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Polarization of Protons Produced in the Stripping of Deuterons by B^{10},Si^{28}, and Ca^{40}†

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The angular dependences of the polarization of protons from the $B^{10}(d,p)B^{11}{}_{\text{g.s.}}$, $Si^{28}(d,p)Si^{29}{}_{\text{g.s.}}$, and $Ca^{40}(d,p)Ca^{41}{}_{\text{g.s.}}$ reactions have been measured using 10-MeV deuterons. The polarizations in all of these reactions were found to exceed the limits set by the central potential distorted wave Born approximation. The polarizations of protons resulting from the $B^{10}(d,p)B^{11}{}_{2.14\text{ MeV}}$ and $B^{10}(d,p)B^{11}{}_{4.46-5.03\text{ MeV}}$ reactions have also been measured at a few angles.

INTRODUCTION

THE distorted-wave Born approximation (hereafter abbreviated DWBA) has been developed in recent years in an effort to understand the deuteron stripping process and to provide a more complete description of it than was possible with the plane wave or Butler theory. Instead of neglecting the interactions of the deuteron and proton with the nucleons, as did Butler, it approximates them through the use of complex optical-model potentials. Many computer calculations based on this theory have been performed to date, and usually a set of optical parameters has been found which gives a better description of the differential cross section than does the Butler theory.[1] Unfortunately, it has also become apparent that the theory is less than completely satisfactory. The shortcomings of the theory are not surprising, since it is a greatly simplified treatment which neglects all processes competing with stripping, such as compound nucleus formation, exchange stripping, etc. Even so, the general DWBA is still very complicated and most calculations have been made using only local central potentials, ignoring spin-dependent or velocity-dependent terms. In order to understand the mechanism of deuteron stripping, as well as other direct interactions, it is necessary to evaluate the nature and magnitude of the effects caused by these additional complications.

The central potential DWBA suffers from several characteristic defects; for example, it usually overemphasizes the minima of the differential cross section, and is often in error by a factor of two or more in predicting the absolute cross section. Its most pronounced defect, however, has been its inability to predict correctly the polarization P of the outgoing protons in the few cases for which this has been measured. For example, the theory predicts an upper limit[2] on the magnitude of the polarization which has already been found to be exceeded by the experimental values.[3] The existence of a limit is a consequence of the use of central distorting potentials which can orient the proton's spin only indirectly. Distortion of the proton and deuteron waves destroys the symmetry of the matrix elements describing the reaction, and thereby orients the angular momentum of the captured neutron. A fraction of this orientation is transferred to the proton's spin through the couplings of the various angular momenta in the final nucleus and in the deuteron ground state; the result is

$$\begin{aligned} P&=-\tfrac{1}{3}\langle m\rangle/l\,, && (j=l-\tfrac{1}{2})\,,\\ P&=+\tfrac{1}{3}\langle m\rangle/(l+1)\,, && (j=l+\tfrac{1}{2})\,. \end{aligned} \tag{1}$$

Here, l and j are the orbital and total angular momenta of the captured neutron, while $\langle m\rangle$ is the expectation value of $\mathbf{L}_z$ given by the theory. Since $\langle m\rangle\leq l$, the polarization magnitude is limited to $\frac{1}{3}$ when $j=l-\frac{1}{2}$, and to $l/3(l+1)$ when $j=l+\frac{1}{2}$.

One of the original reasons for measuring polarization was to obtain nuclear spectroscopic information. The angular distribution in (d,p) reactions yields the momentum transfer l. In order to obtain j it is necessary to know how the spin of the stripped nucleon couples to l, and this, from Eq. (1), should be given by a polarization measurement, assuming the sign of $\langle m\rangle$ is obtainable from theory. The semiclassical theory indicated that the sign of $\langle m\rangle$ should always be the same (positive) in the stripping peak. Early measurements tended to confirm this prediction and led to the postulation of a "sign rule"; however, several exceptions to the rule have been found in more recent measurements.[4]

The failure of the theory to predict the polarization correctly suggests that spin-dependent terms must be included in the distorting potentials. This explanation is strengthened by the experience gained from optical model studies of elastic scattering, in which spin-orbit terms, added in order to explain the polarization, also cured that theory's tendency to overestimate the depths of minima in the differential cross section.

Other reaction mechanisms may be important, how-

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[1] There have been a great number of calculations reported; for example, see W. Tobocman, Phys. Rev. **115**, 98 (1959); W. R. Smith and E. V. Ivash, *ibid.* **128**, 1175 (1962).

