different pqr. This is brought out more clearly in the next section. The $\tau^{(i)}$ have, throughout, been evaluated from transmission coefficients for a Perey-Buck nonlocal optical potential,[41] as taken from a private tabulation kindly made available by the above authors and "meaned" to obtain T_l's which correspond with neglect of spin-orbit interaction.

3. CORRELATION FORMULAS FOR TARGET NUCLEI HAVING A 0+ GROUND-STATE SPIN

Nuclei having a ground-state spin $J_0\pi_0=0+$ represent the largest class among those target nuclei suitable for experimental correlation studies. Reference 12 deals with investigations in the range $24\leqslant A\leqslant 68$ carried out to date, for which analysis reveals CN correlation theory to be in good agreement with experiment at energies similar to those selected here and for target nuclei beyond $A\approx 40$.

A. Scattering to the First Level ($0+\rightarrow J_1\pi_1\rightarrow 2+\rightarrow 0+$ Sequence)

The experiments analyzed in Ref. 12 all involved inelastic nucleon scattering to the first level ($J_2\pi_2=2+$) of *e-e* target nuclei, followed by deexcitation γ radiation to the ground state ($J_3\pi_3=0+=J_0\pi_0$). Substitution of these values and $L_2=L_2'=2$ causes the correlation to assume the form (16) with $J_2=L_2=2$. Parity considerations require that (l_1+l_2) be even, and the further arbitrary momentum cutoff l_1, $l_2\leqslant 2$ restricts the number of pairs of values J_1, j_2 permitted by momentum selection rules to 11, each of which is linked with an associated $\tau^{(i)}$. In the present case, i runs from 1 to 5 [Eq. (43) of the present paper and Eq. (63) of Ref. 12] and μ, ν, λ are each confined to the values 0, 2, or 4 within the restrictions of triangle relations (essentially, $\mu\leqslant 2J_1$, $\nu\leqslant 2j_2$ and $|\mu-\nu|\leqslant\lambda\leqslant\mu+\nu$), which causes the summation to extend over 59 sets of J_1, j_2, μ, ν, λ combinations. The intermediate correlation formula so obtained is cited as Eq. (66) in Ref. 12, and the final formula in the desired form (35) as Eq. (67) of Ref. 12 for $\varphi=0°$ and Eq. (68) of Ref. 12 for $\varphi=90°$.

It may also be mentioned that calculations have been undertaken for the $0+\rightarrow J_1\pi_1\rightarrow 2+\rightarrow 0+$ correlation which go beyond the "two-channel approximation," though still restricted to orbital momenta l_1, $l_2\leqslant 2$. Numerical results have been evaluated in particular for Fe^{56} as target nucleus. In the case of inelastic *proton* scattering at a lab energy of 5.8 MeV, which exceeds the (p,n) threshold, the CN may decay by several channels, e.g., by proton emission to the ground state, first level, or higher levels of Fe^{56}, or by neutron emission to the 4+ ground state in Co^{56} (or to higher levels of unknown spin). It has been experimentally found that at 5.8 MeV, inelastic proton scattering occurs almost exclusively to the first level only, but that neutron emission to the 4+ state of Co^{56} could be appreciable; the correlation has accordingly been evaluated for this "three-channel approximation" and is cited in the Appendix to the paper of Gobbi *et al.*[42] Quantitatively, it was found that the correlation in the "three-channel approximation" was practically identical with that in the "two-channel approximation" in structure, but reduced in absolute magnitude by about 20% when transmission coefficients for a Perey proton potential[43] and a Perey-Buck neutron potential[41] were used. This finding was also observed in a series of unpublished calculations for n'-γ correlations when 3.2-MeV neutrons are incident on Fe^{56} whose levels have a spin sequence 0+, 2+, 4+, 2+, $\cdots$. The influence upon the double-differential cross section for scattering to the first level (2+) followed by γ decay to the ground state when neutron decay of the CN can also occur to the second (4+) and third (2+) levels of Fe^{56} has been found to be similar. In presence of the one additional open channel to the 4+ level, the cross section is reduced by 10%, as against a 23% reduction in presence of an extra decay channel to the upper 2+ state, and a 28% reduction for *both* these additional channels. In all instances, the *structure* of the correlation function in the $\varphi=0°$ plane plotted against θ_2 remains practically unaltered (e.g., the peak-to-valley ratio of the curves for $\theta_1=0°, 45°, 90°$ throughout remains at 2.0, 1.4, and 1.1, respectively).

[41] F. Perey and B. Buck, Nucl. Phys. **32**, 353 (1962).

B. Scattering to the Second Level

The spin of the second excited state of *e-e* nuclei has in practice been found to be 0+, 2+, 3− or 4+. Each of these possibilities is considered in the present section, which first treats γ decay involving an *observed* transition from the second to the first level and then goes on to consider γ cascades in which the transition from the second to the first level is *unobserved* but that from the first level to the ground state is *observed*. For clarity, the section is subdivided into separate portions for each spin sequence, arranged in increasing order of spins $J_2\pi_2$.

(i). $0+\rightarrow J_1\pi_1\rightarrow 0+\rightarrow 2+$ Sequence

The isotropy of γ decay from a 0+ state renders this correlation essentially a particle distribution, with $\lambda\leqslant 2J_2=0$. Summing Eq. (16) over 9 terms after setting $l_1=l_2\leqslant 2$, $J_1=j_2$, $J_2=0$, and $\mu=\nu$ yields

$$\frac{d^2\sigma}{d\Omega_1 d\Omega_2}=\frac{1}{4\pi}\frac{d\sigma}{d\Omega_1}=\frac{\lambda\!\!\!^{-2}}{32\pi}\{2\tau^{(1)}+\tau^{(2)}[6+4P_2(x)]+\tau^{(3)}\times[10+10.85714P_2(x)+5.14286P_4(x)]\},\quad(37)$$

[42] B. Gobbi, R. E. Pixley and E. Sheldon, Nucl. Phys. (to be published).

[43] F. G. Perey, in *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach Publishers, Inc., New York, 1963), p. 125.

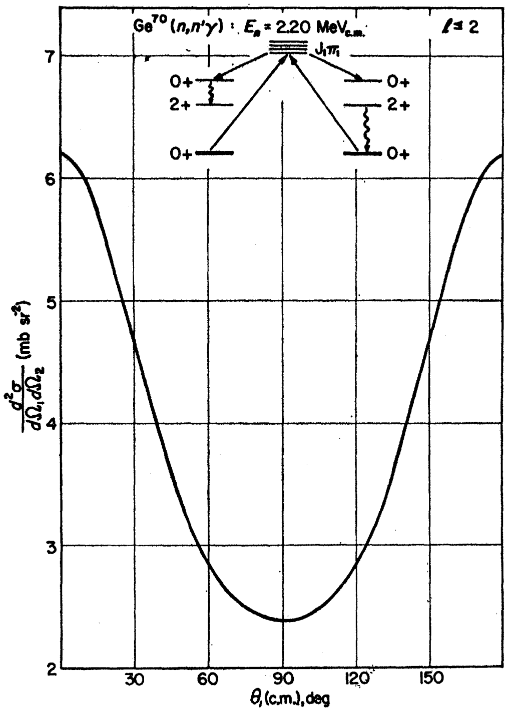

FIG. 1. Correlation function (isotropic in the γ-emission angle θ_2) for inelastic neutron scattering (described throughout by a Perey-Buck nonlocal optical potential, Ref. 41) at 2.20 MeV (c.m.) to the second excited state of Ge^{70}, illustrating the θ_1 dependence of the CN double-differential cross section for a $0+\to J_1\pi_1\to 0+\to 2+$ and a $0+\to J_1\pi_1\to 0+ \twoheadrightarrow 2+\to 0+$ transition sequence; essentially a particle scattering distribution (divided by 4π).

with

$$\tau^{(1)}\equiv\frac{T_0(E_1)T_0(E_2)}{T_0(E_1)+T_0(E_2)},\quad \tau^{(2)}\equiv\frac{T_1(E_1)T_1(E_2)}{T_1(E_1)+T_1(E_2)},$$
$$\tau^{(3)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_2(E_1)+T_2(E_2)}. \qquad (38)$$

In final form, employing the relation

$$\frac{\lambda\!\!\!^{-2}}{32\pi}=\frac{2.063009}{E_1}, \qquad (39)$$

for $\lambda\!\!\!^{-}$ in cm and E_1 in MeV, Eq. (37) may be rewritten as

$$\begin{aligned}d^2\sigma/d\Omega_1 d\Omega_2 &= (E_1)^{-1}\{x^4[46.41770\tau^{(3)}]\\ &+x^2[12.37805\tau^{(2)}-6.18903\tau^{(3)}]+[4.12602\tau^{(1)}\\ &+8.25204\tau^{(2)}+13.40956\tau^{(3)}]\}\ \text{mb sr}^{-2}. \qquad (40)\end{aligned}$$

Apart from Ar^{40}, the nuclei S^{32} or Ge^{70} would appear to be suitable as targets, the last named having the advantage of being a heavy nucleus which, even for low incident energies, would form a compound system of high level density. Its isotopic abundance is reasonable (20.5%) and a suitable neutron energy for population of second (but not higher) levels would be 2.2 MeV (c.m.), which would readily be obtainable from the *d*-*d* reaction. The correlation has accordingly been evaluated for $Ge^{70}(n,n'\gamma)$ at $E_n=2.2$ MeV (c.m.) and is shown in function of θ_1 in Fig. 1. This shows a pronounced dip at $\theta_1=90°$, the peak-to-valley ratio being large (2.6), and the cross section being reasonable in magnitude. By contrast, it may be mentioned that the corresponding correlation for neutrons going to the *first* level of Ge^{70} under the above conditions *peaks* toward $\theta_1=90°$; the maximum peak-to-valley ratio, occurring for $\theta_2=90°$, is 1.8 and the double-differential cross section rises from 2.85 mb sr^{-2} at $\theta_1=0°$ to 5.13 mb sr^{-2} at $\theta_1=90°$.

(*ii*). $0+\to J_1\pi_1\to 0+ \twoheadrightarrow 2+\to 0+$ *Sequence*

Since this case fulfils the condition $J_4=0$, $L_2=L_2'=L_3=L_3'$, it follows from the discussion of Sec. 2B that the double-differential cross section is again independent of θ_2 and φ, and is identical with that of (*i*) above, for which reason, Fig. 1 illustrates the correlation (essentially the distribution) for this case also.

Since $0+\to 0+$ γ transitions are strictly forbidden, no direct γ decay can occur from a level $J_2\pi_2=0+$ to the 0+ ground state, and accordingly no correlation expression exists for deexcitation of a 0+ level by γ decay to the ground state.

(*iii*). $0+\to J_1\pi_1\to 2+\to 2+$ *Sequence*

Unlike the two previous cases, the second level can here decay by γ emission either to the first excited state or direct to the ground state; in the latter instance the correlation expression is identical with that for a normal $0+\to J_1\pi_1\to 2+\to 0+$ sequence except insofar as the $\tau^{(i)}$ are changed numerically through new values of $T_l(E_2)$ for the different energy E_2. In the former instance, the γ radiation from the second to the first level can be of mixed multipolarity ($M1+E2$) and the correlation has to be evaluated afresh from Eq. (13). The result may conveniently be expressed in terms of $M_0^{(2)}$, $M_2^{(2)}$, and $M_4^{(2)}$ since λ can assume the values 0, 2, 4, with l_1+l_2 even, and l_1, $l_2\leqslant 2$. From the definition (7), it follows here that

$$\begin{aligned}M_0^{(2)}&=-1/\sqrt{5}=-0.447214;\\ M_2^{(2)}&=(1+\Delta_2^2)^{-1}(0.187083+0.547723\Delta_2-0.057270\Delta_2^2);\\ M_4^{(2)}&=0.136598\Delta_2^2/(1+\Delta_2^2).\end{aligned} \qquad (41)$$

Summation over 57 terms yields on substituting for $M_0^{(2)}$ from (41),

$$\begin{aligned} d^2\sigma/d\Omega_1 d\Omega_2 = (5\lambda^2/32\pi)\{ & \tau^{(1)}[0.8+1.603569M_2^{(2)}P_2(w)-0.478092M_4^{(2)}P_4(w)]+\tau^{(2)}[0.4+0.748333M_2^{(2)}P_2(w)] \\ & +\tau^{(3)}[1.6-0.48P_2(x)+1.496664M_2^{(2)}P_2(y)-0.319992M_2^{(2)}S_{222}] \\ & +\tau^{(4)}[2.0+4.062374M_2^{(2)}P_2(y)-1.434275M_4^{(2)}P_4(y)] \\ & +\tau^{(5)}[4.0-0.571428P_2(x)-0.514284P_4(x)-1.069043M_2^{(2)}P_2(y)+0.717140M_4^{(2)}P_4(y)-1.909014M_2^{(2)}P_2(w) \\ & +0.170748M_4^{(2)}P_4(w)-1.613992M_2^{(2)}S_{222}+0.500666M_2^{(2)}S_{242}-0.053175M_4^{(2)}S_{224}+0.008153M_4^{(2)}S_{244} \\ & +0.610190M_2^{(2)}S_{422}+0.122309M_4^{(2)}S_{424}-0.164093M_2^{(2)}S_{442}-0.280649M_4^{(2)}S_{444}]\}, \end{aligned} \tag{42}$$

with

$$\begin{aligned} &\tau^{(1)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_2)}, \quad \tau^{(2)}\equiv\frac{T_1(E_1)T_1(E_2)}{T_1(E_1)+T_1(E_2)}, \quad \tau^{(3)}\equiv\frac{T_1(E_1)T_1(E_2)}{T_1(E_1)+2T_1(E_2)}, \\ &\tau^{(4)}\equiv\frac{T_2(E_1)T_0(E_2)}{T_2(E_1)+T_0(E_2)+2T_2(E_2)}, \quad \tau^{(5)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_2(E_1)+T_0(E_2)+2T_2(E_2)}, \end{aligned} \tag{43}$$

a set of $\tau^{(i)}$ identical with that for a $0+ \to J_1\pi_1 \to 2+ \to 0+$ spin sequence. For the $\varphi=0°$ plane, Eq. (42) can be transformed into

$$\begin{aligned} d^2\sigma/d\Omega_1 d\Omega_2 = (E_1)^{-1}\{ & x^4y^4[-172.60374M_4^{(2)}\tau^{(1)}-52.39648M_4^{(2)}\tau^{(5)}] \\ & +x^4y^2[172.60374M_4^{(2)}\tau^{(1)}+\tau^{(5)}(177.22194M_2^{(2)}+13.42852M_4^{(2)})]+x^2y^4[172.60374M_4^{(2)}\tau^{(1)}-15.10272M_4^{(2)}\tau^{(5)}] \\ & +x^4[-21.57547M_4^{(2)}\tau^{(1)}+\tau^{(5)}(-23.20885-70.88893M_2^{(2)}-0.44020M_4^{(2)})] \\ & +y^4[-21.57547M_4^{(2)}\tau^{(1)}-64.72640M_4^{(2)}\tau^{(4)}+32.97941M_4^{(2)}\tau^{(5)}] \\ & +x^2y^2[\tau^{(1)}(49.62266M_2^{(2)}-178.76816M_4^{(2)})+23.15726M_2^{(2)}\tau^{(2)} \\ & +13.23237M_2^{(2)}\tau^{(3)}+\tau^{(5)}(-95.69976M_2^{(2)}+42.09425M_4^{(2)})] \\ & +x^2[\tau^{(1)}(-24.81133M_2^{(2)}+24.65768M_4^{(2)})-11.57863M_2^{(2)}\tau^{(2)} \\ & -7.42683\tau^{(3)}+\tau^{(5)}(11.05177+75.61514M_2^{(2)}-2.63308M_4^{(2)})] \\ & +y^2[\tau^{(1)}(-24.81133M_2^{(2)}+24.65768M_4^{(2)})-11.57863M_2^{(2)}\tau^{(2)}+23.15723M_2^{(2)}\tau^{(3)} \\ & +\tau^{(4)}(62.85535M_2^{(2)}+55.47977M_4^{(2)})+\tau^{(5)}(9.45196M_2^{(2)}-25.93438M_4^{(2)})] \\ & +[\tau^{(1)}(8.25204+16.54089M_2^{(2)}-4.93154M_4^{(2)})+\tau^{(2)}(4.12602+7.71909M_2^{(2)}) \\ & +\tau^{(3)}(18.97968-12.12987M_2^{(2)})+\tau^{(4)}(20.63009-20.95179M_2^{(2)}-5.54798M_4^{(2)}) \\ & +\tau^{(5)}(42.21801-35.05091M_2^{(2)}+0.11444M_4^{(2)})]+x^2y^2z[-172.60374M_4^{(2)}\tau^{(1)}-52.39648M_4^{(2)}\tau^{(5)}] \\ & +x^2z[86.30187M_4^{(2)}\tau^{(1)}+\tau^{(5)}(177.22194M_2^{(2)}-12.76997M_4^{(2)})+y^2z[86.30187M_4^{(2)}\tau^{(1)}-41.30121M_4^{(2)}\tau^{(5)}] \\ & +z[\tau^{(1)}(49.62266M_2^{(2)}-49.31535M_4^{(2)})+23.15726M_2^{(2)}\tau^{(2)} \\ & +13.23237M_2^{(2)}\tau^{(3)}+\tau^{(5)}(-113.42246M_2^{(2)}+52.92601M_4^{(2)})]\}\ \text{mb sr}^{-2}. \end{aligned} \tag{44}$$

