these calculations, it was assumed[34] that the π^- was captured at a random position within the nucleus by a proton-proton pair and a neutron-proton pair with equal probability. The latter assumption has been shown experimentally to be invalid.[6] In the earlier work of Puppi *et al.*,[30,31] this effect was taken into account and more captures were assumed on neutron-proton pairs than proton-proton pairs. Our results are shown plotted with the Monte Carlo results[33] as histograms in Figs. 18 through 20. Our data (from Table III) are combined into energy intervals corresponding approximately to those of the Monte Carlo calculations. In Fig. 19, we have plotted our ${}_{48}Cd^{112}$ results with the Monte Carlo results for ${}_{44}Ru^{100}$. The experimental results include the evaporation as well as the "direct" neutrons while the Monte Carlo give only the "direct" neutrons. The evaporation process is, however, negligible above 20 MeV and the spectra can be compared above this energy. The general agreement is good considering the simplicity of the model used in the calculations.

[34] N. Metropolis, R. Bivins, M. Storm, J. M. Miller, G. Friedlander, and A. Turkevich, Phys. Rev. **110**, 204 (1958).

ACKNOWLEDGMENTS

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Neutron Transfer in $N^{14}(N^{14},N^{13})N^{15}$ at Low Energies*

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Formulas concerned with approximate quantum-mechanical corrections to the semiclassical (SC) treatment of the neutron transfer reaction previously contained or implied in the literature are extended and put in a more convenient form for application. New data of McIntyre, Jobes, and Becker in the laboratory energy range 9.0 to 18.0 MeV are compared with theoretical expectation. The accuracy of a previous discussion of 18-MeV data by one of the authors is improved. At 18 and at 12.6 MeV reasonable agreement with experimental angular distributions is found close to 90° in the c.m. system. At smaller angles the experimental values of the cross section are below those calculated at 18 MeV, in agreement with the influence of absorption on the recoils suggested for this case by McIntyre and Jobes; at 12.3 MeV the experimental values are somewhat smaller than the theoretical as would be the case in the presence of virtual Coulomb excitation (VCE). Total transfer cross sections show a systematic increase over expectation by a factor of about 2.9 between 9.0 and 12.8 MeV as though some VCE were present. The calculated ratio of the 90° cross section at 18 MeV to that at 12.6 MeV is about 10 times that observed. Possible explanations of this discrepancy are discussed.

I. INTRODUCTION AND NOTATION

THE neutron tunneling mechanism with special reference to the $N^{14}(N^{14},N^{13})N^{15}$ reaction has been discussed by Breit and Ebel,[1] Ebel,[2] Breit and Ebel,[3] and Breit[4] from related viewpoints. Although the quantum mechanical description of the phenomenon was already used in some of the early formulations such as,[1–3] the formulas in these papers that are useful in direct practical applications are mostly those obtained in the semiclassical (SC) approximation. The magnitude of the corrections for the quantum-mechanical character of the motion of the heavy particles has been estimated employing a δ-function potential as a representation of the effect of one of the nuclei,[4] making use of Ter-Martirosyan's evaluation[5] of an integral occurring in the theory of (n,d) reactions in terms of the hypergeometric function which is very similar to the integral obtained by Biedenharn, Boyer, and Goldstein[6] at about the same time. A treatment of the neutron

* Work supported by the U. S. Atomic Energy Commission and by the U. S. Air Force Office of Scientific Research.
[1] G. Breit and M. E. Ebel, Phys. Rev. **103**, 679 (1956). The abbreviation BE-I for this reference is occasionally used.
[2] M. E. Ebel, Phys. Rev. **103**, 958 (1956).
[3] G. Breit and M. E. Ebel, Phys. Rev. **104**, 1030 (1956). The abbreviation BE-II for this reference is occasionally used.
[4] G. Breit, *Handbuch der Physik* edited by S. Flügge (Springer-Verlag, Berlin, 1959), **41.1**, especially Sec. 48.
[5] K. A. Ter-Martirosyan, Zh. Eksperim. i Teor. Fiz. **29**, 713 (1955) [translation: Soviet Phys.—JETP **2**, 620 (1956)].
[6] L. C. Biedenharn, K. Boyer, and M. Goldstein, Phys. Rev. **104**, 383 (1956).

