The $Ta^{181}(d,p)Ta^{182}$ Reaction and a Band-Mixing Analysis of the Observed Energy Levels*

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The multiple-gap magnetic spectrograph has been used to investigate the $Ta^{181}(d,p)Ta^{182}$ reaction at a bombarding energy of 7.0 MeV. Numerous energy levels up to an excitation energy of about 2.5 MeV in Ta^{182} were observed with an 8-keV resolution width. A ground-state Q value of 3.832 ± 0.008 MeV was measured. The model of a rotator plus two odd nucleons was used in calculations in an attempt to interpret the observed energy levels. The effects of Coriolis band mixing were included in the calculation. An acceptable fit was obtained for the lower excited states under the assumption that the stripped neutron entered the $(510\frac{1}{2}^-)$, $(512\frac{3}{2}^-)$, and $(503\frac{7}{2}^-)$ Nilsson orbits.

INTRODUCTION

THE development of the multiple-gap magnetic spectrograph[1] has made it possible to study heavy nuclei with (d,p) reactions at the 8-MeV bombarding energies available from the MIT-ONR accelerator. An investigation of the $Bi^{209}(d,p)Bi^{210}$ reaction[2] at this laboratory showed that the nuclear shell model gave fairly simple interpretations of nearly all the energy levels observed in the odd-odd nucleus Bi^{210}—presumably because of the proximity of the latter to the doubly magic nucleus Pb^{208}. A similar investigation of a heavy, highly deformed odd-odd nucleus was undertaken with the hope that its level structure as revealed by the (d,p) reaction could be readily understood with the model of a rigid rotator plus two odd nucleons. The odd-odd nucleus Ta^{182} was chosen since it is one proton removed from a deformed nucleus W^{183}, which has been well studied both experimentally[3] and theoretically.[4] The present report describes the work with the $Ta^{181}(d,p)Ta^{182}$ reaction. A report on the investigation of the $W^{182}(d,p)W^{183}$ reaction with the multiple-gap spectrograph is forthcoming.

The individual energy levels in Ta^{182} have previously been studied by techniques which depend on gamma-ray transitions. Axel and Sunyar[5] measured the decay gamma rays and internal-conversion electrons emitted from the 115-day isomer of Ta^{182}. They propose an energy level scheme of Ta^{182} to account for the five gamma-ray transitions which they observed. Bartholomew *et al.*[6] have found energy levels in Ta^{182} by studying the $Ta^{181}(n,\gamma)Ta^{182}$ reaction with a pair spectrometer at 1% resolution width. Many levels near the ground state can be deduced from these data on the capture gamma rays. Harvey[7] and Cohen *et al.*[8] have studied the $Ta^{181}(d,p)Ta^{182}$ reaction with coarse resolution. These (d,p) experiments, however, give no detailed information about the energy-level structure of Ta^{182}.

THEORY

The theoretical level structure of Ta^{182} was calculated in fair detail, as were the theoretical cross sections for the $Ta^{181}(d,p)Ta^{182}$ reaction. These calculations had to take account of the mixing of the various intrinsic states by the Coriolis interaction, since a less sophisticated calculation has little value for Ta^{182} in which the Coriolis mixing produces large changes in the positions and differential cross sections of the energy levels.

In the independent-particle model used, the odd proton and neutron are strongly coupled to the core and move independently in the deformed potential. In this model the energy levels separate naturally into two kinds, the intrinsic excitations and the rotational excitations. Intrinsic excitations occur when the odd particles are excited to higher states in the deformed potential or when the coupling of the intrinsic spins of the odd nucleons is changed. Because of the deformation of the core, the intrinsic spins of the odd proton and odd neutron can couple in only two ways, parallel ($\Sigma=1$) or antiparallel ($\Sigma=0$) along the symmetry axis of the core,

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[1] H. A. Enge and W. W. Buechner, Rev. Sci. Instr. **34**, 155 (1963).

[2] J. R. Erskine, W. W. Buechner, and H. A. Enge, Phys. Rev. **128**, 720 (1962).

[3] J. J. Murray, F. Boehm, P. Marmier, and J. W. M. DuMond, Phys. Rev. **97**, 1007 (1955).

[4] A. K. Kerman, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **30**, No. 15 (1956).

[5] A. W. Sunyar and P. Axel, Phys. Rev. **121**, 1158 (1961).

[6] G. A. Bartholomew, J. W. Knowles, G. Manning, and P. J. Campion, Atomic Energy of Canada Limited, Progress Report No. 517, 1957 (unpublished).

[7] J. A. Harvey, Can. J. Phys. **31**, 278 (1953).

[8] B. L. Cohen, J. B. Mead, R. E. Price, K. S. Quisenberry, and C. Martz, Phys. Rev. **118**, 499 (1960).

Σ being the sum of the intrinsic spins of the odd proton and neutron. Gallagher and Moszkowski[9] have given a rule which states that the more tightly bound of these two possible couplings has parallel intrinsic spins for the ground state (i.e., $\Sigma=1$).

Rotational excitations occur when the nucleus rotates mechanically as a unit. This kind of motion can take place for any of the intrinsically excited states of the nucleus. Each of these intrinsic excitations gives rise to a band of rotational states. If Coriolis mixing is neglected, the energies of states in the rotational band are given by the well-known expression $E=E_0+(\hbar/2\mathcal{I})\times[I(I+1)]$, where $\mathcal{I}$ is the moment of inertia of the core, and I is the total angular momentum of the state.