To illustrate the angular dependence of this correlation, the nucleus Zn^{66} has been chosen as a representative example: since its energy states lie[34] at 0 MeV (0+), 1.04 MeV (2+), 1.87 MeV (2+), 2.37 MeV (0+?), etc., results have been evaluated for neutron scattering at 2.37 MeV, as this cannot lead to population of levels higher than the second excited state. The double-differential cross section has been computed numerically from Eq. (44) for $\theta_1=0°$, 45°, and 90° with $\varphi=0°$ and θ_2 ranging from 0° to 180° in steps of 5°, for coincidences between neutrons scattered to the second level and γ radiation going thence to the first level. The latter has been shown[44] to be of mixed multipolarity with mixing ratio around $\Delta_2=+3$. Figure 2 depicts correlation results computed for the above conditions in the "two-channel approximation" neglecting the influence upon the $\tau^{(i)}$ of neutrons going to the first level of Zn^{66}. The numerical value and sign of the mixing ratio can exercise a decisive influence upon the form of the correlation; subsidiary investigations which have been undertaken to examine the "sensitiveness" of the correlation yielded the following results. The expression for the double-differential cross section assuming $M1$ γ-multipolarity was derived from (41), (43), (44) by setting $\Delta_2=0$ and, as a check, from first principles [summing Eq. (2) over 43 terms with $\lambda=0$, 2 only]. The resultant correlation curves for $\theta_1=0°$, 45°, 90° in function of θ_2 were practically identical with those in Fig. 2, though of slightly larger amplitude, the biggest discrepancy occurring for $\theta_1=0°$, where the peak-to-valley ratio is 1.7 for $\Delta_2=0$ as against 1.4 for $\Delta_2=+3$; the absolute magnitudes were closely comparable. Similarly evaluated curves for S^{34} [$E_n=3.80$ MeV (c.m.)] and Se^{76} [$E_n=1.79$ MeV (c.m.)] with $\Delta_2=0$ were also the same in appearance. However, those ensuing for Zn^{66} at $E_n=2.37$ MeV (c.m.) when one artificially sets $\Delta_2=-3$ for comparison are radically different in character in that they climb to a maximum around $\theta_2\approx 90°$ and have peak-to-valley ratios of 3.1, 1.9, and 1.5, respectively, for $\theta_1=0°$, 45°, 90°.

[44] A. K. Sen Gupta and D. M. Van Patter, Phys. Letters 3, 355 (1963).

(*iv*). $0+ \rightarrow J_1\pi_1 \rightarrow 2+ \twoheadrightarrow 2+ \rightarrow 0+$ *Sequence*

A change in the appearance of correlation curves compared with those of Fig. 2 can also arise on considering the second excited state to decay by a γ cascade in which the first transition is unobserved and the second (pure $E2$ multipolarity to the ground state) is observed. A special case yielding identity in the correlation results when the γ radiation from the second to the first level has pure $E2$ character, for then the condition $J_4=0$, $L_2=L_2'=L_3=L_3'$ discussed toward the end of Sec. 2B is fulfilled and the $0+ \rightarrow J_1\pi_1 \rightarrow 2+ \twoheadrightarrow 2+ \rightarrow 0+$ correlation is identical with that for a $0+ \rightarrow J_1\pi_1 \rightarrow 2+ \rightarrow 2+$ transition sequence. If, however, the γ multipolarity in the $2+ \rightarrow 2+$ step is predominantly $M1$, marked differences are to be expected. Consequently, the calculations presented in this subsection have assumed that $\Delta_2=0$ (pure $M1$), which also greatly simplified their complexity. Assuming further that l_1, $l_2 \leqslant 2$, one finds on summing Eq. (24) over 59 terms with $\lambda=0$, 2 that

$$\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2 =& (25\lambda\!\!\!^{-2}/32\pi)\{\tau^{(1)}[0.16+0.042857P_2(w)+0.030476P_4(w)]+\tau^{(2)}[0.08+0.02P_2(w)] \\
&+\tau^{(3)}[0.32-0.096P_2(x)+0.04P_2(y)-0.008552S_{222}+0.020399S_{224}]+\tau^{(4)}[0.4+0.108571P_2(y)+0.091429P_4(y)] \\
&+\tau^{(5)}[0.8-0.114287P_2(x)-0.102857P_4(x)-0.028571P_2(y)-0.045714P_4(y)-0.051020P_2(w)-0.010885P_4(w) \\
&-0.043136S_{222}+0.023789S_{224}+0.013381S_{242}-0.000520S_{244} \\
&+0.016308S_{422}-0.007797S_{424}-0.004386S_{442}+0.017890S_{444}]\}, \qquad (45)
\end{aligned}$$

with $\tau^{(i)}$ as given by Eq. (43). For the $\varphi=0°$ plane, Eq. (45) can be transformed into

$$\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2 =& (E_1)^{-1}\{x^4y^4[55.0136\tau^{(1)}+16.7003\tau^{(5)}]+x^4y^2[-55.0136\tau^{(1)}+19.4027\tau^{(5)}] \\
&+x^2y^4[-55.0136\tau^{(1)}+33.0089\tau^{(3)}+37.8233\tau^{(5)}]+x^4[6.8767\tau^{(1)}-32.5415\tau^{(5)}] \\
&+y^4[6.8767\tau^{(1)}-16.5041\tau^{(3)}+20.6301\tau^{(4)}-27.0161\tau^{(5)}]+x^2y^2[63.6091\tau^{(1)}+3.0945\tau^{(2)}-28.8828\tau^{(3)}-56.8570\tau^{(5)}] \\
&+x^2[-11.1746\tau^{(1)}-1.5473\tau^{(2)}-4.1260\tau^{(3)}+25.2963\tau^{(5)}] \\
&+y^2[-11.1746\tau^{(1)}-1.5473\tau^{(2)}+19.5986\tau^{(3)}-9.2835\tau^{(4)}+26.0337\tau^{(5)}] \\
&+[12.0342\tau^{(1)}+5.1575\tau^{(2)}+37.7531\tau^{(3)}+19.5986\tau^{(4)}+35.6115\tau^{(5)}] \\
&+x^2y^2z[55.0136\tau^{(1)}+16.7003\tau^{(5)}]+x^2z[-27.5068\tau^{(1)}+27.7529\tau^{(5)}] \\
&+y^2z[-27.5068\tau^{(1)}+33.0089\tau^{(3)}+46.1735\tau^{(5)}] \\
&+z[22.3493\tau^{(1)}+3.0945\tau^{(2)}-12.3781\tau^{(3)}-46.1728\tau^{(5)}]\}\ \text{mb sr}^{-2}. \qquad (46)
\end{aligned}$$

The θ_2 dependence of this expression for $\theta_1=0°$, 45°, and 90° in the $\varphi=0°$ plane is illustrated in Fig. 3 for the $Zn^{66}(n,n'\gamma)$ reaction at $E_n=2.37$ MeV (c.m.) (i.e., with the same $\tau^{(i)}$ as were used in the calculations upon which

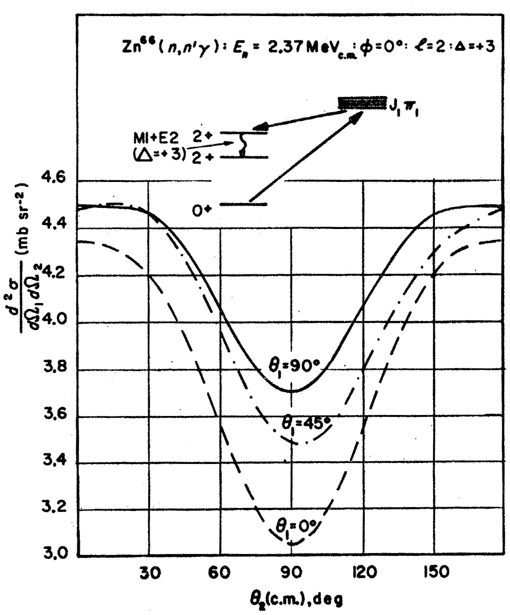

FIG. 2. θ_2 dependence of the CN double-differential cross section for a $0+ \rightarrow J_1\pi_1 \rightarrow 2+ \rightarrow 2+$ transition sequence and for $\theta_1=0°$, 45°, 90° ($\varphi=0°$), illustrated by the $Zn^{66}(n,n'\gamma)$ reaction at 2.37 MeV (c.m.) with a γ multipole mixing ratio $\Delta_2=+3$.

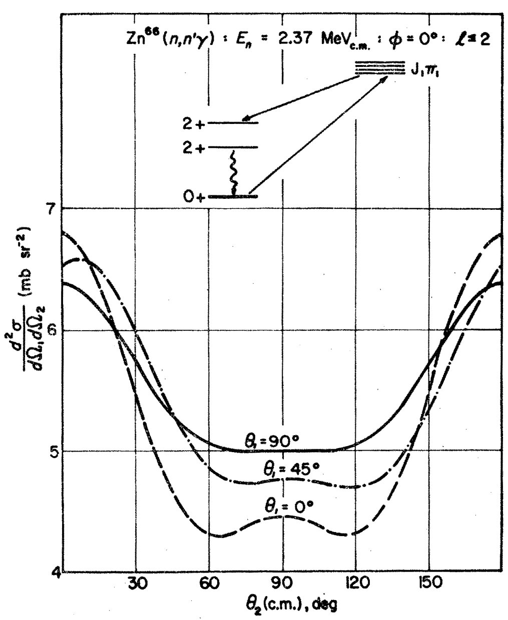

FIG. 3. Effect upon the correlation function depicted in Fig. 2 of observing the second rather than the first γ transition of the cascade from the second level of Zn^{66}. For simplicity, the unobserved γ radiation has been treated as if pure $M1$ (with $\Delta_2=0$).

Fig. 2 is based, but with $\Delta_2=0$). Comparison of Fig. 3 with Fig. 2 not only shows the cross section now to be appreciably larger (it will be recalled that setting Δ_2 to zero had practically no influence upon the magnitude or shape of the curves depicted in Fig. 2 for $\Delta_2=3$), but in addition to have altogether different structure around $\theta_2=90°$, whose form is rather striking.

A (partial) test of the correctness of Eqs. (45) and (42) lies in integrating these to obtain either particle scattering distributions which should be identical with each other and with the expression deduced from first principles, or γ distributions which in each case could be compared with the formulas quoted by Van Patter.[32] These tests, together with an independent check[31] of the full calculations, consistently substantiated the reliability of the present results and indicated the difference in correlation behavior (when the γ radiation is predominantly or purely $M1$) to be a genuine effect in the present instance.

In the case of γ decay occurring from the second (2+) level direct to the ground state, the correlation is the same analytically as for the $0+\rightarrow J_1\pi_1\rightarrow 2+\rightarrow 0+$ sequence evaluated[12] for scattering to the first (2+) level followed by γ decay to the ground state; it is necessary only to insert new values of the transmission coefficients $T_l(E_2)$ into the requisite τ terms. As an example of the respective magnitudes of the double-differential cross section when scattering occurs to the *second* (2+) level rather than to the first (2+), one finds for the reaction $Zn^{66}(n,n'\gamma)$ at $E_n=2.37$ MeV (c.m.) for $\theta_1=\theta_2=90°$, $\varphi=0°$ that $d^2\sigma/d\Omega_1d\Omega_2$ is 3.271 mb sr^{-2} for neutrons going to the second level (at 1.87 MeV) as against 5.233 mb sr^{-2} for neutrons going to the first level (at 1.04 MeV), the amplitude and structure being identical in both cases.

(v). $0+\rightarrow J_1\pi_1\rightarrow 3-\rightarrow 2+$ *Sequence*

The feature just noted renders it interesting to compare correlation behavior without and with an unobserved intermediate γ decay step when nucleon scattering takes place to a 3− second excited level. In the absence of unobserved radiation and taking the multipolarity of the γ transition to the first level to be pure $E1$, the summation of Eq. (13) involves 26 terms (for l_1, $l_2\leqslant 2$ and l_1+l_2 odd, since the parities of ground and second excited states differ) with $\lambda=0$, 2 and yields

$$\begin{aligned}d^2\sigma/d\Omega_1d\Omega_2=&(21\lambda\!\!\!^{-2}/32\pi)\{\tau^{(1)}[+0.095238-0.032653P_2(w)]\\&+\tau^{(2)}[+0.380952-0.081633P_2(x)-0.034286P_2(y)-0.081632P_2(w)+0.005236S_{222}+0.006635S_{242}]\\&+\tau^{(3)}[+0.190476+0.038095P_2(x)-0.045714P_2(y)-0.045714P_2(w)-0.004887S_{222}]\\&+\tau^{(4)}[+0.571428-0.179592P_2(x)-0.151836P_2(y)+0.017143P_2(w)+0.015185S_{222}+0.009952S_{422}]\},\end{aligned}\tag{47}$$

with

$$\tau^{(1)}\equiv\frac{T_1(E_1)T_2(E_2)}{T_1(E_1)+T_2(E_2)},\quad\tau^{(2)}\equiv\frac{T_1(E_1)T_2(E_2)}{T_1(E_1)+2T_2(E_2)},\quad\tau^{(3)}\equiv\frac{T_2(E_1)T_1(E_2)}{T_2(E_1)+T_1(E_2)},\quad\tau^{(4)}\equiv\frac{T_2(E_1)T_1(E_2)}{T_2(E_1)+2T_1(E_2)}.\tag{48}$$

Equation (47) can, in the $\varphi=0°$ plane, be written as

$$\begin{aligned}d^2\sigma/d\Omega_1d\Omega_2=&(E_1)^{-1}\{x^4y^2[+9.0186\tau^{(2)}]+x^4[-4.5093\tau^{(2)}]\\&+x^2y^2[-4.2439\tau^{(1)}-19.8935\tau^{(2)}-5.0926\tau^{(3)}+3.2625\tau^{(4)}]+x^2[+2.1220\tau^{(1)}+4.5093\tau^{(2)}+5.4463\tau^{(3)}-14.1375\tau^{(4)}]\\&+y^2[+2.1220\tau^{(1)}+3.9786\tau^{(2)}-12.3338\tau^{(4)}]+[+2.7114\tau^{(1)}+15.2663\tau^{(2)}+6.1536\tau^{(3)}+34.1369\tau^{(4)}]\\&+x^2z[+9.0186\tau^{(2)}]+z[-4.2439\tau^{(1)}-15.3842\tau^{(2)}-5.0926\tau^{(3)}-3.5012\tau^{(4)}]\}\text{ mb sr}^{-2},\end{aligned}\tag{49}$$

a correlation which has been illustrated for the $Ni^{64}(n,n'\gamma)$ reaction at $E_n=4.40$ MeV (c.m.) in Fig. 4. This is noteworthy in that all three correlation curves rise to a *maximum* around $\theta_2=90°$ (that for $\theta_1=45°$ peaks at $\theta_2=118°$), a hitherto unobserved feature in CN correlation behavior. The $\theta_1=45°$ curve in particular highlights the absence of symmetry about $\theta_2=90°$.