transfer mechanism has been discussed by Greider[7] from the viewpoint of the T matrix. The quantum-mechanical corrections derived in Ref. 4 have been considered more fully in Refs. 8 and 9 without the aid of the δ-function potential. According to Ref. 8, the same quantum-mechanical corrections for the angular distribution and the energy dependence as have been found[4] are expected to apply provided the omission of various terms listed[8] do not lead to too large corrections. In both cases the transfer has been treated as isotropic regarding the orientation of the neutron wave function, a procedure approximately justified for the reaction under consideration by the calculation[1] of the p-shell effects employing plausible but doubtless not completely accurate assignments of nucleon configurations. The applicability of the quantum-mechanical corrections is subject to the validity of a number of simplifying assumptions and approximations mentioned.[8,9] The virtual Coulomb excitation (VCE) process is expected[3] to have some influence on the differential cross section and the energy dependence of the total cross section.

The earlier data[10] have not distinguished between transfers to different nuclear levels. The addition of new measurements[11–13] which differentiate between transitions into different nuclear states makes a reconsideration of previous attempts at interpretation desirable even though a complete quantitative interpretation is not possible at this time since there are many incompletely worked out phases[8,9] of the problem even apart from the VCE.

A few symbols the meaning of which may be hard to ascertain in the text are listed below.

$\alpha=(2M_n|E_s|/\hbar^2)^{\frac{1}{2}}$ = reciprocal range constant of transferred neutron [cf. Eq. (4.1)]; M_n = effective reduced mass of neutron motion; $|E_s|$ = neutron separation energy; θ = scattering angle in center of mass system; E_l = laboratory energy of incident nucleus; r = distance between the two nuclei.

II. THE PHASE FACTOR

On account of the identity of the colliding particles there is an interference effect between the final state waves corresponding to the incident N^{14} having become N^{13} emerging at angle θ in the center of mass system and the target N^{14} having become N^{13} emerging at the same angle while the incident N^{14} has become N^{15} emerging at angle $\pi-\theta$. Equation (16.7) of[1] takes this effect into account. In the interpretation[8] of the 18-MeV data of Jobes and McIntyre,[12] it has been noticed that the low value of the reaction cross section σ at $\theta=90°$ was consistent with the expected factor $\frac{2}{3}$ mentioned in[1] as the ratio of σ at 90° to the value expected in the absence of interference effects, i.e., to the extrapolation of the σ versus θ curve drawn through the nodes of the interference wiggles. It was then also noticed from an approximate estimate of the relative phases of the scattering amplitude $f(\theta)$ at different angles that the data were approximately consistent with the type of curve expected from Eq. (16.7).[1] The phase factor of f in the approximations[4,8] can be evaluated employing the hypergeometric function ${}_2F_1$ occurring.[5,6] This function will be denoted by F below. In Ref. 5 there is available an approximation to $|F|^2$ obtained by the steepest descents method applied to a contour integral representation of F. The approximate phase used in Ref. 8 was obtained by means of the steepest descents approximation and an explicit derivation is presented below for the case of a negligible reaction Q value. The integral entering the matrix element essentially in the notation[4] is

$$I_0(\theta)=1/(4\pi)^{\frac{1}{2}}\int\psi^{(-)*}(\eta_f,\mathbf{k}_f;\mathbf{r})\times\psi(\eta_i,\mathbf{k}_i;\mathbf{r})(e^{-\alpha r}/r)d\mathbf{r}$$
$$=4\pi^{\frac{3}{2}}(\eta/\alpha^2)e^{-2\pi\eta}[1-(2ik/\alpha)]^{2i\eta}\times[F(-i\eta,-i\eta;1;-\zeta)/(1+\zeta)^{2i\eta+1}],\quad(2.1)$$

with

$$\zeta=4k^2\mathbf{s}^2/\alpha^2,\quad \mathbf{s}=\sin(\theta/2)\quad(2.2)$$

and

$$\eta=Z_1Z_2e^2/\hbar v.\quad(2.3)$$

Here Z_1, Z_2 are the charges on the two nuclei, v is the relative velocity, $k/(2\pi)$ is the wave number, and the other symbols have their usual significance. In the steepest descents approximation, as in the work of Ter-Martirosyan,[5]

$$F(i\eta,i\eta;1;-\zeta)=[(1+\zeta)^{-i\eta+\frac{1}{2}}/2(\pi\eta)^{\frac{1}{2}}\zeta^{\frac{1}{4}}]\times\exp[\pi\eta-2\eta\tan^{-1}(\zeta^{-\frac{1}{2}})+i\tan^{-1}(\zeta^{\frac{1}{2}})].\quad(2.4)$$