The excitation energies and differential cross sections were calculated with a program written in FORTRAN for the Argonne IBM-704 computer. The general scheme of the calculation is as follows. The wave functions and energies of the various states were found by calculating and numerically diagonalizing the Hamiltonian matrix. In this way the Coriolis mixing between any number of intrinsic states could be taken into account. Next, a reduced width for each of the states was calculated on the assumption of no mixing of states. Then the mixed wave functions found by diagonalizing the Hamiltonian were used to calculate the mixed reduced width. Finally, the differential cross sections at a particular observation angle were calculated from the mixed reduced widths and a set of intrinsic single-particle cross sections which gave the value of the cross section for different values of orbital angular momentum of the captured neutron. This set of cross sections had previously been obtained from a separate distorted-wave Born approximation (DWBA) stripping calculation.

The energy spectrum and wave functions for Ta^{182} were calculated by use of expressions based on those derived by Kerman[10] in his treatment of the problem of a single particle interacting with a rotator. For odd-odd nuclei, Kerman's calculation can be carried over with the exception that now the odd-particle wave functions χ_K are replaced by two-particle wave functions $\chi_K=\chi_{kp}\chi_{kn}$, where $K=kp+kn$, and the angular momentum J of the particle is replaced by the angular momentum of the neutron-proton system.

The wave functions that form the basis vectors for the calculation are the solutions of the Hamiltonian in which the rotational particle coupling (Coriolis interaction) term has been removed. These wave functions are written

$$\psi^I{}_{MK}=[(2I+1)^{1/2}/4\pi]\times\{\mathfrak{D}^I{}_{MK}\chi_K+(-1)^{I+K}\mathfrak{D}^I{}_{M-K}\mathcal{R}_1\chi_K\}, \quad (1)$$

where $\mathcal{R}_1$ is an operator that produces a rotation about an axis perpendicular to the symmetry axis of the nucleus, I is the total angular momentum, and K is the projection of I on the symmetry axis. By use of these wave functions, the diagonal matrix elements of the Hamiltonian become

$$\langle H\rangle_{KK}=A[I(I+1)+\langle J^2\rangle_{KK}-2K^2]+E(\text{Nilsson})+\langle V(np)\rangle_{KK}, \quad (2)$$

where $A=\hbar^2/2\mathcal{I}$ is the unit of rotational energy, J is the angular momentum of the neutron-proton system, E (Nilsson) are the eigenvalues of the Nilsson Hamiltonian,[12] and $V(np)$ is the interaction between the two odd nucleons. In the computer program, $\langle V(np)\rangle_{KK}$ was not calculated since little is known about the residual interaction in deformed nuclei. Furthermore, the tabulated Nilsson eigenvalues E(Nilsson) were not used since these quantities ordinarily do not agree exactly with the experimental energies. These two terms were replaced by an adjustable parameter C_K which also absorbed the other terms that are constant for a particular rotational band. With this simplification, the diagonal matrix elements become

$$\langle H\rangle_{KK}=AI(I+1)+C_K, \quad (3)$$

where

$$C_K=A[\langle J^2\rangle_{KK}-2K^2]+E(\text{Nilsson})+\langle V(np)\rangle_{KK}. \quad (4)$$

The constant C_K can be adjusted to place the base of the rotational band wherever desired.

The off-diagonal matrix elements of the Hamiltonian are

$$\langle H\rangle_{K,K-1}=\langle H\rangle_{K-1,K}=-A(I_-J_++I_+J_-)=-A[(I+K)(I-K+1)]^{1/2}\langle J_-\rangle_{K-1,K}, \quad (5)$$

where $I_\pm=I_1\pm iI_2$ and $J_\pm=J_1\pm iJ_2$, and where use has been made of the fact that the $J_\pm$ operator connects only those states in which $\Delta K=\pm1$. To carry out the calculation, it was necessary to obtain an expression for $\langle J_-\rangle_{K-1,K}$ in terms of the Nilsson $C_k{}^j$ coefficients which are obtained from the $a_{l\Lambda}$ coefficients tabulated by Nilsson.[11] These two coefficients are related by the Clebsch-Gordan transformation

$$C_k{}^j=\sum_\Lambda a_{l\Lambda}\langle l,\tfrac{1}{2},\Lambda,k-\Lambda|j,k\rangle.$$

The intrinsic wave functions which mix were written as

$$\chi_K=\chi_{k\alpha}\chi_{k\gamma} \quad \text{and} \quad \chi_{K-1}=\chi_{k\beta}\chi_{k\gamma}. \quad (6)$$

The form of these wave functions makes use of the fact that, since J_- is a one-particle operator, the only two-particle states that can mix are the two that have the same Nilsson state $\chi_{k\gamma}$ for either the proton or the neutron. Furthermore, the nature of the J_- operator demands that $k_\alpha=k_\beta+1$ for there to be nonzero matrix

[9] C. J. Gallagher, Jr. and S. A. Moszkowski, Phys. Rev. 111, 1282 (1958).

[10] A. K. Kerman, in *Nuclear Reactions*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Company, Amsterdam, 1959), Vol. I, Chap. X.

[11] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 29, No. 16 (1955).

[12] G. R. Satchler, Ann. Phys. (N. Y.) 3, 275 (1958).