In connection with the cutoff l_1, $l_2\leqslant 2$ employed in deriving Eqs. (47) and (49) it may be pointed out that a subsidiary calculation has been performed in which not only S, P, and D waves were considered, but also the additional incident and outgoing pair of waves with $l_1=3$, $l_2=0$ to ascertain whether the presence of an F wave in the incident channel radically influences the correlation. Inclusion of this extra pair of waves simply involved an additional 4 terms in the summation, with $\lambda=0$, 2 and $\tau^{(5)}\equiv T_3(E_1)\cdot T_0(E_2)/[T_3(E_1)+T_0(E_2)]$. This led to a term $+\tau^{(5)}[+0.666667-0.234014P_2(y)]$ as an appendage to Eq. (47) and consequently a term $(-15.207332y^2+33.951240)\tau^{(5)}$ as an appendage to Eq. (49).

The influence upon the $Ni^{64}(n,n'\gamma)$, $E_n=4.40$ MeV (c.m.) correlation of these additional terms is to effect an appreciable increase in the cross section (which now ranges from around 2 to about 4 mb sr^{-2} as θ_2 goes from 0° to 90°) and a slight increase in the amplitude (the peak-to-valley ratio for the $\theta_1=0°$ curve remains unchanged at 1.8 but that for the $\theta_1=90°$ curve becomes 1.5, as against 1.2 when l_1, $l_2\leqslant 2$). This influence should therefore become

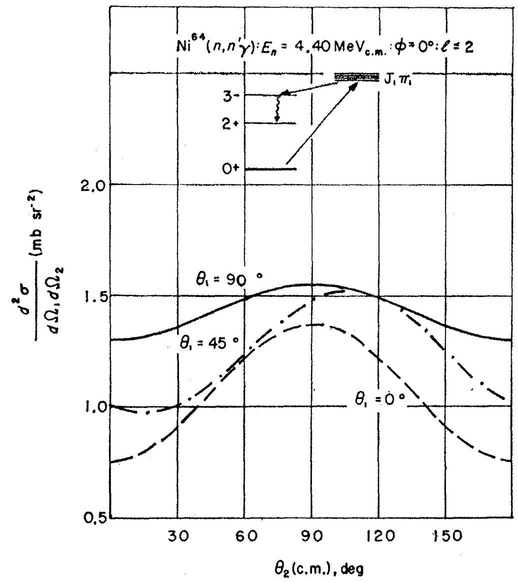

FIG. 4. Peaking of the correlation in function of the γ-emission angle θ_2 around $\theta_2=90°$, illustrated for a $0+\rightarrow J_1\pi_1\rightarrow 3-\rightarrow 2+$ transition sequence in the case of inelastic scattering of 4.4-MeV neutrons to the second level of Ni^{64} followed by an observed pure $E1$ γ transition to the first level.

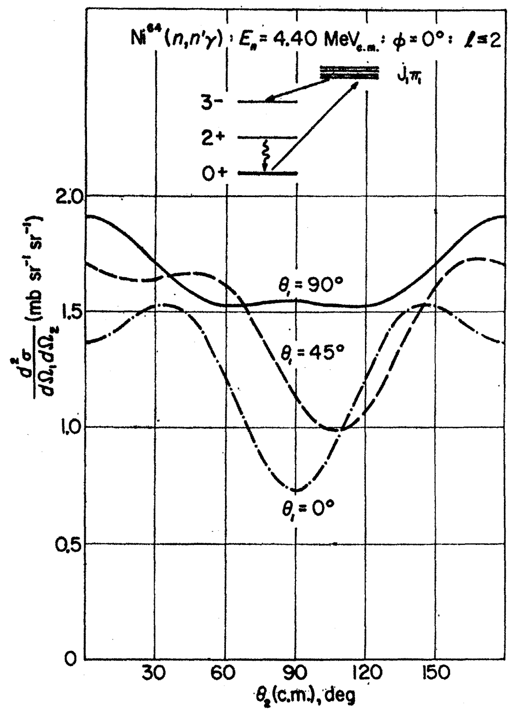

FIG. 5. As Fig. 4, but for an *unobserved* $E1$ γ transition followed by observed $E2$ γ decay from the first level to the ground state.

perceptible when *absolute* measurements are carried out, but represents too slight a change in structure to become perceptible for relative coincidence measurements (in arbitrary units).

(vi). $0+\rightarrow J_1\pi_1\rightarrow 3-\twoheadrightarrow 2+\rightarrow 0+$ Sequence

The fact that in this case L_2 and L_3 are both pure but not the same, suggests that the correlation will differ from that evaluated in the previous subsection. The summation here is also slightly more extensive in that 36 terms are involved since λ may now assume the values 0, 2, *and* 4. The $\tau^{(i)}$ are of course unchanged from those defined in Eq. (48), but the correlation becomes

$$\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2 = {} & (35\lambda^2/32\pi)\{\tau^{(1)}[0.057143+0.027988P_2(w)-0.017104P_4(w)] \\
&+\tau^{(2)}[0.228572-0.048980P_2(x)+0.029388P_2(y)+0.069971P_2(w) \\
&+0.017104P_4(w)-0.004488S_{222}-0.016354S_{224}-0.005687S_{242}+0.006497S_{244}] \\
&+\tau^{(3)}[0.114286+0.022857P_2(x)+0.039184P_2(y)+0.039184P_2(w)+0.004189S_{222}-0.011448S_{224}] \\
&+\tau^{(4)}[0.342858-0.107755P_2(x)+0.130146P_2(y)-0.025656P_4(y) \\
&-0.014694P_2(w)-0.013015S_{222}-0.008531S_{422}-0.007359S_{224}+0.009746S_{424}]\}, \qquad (50)
\end{aligned}$$

and in the $\varphi=0°$ plane reduces to

$$\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2 = {} & (E_1)^{-1}\{x^4y^4[-43.2250\tau^{(1)}+3.9312\tau^{(2)}]+x^4y^2[43.2250\tau^{(1)}-21.0247\tau^{(2)}] \\
&+x^2y^4[43.2250\tau^{(1)}-45.1900\tau^{(2)}-25.9346\tau^{(3)}-22.9866\tau^{(4)}]+x^4[-5.4031\tau^{(1)}+9.0382\tau^{(2)}] \\
&+y^4[-5.4031\tau^{(1)}+21.1208\tau^{(2)}+12.9673\tau^{(3)}-3.9794\tau^{(4)}] \\
&+x^2y^2[-38.7061\tau^{(1)}+77.1176\tau^{(2)}+31.3575\tau^{(3)}+13.5258\tau^{(4)}]+x^2[3.1437\tau^{(1)}-27.8016\tau^{(2)}-4.3618\tau^{(3)}-9.7416\tau^{(4)}] \\
&+y^2[3.1437\tau^{(1)}-28.9799\tau^{(2)}-12.9673\tau^{(3)}+22.5466\tau^{(4)}]+[4.9119\tau^{(1)}+26.7209\tau^{(2)}+10.7276\tau^{(3)}+20.1586\tau^{(4)}] \\
&+x^2y^2z[-43.2250\tau^{(1)}+3.9312\tau^{(2)}]+x^2z[21.6125\tau^{(1)}-19.0591\tau^{(2)}] \\
&+y^2z[21.6125\tau^{(1)}-43.2244\tau^{(2)}-25.9346\tau^{(3)}-22.9866\tau^{(4)}] \\
&+z[-6.2873\tau^{(1)}+44.9930\tau^{(2)}+18.3901\tau^{(3)}+14.8533\tau^{(4)}]\}\ \text{mb sr}^{-2}. \qquad (51)
\end{aligned}$$

Again, the scattering cross section $d\sigma/d\Omega_1$ is the same when derived from Eq. (50) or from (47) by integrating over the γ-emission angle. The respective γ distributions $d\sigma/d\Omega_2$ obtained by integrating over Ω_1 agree with those cited by Van Patter.[32]

The drastic alteration in the θ_2 dependence of the correlation as compared with Fig. 4 is shown by Fig. 5, which was derived using the same $\tau^{(i)}$ for the $Ni^{64}(n,n'\gamma)$ reaction at $E_n=4.40$ MeV (c.m.) with $l_1, l_2\leqslant 2$ as were employed for Fig. 4, but now illustrates the correlation when an unobserved γ transition intervenes. The absolute magnitude of the cross section remains rather small; inclusion of higher partial waves appears slightly to augment the magnitude without appreciably altering the structure.

(vii). 0+ → $J_1\pi_1$ → 3− → 0+ Sequence

When the observed γ transition is that from a level $J_2\pi_2=3-$ direct to the 0+ ground state, the correlation is derived theoretically by summing Eq. (13) over 38 terms with $\lambda=0$, 2, 4, 6 and $l_1, l_2\leqslant 2$ (such that l_1+l_2 is odd) to obtain

$$\begin{aligned}d^2\sigma/d\Omega_1d\Omega_2=&(7\lambda\!\!\!^{-2}/32\pi)\{\tau^{(1)}[+0.285714+0.244898P_2(w)+0.040816P_4(w)]\\&+\tau^{(2)}[+1.142858-0.244900P_2(x)+0.257143P_2(y)+0.612245P_2(w)-0.040816P_4(w)\\&-0.039271S_{222}+0.039027S_{224}-0.049761S_{242}-0.015505S_{244}-0.092299S_{246}]\\&+\tau^{(3)}[+0.571428+0.114285P_2(x)+0.342857P_2(y)+0.342857P_2(w)+0.036653S_{222}+0.027319S_{224}]\\&+\tau^{(4)}[+1.714286-0.538777P_2(x)+1.138775P_2(y)+0.061224P_4(y)-0.128570P_2(w)\\&-0.113887S_{222}+0.017563S_{224}-0.074641S_{422}-0.023257S_{424}-0.138449S_{426}]\},\end{aligned}\tag{52}$$

with the $\tau^{(i)}$ as defined in Eq. (48).

The two hyperpolynomials S_{246} and S_{426} in the above expression have not hitherto been published; in the $\varphi=0°$ plane, their respective values are

$$\begin{aligned}S_{246}=&+522.368275x^4y^6-522.368275x^2y^6+65.296032y^6-759.808401x^4y^4+271.077475x^4y^2+771.680404x^2y^4\\&-13.850674x^4-100.912047y^4-280.970812x^2y^2+14.698674x^2+39.573351y^2-2.261334+522.368275x^2y^4z\\&-261.184095y^4z-498.624172x^2y^2z+87.061365x^2z+261.184100y^2z-47.488019z,\end{aligned}\tag{53}$$

$$\begin{aligned}S_{426}=&+130.592069x^2y^6-65.296036y^6-184.016097x^2y^4+94.976051y^4+63.317366x^2y^2\\&-3.109335x^2-33.637351y^2+1.696001+130.592073y^4z-118.720067y^2z+19.786678z.\end{aligned}\tag{54}$$

Equations (53) and (54) can be substituted in Eq. (52) and the latter reduced to a polynomial for the $\varphi=0°$ plane:

$$\begin{aligned}d^2\sigma/d\Omega_1d\Omega_2=&(E_1)^{-1}\{x^4y^6[-696.2624\tau^{(2)}]+x^2y^6[+696.2624\tau^{(2)}-261.0990\tau^{(4)}]+y^6[-87.0328\tau^{(2)}+130.5495\tau^{(4)}]\\&+x^4y^4[+20.6299\tau^{(1)}+1010.8702\tau^{(2)}]+x^4y^2[-20.6299\tau^{(1)}-379.9794\tau^{(2)}]\\&+x^2y^4[-20.6299\tau^{(1)}-1007.0024\tau^{(2)}+12.3778\tau^{(3)}+378.8838\tau^{(4)}]+x^4[+2.5787\tau^{(1)}+28.4954\tau^{(2)}]\\&+y^4[+2.5787\tau^{(1)}+124.4248\tau^{(2)}-6.1889\tau^{(3)}-187.9914\tau^{(4)}]\\&+x^2y^2[+31.9765\tau^{(1)}+400.9965\tau^{(2)}+1.2380\tau^{(3)}-143.4305\tau^{(4)}]\\&+x^2[-8.2520\tau^{(1)}-45.3863\tau^{(2)}-3.7134\tau^{(3)}+1.5085\tau^{(4)}]+y^2[-8.2520\tau^{(1)}-51.5752\tau^{(2)}+6.1889\tau^{(3)}+95.7365\tau^{(4)}]\\&+[+8.2520\tau^{(1)}+26.8192\tau^{(2)}+9.9024\tau^{(3)}+11.8443\tau^{(4)}]+x^2y^4z[-696.2624\tau^{(2)}]\\&+y^4z[+348.1312\tau^{(2)}-261.0990\tau^{(4)}]+x^2y^2z[+20.6299\tau^{(1)}+662.7389\tau^{(2)}]+x^2z[-10.3150\tau^{(1)}-135.6427\tau^{(2)}]\\&+y^2z[-10.3150\tau^{(1)}-327.5016\tau^{(2)}+12.3778\tau^{(3)}+248.3343\tau^{(4)}]\\&+z[+16.5040\tau^{(1)}+90.7724\tau^{(2)}+7.4269\tau^{(3)}-35.5094\tau^{(4)}]\}\ \text{mb sr}^{-2}.\end{aligned}\tag{55}$$

The correlation curves in function of θ_2 as given by Eq. (55) for inelastic scattering of 4.40-MeV neutrons to the second level (3−) of Ni^{64} in coincidence with 1.34-MeV de-excitation γ radiation are depicted in Fig. 6, which may be compared with Figs. 4 and 5.

(viii). 0+ → $J_1\pi_1$ → 4+ → 2+ Sequence

An otherwise lengthy summation in Eq. (13) can be confined to but 33 terms on restricting the orbital momenta to $l_1, l_2\leqslant 2$ with l_1+l_2 even, and $\lambda=0$, 2, 4. In the present case, the additional operation of momentum selection rules curbs the incident and outgoing radiation to D waves only, associated with $\frac{3}{2}+$ and $\frac{5}{2}+$ levels in the CN (it is thus possible that incorporation of higher orbital momenta than $l=2$ might appreciably affect the correlation). Thence, with

$$\tau^{(1)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_2(E_1)+T_2(E_2)},\qquad\tau^{(2)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_2(E_1)+2T_2(E_2)},\tag{56}$$

the correlation ensues as

$$d^2\sigma/d\Omega_1 d\Omega_2 = (45\lambda\!\!\!^{-2}/32\pi)\{\tau^{(1)}[+0.089+0.025397P_2(x)+0.024943P_2(y)+0.035633P_2(w)-0.012353P_4(w)$$
$$+0.003809S_{222}-0.005906S_{224}+0.000852S_{242}-0.000939S_{244}]$$
$$+\tau^{(2)}[+0.266667-0.038095P_2(x)-0.019047P_4(x)+0.074830P_2(y)+0.058795P_2(w)+0.018529P_4(w)$$
$$-0.000816S_{222}-0.016452S_{224}-0.006571S_{242}+0.004424S_{244}$$
$$-0.005293S_{422}+0.003016S_{424}-0.001856S_{442}+0.001775S_{444}]\}, \quad (57)$$

which, for the $\varphi=0°$ plane can be expressed as

$$d^2\sigma/d\Omega_1 d\Omega_2 = (E_1)^{-1}\{x^4y^4[-32.8362\tau^{(1)}+31.8946\tau^{(2)}]+x^4y^2[+36.1001\tau^{(1)}-50.9983\tau^{(2)}]$$
$$+x^2y^4[+16.4162\tau^{(1)}-83.8376\tau^{(2)}]+x^4[-5.7365\tau^{(1)}+8.3679\tau^{(2)}]+y^4[+4.1046\tau^{(1)}+27.7881\tau^{(2)}]$$
$$+x^2y^2[-13.7331\tau^{(1)}+112.9671\tau^{(2)}]+x^2[+3.5684\tau^{(1)}-29.0348\tau^{(2)}]+y^2[-4.3685\tau^{(1)}-27.3935\tau^{(2)}]$$
$$+[+9.2515\tau^{(1)}+33.4240\tau^{(2)}]+x^2y^2z[-32.8362\tau^{(1)}+31.8946\tau^{(2)}]+x^2z[+19.6820\tau^{(1)}+35.0510\tau^{(2)}]$$
$$+y^2z[-67.8903\tau^{(2)}]+z[+4.3159\tau^{(1)}+60.6268\tau^{(2)}]\}\ \text{mb sr}^{-2}. \quad (58)$$

In Fig. 7, this is illustrated in function of θ_2 for $\theta_1=0°$, 45°, and 90° using $\tau^{(i)}$ for the $Fe^{56}(n,n'\gamma)$ reaction at $E_n=2.60$ MeV (c.m.). The θ_2 dependence is again rather novel, particularly when $\theta_1=90°$, but the somewhat low double-differential cross section may make this transition sequence difficult to study experimentally. Calculations for an alternative possible target nucleus, Ti^{46} at $E_n=2.80$ MeV (c.m.), yielded similar structure and magnitude for the correlation curves.