Combining (2.4) with (2.1) a short calculation gives

$$\arg I_0(\theta)=-\pi/2+\Phi,\quad(2.5)$$

$$\Phi=\eta\ln[(\alpha^2+4k^2)/(\alpha^2+4k^2\mathbf{s}^2)]+\tan^{-1}(\alpha/2k\mathbf{s}).\quad(2.6)$$

For $\alpha\rightarrow0$ this gives

$$\arg I_0(\theta)\rightarrow-\pi/2-\eta\ln\mathbf{s}^2;\ (\alpha=0).\quad(2.7)$$

In this limit the relative phases for different angles are thus such as would be obtained for Coulomb scattering,

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[7] K. R. Greider, Phys. Rev. Letters **9**, 392 (1962); in *Proceedings of the International Symposium on Direct Interactions and Nuclear Reaction Mechanisms, Padua, September 1962* (Gordon and Breach, New York, 1963); in Third Conference on Reactions Between Complex Nuclei, Asilomar, April 1963.

[8] G. Breit, in *Proceedings of the International Symposium on Direct Interactions and Nuclear Reaction Mechanisms, Padua, September 1962* (Gordon and Breach, New York, 1963).

[9] G. Breit, in Third Conference on Reactions Between Complex Nuclei, Asilomar, April 1963.

[10] H. L. Reynolds and A. Zucker, Phys. Rev. **101**, 166 (1956).

[11] K. S. Toth, Phys. Rev. **121**, 1190 (1961); **123**, 582 (1961).

[12] F. C. Jobes and J. A. McIntyre (private communication); cf. also L. C. Becker, F. C. Jobes, and J. A. McIntyre, in Third Conference on Reactions between Complex Nuclei, Asilomar, April 1963 (unpublished).

[13] L. C. Becker and J. A. McIntyre (private communication); cf. also last reference under Ref. 12.

in agreement with the discussion[1] of the connection of the classical and quantum treatments of the motion of the heavy aggregates. An expression for the phase of the amplitude equivalent to that in (2.6) has been stated by Greider in the last of the references in Ref. 7 for a different though somewhat similar approximation.

III. EQUATIONS FOR THE CROSS SECTION

In Ref. 1 the calculations in the SC approximation for nonidentical particles the results are expressed first by means of the exact result of integrating over the classical orbit as in Eq. (23) and then by means of an approximation to the Bessel function of imaginary argument of the second kind, K_0, as in (32.1) leading, respectively, to the two forms for the transfer probability in Eq. (25). The second of these corresponds to (23.1) and is convenient in numerical work. Equation (25.2) of Ref. 1 corresponds to this approximation to the SC approach. It will be convenient to work below in this simplified approximation. Since, in what follows, the SC approach plays only the role of a reference standard it is immaterial which of the two forms in Eq. (25) of Ref. 1 is used provided the final angular distribution corresponds to the quantum-mechanical calculation.

The formulas for the cross section can then be expressed in terms of quantities A, A_{SC} representing, respectively, on the same relative scale the absolute values of the scattering amplitudes on the quantum-mechanical (QM) and the SC viewpoints. Here

$$A_{\text{SC}}=(1/\mathbf{s}^{\frac{3}{2}})\exp(-\alpha a'/\mathbf{s}) \tag{3.1}$$

$$A/A_{\text{SC}}=(1+u^2)^{-\frac{3}{4}}\exp[2\eta(u-\tan^{-1}u)], \tag{3.2}$$

where

$$u=\alpha/(2k\mathbf{s})=\zeta^{-\frac{1}{2}} \tag{3.3}$$

and

$$a'=\eta/k \tag{3.4}$$

is one half of the classical distance of closest approach in a head-on collision. For nonidentical particles

$$\sigma/\sigma_{\text{SC}}=(A/A_{\text{SC}})^2 \quad \text{(nonidentical particles)}, \tag{3.5}$$

in agreement with Eq. (48.33) of Ref. 4 and the approximation to that equation in (48.34) of the same reference within the limits of validity of the latter.