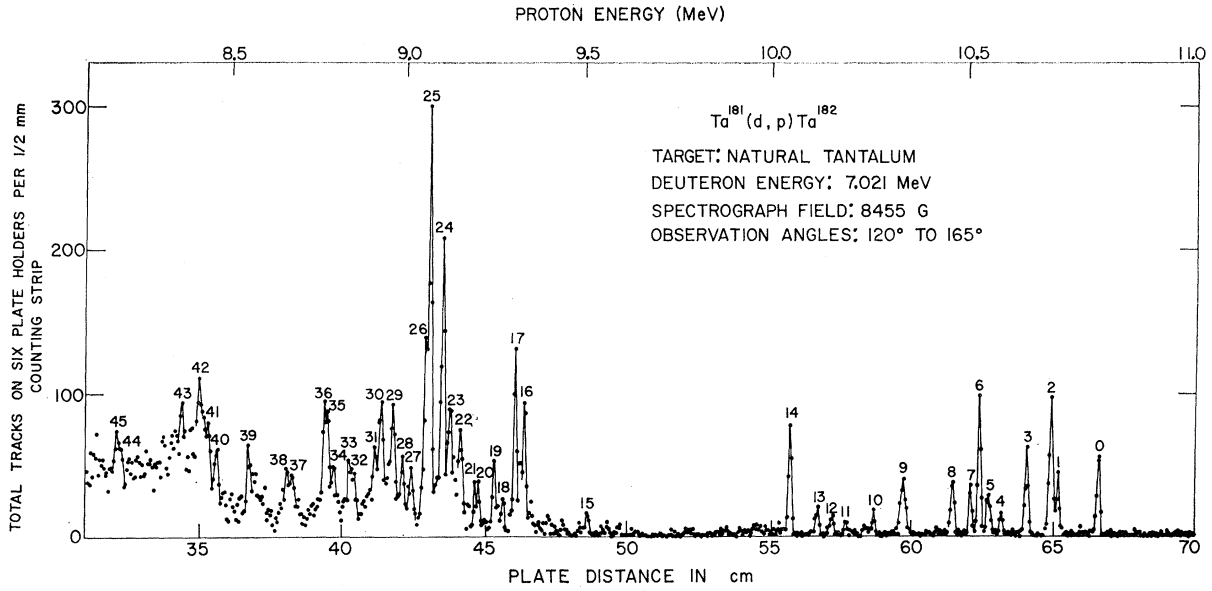

FIG. 1. Spectrum of protons observed from a natural tantalum target bombarded with 7.0-MeV deuterons. Data from six plate holders have been superimposed to enhance the yield.

elements. Use of Eqs. (1), (5), and (6) leads to

$$\langle H\rangle_{K,K-1}=\langle H\rangle_{K-1,K}=-A[(I+K)(I-K+1)]^{1/2} \times\sum_j[(j+k_\alpha)(j-k_\alpha+1)]^{1/2}C_{k\alpha}{}^jC_{k\beta}{}^j, \quad (7)$$

where

$$I\geqslant K, \quad k_\alpha=k_\beta+1, \quad \text{and} \quad K>0.$$

For values of $k<0$, it is helpful to note that the $C_k{}^j$ are related by

$$C_{-k}{}^j=(-1)^{j+l-\frac{1}{2}}C_k{}^j. \quad (8)$$

Equation (7) was the expression used by the computer to calculate the off-diagonal matrix elements.

The reduced-width amplitudes θ^j for each energy level were calculated from the expression given by Satchler[12] [his Eq. (22)]. This expression gives the reduced-width amplitudes with no Coriolis mixing.

The reduced widths which include the effects of band mixing were obtained from the mixed wave function ψ_{mix} produced by the diagonalization of the Hamiltonian matrix, i.e., from

$$\psi_{\text{mix}}=\sum_i a_i\psi_i, \quad (9)$$

where the ψ_i are the basis vectors described in Eq. (3), and the a_i are the amplitudes. The mixed reduced-width amplitude is then

$$\theta_{\text{mix}}{}^j=\sum_i a_i\theta_i{}^j. \quad (10)$$

The differential cross section is

$$d\sigma/d\Omega=[(2I_2+1)/(2I_1+1)]\sum_j(\theta_{\text{mix}}{}^j)^2\phi_l, \quad (11)$$

where I_1 and I_2 are the spins of the target and residual nucleus, respectively, and ϕ_l is the intrinsic single-particle differential cross section given by a DWBA stripping calculation.

EXPERIMENTAL PROCEDURE

The MIT multiple-gap magnetic spectrograph[1] was used to record the proton spectra produced by the bombardment of tantalum with 7.0-MeV deuterons. One bombardment of 5000 μCoul was made with nuclear-track plates loaded in all gaps from 90° to 172.5°. Aluminum foil was used in front of each plate holder except one of the two at 90° in order to eliminate elastically scattered deuterons from the target. The elastically scattered deuterons observed in the gap with no foil were used to measure the target thickness and to check target composition.

The Ta^{181} target was prepared from natural tantalum metal, which is 99.988% monoisotopic. An electron-bombardment evaporator deposited the tantalum metal onto a carbon film which had been laid down on a glass microscope slide. After the evaporation, the layer of tantalum and carbon was floated off the slide and picked up on a target frame which had been covered with several layers of Formvar. The Formvar gave support to the carbon backing which had been weakened by the very high temperature of the evaporation. The thickness of the tantalum layer was measured to be 71 μg/cm^2. This measurement of thickness depends on the assumption that the elastic scattering process is pure Rutherford scattering. Elbek and Brockelman[13] tested this assumption for 7-MeV protons which had been elastically scattered off gold and found that the assumption is good to better than 5%. In the present experiment

[13] B. Elbek and C. K. Bockelman, Phys. Rev. **105**, 657 (1957).

the scattering cross section is probably within 10% of that for pure Rutherford scattering.