Integration of Eq. (57) over Ω_1 yields a γ distribution $d\sigma/d\Omega_2$ which agrees perfectly with Van Patter's expression[32] and thereby confirms his emendation of an incorrect value in the formula published by Hosoe and Suzuki.[45]

(ix). $0+\rightarrow J_1\pi_1\rightarrow 4+ \twoheadrightarrow 2+\rightarrow 0+$ Sequence

This correlation tallies identically with that above, since $L_2=L_2'=L_3=L_3'$, and $J_4=0$. Figure 7 accordingly depicts results for the spin sequences of both subsections *(viii) and (ix)*.

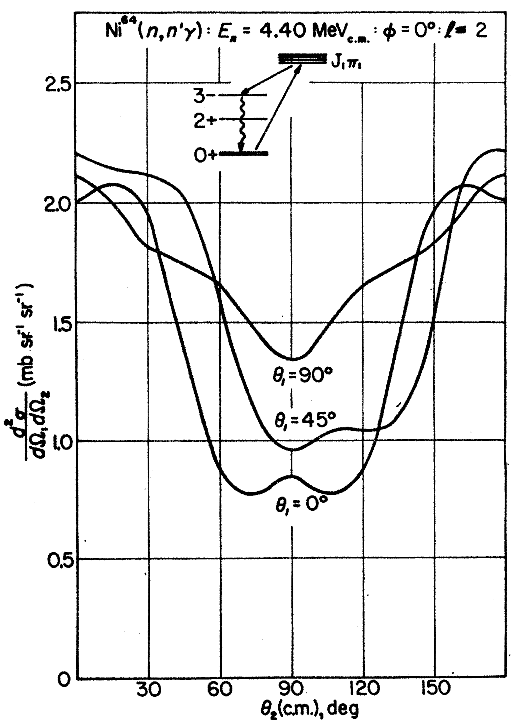

FIG. 6. As Figs. 4 and 5, but for direct crossover $E3$ γ radiation from the second level to the ground state.

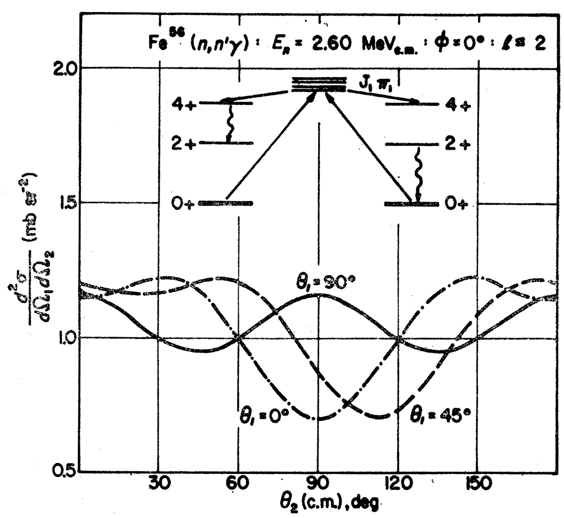

FIG. 7. Identity of the double-differential cross sections for $0+\rightarrow J_1\pi_1\rightarrow 4+\rightarrow 2+$ and $0+\rightarrow J_1\pi_1\rightarrow 4+\twoheadrightarrow 2+\rightarrow 0+$ transition sequences, illustrated by the $Fe^{56}(n,n'\gamma)$ reaction at $E_n=2.60$ MeV (c.m.).

[45] M. Hosoe and S. Suzuki, J. Phys. Soc. Japan 14, 699 (1959).

As for scattering to a state $J_2\pi_2=4+$ followed by direct γ decay to the $0+$ ground state, the high multipole order ($E4$) indicates the γ transition probability to be so low as to preclude its application for correlation studies. The results of unpublished calculations [which, when applied to the $Fe^{56}(n,n'\gamma)$ reaction at $E_n=2.60$ MeV (c.m.) yield correlation curves rather similar to those shown in Fig. 6] are therefore omitted from the present account.

4. CORRELATION FORMULAS FOR TARGET NUCLEI HAVING NONZERO GROUND-STATE SPIN

The present section collates theoretical correlation expressions for some transition sequences which are most likely to be conducive to experimental investigation with target nuclei in the range $29\leqslant A\lesssim 100$. The evaluations accordingly cover nuclei with $J_0\pi_0=\frac{1}{2}\pm$, $1+$, $\frac{3}{2}\pm$, $\frac{5}{2}+$, $\frac{7}{2}-$, and are illustrated graphically for neutron scattering (assuming l_1, $l_2\leqslant 2$) upon representative targets.

A. Scattering to the First Level

(i). $\frac{1}{2}+\rightarrow J_1\pi_1\rightarrow\frac{3}{2}+\rightarrow\frac{1}{2}+$ *Sequence*

Provision for mixed multipolarity ($M1+E2$) in the deexcitation γ radiation was made by expressing the correlation in terms of the quantities, $M_\lambda^{(2)}$ defined in Eq. (7). Since the ground-state spin is nonzero, the general correlation formula (2) has to be employed instead of (13) as heretofore. The summation in the present instance extends over 84 terms with $\lambda=0$, 2, and l_1, $l_2\leqslant 2$ (such that l_1+l_2 is even) to yield the result

$$\begin{aligned}
d^2\sigma/d\Omega_1d\Omega_2= (2\lambda\!\!\!^{-2}/32\pi)\{&\tau^{(1)}[+0.5M_0^{(2)}-0.5M_2^{(2)}P_2(w)]+\tau^{(2)}[+1.5M_0^{(2)}]\\
&+\tau^{(3)}[+3.0M_0^{(2)}-1.5M_2^{(2)}P_2(w)]+\tau^{(4)}[+0.5M_0^{(2)}-0.5M_2^{(2)}P_2(w)]\\
&+\tau^{(5)}[+6.0M_0^{(2)}-0.6M_0^{(2)}P_2(x)-0.15M_2^{(2)}P_2(y)-0.6M_2^{(2)}P_2(w)+0.2245M_2^{(2)}S_{222}]\\
&+\tau^{(6)}[+5.0M_0^{(2)}-1.75M_2^{(2)}P_2(y)+1.5M_2^{(2)}P_2(w)+0.37417M_2^{(2)}S_{222}]+\tau^{(7)}[+1.5M_0^{(2)}-0.75M_2^{(2)}P_2(y)]\\
&+\tau^{(8)}[+3.0M_0^{(2)}+0.45M_2^{(2)}P_2(y)-1.5M_2^{(2)}P_2(w)+0.16036M_2^{(2)}S_{222}-0.21514M_2^{(2)}S_{242}]\\
&+\tau^{(9)}[+5.0M_0^{(2)}-3.75M_2^{(2)}P_2(y)]\\
&+\tau^{(10)}[+10.0M_0^{(2)}+1.53062M_0^{(2)}P_2(x)-0.81633M_0^{(2)}P_4(x)+2.67857M_2^{(2)}P_2(y)+3.57143M_2^{(2)}P_2(w)\\
&+1.55448M_2^{(2)}S_{222}-0.54883M_2^{(2)}S_{242}-0.12193M_2^{(2)}S_{422}+0.17055M_2^{(2)}S_{442}]\\
&+\tau^{(11)}[+7.0M_0^{(2)}+4.28571M_0^{(2)}P_2(x)-0.78571M_0^{(2)}P_4(x)-1.8M_2^{(2)}P_2(y)+1.5M_2^{(2)}P_2(w)\\
&+0.27490M_2^{(2)}S_{222}+0.34832M_2^{(2)}S_{242}-0.75129M_2^{(2)}S_{422}+0.29847M_2^{(2)}S_{442}]\}\,. \qquad (59)
\end{aligned}$$

The M terms here assume the values

$$M_0^{(2)}=0.5\,,\quad M_2^{(2)}=(1+\Delta_2^2)^{-1}(0.25-0.86603\Delta_2-0.25\Delta_2^2)\,, \qquad (60)$$

and the τ terms are defined as

$$\begin{aligned}
&\tau^{(1)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+T_2(E_2)},\quad \tau^{(2)}\equiv\frac{T_0(E_1)T_0(E_2)}{T_0(E_1)+T_2(E_1)+T_0(E_2)+2T_2(E_2)},\quad \tau^{(3)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+T_2(E_1)+T_0(E_2)+2T_2(E_2)},\\
&\tau^{(4)}\equiv\frac{T_1(E_1)T_1(E_2)}{T_1(E_1)+T_1(E_2)},\quad \tau^{(5)}\equiv\frac{T_1(E_1)T_1(E_2)}{2T_1(E_1)+2T_1(E_2)},\quad \tau^{(6)}\equiv\frac{T_1(E_1)T_1(E_2)}{T_1(E_1)+2T_1(E_2)},\\
&\tau^{(7)}\equiv\frac{T_2(E_1)T_0(E_2)}{T_0(E_1)+T_2(E_1)+T_0(E_2)+2T_2(E_2)},\quad \tau^{(8)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+T_2(E_1)+T_0(E_2)+2T_2(E_2)},\\
&\tau^{(9)}\equiv\frac{T_2(E_1)T_0(E_2)}{2T_2(E_1)+T_0(E_2)+2T_2(E_2)},\quad \tau^{(10)}\equiv\frac{T_2(E_1)T_2(E_2)}{2T_2(E_1)+T_0(E_2)+2T_2(E_2)},\quad \tau^{(11)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_2(E_1)+2T_2(E_2)}.
\end{aligned} \qquad (61)$$

For the $\varphi=0°$ plane, Eq. (59) can be transformed into the following expression:

$$
\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2 = (E_1)^{-1}\{ & x^4y^2[-27.8506M_2^{(2)}\tau^{(8)}-77.3628M_2^{(2)}\tau^{(10)}+34.0396M_2^{(2)}\tau^{(11)}] \\
& +x^4[+13.9253M_2^{(2)}\tau^{(8)}+(-14.7358M_0^{(2)}+31.3136M_2^{(2)})\tau^{(10)}+(-14.1832M_0^{(2)}-29.9136M_2^{(2)})\tau^{(11)}] \\
& +x^2y^2[-6.1890M_2^{(2)}\tau^{(1)}-18.5671M_2^{(2)}\tau^{(3)}-6.1890M_2^{(2)}\tau^{(4)}-11.1402M_2^{(2)}\tau^{(5)} \\
& +12.3781M_2^{(2)}\tau^{(6)}+4.6417M_2^{(2)}\tau^{(8)}+82.8896M_2^{(2)}\tau^{(10)}-49.5122M_2^{(2)}\tau^{(11)}] \\
& +x^2[+3.0945M_2^{(2)}\tau^{(1)}+9.2835M_2^{(2)}\tau^{(3)}+3.0945M_2^{(2)}\tau^{(4)}+3.7134(-M_0^{(2)}+M_2^{(2)})\tau^{(5)}-9.2835M_2^{(2)}\tau^{(6)} \\
& -4.6418M_2^{(2)}\tau^{(8)}+(+22.1035M_0^{(2)}-49.7338M_2^{(2)})\tau^{(10)}+(+38.6814M_0^{(2)}+34.0396M_2^{(2)})\tau^{(11)}] \\
& +y^2[+3.0945M_2^{(2)}\tau^{(1)}+9.2835M_2^{(2)}\tau^{(3)}+3.0945M_2^{(2)}\tau^{(4)}+2.7851M_2^{(2)}\tau^{(5)}-20.1143M_2^{(2)}\tau^{(6)}-4.6418M_2^{(2)}\tau^{(7)} \\
& +9.2835M_2^{(2)}\tau^{(8)}-23.2089M_2^{(2)}\tau^{(9)}-11.0524M_2^{(2)}\tau^{(10)}-6.1890M_2^{(2)}\tau^{(11)}] \\
& +[+2.0630(M_0^{(2)}-M_2^{(2)})\tau^{(1)}+6.1890M_0^{(2)}\tau^{(2)}+(+12.3781M_0^{(2)}-6.1890M_2^{(2)})\tau^{(3)}+2.0630(M_0^{(2)}-M_2^{(2)})\tau^{(4)} \\
& +(+25.9939M_0^{(2)}-0.9284M_2^{(2)})\tau^{(5)}+(+20.6301M_0^{(2)}+11.8623M_2^{(2)})\tau^{(6)}+(+6.1890M_0^{(2)}+1.5473M_2^{(2)})\tau^{(7)} \\
& +(+12.3781M_0^{(2)}-4.6418M_2^{(2)})\tau^{(8)}+(+20.6301M_0^{(2)}+28.3664M_2^{(2)})\tau^{(9)} \\
& +(+36.8395M_0^{(2)}+20.2619M_2^{(2)})\tau^{(10)}+(+18.8250M_0^{(2)}+3.0925M_2^{(2)})\tau^{(11)}] \\
& +x^2z[-27.8506M_2^{(2)}\tau^{(8)}-77.3628M_2^{(2)}\tau^{(10)}+34.0396M_2^{(2)}\tau^{(11)}] \\
& +z[-6.1890M_2^{(2)}\tau^{(1)}-18.5671M_2^{(2)}\tau^{(3)}-6.1890M_2^{(2)}\tau^{(4)}-11.1402M_2^{(2)}\tau^{(5)} \\
& \qquad +12.3781M_2^{(2)}\tau^{(6)}-9.2835M_2^{(2)}\tau^{(8)}+52.4235M_2^{(2)}\tau^{(10)}+21.6616M_2^{(2)}\tau^{(11)}]\}\ \text{mb sr}^{-2}. \qquad (62)
\end{aligned}
$$

A suitable reaction to illustrate the correlation expression (62) for the $\frac{1}{2}+\to J_1\pi_1\to\frac{3}{2}+\to\frac{1}{2}+$ sequence is $Si^{29}(n,n'\gamma)$ at $E_n=2.0$ MeV (c.m.). The mixing ratio of the deexcitation γ radiation has been determined by groups at Chalk River[46,47] to be either $\Delta_2=+3.4$ or -0.23. Both of these possibilities, as also the special case $\Delta_2=0$ (pure $M1$), have been subjected to numerical computation, the resultant correlation curves for $\theta_1=0°, 45°, 90°$ in function of θ_2 being depicted in Fig. 8, which furnishes another instance of the radical change in correlation structure on changing the mixing ratio.

(ii). $\frac{3}{2}\pm\to J_1\pi_1\to\frac{1}{2}\pm\to\frac{3}{2}\pm$ *Sequence*

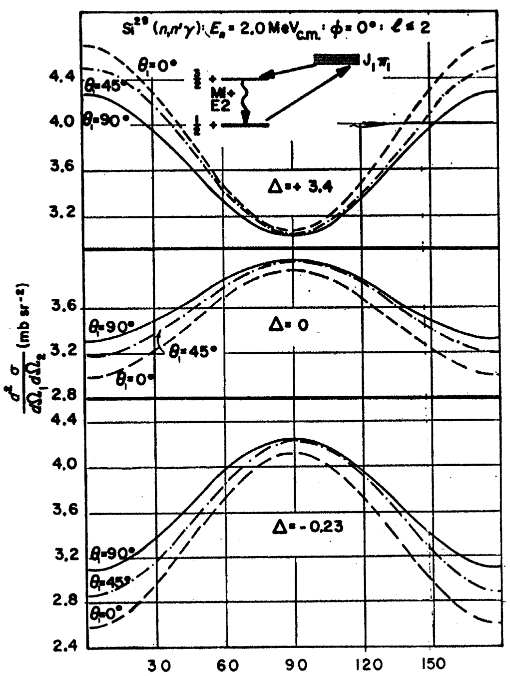

FIG. 8. Influence of the multipole mixing ratio Δ_2 upon the correlation for a $\frac{1}{2}+\to J_1\pi_1\to\frac{3}{2}+\to\frac{1}{2}+$ sequence in the case of inelastic scattering of 2.0-MeV neutrons to the first level of Si^{29}.