[2] R. Huby, M. Y. Refai, and G. R. Satchler, Nucl. Phys. **9**, 94 (1958).

[3] R. G. Allas and F. B. Shull, Phys. Rev. **116**, 996 (1959); **125**, 941 (1962).

[4] See, for example, W. P. Johnson and D. W. Miller, Phys. Rev. **124**, 1190 (1961).

ever, and it has recently been emphasized[5] that resonance processes can have significant effects on reactions in regions where there are a large number of overlapping compound levels (i.e., the continuum region). For light nuclei, at the excitation energies encountered in the present experiment, the density of levels is not necessarily so great that interferences between the compound and direct waves are unimportant. In that case, the compound nucleus contribution cannot be thought of merely as a nearly isotropic unpolarized addition to the cross section, but instead will cause both the differential cross section and polarization to fluctuate rapidly as the energy of the incident particle is varied. Due to the large number of levels participating at any one energy, the energy dependence of the polarization will be quite complicated. While it is difficult to estimate the importance of resonance processes in stripping reactions, the large fluctuations found[6] in $C^{12}(d,p)C^{13}$ caution one against an overly naive use of the DWBA to interpret experimental results.

Since Eq. (1) fails so badly in predicting the polarization and since only a relatively small number of polarizations have been measured, it appeared worthwhile to make additional measurements to see if any systematic features are present and to provide data for more complete theoretical computations. The present paper reports the continuation of earlier work at this laboratory on the polarization of protons produced in (d,p) reactions induced by 10-MeV deuterons.[3,7,8] In order to enhance the value of any theoretical comparisons to them, the measurements have been extended over a greater angular range than has been customary.

EXPERIMENTAL TECHNIQUE

The polarization[9] of the protons produced by the stripping reactions P_1 was determined by the standard technique of observing the left-right asymmetry induced in their scattering from a target (carbon) of known analyzing power. The polarization was then found from the relation

$$P_1 P_{2,\mathrm{eff}} = A - A_\sigma. \tag{2}$$

Here $P_{2,\mathrm{eff}}$ is the effective analyzing power of the polarimeter and A is the left-right asymmetry, given by $A=(N_L-N_R)/(N_L+N_R)$, where N_L and N_R are the numbers detected by the left and right counter telescopes. The factor A_σ is an instrumental asymmetry due to the combined effects of the angular variation of the stripping cross section and the finite size of the carbon target.

The equipment used in this experiment has been described in an earlier paper,[7] but the experimental procedure has been modified and the polarimeter calibrated. Previously, two successive measurements of the asymmetry were made, the polarimeter being rotated through 180° about the proton beam between measurements so that errors due to differences in the counters and their electronics were averaged out. In the present experiment the counting period was broken into about 40 subperiods, with rotation of the polarimeter after each subperiod so that long term drifts in the electronics would not destroy the cancellation of errors.

The polarimeter was calibrated at two energies by using it to analyze a beam of fully polarized protons produced by Rosen's method[10] of scattering an alpha beam with a hydrogen target. The calibration was performed with the 42-MeV alpha beam of the NASA 60-in cyclotron. Using a value of 0.96 ± 0.02 for the effective polarization of the proton beam,[11] the analyzing power of the polarimeter was found to be 0.643 ± 0.021 and 0.642 ± 0.022 at $E_p=16.2$ and 15.4 MeV, respectively. At other energies $P_{2,\mathrm{eff}}$ was assumed to follow the trend of the energy dependence observed by Sanada[12] for the polarization parameter of carbon at 50° (lab). This procedure should be reliable since the angular distribution of the polarization is almost invariant with energy.[13] A complete calibration at all energies was not performed because the large energy spread of the proton beam used for calibration would have given incorrect results in the vicinity of $E_p=12$ MeV, where the polarization drops off rapidly.

EVALUATION OF THE DATA

The asymmetry A was first evaluated for each counter separately, using as N_L and N_R the numbers of protons detected by that counter when in its normal and rotated positions, respectively. The two resulting values for A were averaged to find $\bar{A}$, from which the polarization was then determined through Eq. (2).

As a preliminary step, however, the left and right pulse-height spectra from one counter or the other were added to form a composite spectrum, several of which are shown in Fig. 1. From the composite spectrum, the locations of an upper and a lower cutoff point were determined, shown by arrows in Fig. 1. The same cutoff points were then used with the left and right spectra to determine N_L and N_R for that counter. This procedure inevitably includes a few counts from adjacent proton

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[5] T. Ericson, *Proceedings of the International Conference on Nuclear Structure, Kingston* (University of Toronto Press, Toronto, 1960).