The Q values were extracted from the plate-counting data by the methods described in the report on the $Bi^{209}(d,p)Bi^{210}$ reaction.[2] The ground-state proton group from the $C^{13}(d,p)C^{14}$ reaction appeared near the tantalum ground-state group and was used as a reference group to eliminate uncertainties in the spectrograph calibration and incident-deuteron energies. A ground-state Q value of 5.951 MeV was assumed for the $C^{13}(d,p)C^{14}$ reaction. All tantalum Q values are based on this number.

TABLE I. Measured excitation energies, Q values, and differential cross sections of levels in Ta^{182} formed through the $Ta^{181}(d,p)Ta^{182}$ reaction.

Level	E_x[a] (MeV)	Q[b] (MeV)	Differential cross section[c]	Suggested quantum numbers I	Predominant K
0	0	3.832	10.8	3	3
1	0.099	3.733	7.1	5	5
2	0.115	3.717	18.5	4	3
3	0.174	3.658	10.4	4	4
4	0.235	3.597	2.6		
5	0.269	3.563	5.8		
6	0.292	3.540	15.1	5	4
7	0.315	3.517	5.4		
8	0.358	3.474	7.8		
9	0.477	3.355	9.2		
10	0.555	3.277	2.5		
11	0.625	3.207	1.4		
12	0.660	3.172	2.5		
13	0.700	3.132	3.5		
14	0.771	3.061	12.6	7	7
15	1.308	2.524			
16	1.484	2.348	16.5		
17	1.511	2.321	20.5		
18	1.544	2.288			
19	1.568	2.264			
20	1.613	2.219			
21	1.624	2.208			
22	1.660	2.172			
23	1.693	2.139			
24	1.713	2.119	33.9		
25	1.750	2.082	55.2		
26	1.764	2.068			
27	1.803	2.029			
28	1.827	2.005			
29	1.853	1.979			
30	1.888	1.944			
31	1.908	1.924			
32	1.963	1.869			
33	1.984	1.848			
34	2.027	1.805			
35	2.043	1.789			
36	2.055	1.777			
37	2.146	1.686			
38	2.166	1.666			
39	2.274	1.558			
40	2.369	1.463			
41	2.394	1.438			
42	2.420	1.412			
43	2.478	1.354			
44	2.659	1.173			
45	2.674	1.158			

[a] The estimated uncertainty is 3 keV for levels Nos. 1 through 14, and 7 keV for the other levels.
[b] The estimated uncertainty is 10 keV.
[c] These numbers are averages of the differential cross sections observed at angles from 120° to 165°. The absolute differential cross section is given directly in microbarns per steradian with an estimated uncertainty of 20%. The accuracy of these differential cross sections relative to each other is somewhat better, having an uncertainty of about 10% for the intense groups.

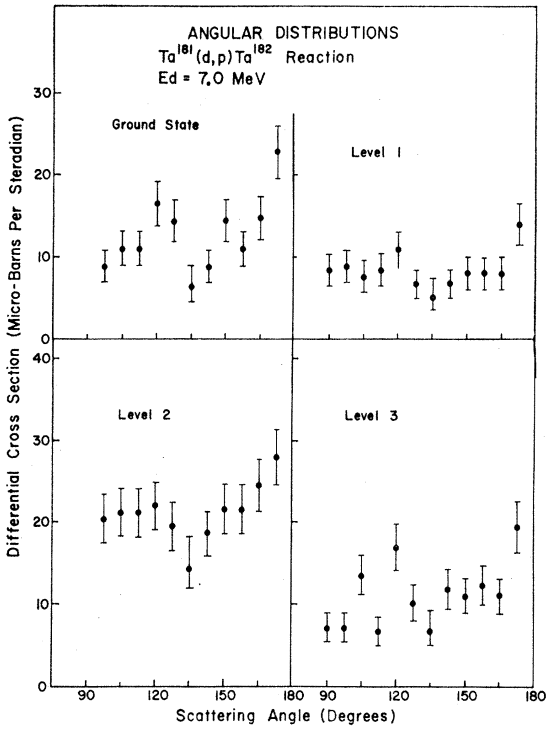

FIG. 2. Angular distribution of proton groups leading to the ground state and first three excited states in the $Ta^{181}(d,p)Ta^{182}$ reaction.

To increase the yield of the proton groups, the plate-counting data from 6 angles were superimposed to give an effective bombardment of 30 000 μCoul. Observation of the position of the strong groups enabled the experimenter to adjust the plate distance scales of the various plate holders to bring all groups into proper register. Differences in the calibration of individual plate holders made this shifting of the data necessary. This technique allowed observation of weak groups which were very difficult to find on a single plate holder. Furthermore, this treatment of the data tended to discriminate against proton groups arising from impurities in the target since the energy dependence of the contaminant groups is a different function of the angle of observation.

