Pion-Proton Interaction at 4.13 BeV/c and Backward Scattering*

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The experimental data for $\pi^- - p$ scattering at 4.13 BeV/c are analyzed. The main results are the following: (i) There is an upper limit for the backward peak owing to the unitarity of S matrix. (ii) In spite of the fact that the total elastic cross section comes almost entirely from the forward peak, the S matrix for the partial wave is markedly affected by the existence of a small backward peak.

1. INTRODUCTION

RECENTLY, the experimental data for $\pi^- - p$ elastic scattering at 4.13 BeV/c have been reported by Perl *et al.*[1] In this paper we try to analyze the data and study the backward scattering. In such high-energy phenomena, the real part of elastic scattering amplitude is so small that it may be neglected compared with its imaginary part. Then, as is well known, the elastic scattering amplitude $f(\theta)$ for $\pi - N$ scattering can be expressed in the following form when the spin dependence of the S matrix is neglected:

$$f(\theta)=(i/2k)\sum_l (2l+1)(1-\eta_l)P_l(\cos\theta)$$
$$=(i/2)\sum_l (2l+1)\xi_l P_l(\cos\theta)\,, \quad (1)$$

$$\xi_l=(1-\eta_l)/k\,, \quad (2)$$

where $0\leq\eta_l\leq 1$.

According to the experimental results[1] for $\pi - N$ scattering, the forward peak contains at least (90–95)% of the total elastic cross section up to 32° and can be expressed fairly well by

$$d\sigma/d\Omega=\exp(A_0+A_1t)\ \text{mb/sr}\,, \quad (3)$$

with

$$A_0=3.32\,,\quad A_1=8.4\ (\text{BeV}/c)^{-2}\,, \quad (4)$$

where $t=-2q^2(1-\cos\theta)$ and q is the magnitude of pion momentum in the center-of-mass system. As is shown in Fig. 1, however, $d\sigma/d\Omega$ in the region of large $|t|$ cannot be expressed by Eq. (3). It is also said that the forms $d\sigma/d\Omega=\exp(A_0+A_1t+A_2t^2)$ and $d\sigma/d\Omega=\exp(A_0+A_1t+A_3t^3)$ with three parameters are very bad fits for large-angle scattering.[1] Perl *et al.*[1] made a weighted least-square fit for the entire range of $|t|$ to the equation

$$d\sigma/d\Omega=\exp(A_0+A_1t+A_2t^2+A_3t^3+A_4t^4)\ \text{mb/sr}\,. \quad (5)$$

As another approach to the formulation of $f(\theta)$, we now adopt the following expression:

$$f(\theta)=i(\exp[\tfrac{1}{2}(A_0+A_1t)]+C \pm\exp\{\tfrac{1}{2}[B_0+B_1(u-u_0)]\})\,, \quad (6)$$

$$u=[(m^2-\mu^2)^2/s]-2q^2(1+\cos\theta)\,, \quad (7)$$

where m and μ are the masses of proton and pion, respectively, s is the square of the total energy in the center-of-mass system, and u_0 is the value of u at 180°. The parameter C is determined so that $d\sigma/d\Omega$ at 90° may have the same value with the observed one. The second term iC in Eq. (6) may be interpreted as an effect due to the inelastic scattering which might be described in terms of the statistical model. The first and third terms are responsible, respectively, for the forward peak and the backward peak, if the latter exists. In describing the forward peak, we adopt throughout this paper the values $A_0=3.32$ and $A_1=8.4$ (BeV/c)$^{-2}$ which were estimated by Perl *et al.*[1] For backward scattering there is no detailed experimental data at the present, so the following cases are taken into consideration.

Case (I): There is no backward peak.
Case (II): There is a pronounced backward peak.

In Sec. 2 we show how to determine the parameters C, B_0, and B_1. For comparison we also consider the following case:

Case (III): Only the forward peak expressed by $\exp(A_0+A_1t)$ is taken into account. That is, the second and third terms are neglected.

In Sec. 3, a phenomenological analysis for $\pi^- - p$ scattering at 4.13 BeV/c is made, and the elastic and inelastic cross sections due to the lth partial wave are estimated. In Sec. 4, the conclusions derived from our analysis are summarized. In Sec. 5, some discussions about our method are made.

2. ON THE EXPRESSION FOR THE SCATTERING AMPLITUDE

In the region of small $|t|$, the second and third terms in Eq. (6) are negligibly small compared with the first term. Therefore, the forward peak can be expressed approximately by $\exp(A_0+A_1t)$. For our description of elastic scattering in the neighborhood of 90°, we have

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[1] M. L. Perl, L. W. Jones, and C. C. Ting, Phys. Rev. **132**, 1252 (1963).