[6] J. E. Evans, J. A. Kuehner, and E. Almqvist, Bull. Am. Phys. Soc. **7**, 60 (1962).

[7] R. G. Allas, R. W. Bercaw, and F. B. Shull, Phys. Rev. **127**, 1252 (1962), and Ref. 3.

[8] Preliminary results for boron and silicon have been reported by R. W. Bercaw and F. B. Shull, Bull. Am. Phys. Soc. **7**, 269 (1962).

[9] The sign of P is taken to be positive if it is parallel to $\mathbf{k}_d\times\mathbf{k}_p$ in accordance with the Basel convention. Here, $\mathbf{k}_d$ and $\mathbf{k}_p$ are the momenta of the deuteron and proton.

[10] L. Rosen and J. E. Brolley, Jr., Phys. Rev. **107**, 1454 (1957).

[11] E. Boschitz, Nucl. Phys. **30**, 468 (1962).

[12] J. Sanada, Helv. Phys. Acta, Suppl. **6**, 249 (1961).

[13] For distributions at 12, 14, 16, and 18 MeV see L. Rosen, J. E. Brolley, and L. Stewart, Phys. Rev. **121**, 1423 (1961); S. Yamabe, M. Kondo, S. Kato, T. Yamazaki, and J. Ruan, J. Phys. Soc. Japan **15**, 2154 (1960); and Ref. 11.

TABLE I. Polarization of protons from $\mathrm{Si}^{28}(d,p)\mathrm{Si}^{29}{}_{\mathrm{g.s.}}$ at $E_d=10$ MeV.[a]

θ_{lab} (deg)	$\theta_{\mathrm{c.m.}}$ (deg)	$\bar{A}$	A_σ	E_p (MeV)	$-P_{2,\mathrm{eff}}$	P_{av}
+7	7.3	+0.006±0.009	−0.017	12.7	0.568	−0.019±0.027
+10	10.3	−0.049±0.012	−0.039	12.7	0.568	+0.013±0.024
+10	10.3	+0.035±0.016	−0.039	13.3	0.594	
+12	12.5	+0.028±0.012	−0.052	12.7	0.568	−0.042±0.034
+15	15.6	−0.059±0.020	−0.065	13.1	0.584	+0.024±0.028
+15	15.6	−0.091±0.018	−0.060	14.0	0.618	
+18	18.7	−0.112±0.022	−0.073	13.1	0.584	+0.075±0.032
+18	18.7	−0.116±0.022	−0.067	13.9	0.611	
+21	21.8	−0.104±0.034	−0.055	14.0	0.618	+0.073±0.045
−21	21.8	−0.080±0.031	−0.055	14.0	0.618	
−25	25.9	+0.116±0.035	−0.018	14.9	0.636	−0.211±0.059
+30	31.1	+0.045±0.030	+0.012	13.9	0.611	−0.041±0.041
−30	31.1	+0.027±0.043	+0.018	15.2	0.636	
+34.5	35.9	+0.117±0.031	0.000	14.4	0.631	−0.186±0.053
39	40.6	+0.079±0.031	−0.021	15.0	0.636	−0.157±0.052
−45	46.7	+0.028±0.064	−0.060	14.7	0.636	−0.050±0.103
−58	59.8	−0.098±0.061	+0.037	14.5	0.636	+0.212±0.099
+65	66.9	−0.082±0.050	+0.012	14.5	0.636	+0.132±0.053
−65	66.9	−0.065±0.043	+0.012	14.5	0.636	
−72.5	74.6	−0.048±0.055	−0.011	14.3	0.626	+0.059±0.089
+80	82.2	+0.029±0.051	−0.034	14.2	0.626	−0.101±0.086

[a] Polarization is defined to be positive when it is parallel to $\mathbf{k}_d \times \mathbf{k}_p$.

TABLE II. Polarization from $\mathrm{B}^{10}(d,p)\mathrm{B}^{11}$: ground state.