On account of the identity of the colliding N^{14} nuclei there enters in addition the previously mentioned interference effect as in Eq. (16.7).[1] In the notation used here there corresponds to that equation the replacement

$$A^2(\theta)+A^2(\pi-\theta)\rightarrow A^2(\theta)+A^2(\pi-\theta) -\tfrac{2}{3}A(\theta)A(\pi-\theta)\cos\Delta\Phi, \tag{3.6}$$

where

$$\Delta\Phi=\Phi(\theta)-\Phi(\pi-\theta). \tag{3.7}$$

The quantity on the left side of Eq. (3.6) is proportional to the effective cross section for observing together, in the case of nonidentical particles, the direct transfers giving N^{13} at scattering angle θ and the recoils in which the target nucleus has become N^{13} while the projectile has become N^{15}, the latter emerging at angle $\pi-\theta$. It gives the angular distribution of the N^{13} nuclei in the center-of-mass system.

IV. COMPARISON WITH EXPERIMENT

The calculations for the comparison with experiment have been made employing for the separation energies of the neutron in N^{14} and N^{15} the values 10.54 and 10.83 MeV, respectively. In obtaining the reduced mass for the motion of the neutron the masses of the neutron and of the neutral N^{13} and N^{14} atoms were used as 1.00898, 13.009858, and 14.007515 amu, respectively, and the mass of 7 electrons, allowing 0.51098 MeV per electron, was subtracted to give the nuclear masses in the latter two cases. If the values of α corresponding to the initial and final states of the neutron are referred to as a^i and a^f, respectively, the value of α used was

$$\alpha=(\alpha^i+\alpha^f)/2=0.69267\ F^{-1}. \tag{4.1}$$

Fundamental constants, other than those mentioned, involved in the calculation of α enter in the combinations $\hbar/mc=3.8615\times10^{-11}$ cm and 1 amu$=931.16$ MeV. These numbers were used accurately as a matter of definiteness. It is realized, however, that neither theory nor experiment can lead to values of comparable accuracy at this time. The employment of an arithmetic mean between α^i and α^f is itself an approximation connected with the use of a simplified form of the theory in which the reaction Q value is set equal to zero. Furthermore, as is readily seen,[3] the employment of the reduced mass, i.e., taking into account the nonvanishing value of the ratio of the neutron mass to that of the residual nucleus, necessitates the introduction of additional terms in the effective Hamiltonian which have not been taken into account. It is, nevertheless, believed that these approximations do not invalidate the qualitative features of the comparison. On account of the inclusion of the reduced mass effect the value of α in (4.1) is smaller than that used for the same reaction in Refs. 1, 2, and 4.

In Fig. 1 is shown the comparison of the data of Jobes and McIntyre[12] with Eq. (3.6). In this comparison the vertical scale factor of the calculated curve was adjusted in such a way as to give 0.230 mb/sr for the sum of the first two terms in (3.6), i.e., neglecting the interference effect. The number 0.230 mb/sr was arrived at by employing the Jobes-McIntyre analysis of their data into effects of "directs" and "recoils" with which they supply a derived sum of these two effects. On the basis of their curve for the sum it appears probable that the value at 90° is about as used. If the theory were good enough to be sure of the theoretically expected shape of the angular distribution curve the

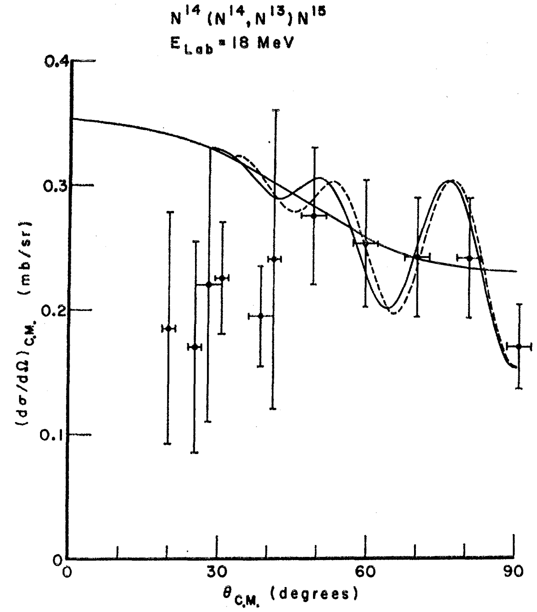

FIG. 1. Comparison of calculated and observed[12] differential cross sections at bombarding energy 18 MeV. The full curve is for α as in Eq. (4.1). The dashed curve is for $\alpha=0$. Nuclear absorption and VCE are neglected. The transfer of experimental points from the graphs supplied by the experimentalists in this and the following figure may contain a few small inaccuracies which do not affect the substance of this paper.