The excitation of energies of levels in Ta^{182} were obtained by two methods which cross checked each other. The excitation energies could be calculated from either the superimposed data from all plate holders or from the data of each plate holder separately. The uncertainties in the excitation energy were estimated by noting the consistency between the various measurements.

EXPERIMENTAL RESULTS

The proton spectra observed in the $Ta^{181}(d,p)Ta^{182}$ reaction are shown in Fig. 1. This figure shows the super-

imposed data from six plate holders. The proton groups have a width of about 8 keV at half-maximum. The excitation energies and Q values of the observed levels, and also an estimate of the standard error, are given in Table I. The differential cross sections and suggested spins for some of the levels are also given. The accuracy of the cross sections as absolute measurements is about 15% for the strong groups. Their relative accuracy is somewhat better. For the region above 1.3-MeV excitation energy, only the most intense and clearly resolved groups are listed. The data show a continuum here which presumably is due to many unresolved groups. The Q value of the $Ta^{181}(d,p)Ta^{182}$ ground-state reaction was measured to be 3.832 MeV by comparison with the $C^{13}(d,p)C^{14}$ ground-state group. If the 3-keV error in the Q value of the carbon reaction is included, the standard error on the Q value of the reaction leading to the ground state of tantalum is 8 keV.

Angular distributions of eight proton groups near the ground-state transition are shown in Figs. 2 and 3. The angular distributions of the other weaker groups are similar. Angular distributions were taken only between 90° and 172.5° from the beam direction. The error bars in Figs. 2 and 3 show the magnitude of the statistical error, the principal source of uncertainty.

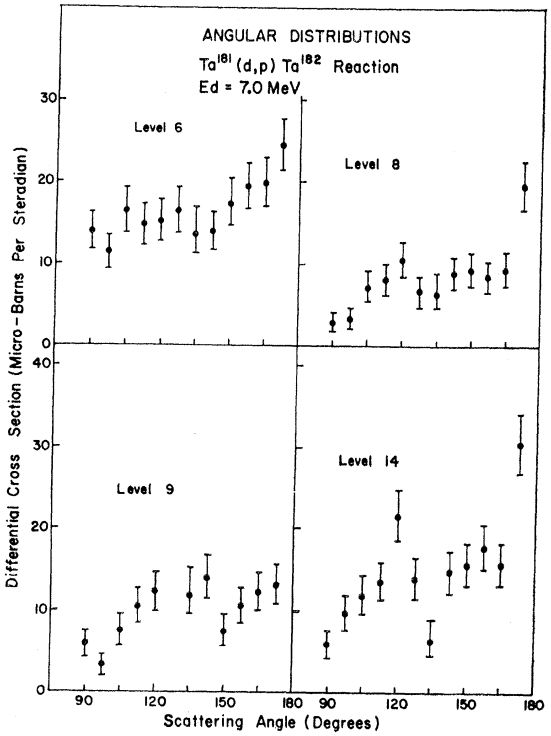

FIG. 3. Angular distribution of proton groups leading to levels 6, 8, 9, and 14 in the $Ta^{181}(d,p)Ta^{182}$ reaction.

DISCUSSION

Comparison with Earlier Studies

The (d,p) reaction has special advantages over gamma-ray techniques for investigating energy-level structure. Perhaps the most important advantage is the unambiguous determination of the relative positions of the energy levels. Another advantage is that the selection rules are less restrictive; certain states observed in (d,p) work cannot be reached in gamma-ray studies.

These advantages are somewhat offset by the fact that the (d,p) reaction has its own kind of selection rules which limit the levels which can be observed. Levels that require large values of angular momentum for the captured neutron are not easily observed. Another limitation is that the reaction mechanism does not easily excite levels in odd-odd nuclei in which the odd proton is in a different state than in the target nucleus, since this requires a two-step process. This limitation of the (d,p) reaction mechanism is probably the reason for the poor agreement between the level scheme observed in the present work and various level schemes reported for Ta^{182} in the gamma-ray studies.

The most detailed of the earlier studies of Ta^{182} is perhaps that of Sunyar and Axel.[5] They observed the gamma rays and conversion electrons emitted by metastable Ta^{182}. Figure 4 compares their proposed level scheme with the levels observed in the present (d,p) study. The uncertainties of the gamma-ray energies are all reported as $\leqslant 3$ keV. Only one of their levels (the 319-keV level) agrees within the stated errors with a level in the (d,p) scheme. It is not surprising that their 503-keV level is not observed with the (d,p) reaction, since they assign a spin of 8 or 9 to this level. For these spins, the capture probability is small because the neutron would have to be captured with a large angular momentum. The 503-keV level is also assigned to be even parity. This may mean that it arises from an excited proton configuration and, for this reason alone, cannot easily be excited by a (d,p) reaction. Similarly, the failure of the 147- and 319-keV levels to appear in the (d,p) data would be expected if they arise from an excited proton configuration. On the other hand, it is interesting to note that the five gamma-ray transitions observed by Sunyar and Axel fit nicely into the (d,p) energy level scheme if only the energy differences are considered. Only the 174-, 315-, and 358-keV states observed in the (d,p) study are needed to account for all the transitions. However, this interpretation seems to be inconsistent with the other data on these transitions reported by Sunyar and Axel.

The neutron-capture gamma rays observed by Bartholomew *et al.*[6] show the presence of a large number of energy levels in Ta^{182}. The levels near the ground state are shown in Fig. 5. The resolution of the pair spectrometer which they used is about 60 keV in this region. Consequently, it is to be expected that some of their peaks represent unresolved levels. The comparison of the two level schemes in Fig. 5 shows little correlation between the present (d,p) level shceme and the levels

of Bartholomew *et al.* except for the level at 99 keV.