θ_{lab} (deg)	$\theta_{\mathrm{c.m.}}$ (deg)	$\bar{A}$	A_σ	E_p (Mev)	$-P_{2,\mathrm{eff}}$	P_{av}
+ 11	12.0	−0.063±0.030	+0.028	16.5	0.610	0.157±0.038
+ 11	12.0	−0.068±0.026	+0.028	17.1	0.585	
+ 13	14.2	−0.097±0.030	+0.024	17.1	0.585	+0.170±0.029
+ 13	14.2	−0.064±0.029	+0.024	16.5	0.610	
+ 13	14.2	−0.079±0.028	+0.024	17.1	0.585	
+ 13	14.2	−0.077±0.029	+0.024	15.9	0.636	
+ 15	16.4	−0.093±0.021	+0.016	17.8	0.559	+0.191±0.035
+ 15	16.4	−0.093±0.028	+0.016	17.1	0.585	
+ 19	20.7	−0.104±0.048	+0.003	17.7	0.559	+0.172±0.032
+ 19	20.7	−0.082±0.024	+0.003	17.7	0.559	
+ 22.5	24.5	−0.084±0.029	−0.009	18.0	0.542	+0.138±0.056
+ 27	29.4	−0.127±0.022	−0.026	17.5	0.567	+0.180±0.043
+ 30	32.7	−0.142±0.020	−0.030	17.9	0.551	+0.205±0.043
+ 37.5	40.8	−0.195±0.041	−0.029	17.9	0.551	+0.302±0.080
+ 45	48.8	−0.171±0.029	−0.007	17.4	0.585	+0.288±0.054
+ 52.5	56.8	−0.146±0.040	+0.011	16.9	0.593	+0.248±0.050
− 52.5	56.8	−0.126±0.036	+0.011	17.0	0.585	
− 60	64.6	−0.117±0.028	+0.006	16.6	0.602	+0.204±0.050
− 67.5	72.5	−0.090±0.031	−0.006	16.2	0.618	+0.136±0.054
− 75	80.2	−0.052±0.034	−0.016	15.8	0.636	+0.057±0.055
+ 90	95.4	−0.013±0.034	0.000	15.0	0.636	+0.020±0.055
+105	110.2	+0.034±0.037	0.000	14.5	0.636	−0.053±0.060
+112.5	117.5	+0.007±0.049	0.000	13.2	0.593	−0.012±0.084
+120	124.6	−0.080±0.059	0.000	12.9	0.577	+0.060±0.069
+120	124.6	0.000±0.052	0.000	13.0	0.585	
+130	134.0	−0.048±0.055	0.000	12.6	0.559	+0.086±0.100

TABLE III. Polarization from $\mathrm{B}^{10}(d,p)\mathrm{B}^{11}$: first excited state (2.14 MeV).[a]

θ_{lab} (deg)	$\theta_{\mathrm{c.m.}}$ (deg)	$\bar{A}$	A_σ	E_p (MeV)	$-P_{2,\mathrm{eff}}$	P_{av}
−60	65.0	+0.076±0.055	+0.006	14.5	0.636	−0.110±0.118
−67.5	72.8	+0.074±0.058	+0.002	14.2	0.626	−0.115±0.123
−75	80.5	+0.147±0.054	−0.002	13.7	0.610	−0.244±0.124

Statistical uncertainty includes +0.010 to account for the contamination of the proton peak by neighboring peaks.

TABLE IV. Polarization from $B^{10}(d,p)B^{10}$: unresolved second and third excited states (4.46, 5.03 MeV).[a]

θ_{lab} (deg)	$\theta_{\text{c.m.}}$ (deg)	$\bar{A}$	A_σ	E_p (MeV)	$-P_{2,\text{eff}}$	P_{av}
+22.5	24.9	−0.059±0.025	−0.006	13.2	0.593	+0.092±0.060
+27	29.7	−0.100±0.018	−0.010	12.7	0.568	+0.158±0.054
+30	33.1	−0.086±0.016	−0.022	13.3	0.593	+0.107±0.050
+37.5	41.3	−0.091±0.032	−0.028	13.3	0.593	+0.106±0.075
+45	49.4	−0.084±0.025	−0.015	12.8	0.576	+0.119±0.063
−52.5	57.4	−0.036±0.029	−0.007	12.5	0.559	+0.050±0.067

[a] Correction for contamination by ground-state protons via $C^{12}(p,p')C^{12}$ is included in A_σ.

TABLE V. Polarization from $Ca^{40}(d,p)Ca^{41}{}_{\text{g.s.}}$.