Jobes-McIntyre decomposition of the observed numbers into those due to directs and recoils could be improved on, the latter having been possible because of the fortuitous circumstance already apparent[8] that the second, third, and fourth experimental points below 90° fall at approximately the nodes of the last term in (3.6). But quite independently of the Jobes-McIntyre analysis, one could consider the theoretical curves shown in the figure as an approximate fit to the experimental points between 50° and 90°. The expected drop in $d\sigma/d\Omega$ at 90° by a factor $\frac{2}{3}$ below the curve for no interference is qualitatively confirmed by the observations. It may be mentioned that Eq. (16.7) of Ref. 1, from which (3.6) has been derived, was obtained making use of special nucleon configurations. In the general case the ratios $1:1:1:\frac{3}{2}$ for the three substates with $I=1$ and the state with $I=0$ may be modified and there would then be a corresponding change in the factor $\frac{2}{3}$.

In Fig. 1 there is drawn in a second theoretical curve represented by dashes which has been obtained by letting $\alpha=0$ in the computation of $\Delta\Phi$, i.e., by means of Eq. (2.7), rather than (2.5) and (2.6). This corresponds to the procedure used for the curve shown in Ref. 8. The phase varies somewhat more rapidly in this approximation in agreement with values readily obtainable from

$$(d\Phi/ds)/(d\Phi/ds)_{\alpha=0}=[1+u/(2\eta)]/(1+u^2)\,, \qquad (4.2)$$

which follows from (2.6). The reason for considering $\alpha=0$ is the possible presence of the VCE which puts the neutron into an excited state and thereby, in a sense, decreases α or else makes it imaginary. From the available data it is not possible to draw a definite conclusion concerning whether the full or the dashed curve give a better representation of the data. The points at 49° and 60° agree perhaps somewhat better with the curve for $\alpha=0$ but the onset of absorption which is doubtless taking place from 40° down to smaller angles could be responsible for the low position of the point at 49° which furthermore is in agreement with both calculated curves within practically the experimental error.

It may also be mentioned, as has been done,[8] that the observation of interference wiggles should prove to be valuable both in connection with obtaining information on VCE and of nuclear absorption. The decrease in the amplitude of the wiggles in going toward smaller angles from $\theta=90°$ is caused partly by the decrease in $A(\theta)$. This may be expected to be less rapid in the presence of VCE. On the other hand, the effect of competing reactions which gives rise to the absorption may be expected to decrease $A(\pi-\theta)$ at small θ and thus also to decrease the amplitude of the wiggles. It is probable that $A(\pi-\theta)>A(\theta)$ in the angular range 25–50°. If so then a first guess regarding the decrease in $A(\pi-\theta)$ should be obtainable from the curve through the nodal points and both $A(\pi-\theta)$ and $A(\theta)$ should thus be determinable. Except for the identification of the two, both of these quantities are determined by the values of $A^2(\theta)+A^2(\pi-\theta)$ and of $A(\theta)A(\pi-\theta)$. In the presence of absorption, the formula for Φ needs of course modification and the discussion of the differences between the curves for α as in (4.1) and $\alpha=0$ made above in relation to the points at the higher angles may be affected thereby. For a discussion of the bearing of evidence concerning absorption in the data of Jobes and McIntyre as well as the older measurements of angular distributions by Reynolds and Zucker[10] at $E_l=16.3$ and 19.2 MeV on the interpretation of elastic scattering data by McIntosh, Rawitscher, and Park,[14] reference is made to the Padua Conference report.[8] It may be stated, however, that although in the transfer data there are indications of absorption at rather large internuclear separations the discussion in Ref. 8 did not indicate a contradiction to the elastic scattering data. To the qualifications regarding the validity of the comparisons[8] one may add that elastic scattering of N^{13} by N^{15} has as much bearing on the transfer process as that of N^{14} by N^{14} and that data on the former scattering process are not available.