The most energetic γ ray observed by Bartholomew *et al.* has an energy of 6.060±0.008 MeV. This is in excellent agreement with the neutron binding energy of 6.057±0.008 MeV for Ta^{182} predicted by the (d,p) ground-state Q value measured in the present work. (A deuteron binding energy of 2.225 MeV has been used.) This close agreement gives one confidence that the Ta^{182} ground-state group is being observed in the present work.

Interpretation of Ta^{182} as a Rotator Plus Two Odd Nucleons

The energies of the low-lying states in Ta^{182} and their differential cross sections in the $Ta^{181}(d,p)Ta^{182}$ reaction were calculated with the band-mixing computer code described above. (No attempt was made to calculate and interpret those states that lie above 1.0-MeV excitation.) The free parameters were varied widely in a search for the best fit to the experimental proton spectra. In these calculations, the Nilsson states of the odd neutron were the same as those found necessary to fit the observed proton spectra in the $W^{182}(d,p)W^{183}$ reaction in which presumably the same neutron states are involved. The (510 1/2−), (512 3/2−), and (503 7/2−) Nilsson states[14] were used for the odd neutron. In the calculations, the odd proton was for the most part treated as remaining in the same Nilsson state (404 7/2+) as in the target nucleus.

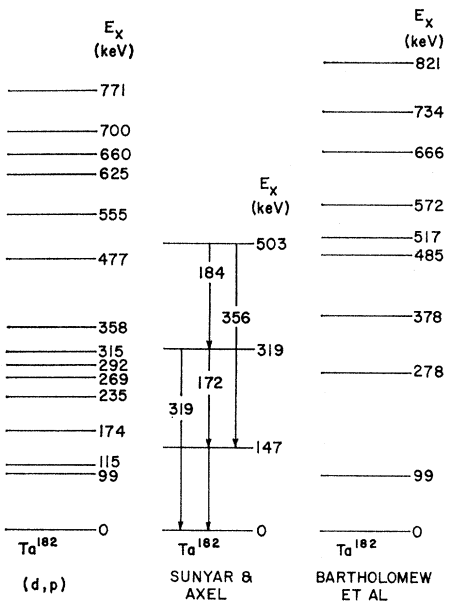

FIG. 4. Energy levels in Ta^{182}. Our observations with the (d,p) reaction are compared with the energy levels reported by earlier investigations.

[14] The intrinsic states are described by a symbol of the form $(Nn_z\Lambda K\pi)$, where N, n_z, and Λ are the "asymptotic" quantum numbers, K is the component of the total angular momentum along the symmetry axis, and π is the parity.

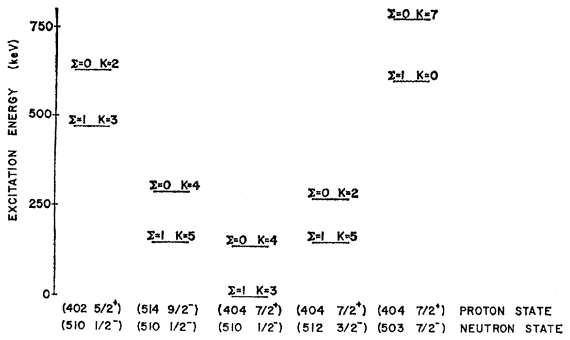

FIG. 5. Intrinsic states produced in Ta^{182} by the odd proton and neutron. The quantum numbers and notation used to label these states are discussed in the text.

Only excited neutron configurations were included in the calculation since the exact locations of the excited proton configurations are unknown. A band-mixing calculation was performed to see how these excited proton configurations affect the excited neutron states. Two intrinsic states of the odd proton were considered, the (514 9/2−) and the (402 5/2+) states. Mottelson and Nilsson[15] have reported these states to be at excitation energies of 152 and 480 keV, respectively in Ta^{181}. The former of these two states should give rise to two series of rotational levels in Ta^{182} with a [(514 9/2−), (510 1/2−)][16] configuration. However, this group o states has different parity than most of the low-lying states seen with the (d,p) reaction and consequently cannot mix with them. (These two intrinsic states in Ta^{182} as well as other states discussed below, are illustrated in Fig. 5.) The other proton state should produce the [(402 5/2+), (510 1/2−)] configuration near 480-keV excitation in Ta^{182}. Our calculation showed that this excited proton configuration lowers the excitation energy of the low-lying $J=3$, 4, and 5 states from the excited neutron configurations by only 2–6 keV, and produces less than a 1% change in the amplitude of the parts of the wave function that come from the excited neutron configurations. These changes are so small that the neglect of the excited proton configuration does not greatly disturb the results calculated with only the excited neutron configurations.

The following procedure was used to find the set of parameters that gave the best fit to the observed spectrum. First, all the parameters were varied one by one in order to test the sensitivity of the calculated spectrum to the various parameters. The choice of excitation energies of the intrinsic states had by far the largest effect on the calculated spectrum; changes in the moments of inertia, deformation parameter η, and the intrinsic single-particle cross section produced much smaller effects. Next, the values of all the parameters except those that set the positions of the intrinsic

[15] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter **1**, No. 8 (1959).