θ_{lab} (deg)	$\theta_{\text{c.m.}}$ (deg)	$\bar{A}$	A_σ	E_p (MeV)	$-P_{2,\text{eff}}$	P_{av}
+ 15	15.4	+0.019±0.038	+0.018	14.5	0.636	−0.002±0.063
− 20	20.5	+0.053±0.036	+0.019	14.5	0.636	−0.053±0.060
− 25	25.6	+0.087±0.029	+0.018	14.4	0.631	−0.109±0.051
+ 30	30.8	+0.081±0.046	+0.011	15.1	0.636	−0.110±0.074
+ 37.5	38.5	+0.055±0.039	+0.003	15.0	0.636	−0.028±0.038
− 37.5	38.5	−0.006±0.030	+0.003	14.9	0.636	
+ 45	46.1	+0.039±0.034	−0.017	14.3	0.626	−0.059±0.044
− 45	46.1	0.000±0.036	−0.017	14.8	0.636	
− 52	53.7	+0.035±0.030	−0.019	14.9	0.636	−0.085±0.058
− 60	61.4	+0.076±0.046	−0.008	14.7	0.636	−0.132±0.076
+ 63.7	64.1	+0.095±0.041	−0.002	13.9	0.611	−0.159±0.069
+ 67.5	69.0	+0.084±0.037	0.000	14.5	0.636	−0.185±0.048
− 67.5	69.0	+0.171±0.048	0.000	14.6	0.636	
+ 71	72.5	−0.075±0.035	+0.002	14.4	0.631	−0.116±0.066
+ 75	76.5	+0.087±0.041	−0.002	14.3	0.626	−0.149±0.045
− 75	76.5	+0.096±0.032	−0.002	14.5	0.636	
− 80	81.6	+0.086±0.035	−0.006	14.3	0.626	−0.147±0.068
+ 90	91.6	+0.089±0.036	−0.009	13.6	0.603	−0.178±0.050
+ 90	91.6	+0.124±0.054	−0.009	14.1	0.618	
+ 97.5	99.1	+0.216±0.036	−0.009	14.2	0.626	−0.359±0.071
+105	106.5	+0.202±0.043	−0.005	13.9	0.618	−0.335±0.075
−112.5	114.0	+0.217±0.054	−0.005	13.6	0.594	−0.365±0.095
+120	121.4	+0.113±0.047	−0.003	13.7	0.593	−0.195±0.082
+129	130.3	−0.037±0.054	0.000	13.3	0.594	+0.063±0.093

peaks, but the effect of this contamination was found to be a small fraction of the statistical error in several test cases which were evaluated by making Gaussian fits to the peaks. The values found for the polarization are listed in Tables I through V, as well as being given in the figures. Errors given in the tables were determined from

$$\delta P_1 = [(\delta\bar{A})^2 + \Delta^2 + (P_1 \delta P_2)^2]^{1/2} / P_{2,\text{eff}},$$

where $\delta P_2 = \pm 0.030$ and $\delta\bar{A}$ is the statistical fluctuation of $\bar{A}$. The additional factor Δ is included to account for miscellaneous uncertainties, such as those occurring in the determination of A_σ. It was estimated to be ±0.01, except in the measurement of $B^{10}(d,p)B^{11}{}_{2.14\text{ MeV}}$, where the estimate was increased to ±0.02 to account for contamination of the peak from the much larger ground-state and second excited state peaks.

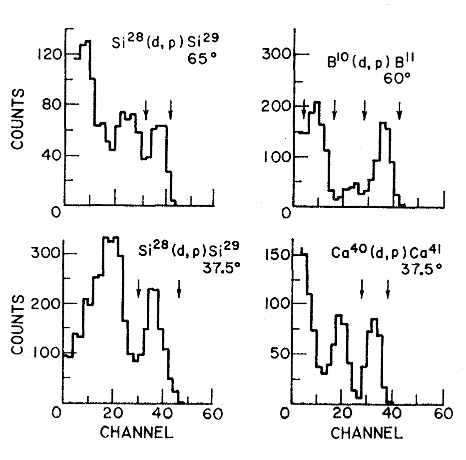

FIG. 1. Typical pulse-height spectra produced by the polarimeter. The vertical arrows are at the cutoff points.

The relative differential cross sections of the reactions are also shown in the figures for reference. These were taken with the same equipment used to measure polarization,[14] except that a single counter replaced the polarimeter, and a monitor counter was used to determine the counting intervals.

Although the experiment was designed to measure

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[14] The cross section for $B^{10}(d,p)B^{11}$ is taken from B. Zeidman and J. M. Fowler, Phys. Rev. **112**, 2020 (1958).

the ground state polarizations, it was possible at the same time to measure the polarization at a few angles for the $B^{10}(d,p)B^{11}$ reaction to the 2.14-MeV state and to the unresolved 4.46- and 5.03-MeV states. In evaluating the latter polarization a correction had to be applied because the peak was contaminated with protons which resulted from $B^{10}(d,p)B^{11}{}_{g.s.}$ and then were inelastically scattered by the carbon analyzer. This process was important because the inelastic scattering cross section to the 4.43-MeV state of carbon is almost one-half the elastic cross section. Fortunately, both the cross section and polarization of this inelastic scattering are known, allowing its contribution to be subtracted. Due to the similarities of the polarizations from the different levels of B^{11} and the similarities of the polarization from the carbon elastic and inelastic scatterings the corrections were less than 0.01 except at one angle, where it was 0.02.