Internuclear separations are appreciably greater at the laboratory energy 12.3 MeV than at 18 MeV, the classical distance of closest approach in a head-on

[14] J. S. McIntosh, S. C. Park, and G. H. Rawitscher, *Proceedings of the Second Conference on Reactions between Complex Nuclei, Gatlinburg, Tennessee, 1960* (John Wiley & Sons, Inc., New York, 1960), Paper C-1, p. 127, and references therein.

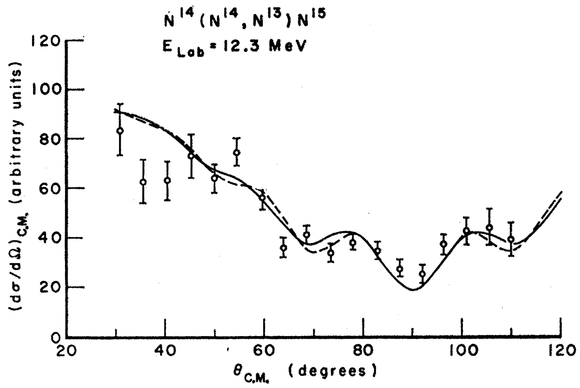

FIG. 2. Comparison of calculated and observed[13] differential cross sections at bombarding energy of 12.3 MeV. Full and dashed curve designations as in Fig. 1.

collision being about 11.4F. There is presumably no chance of appreciable absorption at this large distance. In Fig. 2 are shown the experimental points of McIntyre and Becker and two calculated curves. The vertical scale factor for the latter was adjusted to give an approximate overall fit to the data. The full curve corresponds to the use of (2.6) in (3.6), the dashed curve to the approximation (2.7) to Φ. Had the scale factor been increased to fit the two points closest to 90° more accurately, there would have been a larger difference between measured and calculated values at the smaller angles. On the whole the calculated ratio of small angle to 90° cross sections is larger than the measured ratio. Such a deviation would take place if the VCE participated in the process. It has been noted by one of the authors (G. B.) in a discussion at the Padua Conference that the experimental data shown are reasonably consistent regarding the fluctuations around a smooth curve with the phase variation for $\alpha=0$ but caution was expressed regarding the fit so obtained being possibly fortuitous. Later data by Becker and McIntyre indicate that the fluctuations may indeed have been statistical. There is thus as yet no definite evidence from the wiggliness of the curves for or against the participation of VCE.

New measurements of the total cross section σ_T at low energies have been made by Becker and McIntyre and compared by them with the SC tunneling formula employing for the calculation of α the neutron mass directly as in BE-I. In the energy range 9.00 MeV $<E_l<$12.8 MeV the observed decrease in σ_T is not as rapid as that calculated in the manner just mentioned, all experimental points being consistently above the SC approximation by various factors ranging between 1.8 at $E_l=9.1$ MeV to 18 at 9.5 MeV. Taking the logarithms of these factors, extrapolating these linearly to 9.0 MeV on the assumption that there is agreement at 12.8 MeV and taking the mean of 7 values between $E_l=9.1$ MeV and 10.4 MeV and taking the antilogarithm of this mean an equivalent logarithmic mean of the extrapolated factor of 4.8 results at 9.0 MeV. The uncertainty in this factor is hard to estimate accurately. It appears improbable that it is less than 2.2, especially because the factor 4.8 does not account for the high measured value at $E_l=8.5$ MeV, which was not included among those averaged and because some of the experimental points with relatively small uncertainties are definitely too high.