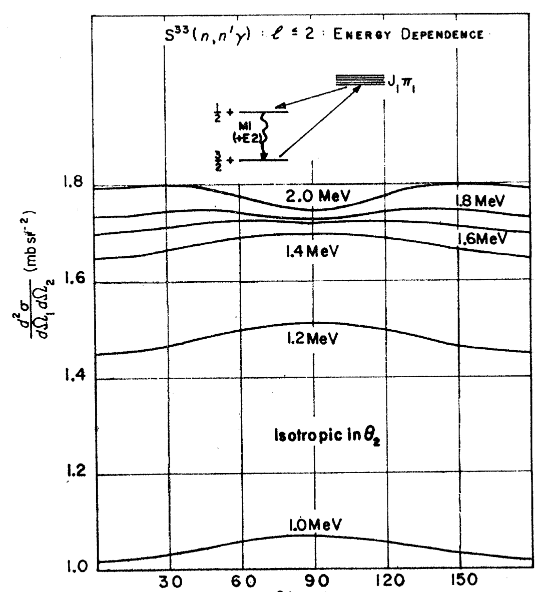

FIG. 9. Influence of the incident neutron energy upon the $\frac{3}{2}+\to J_1\pi_1\to\frac{1}{2}+\to\frac{3}{2}+$ correlation (scattering distribution$\div 4\pi$) for S^{33}.

[46] D. A. Bromley, H. E. Gove, E. B. Paul, A. E. Litherland, and E. Almqvist, Can. J. Phys. **35**, 1042 (1957).
[47] G. J. McCallum and A. E. Litherland, Bull. Am. Phys. Soc. **5**, 56 (1960).

The transition sequences with either positive or negative parity throughout for π_0, π_2, π_3 have been considered by Sheldon.[48] They represent a rather special case in that γ emission from a state of spin $J_2=\frac{1}{2}$ is isotropic and in consequence the correlation reduces essentially to a scattering distribution. It is independent not only of θ_2 and φ but also of the mixing ratio Δ_2, being given by

$$d^2\sigma/d\Omega_1 d\Omega_2=(1/4\pi)(d\sigma/d\Omega_1)=(1/4)(\lambda\!\!\!^{-2}/32\pi)\{3\tau^{(1)}+3\tau^{(2)}+10\tau^{(3)}+\tau^{(4)}[12-1.2P_2(x)]+10\tau^{(5)}+\tau^{(6)}+\tau^{(7)}+6\tau^{(8)}$$
$$+6\tau^{(9)}+\tau^{(10)}[20+3.06122P_2(x)-1.63265P_4(x)]+\tau^{(11)}[14+8.57143P_2(x)-1.57143P_4(x)]\}. \quad (63)$$

The $\tau^{(i)}$ are defined in Eq. (2) of Ref. 48 and the final expression is cited as Eq. (1) in that publication. The noteworthy feature of this result is that in addition to the above-mentioned θ_2 isotropy, the θ_1 dependence is strikingly weak at the fairly low incident energies considered [$E_n=1.90$ MeV (c.m.) for the $(n,n'\gamma)$ reaction on S^{33} and 0.90 MeV (c.m.) on Cu^{63}; see Fig. 1 of Ref. 48]; there would thus seem to be virtual isotropy over all emission directions in space. To ascertain whether this quasi-isotropy (associated with a peak-to-valley ratio of 1.01) was a consequence simply of choice of rather low incident and emergent energy, the energy dependence of the S^{33} ($\pi=+$) correlation was elucidated for E_n ranging from 1 to 2 MeV. This is shown in Fig. 9, which indicates the optimal energy for quasi-isotropy to lie around 1.7 MeV, the curves for energies above and below this value having more appreciable an undulation. It may perhaps also be mentioned that the correlation for the $Fe^{56}(n,n'\gamma)$ reaction with $E_n=1.1$ MeV (c.m.), $E_{n'}\cong 0.3$ MeV (c.m.) (a $0+\rightarrow J_1\pi_1\rightarrow 2+\rightarrow 0+$ transition sequence) displays a much larger amplitude; in function of θ_1, the peak-to-valley ratio is 1.20 and in function of θ_2 it is as much as 2.22 when $\theta_1=0°$.

Though small, the cross sections in Fig. 9 should lie within the bounds of feasible measurement; it will be shown later that for scattering to second levels with $J_2\pi_2=\frac{1}{2}-$ of nuclei having ground-state spin $J_0\pi_0=\frac{7}{2}-$, a similar situation of quasi-isotropy exists, but that cross sections are roughly only one-quarter of those in Fig. 9 and thereby rather too small for straightforward measurement.

This phenomenon of quasi-isotropy commends itself for investigations which seek to elucidate the admixture of direct interaction (DI) to the scattering mechanism at low incident energies, for the CN component would constitute a constant "background" in the measured correlation (or scattering distribution) of known magnitude. An approach from this direction might well experimentally shed light upon CN/DI mixing and interference, a problem which has recently formed the subject of considerable discussion.[49–54]

(iii). $\frac{7}{2}-\rightarrow J_1\pi_1\rightarrow\frac{3}{2}-\rightarrow\frac{7}{2}-$ *Sequence*

Even though here $\lambda=0, 2$, the γ transition being of pure multipolarity ($E2$) and the orbital momenta being restricted to $l_1, l_2\leqslant 2$ (with l_1+l_2 even), the summation in Eq. (2) is quite lengthy, involving as it does 104 terms. The ensuing correlation formula is

$$\begin{aligned}d^2\sigma/d\Omega_1 d\Omega_2=&(\lambda\!\!\!^{-2}/32\pi)(5/2)\{\tau^{(1)}[0.7-0.021429P_2(w)]+\tau^{(2)}[0.45+0.018367P_2(w)]\\
&+\tau^{(3)}[0.7-0.14P_2(x)-0.02P_2(y)+0.02P_2(w)-0.002139S_{222}]\\
&+\tau^{(4)}[0.5+0.007143P_2(y)-0.021429P_2(w)-0.001527S_{222}]+\tau^{(5)}[0.5+0.010204P_2(y)]\\
&+\tau^{(6)}[1.0-0.029155P_2(x)+0.015549P_4(x)+0.007289P_2(y)\\
&-0.051020P_2(w)+0.004230S_{222}-0.001493S_{242}-0.000332S_{422}+0.000464S_{442}]\\
&+\tau^{(7)}[1.4-0.464286P_2(x)-0.052381P_4(x)-0.027857P_2(y)\\
&-0.042857P_2(w)+0.004254S_{222}+0.005391S_{242}+0.007155S_{422}-0.002843S_{442}]\\
&+\tau^{(8)}[0.9-0.177114P_2(x)-0.034111P_4(x)-0.017711P_2(y)\\
&+0.036735P_2(w)-0.002705S_{222}-0.000605S_{242}-0.002330S_{422}-0.000370S_{442}]\\
&+\tau^{(9)}[0.15+0.003061P_2(y)]+\tau^{(10)}[0.3-0.001837P_2(y)+0.021429P_2(w)-0.000655S_{222}+0.000878S_{242}]\}, \quad (64)\end{aligned}$$

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[48] E. Sheldon, Phys. Letters 5, 157 (1963).
[49] M. Sano, S. Yoshida, and T. Terasawa, Nucl. Phys. 6, 20 (1958).
[50] S. Yoshida, in *Proceedings of the Kingston International Conference on Nuclear Structure*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, and North-Holland Publishing Company, Amsterdam, 1960), p. 336.
[51] L. S. Rodberg, Phys. Rev. 124, 210 (1961).
[52] N. Austern, in *Selected Topics in Nuclear Theory*, edited by F. Janouch (International Atomic Energy Agency, Vienna, 1963).
[53] N. Austern, in Proceedings of the Topical Conference on Compound Nuclear States, Gatlinburg, 1963 (unpublished).
[54] K. K. Seth, in *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach Publishers, Inc., New York, 1963), p. 267.

with

$$\tau^{(1)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+2T_2(E_2)},\quad \tau^{(2)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+T_2(E_2)},\quad \tau^{(3)}\equiv\frac{T_1(E_1)T_1(E_2)}{2T_1(E_1)+T_1(E_2)},$$
$$\tau^{(4)}\equiv\frac{T_1(E_1)T_1(E_2)}{T_1(E_1)+2T_1(E_2)},\quad \tau^{(5)}\equiv\frac{T_2(E_1)T_0(E_2)}{2T_2(E_1)+T_0(E_2)},\quad \tau^{(6)}\equiv\frac{T_2(E_1)T_2(E_2)}{2T_2(E_1)+2T_2(E_2)},$$
$$\tau^{(7)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+2T_2(E_2)},\quad \tau^{(8)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+T_2(E_2)},\quad \tau^{(9)}\equiv\frac{T_2(E_1)T_0(E_2)}{T_2(E_1)+T_0(E_2)},$$
$$\tau^{(10)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_2(E_1)+2T_2(E_2)}. \tag{65}$$

For the $\varphi=0°$ plane, Eq. (64) can be expressed as

$$\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2={}&(E_1)^{-1}\{x^4y^2[-0.263139\tau^{(6)}+1.003871\tau^{(7)}-0.080747\tau^{(8)}+0.142095\tau^{(10)}]\\
&+x^4[0.457358\tau^{(6)}-1.530369\tau^{(7)}-0.709316\tau^{(8)}-0.071048\tau^{(10)}]\\
&+x^2y^2[-0.331555\tau^{(1)}+0.284190\tau^{(2)}+0.353668\tau^{(3)}-0.299978\tau^{(4)}-0.657848\tau^{(6)}-1.303176\tau^{(7)}\\
&+0.605528\tau^{(8)}+0.213146\tau^{(10)}]\\
&+x^2[0.165777\tau^{(1)}-0.142095\tau^{(2)}-1.237805\tau^{(3)}+0.165777\tau^{(4)}-0.225546\tau^{(6)}-2.058406\tau^{(7)}\\
&-1.023042\tau^{(8)}-0.094730\tau^{(10)}]\\
&+y^2[0.165777\tau^{(1)}-0.142095\tau^{(2)}-0.309451\tau^{(3)}+0.221037\tau^{(4)}+0.078941\tau^{(5)}+0.432299\tau^{(6)}+0.087494\tau^{(7)}\\
&-0.393300\tau^{(8)}+0.023682\tau^{(9)}-0.165781\tau^{(10)}]\\
&+[3.499747\tau^{(1)}+2.415615\tau^{(2)}+4.111279\tau^{(3)}+2.439298\tau^{(4)}+2.552448\tau^{(5)}+5.018435\tau^{(6)}+8.215191\tau^{(7)}\\
&+5.240907\tau^{(8)}+0.765734\tau^{(9)}+1.649881\tau^{(10)}]\\
&+x^2z[-0.263139\tau^{(6)}+1.003871\tau^{(7)}-0.080747\tau^{(8)}+0.142095\tau^{(10)}]\\
&+z[-0.331554\tau^{(1)}+0.284190\tau^{(2)}+0.353668\tau^{(3)}-0.299978\tau^{(4)}-0.751826\tau^{(6)}-1.445930\tau^{(7)}\\
&+0.745081\tau^{(8)}+0.284194\tau^{(10)}]\}\ \text{mb sr}^{-2}.
\end{aligned} \tag{66}$$

The θ_2 dependence of this correlation is illustrated for the $Co^{57}(n,n'\gamma)$ reaction at $E_n=1.60$ MeV (c.m.) in Fig. 10 on a rather exaggerated vertical scale. With a peak-to-valley ratio of only 1.02, these curves also display near-isotropy in terms of θ_1 or θ_2.

(iv). $\frac{7}{2}-\rightarrow J_1\pi_1\rightarrow\frac{5}{2}-\rightarrow\frac{7}{2}-$ *Sequence*

For this case, the summation of Eq. (2) with $\lambda=0$, 2 and l_1+l_2 even (l_1, $l_2\leqslant 2$) extends over 145 terms on making provision for the γ decay to be of mixed ($M1+E2$) multipolarity, and yields the result

$$\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2={}&(3/4)(\lambda^2/32\pi)\{2.857737M_0^{(2)}\tau^{(1)}+\tau^{(2)}[5.715474M_0^{(2)}+2.945941M_2^{(2)}P_2(w)]\\
&+\tau^{(3)}[7.348471M_0^{(2)}+0.981982M_2^{(2)}P_2(w)]\\
&+\tau^{(4)}[11.430948M_0^{(2)}+0.285774M_0^{(2)}P_2(x)+2.367664M_2^{(2)}P_2(y)+3.360554M_2^{(2)}P_2(w)-0.236784M_2^{(2)}S_{222}]\\
&+\tau^{(5)}[7.348471M_0^{(2)}-0.577382M_0^{(2)}P_2(x)+0.771556M_2^{(2)}P_2(y)-1.963961M_2^{(2)}P_2(w)+0.082483M_2^{(2)}S_{222}]\\
&+\tau^{(6)}[4.0825M_0^{(2)}-0.291606M_0^{(2)}P_2(x)+0.280566M_2^{(2)}P_2(y)+0.218218M_2^{(2)}P_2(w)+0.076651M_2^{(2)}S_{222}]\\
&+\tau^{(7)}[4.0825M_0^{(2)}+0.623479M_2^{(2)}P_2(y)]\\
&+\tau^{(8)}[8.165M_0^{(2)}+0.654625M_0^{(2)}P_2(x)-0.071414M_0^{(2)}P_4(x)-0.241446M_2^{(2)}S_{222}+0.068437M_2^{(2)}S_{242}\\
&+0.015208M_2^{(2)}S_{422}+0.015951M_2^{(2)}S_{442}]\\
&+\tau^{(9)}[5.715474M_0^{(2)}+2.836832M_2^{(2)}P_2(y)]\\
&+\tau^{(10)}[11.430948M_0^{(2)}+0.631813M_0^{(2)}P_2(x)-0.427689M_0^{(2)}P_4(x)+1.465697M_2^{(2)}P_2(y)+5.891882M_2^{(2)}P_2(w)\\
&-1.000072M_2^{(2)}S_{222}+0.268024M_2^{(2)}S_{242}-0.300565M_2^{(2)}S_{422}+0.043421M_2^{(2)}S_{442}]\\
&+\tau^{(11)}[14.696941M_0^{(2)}-1.590744M_0^{(2)}P_2(x)+0.278514M_0^{(2)}P_4(x)+1.893823M_2^{(2)}P_2(y)+1.963961M_2^{(2)}P_2(w)\\
&-0.020663M_2^{(2)}S_{222}-0.166301M_2^{(2)}S_{242}+0.231317M_2^{(2)}S_{422}-0.062207M_2^{(2)}S_{442}]\\
&+\tau^{(12)}[8.981462M_0^{(2)}+3.177117M_0^{(2)}P_2(x)-0.794283M_0^{(2)}P_4(x)-2.971921M_2^{(2)}P_2(y)-3.429138M_2^{(2)}P_2(w)\\
&-0.453874M_2^{(2)}S_{222}-0.101492M_2^{(2)}S_{242}+0.507446M_2^{(2)}S_{422}+0.080638M_2^{(2)}S_{442}]\\
&+\tau^{(13)}[2.449491M_0^{(2)}-0.124974M_0^{(2)}P_2(x)+0.018704M_2^{(2)}P_2(y)-1.776916M_2^{(2)}P_2(w)\\
&+0.032847M_2^{(2)}S_{222}+0.022995M_2^{(2)}S_{242}]\},
\end{aligned} \tag{67}$$

with

$$\tau^{(1)}\equiv\frac{T_0(E_1)T_0(E_2)}{T_0(E_1)+2T_2(E_1)+T_0(E_2)+2T_2(E_2)},\quad \tau^{(2)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+T_0(E_2)+2T_2(E_2)},$$
$$\tau^{(3)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+2T_2(E_2)},\quad \tau^{(4)}\equiv\frac{T_1(E_1)T_1(E_2)}{2T_1(E_1)+2T_1(E_2)},\quad \tau^{(5)}\equiv\frac{T_1(E_1)T_1(E_2)}{2T_1(E_1)+T_1(E_2)},$$
$$\tau^{(6)}\equiv\frac{T_1(E_1)T_1(E_2)}{T_1(E_1)+2T_1(E_2)},\quad \tau^{(7)}\equiv\frac{T_2(E_1)T_0(E_2)}{T_0(E_1)+2T_2(E_1)+2T_2(E_2)},\quad \tau^{(8)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+2T_2(E_2)},\tag{68}$$
$$\tau^{(9)}\equiv\frac{T_2(E_1)T_0(E_2)}{T_0(E_1)+2T_2(E_1)+T_0(E_2)+2T_2(E_2)},\quad \tau^{(10)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+T_0(E_2)+2T_2(E_2)},$$
$$\tau^{(11)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+2T_2(E_2)},\quad \tau^{(12)}\equiv\frac{T_2(E_1)T_2(E_2)}{2T_2(E_1)+T_2(E_2)},\quad \tau^{(13)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_2(E_1)+2T_2(E_2)},$$

and the $M_\lambda^{(2)}$, defined by Eq. (7) here taking on the values

$$M_0^{(2)}=0.408366,\quad M_2^{(2)}=(1+\Delta_2^2)^{-1}(0.054555+0.566947\Delta_2+0.132489\Delta_2^2).\tag{69}$$