DISCUSSION OF RESULTS

$Si^{28}(d,p)Si^{29}$

The polarization values determined for $Si^{28}(d,p)^{29}{}_{g.s.}$, given in Table I and Fig. 2, are nonzero over the entire angular range. Since the neutron is captured into an $l=0$ state, this result is in immediate disagreement with the central potential DWBA, which predicts zero polarization for such transitions. The most likely reason for the disagreement here is the need for spin-orbit terms in the distorting potentials. Although it is not possible to eliminate compound nucleus formation completely as a factor because of the fixed energy of the cyclotron beam, most evidence indicates that it is unimportant. Kuehner *et al.*,[15] measured the energy dependence of the same reaction with high resolution, and estimate that compound nucleus formation cannot contribute more than about 15% to this reaction. In addition, there is a considerable amount of averaging over the resonances because of our low energy resolution. The combined widths of the target and beam is about 500 keV, considerably larger than the correlation width of the compound levels measured by Kuehner (<100 keV). Additional evidence supporting the neglect of compound nucleus formation comes from a comparison of the data with a similar measurement by Isoya *et al.*,[16] shown in Fig. 2. It would be surprising to find the distributions so similar if interferences are significant.

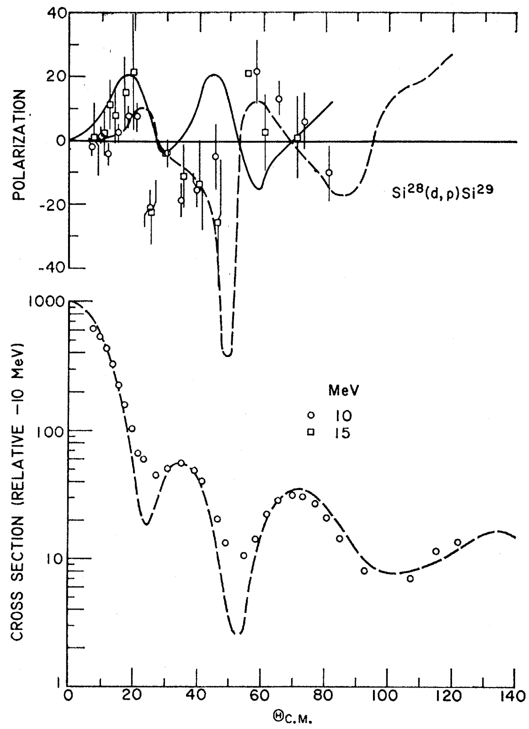

FIG. 2. Polarization of protons from the reaction $Si^{28}(d,p)Si^{29}{}_{g.s.}$ at $E_d=10$ and 15 MeV (the latter from Ref. 16). The solid line is the logarithmic derivative of the relative cross section at 10 MeV. The dashed lines are calculated on the basis of the DWBA using spin-orbit terms in both the proton and deuteron potentials. They are taken from Ref. 17.

The polarization is also compared in Fig. 2 with several theories incorporating spin-orbit potentials. The result of the most exact and most successful of these is shown by the dashed curves. It is a computer computation of both the cross section and the polarization by W. R. Smith,[17] who uses tapered wells, similar to the Woods-Saxon form, for the central real and imaginary potentials, and a real spin-orbit term of the Thomas-Fermi type. The potentials have the following parameters: $V_p=46.0$ MeV, $V_d=96.0$ MeV, $W_p=2.0$ MeV, $W_d=6.0$ MeV, $V_{so,p}=3.0$ MeV, $V_{so,d}=15.0$ MeV, $R_p=3.793$ F, $R_d=R_n=4.25$ F, $a_p=a_d=0.5$ F, $a_n=0.7$ F, where V, W, V_{so} are respectively the depths of the real and imaginary central potentials and the real spin-orbit potential, while R and a are their radii and diffuseness parameters, in fermis. As can be seen, the cross section is well described, but there is only a general similarity between the experimental and theoretical polarization distributions.