Employment of the reduced mass accounts for a factor of only 1.26(7). The ratio of the QM value of σ_T to its SC value was calculated using Eqs. (48.33), (48.35)[4] which correspond to the steepest descents approximation to $I_0(\theta)$. This ratio is then

$$\sigma_{T,\mathrm{QM}}/\sigma_{T,\mathrm{SC}}=\exp[8\eta(u_0-\tan^{-1}u_0)], \qquad (4.3)$$

where

$$u_0=\alpha/2k \qquad (4.4)$$

as in Eq. (48.36).[4] This equation does not take into account the interference effect represented by the last term in Eq. (3.6). To take it into account numerical quadratures of angular distributions with and without that term have been made. For $E_l=12.3$ MeV the effect of including the interference term employing $\Delta\Phi$ for $\alpha=0$ was found to be -0.3%. At 10.5 MeV employment of α as in (4.7) gave an effect less than 0.4% in absolute value. These effects are too small to be of interest. Employing the value of α with the reduced mass effect included, as in (4.1), the calculated values of the ratio given by (4.2) at $E_l=9.0$ and 12.8 MeV are 1.73(4) and 1.32(0), respectively. This effect accounts then for a relative increase by the factor 1.734/1.320=1.31(4). Combining this factor with that for the different α, an overall factor 1.66 is obtained for multiplication of the ratio of the σ_T at 9.0 MeV to its value at 12.8 MeV. It is definitely smaller than the corresponding number 4.8 derived from comparison with experiment and also smaller than the estimated lower limit 2.2 of that number. The direction of the discrepancy is that expected for VCE and is the same as suggested by the comparison of calculated and observed angular distributions at 12.6 MeV. The discrepancies are in the same direction as those discussed in BE-II and Ref. 4 but the effects are smaller the probable value of the factor to be accounted for being 2.9. Qualitatively this situation is in agreement with the decrease in the effect of VCE expected as a result of a reconsideration[15] of the estimates employing actual photodisintegration data rather than sum rules. However, the effect of VCE has not been sufficiently quantitatively treated to be able to speak of an agreement with expectation.

Additional information is contained in the ratio of values of the differential cross sections at $E_l=18$ and 12.3 MeV in the vicinity of 90°. By means of the steepest descents approximation the ratio of differential cross sections at $\theta=90°$ for $E_l=18$ to that for $E_l=12.3$ MeV

[15] G. Breit, *Proceedings of the Second Conference on Reactions between Complex Nuclei, Gatlinburg, Tennessee, 1960* (John Wiley & Sons, Inc., New York, 1960). Paper A-1, p. 1.

may be calculated to be ≈ 157. The experimental value of this ratio employing the directly measured value at 18 MeV and an extrapolated mean value at 12.3 MeV is 1.70×10^{-28} cm^2/1.10×10^{-29} cm^2 = 15.(4). The extrapolation at 12.3 MeV is from 87.5°. It could conceivably be made to give a value 10% higher than that used, changing the ratio to a still smaller value. The absolute differential cross section values of McIntyre and Becker have been obtained employing measurements at 12.18, 12.30, and 12.40 MeV. The central energy of this group corresponds to the relative values used in Fig. 2. Comparison of the calculated ratio 157 with the observed ratio 15.(4) indicates a discrepancy by a factor of ≈ 10. The energy $E_l = 18$ MeV being reasonably high, part of the effect might be caused by absorption or wave distortion. At 90° the value of $r_{\min}$, the minimum distance between nuclear centers along the Rutherford orbit, is 9.4(6)F at 18 MeV and 13.8F at 12.3 MeV. The relative smallness of the former of these distances in comparison with the latter does not exclude this possibility but it appears somewhat unlikely that the whole effect could be due to absorption, the distance 9.46F corresponding in terms of $r_0(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})$ to $r_0 = 1.96F$. If one is to explain all of the effect by absorption and wave distortion, the latter caused primarily by the real parts of the phase shifts, it would also be necessary to account for the approximate agreement of the calculated and observed angular distributions between 60 and 90°. At 60°, the values of $r_{\min}$ for the recoil are 8.4(4)F at 18 MeV and 12.4F at 12.3 MeV. If absorption is appreciable at 9.46F it would be expected to be stronger at 8.4(4)F and the agreement with the theoretical shape of the angular distribution requires further consideration. It is possible perhaps that the increased absorption for the recoil is compensated by a decreased absorption for the directly deflected particle for which at 60° and 18 MeV the $r_{\min}$ is 11.8F. In addition it may be argued that below 40° there are definite observed effects suggesting absorption or wave distortion and that the Rutherford orbit picture is too rough. Nevertheless the discrepancy by a factor of roughly 10 should be easier to explain employing VCE in addition to the other effects. A greater number of accurately measured points at 18 MeV and other energies could supply more rigid tests that would help to distinguish between the theoretical possibilities.

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