For the $\varphi=0°$ plane, Eq. (67) becomes

$$\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2 =\ & (E_1)^{-1}\{x^4y^2[3.100764M_2^{(2)}\tau^{(8)}+12.408262M_2^{(2)}\tau^{(10)}\\
&-7.209282M_2^{(2)}\tau^{(11)}-6.046526M_2^{(2)}\tau^{(12)}+1.116288M_2^{(2)}\tau^{(13)}]\\
&+x^4[(-0.197412-1.808791M_2^{(2)})\tau^{(8)}+(-1.182273-6.907557M_2^{(2)})\tau^{(10)}+(0.769905+4.612392M_2^{(2)})\tau^{(11)}\\
&+(-2.195668+1.716926M_2^{(2)})\tau^{(12)}-0.558143M_2^{(2)}\tau^{(13)}]\\
&+x^2y^2[13.674381M_2^{(2)}\tau^{(2)}+4.558125M_2^{(2)}\tau^{(3)}+17.067652M_2^{(2)}\tau^{(4)}-9.627884M_2^{(2)}\tau^{(5)}+0.537463M_2^{(2)}\tau^{(6)}\\
&-1.291990M_2^{(2)}\tau^{(8)}+17.768250M_2^{(2)}\tau^{(10)}+19.418590M_2^{(2)}\tau^{(11)}-1.360854M_2^{(2)}\tau^{(12)}-9.488335M_2^{(2)}\tau^{(13)}]\\
&+x^2[-6.837191M_2^{(2)}\tau^{(2)}-2.279063M_2^{(2)}\tau^{(3)}+(0.270848-7.799458M_2^{(2)})\tau^{(4)}+(-0.547224+4.558127M_2^{(2)})\tau^{(5)}\\
&+(-0.276376-0.506459M_2^{(2)})\tau^{(6)}+(0.789644+1.808794M_2^{(2)})\tau^{(8)}+(1.612189-5.106769M_2^{(2)})\tau^{(10)}\\
&+(-2.167576-10.581374M_2^{(2)})\tau^{(11)}+(4.893172+4.151542M_2^{(2)})\tau^{(12)}+(-0.118447+4.682161M_2^{(2)})\tau^{(13)}]\\
&+y^2[-6.837191M_2^{(2)}\tau^{(2)}-2.279063M_2^{(2)}\tau^{(3)}-2.304376M_2^{(2)}\tau^{(4)}+6.348820M_2^{(2)}\tau^{(5)}+0.144704M_2^{(2)}\tau^{(6)}\\
&+1.447025M_2^{(2)}\tau^{(7)}+0.258398M_2^{(2)}\tau^{(8)}+6.583964M_2^{(2)}\tau^{(9)}-7.512457M_2^{(2)}\tau^{(10)}-2.093022M_2^{(2)}\tau^{(11)}\\
&-1.894925M_2^{(2)}\tau^{(12)}+4.279056M_2^{(2)}\tau^{(13)}]\\
&+[1.805652\tau^{(1)}+(3.611305+4.558127M_2^{(2)})\tau^{(2)}+(4.643108+1.519377M_2^{(2)})\tau^{(3)}+(7.132326+2.878365M_2^{(2)})\tau^{(4)}\\
&+(4.825516-3.465109M_2^{(2)})\tau^{(5)}+(2.671640+0.279071M_2^{(2)})\tau^{(6)}+(2.579515-0.482342M_2^{(2)})\tau^{(7)}\\
&+(4.935299-0.689061M_2^{(2)})\tau^{(8)}+(3.611305-2.194654M_2^{(2)})\tau^{(9)}+(6.921667+4.459635M_2^{(2)})\tau^{(10)}\\
&+(9.854758+2.596885M_2^{(2)})\tau^{(11)}+(4.482983-2.767717M_2^{(2)})\tau^{(12)}+(1.587185-2.759693M_2^{(2)})\tau^{(13)}]\\
&+x^2z[3.100764M_2^{(2)}\tau^{(8)}+12.408261M_2^{(2)}\tau^{(10)}-7.209282M_2^{(2)}\tau^{(11)}-6.046526M_2^{(2)}\tau^{(12)}+1.116288M_2^{(2)}\tau^{(13)}]\\
&+z[13.674381M_2^{(2)}\tau^{(2)}+4.558125M_2^{(2)}\tau^{(3)}+17.067652M_2^{(2)}\tau^{(4)}-9.627884M_2^{(2)}\tau^{(5)}+0.537463M_2^{(2)}\tau^{(6)}\\
&+31.569266M_2^{(2)}\tau^{(10)}+9.767431M_2^{(2)}\tau^{(11)}-16.141189M_2^{(2)}\tau^{(12)}-8.930193M_2^{(2)}\tau^{(13)}]\}\ \text{mb sr}^{-2}.
\end{aligned}\tag{70}$$

This correlation is illustrated in Fig. 11 for scattering of 1.3-MeV neutrons on Co^{59} when the deexcitation γ radiation is taken to be pure $M1$, as suggested by the investigations of Metzger.[55] Setting $\Delta_2=0$ in (69) yields $M_0^{(2)}=0.408366$, $M_2^{(2)}=0.054555$ for substitution in Eq. (70); the resultant correlation shows but little structure (the vertical scale in Fig. 11 is rather extended) and conventional shape.

B. Scattering to the Second Level, Followed by Cascade γ Radiation

It is convenient to separate those cases in which the second level de-excites by γ emission direct to the ground state (Sec. 4C) and those in which the decay occurs by stopover γ radiation in two steps, either of which may be observed in coincidence with particles going to the second level. The two latter possibilities are considered in the calculations comprising this section.

[55] F. R. Metzger, Phys. Rev. 88, 1360 (1952).

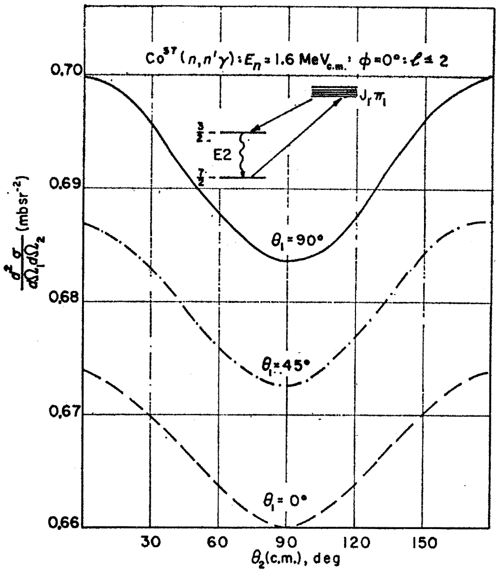

FIG. 10. Correlation for a $\frac{7}{2}- \to J_1\pi_1 \to \frac{3}{2}- \to \frac{7}{2}-$ sequence, illustrated by the $Co^{57}(n,n'\gamma)$ reaction at 1.6 MeV (c.m.) assuming the deexcitation γ radiation from decay of the first level to be pure $E2$.

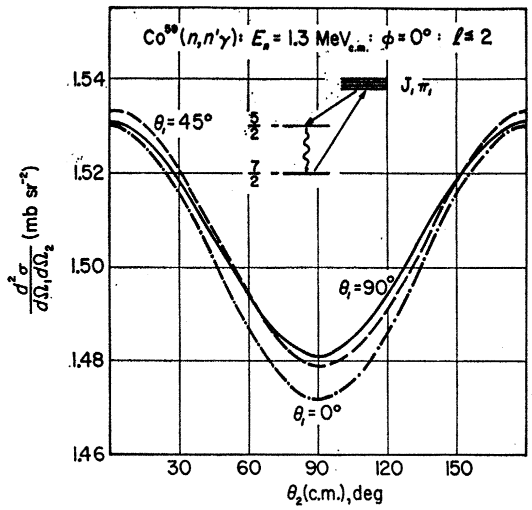

FIG. 11. Correlation for a $\frac{7}{2}- \to J_1\pi_1 \to \frac{5}{2}- \to \frac{7}{2}-$ sequence, illustrated by the $Co^{59}(n,n'\gamma)$ reaction at 1.3 MeV (c.m.) assuming the deexcitation γ radiation from decay of the first level to be pure $M1$.

(i). $\frac{1}{2}+ \to J_1\pi_1 \to \frac{5}{2}+ \to \frac{3}{2}+$ *and* $\frac{1}{2}+ \to J_1\pi_1 \to \frac{5}{2}+ \twoheadrightarrow \frac{3}{2}+ \to \frac{1}{2}+$ *Sequences*

These transitions have been evaluated, but are omitted for reasons akin to those prompting the omission of the $\frac{1}{2}- \to J_1\pi_1 \to \frac{9}{2}+ \to \frac{1}{2}-$ sequence from Sec. 4A (in the latter, the $M4$ transition probability is too low to permit coincidence measurements of sufficient accuracy). In the present instance, the only suitable nuclei with $A<100$ would apparently be Si^{29} or P^{31} for which, however, experimental evidence[46,56] respectively indicates the $\frac{5}{2}+ \to \frac{3}{2}+$ γ transition to be so inhibited as compared with the direct $\frac{5}{2}+ \to \frac{1}{2}+$ γ transition to the ground state (branching ratio $<3\%$) as to preclude its use for correlation studies. It might perhaps be mentioned that both for the $\frac{1}{2}+ \to J_1\pi_1 \to \frac{5}{2}+ \to \frac{3}{2}+$ and $\frac{1}{2}+ \to J_1\pi_1 \to \frac{5}{2}+ \twoheadrightarrow \frac{3}{2}+ \to \frac{1}{2}+$ transition schemes, the correlation curves as evaluated for the $P^{31}(n,n'\gamma)$ reaction at $E_n=3.0$ MeV in function of θ_2, peak around $\theta_2=90°$ and display considerable amplitudes.

(ii). $\frac{7}{2}- \to J_1\pi_1 \to \frac{1}{2}- \to \frac{3}{2}-$ *and* $\frac{7}{2}- \to J_1\pi_1 \to \frac{1}{2}- \twoheadrightarrow \frac{3}{2}- \to \frac{7}{2}-$ *Sequences*

These two transition schemes can be treated jointly by virtue of the restriction $\lambda=0$ which ensues from the triangle relation for the triad $(J_2J_2\lambda)$. This essentially reduces the problem to that of a particle *distribution* and renders it independent of the multipole mixing ratios Δ_2 and Δ_3, as also of course isotropic in θ_2.

Summation of Eq. (2) is confined to 20 terms when $l_1, l_2 \leqslant 2$ (and l_1+l_2 is even), and yields the result

$$d^2\sigma/d\Omega_1 d\Omega_2 = (1/4\pi)(d\sigma/d\Omega_1) = (1/8)(\lambda^2/32\pi)\{7\tau^{(1)}+\tau^{(2)}[5+P_2(x)]+\tau^{(3)}[20-3.061226P_2(x)-0.272109P_4(x)] +\tau^{(4)}[14-7.428571P_2(x)+2.095238P_4(x)]+3\tau^{(5)}+\tau^{(6)}[3+0.428571P_2(x)]\}, \quad (71)$$

with

$$\tau^{(1)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+T_2(E_2)}, \quad \tau^{(2)}\equiv\frac{T_1(E_1)T_1(E_2)}{T_1(E_1)+T_1(E_2)}, \quad \tau^{(3)}\equiv\frac{T_2(E_1)T_2(E_2)}{2T_2(E_1)+2T_2(E_2)},$$

$$\tau^{(4)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+T_2(E_2)}, \quad \tau^{(5)}\equiv\frac{T_2(E_1)T_0(E_2)}{T_2(E_1)+T_0(E_2)+T_2(E_2)}, \quad \tau^{(6)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_2(E_1)+T_0(E_2)+T_2(E_2)}. \quad (72)$$

Equation (71) can be transformed to

$$d^2\sigma/d\Omega_1 d\Omega_2 = (E_1)^{-1}\{x^4[-0.306996\tau^{(3)}+2.363864\tau^{(4)}] +x^2[0.386814\tau^{(2)}-0.920986\tau^{(3)}-4.899646\tau^{(4)}+0.165777\tau^{(6)}] +[1.805133\tau^{(1)}+1.160442\tau^{(2)}+5.525917\tau^{(3)}+4.770708\tau^{(4)}+0.773628\tau^{(5)}+0.718369\tau^{(6)}]\}\text{ mb sr}^{-2}, \quad (73)$$

[56] A. E. Litherland, E. B. Paul, G. A. Bartholomew, and H. E. Gove, Can. J. Phys. 37, 53 (1959).

an expression whose rather slight θ_1 dependence is depicted in Fig. 12 for 1.9-MeV neutrons incident on Co^{57}. Since this would seem to offer another example of over-all "quasi-isotropy," it is interesting to investigate its energy dependence in this region. The results shown in Fig. 13 indicate the isotropy to be most pronounced around an incident energy of 1.65 MeV (c.m.).

C. Scattering to the Second Level, Followed by γ Decay Direct to the Ground State

(*i*). $\frac{1}{2}- \rightarrow J_1\pi_1 \rightarrow \frac{1}{2}- \rightarrow \frac{1}{2}-$ *Sequence*

Since $J_2=\frac{1}{2}$, it follows that $\lambda=0$ and the correlation accordingly reduces essentially to a particle *distribution* which is independent of θ_2 or φ. The present instance is noteworthy in that it serves to dispel the impression (which might otherwise have arisen from similar special $\lambda=0$ cases treated earlier) of "quasi-isotropy" being a general feature of such correlations at the rather low energies considered. In the present case, the correlation (i.e., the distribution) depends strongly on the particle scattering angle θ_1.