[16] In this notation, the Nilsson state of the odd proton is on the left and the state of the odd neutron is on the right.

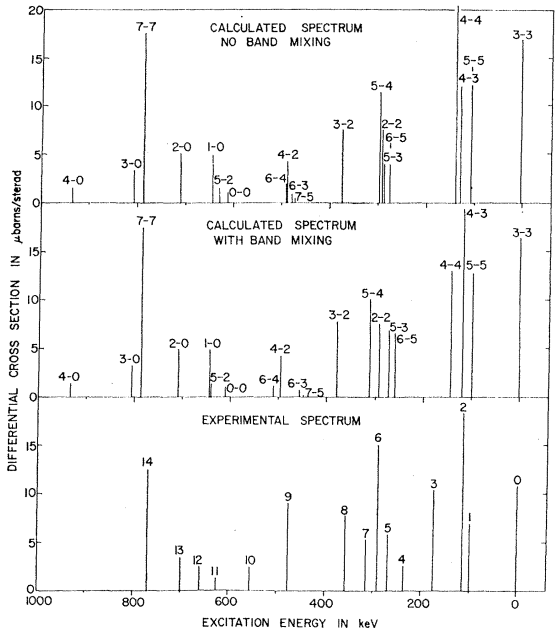

FIG. 6. A comparison of experimentally observed and theoretically calculated proton spectra of the $Ta^{181}(d,p)Ta^{182}$ reaction. Calculated spectra are shown with and without the effects of the Coriolis band mixing. Each calculated state is labeled with a symbol I-K which specifies the total angular momentum I and the projection of I on the symmetry axis K.

states were fixed at the best estimates for these quantities. The moments of inertia used were those found by Kerman[4] for W^{183}. The deformation was fixed at $\eta=4$. The set of single-particle cross sections caluclated by the DWBA stripping program were adopted. The matrix element $\langle K|J_-|K+1\rangle$ that governs the amount of mixing between the various states was not treated as a free parameter. The values of this matrix element were always calculated from the Nilsson wave functions.

At this stage of the parameter search, all possible arrangements of the intrinsic states were tried for the best fit. In particular, first the $[(404\ 7/2^+),\ (512\ 3/2^-)]_{K=5}$, then the $[(404\ 7/2^+),\ (510\ 1/2^-)]_{K=4}$, and finally the $[(404\ 7/2^+),\ (512\ 3/2^-)]_{K=2}$ intrinsic states were tried as the ground state. None of these states showed any promise of giving a calculated proton spectrum that corresponded to the experimental data. The difficulty was that the positions and intensities of the resulting states were too different to expect minor adjustment to bring them into agreement with the experimental spectrum. However, the $[(404\ 7/2^+),\ (510\ 1/2^-)]_{K=3}$ intrinsic state did fit better than the others for the ground state. Consequently, the $K=3$ state was then adopted as the ground state for subsequent calculation. Next, a number of things were tried to ascertain the character of the 99- and 115-keV excited states. One of these states is most probably the $(I=4,\ K=3)$ state from the ground-state rotational band. However, the other of these two states could be either the $[(404\ 7/2^+),\ (510\ 1/2^-)]_{K=4}$ or $[(404\ 7/2^+),\ (512\ 3/2^-)]_{K=5}$ intrinsic states. {The $[(404\ 7/2^+),\ (512\ 3/2^+)]_{K=2}$ states does not fit as well here because of its smaller intensity.} The $K=4$ intrinsic state was ruled out on the basis of its calculated intensity relative to the calculated intensity of the $(I=4,\ K=3)$ state. No matter whether the $K=4$ level was placed at a higher or lower excitation energy than the $K=3$ level, the level at the lower excitation always was much the more intense—contrary to the experimental data. This interesting behavior is due to the special character of the strong band mixing between these two levels. The next choice for the intrinsic state at 99 or 115 keV was the $[(404\ 7/2^+),\ (512\ 3/2^-)]_{K=5}$ state. The calculated intensity of this state fits the 99-keV level best. Finally, an acceptable fit could be obtained for the relative intensities of the 115- and 174-keV levels by identifying the 174-keV level as the $[(404\ 7/2^+),\ (510\ 1/2^-)]_{K=4}$ intrinsic state. Without band mixing the relative intensities of the $(I=4,\ K=3)$ and $(I=4,\ K=4)$ states do not correspond to the observed intensities of the 115- and 174-keV states. However, by placing the $(I=4,\ K=4)$ state close enough to the $(I=4,\ K=3)$ state, the band mixing does reverse the relative intensities. This behavior can be seen in Fig. 6. Unfortunately, the calculated position of the $(I=4,\ K=4)$ state is at 143 keV, not at 174 keV as required by the experimental data. This discrepancy could be explained if the value of the matrix element $\langle K=3|J_-|K=4\rangle$ calculated from the Nilsson wave functions were smaller than the value of the actual matrix element for the real nucleus. This may be happening since Kerman found[4] that for W^{183} the best empirical value for $\langle K=\frac{1}{2}|J_-|K=\frac{3}{2}\rangle$ was 1.371 rather than the value 0.913 calculated directly from the Nilsson wave functions.