An approximate theory based on general reaction theory has been proposed by Biedenharn and Satchler.[18] They ignore the deuteron spin-orbit potential and consider the phase shifts produced in the numerous partial waves by the proton spin-orbit potential. Arguing that it is plausible that the phase shifts should be nearly the same for all the waves, they show that the polarization should then be proportional to the logarithmic derivative of the differential cross section, $[d\sigma(\theta)/d\theta]/\sigma(\theta)$. This function is shown as a solid line in Fig. 2. Although it

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[15] J. A. Kuehner, E. Almqvist, and D. A. Bromley, Nucl. Phys. **21**, 555 (1960).

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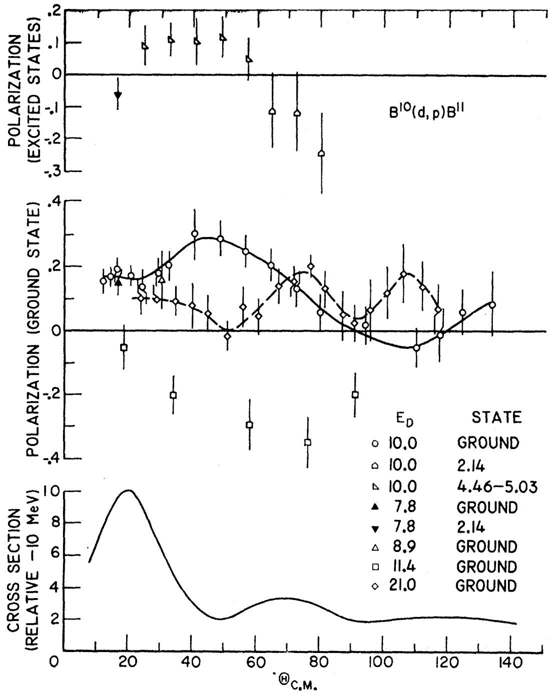

FIG. 3. The top part of the figure shows the polarization of protons from the $B^{10}(d,p)^{11}$ reaction to the 2.14- and 4.46–5.03-MeV levels of B^{11}, while the middle section shows the energy dependence of the polarization of the protons from $B^{10}(d,p)B^{11}{}_{g.s.}$. The lines through the data points are guides to the eye. References are given in the text. The cross section at 10 MeV, shown in the lower section, is taken from Ref. 14.

compares well with the data in the extreme forward direction, the agreement does not hold up at larger angles and is generally poor.

Another approximate theory based on the DWBA by R. C. Johnson[19] agrees somewhat better with the data. He also neglects the deuteron spin-orbit potential, but uses Green's function techniques to expand the spin-dependent wave functions in terms of the central potential wave functions. The final expression is quite complicated, involving several overlap integrals containing distorted wave functions; however, if central distortions are also ignored and cutoff plane waves are used, the expression reduces to a term which is slowly varying with the angle (for low and medium energy stripping reactions) divided by the Butler amplitude. Therefore the polarization distribution should roughly follow the inverse of the Butler amplitude, or $[\sigma(\theta)]^{-1/2}$, rather than $[d\sigma(\theta)/d\theta]/\sigma(\theta)$. In addition, it can be shown that the slowly varying term cannot have any zeros for angles less than 65°, so that the polarization can change sign only at the minima of the cross section, where the Butler amplitude reverses sign. It can be seen from Fig. 2 that the polarization does indeed change sign at 22° and 58°, near the minima of the cross section; there is perhaps also some indication that it varies as $[\sigma(\theta)]^{-1/2}$.

$B^{10}(d,p)B^{11}$

The neutron is captured into a $j=\frac{3}{2}$, $l=1$ state in the ground-state reaction, so that the central potential DWBA would limit the magnitude of the polarization to $\frac{1}{6}$. The data, listed in Table II, are in disagreement with that theory since the limit is exceeded at a number of points outside the stripping peak. It is also exceeded at points inside the peak but by less than the statistical error. All available polarization measurements at various energies are shown in Fig. 3 in order to investigate the reasons for the failure of the theory. As can be seen, the present data are in excellent agreement with isolated points at[20] 7.8 MeV and[21] 11.4 MeV. The data at[22] 21.6 MeV have the same sign as that of the present work, but at twice the energy show more structure, as expected on the basis of the DWBA. The above results, together with Satchler's[23] successful fitting of the energy variation of the differential cross section using the DWBA, indicate that the polarization should be describable with the DWBA including spin-dependent terms. There are, however, the data at[24] 11.4 MeV which are similar to the present polarizations but are reversed in sign. Since the DWBA cannot give such rapid energy variations, resonance effects must be important near 11 MeV. This is a surprising result even in so light a nucleus, since the excitation energy of the compound nucleus is over 30 MeV. It would be interesting to check the energy dependence of the reaction in this energy region at one or more angles.