The result of summing Eq. (2) over 26 terms with $\lambda=0$ and l_1+l_2 even (where l_1, $l_2\leqslant 2$) is

$$\begin{aligned}d^2\sigma/d\Omega_1 d\Omega_2 = (1/4\pi)(d\sigma/d\Omega_1) = (1/2)(\lambda^2/32\pi)\{&\tau^{(1)}+3\tau^{(2)}+3\tau^{(3)}+\tau^{(4)}[6+3.5P_2(x)] \\ &+\tau^{(5)}[12+1.5P_2(x)]+3\tau^{(6)}+\tau^{(7)}[3+1.5P_2(x)]+\tau^{(8)}[20+16.071428P_2(x)+1.428571P_4(x)] \\ &+\tau^{(9)}[7+6.857143P_2(x)+3.142859P_4(x)]\}, \end{aligned} \tag{74}$$

with

$$\begin{aligned}&\tau^{(1)}\equiv\frac{T_0(E_1)T_0(E_2)}{T_0(E_1)+T_0(E_2)}, \quad \tau^{(2)}\equiv\frac{T_0(E_1)T_0(E_2)}{T_0(E_1)+T_2(E_1)+T_0(E_2)+T_2(E_2)}, \quad \tau^{(3)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+T_2(E_1)+T_0(E_2)+T_2(E_2)}, \\ &\tau^{(4)}\equiv\frac{T_1(E_1)T_1(E_2)}{T_1(E_1)+T_1(E_2)}, \quad \tau^{(5)}\equiv\frac{T_1(E_1)T_1(E_2)}{2T_1(E_1)+2T_1(E_2)}, \quad \tau^{(6)}\equiv\frac{T_2(E_1)T_0(E_2)}{T_0(E_1)+T_2(E_1)+T_0(E_2)+T_2(E_2)}, \\ &\tau^{(7)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+T_2(E_1)+T_0(E_2)+T_2(E_2)}, \quad \tau^{(8)}\equiv\frac{T_2(E_1)T_2(E_2)}{2T_2(E_1)+2T_2(E_2)}, \quad \tau^{(9)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_2(E_1)+T_2(E_2)}.\end{aligned} \tag{75}$$

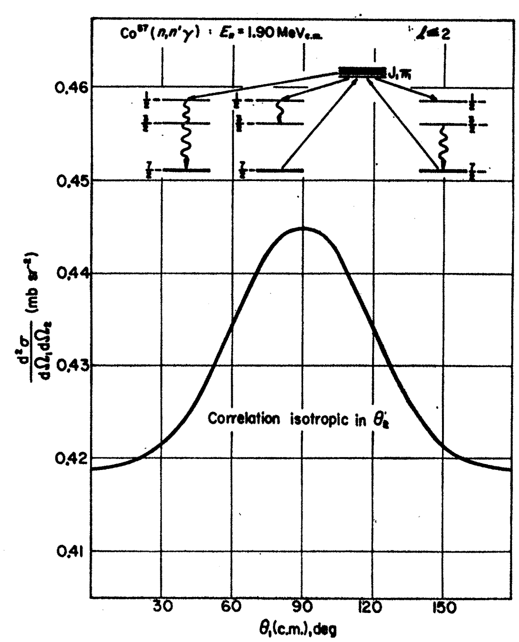

FIG. 12. The correlation (essentially a distribution) for $\frac{7}{2}- \rightarrow J_1\pi_1 \rightarrow \frac{1}{2}- \rightarrow \frac{3}{2}-$, $\frac{7}{2}- \rightarrow J_1\pi_1 \rightarrow \frac{1}{2}- \rightarrow \frac{3}{2}- \rightarrow \frac{7}{2}-$ and $\frac{7}{2}- \rightarrow J_1\pi_1 \rightarrow \frac{1}{2}- \rightarrow \frac{7}{2}-$ transition sequences, illustrated by the $Co^{57}(n,n'\gamma)$ reaction at 1.9 MeV (c.m.), the γ emission being isotropic irrespective of multipolarity.

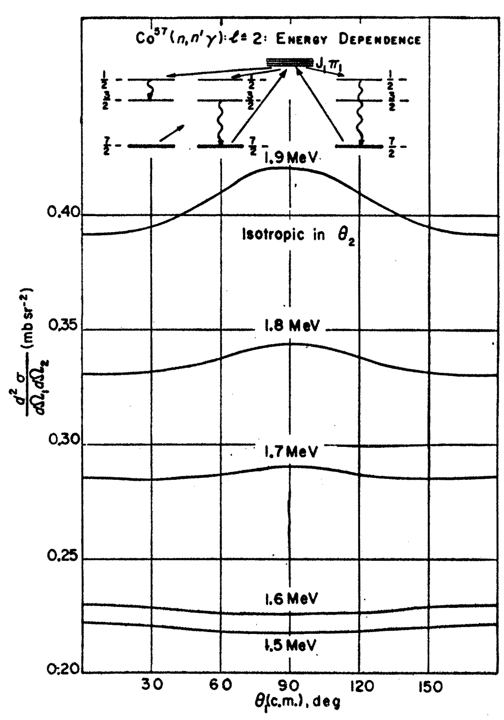

FIG. 13. Energy dependence of the correlation depicted in Fig. 12.

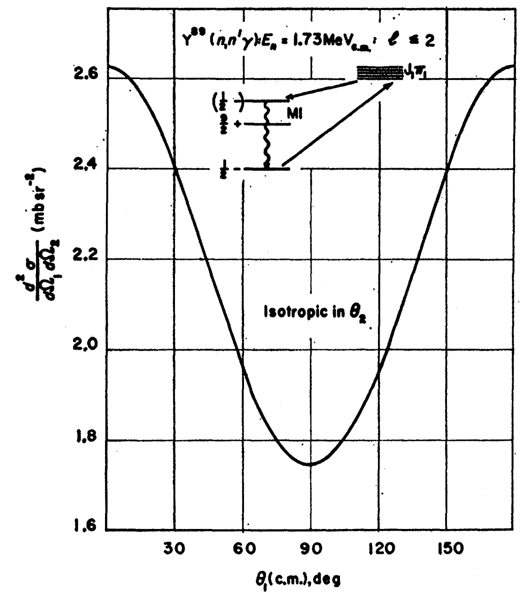

FIG. 14. As Fig. 12, but for a $\frac{1}{2}- \rightarrow J_1\pi_1 \rightarrow \frac{1}{2}- \rightarrow \frac{1}{2}-$ sequence, illustrated by the $Y^{89}(n,n'\gamma)$ reaction at $E_n=1.73$ MeV (c.m.).

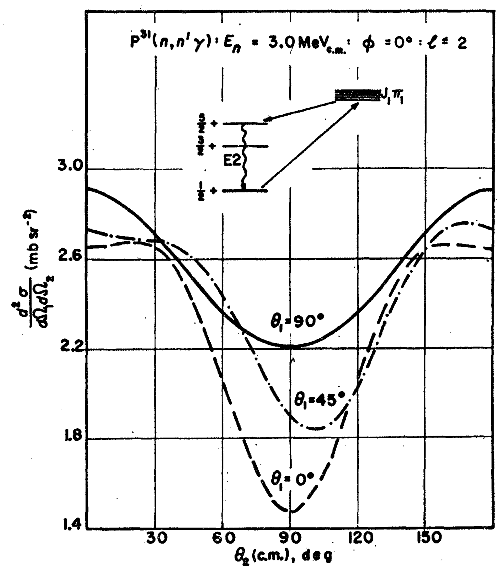

FIG. 15. Correlation for inelastic scattering of 3.0-MeV neutrons to the second level $(\frac{5}{2}+)$ of P^{31}, followed by pure $E2$ γ decay direct to the $(\frac{1}{2}+)$ ground state.

The reduction to a polynomial in x yields

$$
\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2 = (E_1)^{-1}\{&x^4[6.446903\tau^{(8)}+14.183187\tau^{(9)}] \\
&+x^2[5.415399\tau^{(4)}+2.320885\tau^{(5)}+2.320885\tau^{(7)}+19.340709\tau^{(8)}-1.547257\tau^{(9)}] \\
&+[1.031505\tau^{(1)}+3.094514\tau^{(2)}+3.094514\tau^{(3)}+4.383894\tau^{(4)}+11.604426\tau^{(5)}+3.094514\tau^{(6)}+2.320885\tau^{(7)} \\
&+12.893806\tau^{(8)}+4.899646\tau^{(9)}]\}\ \text{mb sr}^{-2}, \qquad (76)
\end{aligned}
$$

which is illustrated in Fig. 14 for inelastic scattering of 1.73-MeV neutrons on Y^{89}. The curve's high peak-to-valley ratio of 1.5 recalls that of Fig. 1 for the $0+ \rightarrow J_1\pi_1 \rightarrow 0+ \rightarrow 2+$ sequence, when compared with the ratios for the other $\lambda=0$ cases (Figs. 9, 12, and 13). The curve amplitude is obviously large for *low* ground-state spins, a situation which prevails in general for angular *distributions.*

(*ii*). $\frac{1}{2}+ \rightarrow J_1\pi_1 \rightarrow \frac{5}{2}+ \rightarrow \frac{1}{2}+$ *Sequence*

As stated in Sec. 4B(*i*), the $\frac{5}{2}+$ second level of nuclei such as P^{31}, which has been chosen to illustrate the present case, decays practically exclusively to the $\frac{1}{2}+$ ground state by direct γ transition rather than by cascade radiation.[56] With pure $E2$ γ multipolarity, the summation of Eq. (2) is simplified; for $l_1, l_2 \leqslant 2$ (with l_1+l_2 even) and $\lambda=0, 2, 4$ it nevertheless has to be carried over 110 terms to yield

$$
\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2 = (15\lambda^2/32\pi)\{&\tau^{(1)}[0.033333+0.019048P_2(w)-0.019048P_4(w)] \\
&+\tau^{(2)}[0.2+0.077551P_2(w)+0.008163P_4(w)] \\
&+\tau^{(3)}[0.2+0.01P_2(x)+0.02P_2(y)+0.08P_2(w)+0.002138S_{222}-0.009562S_{224}] \\
&+\tau^{(4)}[0.333333-0.083333P_2(x)+0.090410P_2(y)-0.009524P_2(w)-0.011709S_{222}-0.011383S_{224}] \\
&+\tau^{(5)}[0.2-0.035714P_2(x)-0.002857P_2(y)+0.077551P_2(w)+0.008163P_4(w)-0.005018S_{222} \\
&-0.004878S_{224}-0.003513S_{242}+0.006822S_{244}] \\
&+\tau^{(6)}[0.333333+0.142790P_2(y)-0.031746P_4(y)] \\
&+\tau^{(7)}[0.666667-0.280612P_2(x)+0.030613P_4(x)+0.015873P_4(y)+0.095238P_4(w)-0.055322S_{222}-0.045301S_{224} \\
&+0.015681S_{242}+0.012182S_{244}+0.003484S_{422}+0.002707S_{424}+0.003654S_{442}-0.006212S_{444}] \\
&+\tau^{(8)}[0.233333+0.114286P_2(y)-0.069841P_4(y)] \\
&+\tau^{(9)}[0.466667-0.047619P_2(x)-0.052381P_4(x)+0.059048P_2(y)+0.069841P_4(y)-0.128568P_2(w) \\
&-0.060317P_4(w)-0.040289S_{222}+0.001084S_{224}+0.107977S_{242}-0.012128S_{244}+0.019677S_{422}+0.017055S_{424} \\
&-0.002843S_{442}-0.008697S_{444}]\}, \qquad (77)
\end{aligned}
$$

with

$$
\begin{aligned}
&\tau^{(1)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+T_2(E_2)},\quad \tau^{(2)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+T_2(E_1)+2T_2(E_2)},\quad \tau^{(3)}\equiv\frac{T_1(E_1)T_1(E_2)}{2T_1(E_1)+T_1(E_2)},\\
&\tau^{(4)}\equiv\frac{T_1(E_1)T_1(E_2)}{T_1(E_1)+2T_1(E_2)},\quad \tau^{(5)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+T_2(E_1)+2T_2(E_2)},\quad \tau^{(6)}\equiv\frac{T_2(E_1)T_0(E_2)}{2T_2(E_1)+T_0(E_2)+2T_2(E_2)},\\
&\tau^{(7)}\equiv\frac{T_2(E_1)T_2(E_2)}{2T_2(E_1)+T_0(E_2)+2T_2(E_2)},\quad \tau^{(8)}\equiv\frac{T_2(E_1)T_0(E_2)}{T_2(E_1)+T_0(E_2)+2T_2(E_2)},\quad \tau^{(9)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_2(E_1)+T_0(E_2)+2T_2(E_2)},
\end{aligned}
\tag{78}
$$

and this reduces for the $\varphi=0°$ plane to

$$
\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2 =\ & (E_1)^{-1}\{x^4y^4[-20.630091\tau^{(1)}+8.841458\tau^{(2)}-8.841458\tau^{(5)}+64.469031\tau^{(7)}-43.839046\tau^{(9)}]\\
&+x^4y^2[20.630091\tau^{(1)}-8.841458\tau^{(2)}+3.536596\tau^{(5)}-56.180166\tau^{(7)}+149.278921\tau^{(9)}]\\
&+x^2y^4[20.630091\tau^{(1)}-8.841458\tau^{(2)}-9.283541\tau^{(3)}-11.051823\tau^{(4)}+2.210383\tau^{(5)}-115.123291\tau^{(7)}+39.970903\tau^{(9)}]\\
&+x^4[-2.578761\tau^{(1)}+1.105183\tau^{(2)}+1.547257\tau^{(5)}+5.986456\tau^{(7)}-65.613752\tau^{(9)}]\\
&+y^4[-2.578761\tau^{(1)}+1.105183\tau^{(2)}+4.641770\tau^{(3)}+5.525919\tau^{(4)}+2.210368\tau^{(5)}-4.297936\tau^{(6)}\\
&+33.769504\tau^{(7)}-9.455458\tau^{(8)}-0.859610\tau^{(9)}]\\
&+x^2y^2[-19.598585\tau^{(1)}+16.356696\tau^{(2)}+15.782019\tau^{(3)}+10.262418\tau^{(4)}+10.830797\tau^{(5)}\\
&+115.123318\tau^{(7)}-148.162842\tau^{(9)}]\\
&+x^2[2.063009\tau^{(1)}-4.862804\tau^{(2)}-4.177593\tau^{(3)}-4.531252\tau^{(4)}-8.067845\tau^{(5)}-25.787612\tau^{(7)}+68.579175\tau^{(9)}]\\
&+y^2[2.063009\tau^{(1)}-4.862804\tau^{(2)}-7.426832\tau^{(3)}-0.887243\tau^{(4)}-7.073179\tau^{(5)}+10.311949\tau^{(6)}\\
&-34.997493\tau^{(7)}+13.409558\tau^{(8)}+22.886649\tau^{(9)}]\\
&+[1.031504\tau^{(1)}+8.841466\tau^{(2)}+8.819363\tau^{(3)}+10.058198\tau^{(4)}+10.057169\tau^{(5)}+7.737315\tau^{(6)}\\
&+28.090080\tau^{(7)}+4.641770\tau^{(8)}+0.152938\tau^{(9)}]\\
&+x^2y^2z[-20.630091\tau^{(1)}+8.841458\tau^{(2)}-8.841458\tau^{(5)}+64.469031\tau^{(7)}-43.839046\tau^{(9)}]\\
&+x^2z[10.315045\tau^{(1)}-4.420726\tau^{(2)}-0.884152\tau^{(5)}-23.945651\tau^{(7)}+127.359403\tau^{(9)}]\\
&+y^2z[10.315045\tau^{(1)}-4.420726\tau^{(2)}-9.283541\tau^{(3)}-11.051832\tau^{(4)}-2.210365\tau^{(5)}-82.888777\tau^{(7)}+18.051385\tau^{(9)}]\\
&+z[-4.126018\tau^{(1)}+9.725608\tau^{(2)}+11.140249\tau^{(3)}+5.304884\tau^{(4)}+11.493912\tau^{(5)}\\
&+46.049354\tau^{(7)}-70.299908\tau^{(9)}]\}\ \text{mb sr}^{-2}.
\end{aligned}
\tag{79}
$$

Its θ_2 dependence for $\theta_1=0°$, $45°$, $90°$ is shown in Fig. 15 for the $P^{31}(n,n'\gamma)$ reaction at $E_n=3.0$ MeV (c.m.).