At this point further adjustment of the parameters did not seem worth while. Perhaps a better fit could have been obtained by adjusting the moments of inertia. However, if that is done, then the values of $\langle J_-\rangle$ should also be adjusted arbitrarily as Kerman did. The precision of the present data does not warrant such a detailed fit. Furthermore, the $[(402\ 5/2^+),\ (510\ 1/2^-)]_{K=2 \text{ and } 3}$ intrinsic states should also be included if a more precise fitting were performed. Lack of information about the

TABLE II. Calculated wave functions, excitation energies, and (d,p) cross sections for the low-lying states in Ta^{182}.

		E (keV)		Wave function[a]				$d\sigma/d\Omega$[b]	
I	K	No mixing	With mixing	A_{K-3}	A_{K-4}	A_{K-5}	A_{K-2}	No mixing	With mixing
2	2	292	292	0.0	0.0	0.0	1.0	7.7	7.7
3	3	3	0	0.996	0.0	0.0	0.088	16.9	16.4
3	2	376	379	0.088	0.0	0.0	−0.996	7.3	7.8
4	3	130	120	0.928	0.350	0.0	0.128	12.3	19.5
4	4	140	143	0.346	−0.937	0.0	0.051	20.6	13.3
4	2	489	496	0.138	−0.003	0.0	−0.990	4.3	4.4
5	3	288	272	0.939	0.285	−0.075	0.176	4.1	7.0
5	4	298	311	0.288	−0.935	0.199	0.061	11.5	10.2
5	5	108	99	0.013	0.211	0.977	0.002	14.3	12.8
5	2	629	642	0.186	−0.006	0.001	−0.983	1.5	1.4

[a] The basis functions are those given in footnote 17.
[b] The cross section is calculated for an observation angle of 180° and is given in units of microbarns/steradian.

TABLE III. Reduced width amplitudes for transitions to the Ta^{182} ground state ($I=3$, $K=3$).

j	l	θ_{jl} unmixed	θ_{jl} mixed
1/2	1	−0.0587	−0.0585
3/2	1	0.5519	0.5325
5/2	3	0.3283	0.3757
7/2	3	−0.1293	−0.1143
9/2	5	−0.0467	−0.0569
11/2	5	0.0054	0.0043

location of these states would make such a fitting very difficult.

The final results of the calculation are shown in Fig. 6 and in Table II. This table lists the excitation energies and differential cross sections with no band mixing, and also the wave functions, energies, and differential cross sections that result when the band mixing is applied. The values of $\hbar^2/2\mathcal{I}$ used in the calculation were 15.853 keV for the [(404 7/2+), (510 1/2−)] bands and 14.050 keV for the [(404 7/2+), (512 3/2−)] bands. The same moments of inertia were used for both $\Sigma=0$ and $\Sigma=1$ bands. The values of the intrinsic single-particle cross sections ϕ_l used were 0.055, 0.019, and 0.0025 mb/sr for $l=1$, 3, and 5, respectively. The calculated values of $\langle J_- \rangle$ which determine the strength of the mixing between the various bands, were $\langle K=3|J_-|K=4\rangle = 0.169$,[17] $\langle K=2|J_-|K=3\rangle = 0.913$, and $\langle K=4|J_-|K=5\rangle = 0.913$. Typical calculated values of the reduced width θ_{jl} are listed in Table III, where the set of θ_{jl} obtained for the ground-state transition is given. The set of unmixed reduced widths is included for comparison. Note that the l value of the captured neutron can be 1, 3, or 5. It is this sort of complexity that makes a machine calculation essential in this problem.

The experimental proton spectrum is compared with the theoretically calculated spectrum in Fig. 6. The agreement is fairly good for the lower excited states. However, the identification of the strong group (No. 6) observed at 292-keV excitation is puzzling. Possibly an accidental superposition of the ($I=5$, $K=4$) state with some other state may give the strong observed intensity for this group. One of the weak groups (No. 5 or 7) probably corresponds to the ($I=2$, $K=2$) state. Groups Nos. 8–14 cannot be specifically identified. A few of these groups may come from the excited proton configurations mentioned earlier. That they are observed at all in the (d,p) reaction would be due to the fact that their wave functions contain some fraction of excited neutron states which are strongly excited in this reaction. It is very likely that the strong group observed at 771-keV excitation is the ($I=7$, $K=7$) level from the [404 7/2+), (503 7/2−)] intrinsic state, since the computer code yields a very large intensity for this state. The other members of this $K=7$ band are too weak to be observed, as is predicted by the computer calculations. The several weak levels between 600- and 700-keV excitation probably are members of the $K=0$ band of the [404 7/2+), (503 7/2−)] intrinsic state although detailed identification is impossible because of their weak intensity.

One thing that is apparent from this work is that a perfect fit to the data cannot be obtained by using the simple rotational model and the values of $\langle K|J_-|K+1\rangle$ given by the Nilsson wave functions. If more information were available (such as some of the quantum numbers for the levels), then perhaps an exact fit could be obtained by empirically adjusting the moments of inertia and values of $\langle K|J_-|K+1\rangle$ as Kerman did for W^{183}. However, the fit obtained by the method described above is fairly good and, consequently, the assigned quantum numbers are probably correct. Nevertheless, it is not clear just how much confidence should be placed in these assignments. One good omen, however, is that the ordering of the intrinsic states conforms to the coupling rule of Gallagher and Moszowski even though this rule was not used to govern the choices of parameters.

[17] In this notation, the $K=2$, 3, 4, and 5 stand for the appropriate wave functions of the [(404 7/2+), (510 1/2−)] and [(404 7/2+), (512 3/2−)] intrinsic states.

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