The polarizations produced by the reactions going to the state at 2.14 MeV and to unresolved states at 4.46 and 5.03 MeV in B^{11} are given in Tables III and IV, and are also presented in the upper part of Fig. 3 for comparison with the ground-state results. In these reactions the neutron is again captured into $l=1$ states, but the j values are not known with certainty. It is interesting to use the sign rule mentioned in the introduction to predict these j values. Even though the central potential DWBA is not valid, there is some justification in applying such a rule to compare stripping reactions proceeding to different levels of the same nucleus when they involve the same l value. If the j values are also the same, the two reactions will be equivalent except for differences caused by the change in the binding energy of the neutron. Unless reasonance effects are important, such

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differences will be small and the polarizations will be quite similar. If the j values are not the same, different partial waves will participate in the reaction and less similarity would be expected in the polarizations. The great similarity between the polarizations of the ground state and 4.46–5.03 MeV state reactions would argue on this basis that the neutron is also captured with $j=\frac{3}{2}$ in the 4.46-MeV level,[25] while the lack of similarity between the ground-state and 2.14-MeV reactions indicates that the neutron is captured with $j=\frac{1}{2}$.

The spin and parity of the 2.14-MeV state of B^{11} have been the subject of a number of papers because the results of stripping experiments were apparently in conflict with the majority of evidence[26] that it was a $J^{\pi}=\frac{1}{2}^{-}$ state. If the neutron were captured into an $l=1$ state, as predicted by the shell model and indicated by experiment,[27] it could carry in a maximum of $\frac{3}{2}$ units of angular momentum. Since the ground state of B^{10} is 3^{+}, this additional angular momentum is insufficient to form a spin $\frac{1}{2}$ state and the reaction cannot proceed. If, on the other hand, the neutron were captured into an $l=2$ state, the parity of the final level would be plus. The neutron would thus have to be captured into an $l=3$ orbital in order to satisfy the assignment. Alternatively, Hensel and Parkinson[20] suggested that some mechanism flips the spin of the outgoing proton, adding another unit of angular momentum to the nucleus, and therefore allowing an $l=1$ stripping to give a $J^{\pi}=\frac{1}{2}^{-}$ final state. Both their measurement at 7.8 MeV and the present measurement give a negative polarization indicating that such a flip occurs. In addition, the general failure of the central potential DWBA removes the justification for assuming that all the angular momentum change must be brought in by the neutron.

$Ca^{40}(d,p)Ca^{41}{}_{g.s.}$

The results of the measurements on $Ca^{40}(d,p)Ca^{41}{}_{g.s.}$ are given in Table V and Fig. 4. Here the neutron is captured into an $l=3$, $j=\frac{7}{2}$ state, so that $j=l+\frac{1}{2}$. The central potential DWBA again fails here, since the observed polarization exceeds the limit of $\frac{1}{4}$ set by Eq. 1. The data are compared in Fig. 4 with other measurements taken at[24] 11.4 MeV and at[22] 21.6 MeV, and seem to be in good agreement with them. As in the case of $B^{10}(d,p)B^{11}{}_{g.s.}$, the 21.6-MeV data show a greater amount of structure, in keeping with the higher energy. Both this agreement and the greater mass of the Ca^{40} nucleus lead one to believe that the DWBA with spin-orbit potentials should be valid. Unfortunately, there have been as yet no attempts to fit any of the distributions with theoretical curves.

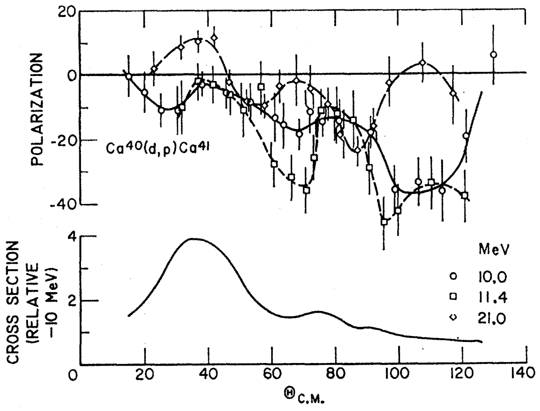

FIG. 4. The energy dependence of the polarization of protons from $Ca^{40}(d,p)Ca^{41}{}_{g.s.}$. The lines through the data points are guides to the eye. References are given in the text. The lower section shows the cross section at 10 MeV.

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