(iii). 1+ → $J_1\pi_1$ → 0+ → 1+ Sequence

As would be expected, this sequence is very simple in that the limitation to $\lambda=0$ restricts the summation in Eq. (2) to but 17 terms for l_1, l_2, $\leqslant 2$ (with l_1+l_2 even) and yields a θ_2-independent correlation (e.g., essentially a particle distribution) of the form

$$
\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2 = (1/4\pi)(d\sigma/d\Omega_1) = (1/3)\cdot(\lambda\!\!\!^{-2}/32\pi)\{&2\tau^{(1)}+4\tau^{(2)}+\tau^{(3)}[12+0.8P_2(x)]+2\tau^{(4)}\\
&+\tau^{(5)}[8+4P_2(x)]+\tau^{(6)}[12+9.306122P_2(x)-0.734693P_4(x)]\},
\end{aligned}
\tag{80}
$$

with

$$
\begin{aligned}
&\tau^{(1)}\equiv\frac{T_0(E_1)T_0(E_2)}{T_0(E_1)+T_2(E_1)+T_0(E_2)},\quad \tau^{(2)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+T_2(E_2)},\quad \tau^{(3)}\equiv\frac{T_1(E_1)T_1(E_2)}{2T_1(E_1)+T_1(E_2)},\\
&\tau^{(4)}\equiv\frac{T_2(E_1)T_0(E_2)}{T_0(E_1)+T_2(E_1)+T_0(E_2)},\quad \tau^{(5)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+T_2(E_2)},\quad \tau^{(6)}\equiv\frac{T_2(E_1)T_2(E_2)}{2T_2(E_1)+T_2(E_2)}.
\end{aligned}
\tag{81}
$$

In terms of x, Eq. (80) becomes

$$
\begin{aligned}
d^2\sigma/d\Omega_1 d\Omega_2 = (E_1)^{-1}\{&x^4[-2.210364\tau^{(6)}]+x^2[0.825203\tau^{(3)}+4.126017\tau^{(5)}+11.493902\tau^{(6)}]\\
&+[1.375339\tau^{(1)}+2.750678\tau^{(2)}+7.976966\tau^{(3)}+1.375339\tau^{(4)}+4.126017\tau^{(5)}+4.862806\tau^{(6)}]\}\ \text{mb sr}^{-2},
\end{aligned}
\tag{82}
$$

which is illustrated for the $P^{32}(n,n'\gamma)$ reaction at $E_n=1.1$ MeV in Fig. 16.

(iv). $\frac{5}{2}+ \to J_1\pi_1 \to \frac{1}{2}+ \to \frac{5}{2}+$ *Sequence*

The condition $\lambda=0$ renders this correlation essentially a distribution independent of θ_2, φ and the γ multipolarity. The summation of Eq. (2) over 30 terms yields

$$d^2\sigma/d\Omega_1 d\Omega_2 = (1/4\pi)(d\sigma/d\Omega_1) = (1/6)(\lambdabar^2/32\pi)\{10\tau^{(1)}+7\tau^{(2)}+\tau^{(3)}[10-2.5P_2(x)] + \tau^{(4)}[6+0.3P_2(x)]+6\tau^{(5)}+\tau^{(6)}[6-1.071428P_2(x)]+\tau^{(7)}[20-8.418367P_2(x)+0.918368P_4(x)] + \tau^{(8)}[14-1.428571P_2(x)-1.571428P_4(x)]+\tau^{(9)}\}, \quad (83)$$

with

$$\tau^{(1)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+2T_2(E_2)},\quad \tau^{(2)}\equiv\frac{T_0(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+T_2(E_2)},\quad \tau^{(3)}\equiv\frac{T_1(E_1)T_1(E_2)}{2T_1(E_1)+T_1(E_2)},$$
$$\tau^{(4)}\equiv\frac{T_1(E_1)T_1(E_2)}{T_1(E_1)+2T_1(E_2)},\quad \tau^{(5)}\equiv\frac{T_2(E_1)T_0(E_2)}{2T_2(E_1)+T_0(E_2)+T_2(E_2)},\quad \tau^{(6)}\equiv\frac{T_2(E_1)T_2(E_2)}{2T_2(E_1)+T_0(E_2)+T_2(E_2)}, \quad (84)$$
$$\tau^{(7)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+2T_2(E_2)},\quad \tau^{(8)}\equiv\frac{T_2(E_1)T_2(E_2)}{T_0(E_1)+2T_2(E_1)+T_2(E_2)},\quad \tau^{(9)}\equiv\frac{T_2(E_1)T_0(E_2)}{T_2(E_1)+T_0(E_2)}.$$

This is equivalent to

$$d^2\sigma/d\Omega_1 d\Omega_2 = (E_1)^{-1}\{x^4[1.381479\tau^{(7)}-2.363864\tau^{(8)}] + x^2[-1.289381\tau^{(3)}+0.154726\tau^{(4)}+0.552592\tau^{(6)}+5.525916\tau^{(7)}+1.289381\tau^{(8)}] + [3.438348\tau^{(1)}+2.406844\tau^{(2)}+3.868142\tau^{(3)}+2.011434\tau^{(4)}+2.063009\tau^{(5)}+2.247206\tau^{(6)} + 8.442373\tau^{(7)}+4.856667\tau^{(8)}+0.343835\tau^{(9)}]\}\ \text{mb sr}^{-2}, \quad (85)$$

and is plotted in Fig. 17 in function of θ_1 for the inelastic scattering of 1.4-MeV neutrons to the second level of Zr^{91}. The peaking of the curve at $\theta_1=90°$ is akin to the correlation (distribution) behavior for a $\frac{7}{2}- \to J_1\pi_1 \to \frac{1}{2}- \to J_3\pi_3$ sequence (cf. Fig. 12) but the amplitude is slightly larger in the present instance.

(v). $\frac{7}{2}- \to J_1\pi_1 \to \frac{1}{2}- \to \frac{7}{2}-$ *Sequence*

The conditions $\lambda=0$ and $\pi_0\pi_2=+$, render this correlation essentially a particle distribution which is identical with that evaluated in Sec. 4B(*ii*) and illustrated in Fig. 12.

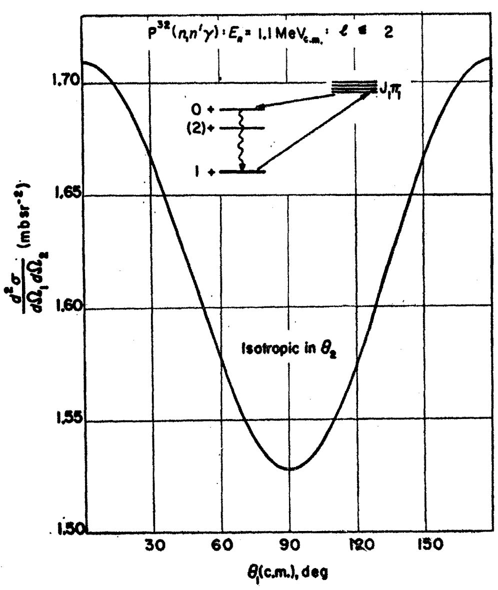

FIG. 16. Double-differential cross section (here equal to $\frac{1}{4}\pi^{-1}d\sigma/d\Omega_1$) for a $1+ \to J_1\pi_1 \to 0+ \to 1+$ transition sequence, illustrated by the $P^{32}(n,n'\gamma)$ reaction at 1.1 MeV (c.m.).

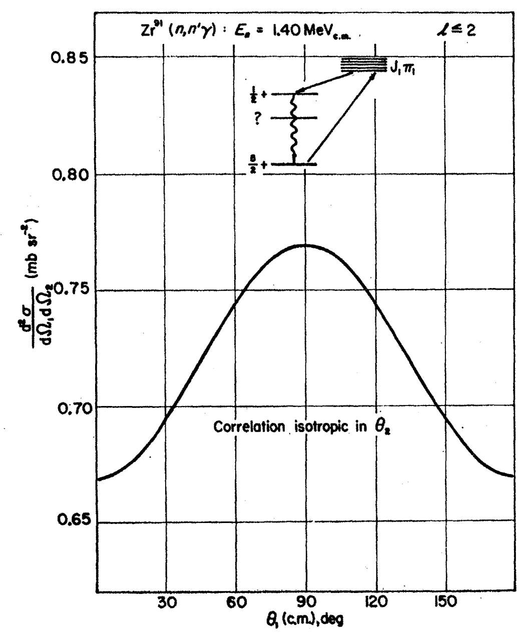

FIG. 17. The correlation (again essentially a distribution) for the $\frac{5}{2}+ \to J_1\pi_1 \to \frac{1}{2}+ \to \frac{5}{2}+$ sequence, illustrated by the $Zr^{91}(n,n'\gamma)$ reaction at 1.4 MeV (c.m.), the γ emission being isotropic irrespective of multipolarity.

5. CONCLUDING REMARKS

Clearly, the results presented in this paper can be applied (as they stand) not only to many other target nuclei than those explicitly cited in the text, but to other levels than have been designated, providing the spin sequence under consideration tallies with one of the cases evaluated. For instance, the formulas derived for a $1+\rightarrow J_1\pi_1\rightarrow 0+\rightarrow 1+$ transition sequence could hold not only for scattering to the *second* level of P^{32} (or, e.g., F^{18}), but in principle also for scattering to the *first* level of P^{30} or N^{14}. In practice, however, the above alternatives would be ruled out experimentally by the short half-lives of F^{18} and P^{30}, or theoretically by the breakdown of the statistical assumption for a nucleus as light as N^{14} bombarded by particles of fairly low energy. The latter preclusion is especially regrettable in view of the rather tantalizingly interesting level scheme of N^{14} (whose levels carry the spin assignments $1+$, $0+$, $1+$, $0-$, $2-$, $1-$, $\cdots$). In fact, in order to obtain a broader spectrum of information on correlation behavior, the requisite calculations *have* been carried through as if the statistical continuum approach were valid, but details of such "*Spielrechnungen*" would be out of place here. As mentioned earlier, other calculations, though performed, have been omitted from the present description since the spin sequences would entail conditions physically unconducive to investigation. Such omissions include the $0+\rightarrow J_1\pi_1\rightarrow 4+\rightarrow 0+$ and $\frac{1}{2}-\rightarrow J_1\pi_1\rightarrow \frac{9}{2}+\rightarrow\frac{1}{2}-$ sequences, respectively involving $E4$ and $M4$ γ multipoles, or $\frac{1}{2}+\rightarrow J_1\pi_1\rightarrow\frac{5}{2}+\rightarrow\frac{3}{2}+$ and $\frac{1}{2}+\rightarrow J_1\pi_1\rightarrow\frac{5}{2}+ \twoheadrightarrow \frac{3}{2}+\rightarrow\frac{1}{2}+$ sequences involving the vanishingly weak $\frac{5}{2}+\rightarrow\frac{3}{2}+$ γ transition in Si^{29} or P^{31}, even though from a theoretical standpoint they offer several features of interest. For example, the last of the above four sequences involves an unobserved and an observed γ transition, both of which may be of mixed multipolarity; on making provision for this, the correlation formula has to be expressed in terms of quantities $M_\lambda^{(2)}M_\lambda^{(3)}$, which provides an opportunity to examine the effect of varying Δ_2 and Δ_3 independently.

The ensemble of data so amassed sheds further light on the manner in which the double-differential cross section for compound inelastic nucleon scattering depends upon diverse physical factors which can vary according to the circumstances of any particular investigation. The basic correlation expression (2) indicates that among other possible parameters, the following can affect the correlation:

(a) inclusion or exclusion of spin-orbit interaction;
(b) the value of the orbital momentum limit $l_{\max}$, which restricts the number of partial waves taken into consideration;
(c) the energies of incident and emergent particles;
(d) the choice of (optical) model;
(e) the azimuth φ under reference;
(f) the existence of additional open exit channels;
(g) the presence of intermediate unobserved transitions (irrespective of their "particle" nature);
(h) the nuclear spins and transition sequence under consideration;
(i) the presence of *mixed* γ multipoles and the magnitude and sign of the mixing ratio.

Of the above, points (a)–(e) have been clarified by the analysis in Ref. 12, albeit exclusively for the $0+\rightarrow J_1\pi_1\rightarrow 2+\rightarrow 0+$ transition sequence. The complementary investigation of the remaining points (f)–(i) constituted the aim of the present work, even though it was clear from the outset that the complexity of the correlation problem would vitiate any attempt to establish an over-all systematic scheme for predicting CN correlation behavior for a given spin sequence and γ multipole mixing ratio. The influence of a given nuclear spin or a certain particle partial wave is too tortuously intertwined within the other variables entering into the correlation calculation for it to retain distinct identity apparent in the final result: "correlations resist correlation."

To summarize some basic results, the detailed comparison of experiment with CN correlation theory in Ref. 12 has not only indicated that the statistical assumption may be made at incident nucleon energies around 5 MeV, providing the target nucleus is not too light ($A\gtrsim 40$), and that CN theory provides remarkably good fits even up to 7 MeV, but that (a) spin-orbit coupling hardly plays any role in the $0+\rightarrow J_1\pi_1\rightarrow 2+\rightarrow 0+$ correlation at these energies, (b) the orbital-momentum cutoff can effectively be taken as $l_{\max}=2$ (or at most 3) under the above conditions, (c) the energy dependence may be appreciable with regard to the magnitude (but not the structure) of the double-differential cross section (Fig. 68 of Ref. 12 and Figs. 9, 13 of the present paper), as may also (d) the dependence upon the optical model parameters (Fig. 69 of Ref. 12). In Ref. 12 was also shown (e) the manner in which the CN correlation varies with azimuth and the similarity (but nonidentity!) was stressed of "perpendicular correlations" (in which either the particle counter or the γ detector is fixed perpendicular to the incident beam, so that $\varphi=90°$) with angular distributions. With regard to the remaining points, the present results indicate (f) a diminution in magnitude but no drastic change in shape to ensue when the double-differential cross section is evaluated on the basis of a "higher-than-two-channel approximation," whereas (g) a radical alteration in both magnitude and shape can occur when an unobserved transition intervenes (for example, Fig. 5 is entirely different from Fig. 4 or from the correlation which results for a $0+\rightarrow J_1\pi_1\rightarrow 2+\rightarrow 0+$ sequence applying to inelastic neutron scattering to the *first* level of the *e-e* target nucleus. Or cf. Fig. 1, in which the correlation is *isotropic* with respect to θ_2). Since the actual nature of the radiation in an unobserved transition does not enter into the treatment of that reaction step, Satchler's theory applies unchanged to consideration of a process such as $0+ \rightarrow J_1\pi_1 \twoheadrightarrow 4+ \rightarrow 2+ \rightarrow 0+$ in which γ-γ cascade coincidences are measured, but the inelastically scattered nucleons are unobserved. Results of such calculations have been quoted in the paper of Broude and Gove[16] and have also been derived independently. The latter unpublished work was applied to the $Fe^{56}(n,n'\gamma)$ reaction at 2.60 MeV; for this case the correlation structure differed very slightly from that deduced by the former authors for Ne^{20}, Mg^{24}, and Si^{28}. Since the Fe^{56} results are absolute, they cannot directly be compared with the latter, which have been expressed in relative units and hence the question as to whether a real discrepancy exists is at present unresolved. Regarding points (h) and (i), the correlation magnitude and structure depend drastically upon the spin transition sequence and upon the γ multipolarity, albeit not in a directly obvious way, and can markedly be influenced by the value and sign of the mixing ratio [see Fig. 8; alternatively, it may be mentioned that a *Spielrechnung* with $\Delta_2=-3$ for the $(M1+E2)$ mixed γ multipolarity in the $0+ \rightarrow J_1\pi_1 \rightarrow 2+ \rightarrow 2+$ sequence applied to $Zn^{66}(n,n'\gamma)$ at $E_n=2.37$ MeV (c.m.) as against the value $\Delta_2=+3$ used in Fig. 2 yielded a correlation which *peaked* around $\theta_2=90°$ and had larger amplitude].

It is thus evident that not only γ-γ but also *particle*-γ correlation studies can yield information on the magnitude and sign of multipole mixing ratios which even now have not fully been established; to a recent compilation,[57] which gives references to earlier work, one might add Ref. 44, and the work by Singhal and Trehan.[58] The latter studies have the additional versatility of furnishing information on reaction mechanism, which may be more sensitive and clear-cut than that offered by angular *distribution* investigations in addition to being potentially capable of indicating the relative admixture of a competing mechanism [see Sec. 4A(*ii*) or Refs. 12, 42, 48 and the "unified reaction" approach of Feshbach *et al.*[59–62]]. It is with the hope of stimulating and clarifying such studies that the present paper has been compiled.

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