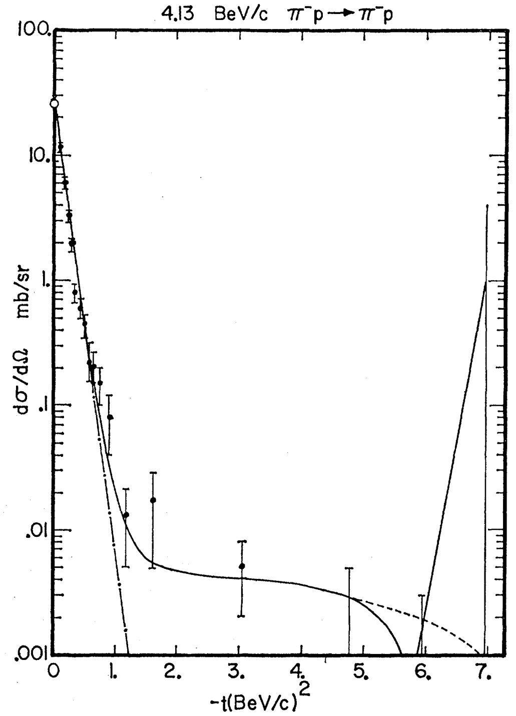

FIG. 1. Differential cross sections for $\pi^- - p$ elastic scattering at 4.13 BeV/c. The quantity $d\sigma/d\Omega$ is expressed by

$$(\exp[\tfrac{1}{2}(A_0+A_1t)]+C-\exp\{\tfrac{1}{2}[B_0+B_1(u-u_0)]\})^2 \text{ mb/sr}$$

and the following cases are taken into consideration:

Case (I): $A_0=3.32$, $A_1=8.4$ (BeV/c)$^{-2}$, $C=0.063$, $B_0=-6.91$, $B_1=1.20$ (BeV/c)$^{-2}$.

Case (II): $A_0=3.32$, $A_1=8.4$ (BeV/c)$^{-2}$, $C=0.063$, $B_0=0$, $B_1=4.46$ (BeV/c)$^{-2}$.

Case (III): Only the forward peak expressed by $\exp(A_0+A_1t)$ with $A_0=3.32$ and $A_1=8.4$ (BeV/c)$^{-2}$ is taken into account.

The behavior of $d\sigma/d\Omega$ in Case (III) is shown by the dash-dot line. For $d\sigma/d\Omega$ in the region (0°–90°), there is almost no difference between Cases (I) and (II). The behavior of $d\sigma/d\Omega$ in these two cases is shown by the solid line. Differential cross sections in the backward direction for Cases (I) and (II) are shown by the dashed line and the solid line, respectively.

only to consider the second term in Eq. (6). According to the experimental data,[1] $(d\sigma/d\Omega)_{90°}\cong 0.004$ mb/sr. Because the third term in Eq. (6) has no large effect on the $d\sigma/d\Omega$ in the forward direction, the difference between the values of $\exp(A_0+A_1t)$ and the observed differential cross sections in the region $|t|=(0.5-3.0)$ (BeV/c)^2 might be explained in terms of a constructive interference between the first and second terms in Eq. (6). This causes C to have a positive sign. Thus, we get

$$C\cong 0.063 \text{ (mb)}^{1/2}. \tag{8}$$

So far as the behavior of $d\sigma/d\Omega$ in the region (0°–90°) is concerned, the differential cross sections estimated by Eq. (6) agree fairly well with the experimental results (cf. the solid line in Fig. 1).

Next let us pay attention to the backward scattering. At the present there is no detailed experimental data for the backward peak with the exception of the following results: (1) A backward peak at 4.13 BeV/c would have to be less than 1/24 the height of the forward peak.[1] (2) The values of $d\sigma/d\Omega$ at 180° are of 0.92 ± 0.47 and 0.38 ± 0.24 mb/sr in the cases of incident π^+ momenta of 3.14 and 4.6 BeV/c, respectively.[2] These results seem to be inconsistent with the theoretical predictions.[3]

In our description for the backward scattering, needless to say, we have only to take into account the second and third terms in Eq. (6). Let us examine the $(\pm)$ sign of the third term. The experimental data[1] seem to suggest destructive interference between the second and third terms. Moreover, when the positive sign is adopted, the unitarity of S matrix cannot be satisfied. More precisely, the quantity $(1-\eta_0)$ for the s wave becomes in our calculation equal to 1.03 even when the

TABLE I. Values of the η_l.

l	η_l Case (I)	Case (II)	Case (III)
0	0.0010	0.2427	0.2368
1	0.2717	0.0512	0.2892
2	0.3897	0.5642	0.3832
3	0.4990	0.3799	0.5009
4	0.6234	0.6945	0.6230
5	0.7337	0.6960	0.7338
6	0.8240	0.8419	0.8240
7	0.8908	0.8832	0.8908
8	0.9363	0.9394	0.9363
9	0.9651	0.9642	0.9651
10	0.9818	0.9825	0.9818
11	0.9913	0.9913	0.9913
12	0.9956	0.9956	0.9956

TABLE II. Elastic and inelastic cross sections due to the lth partial wave.

	Case (I)		Case (II)		Case (III)	
l	σ_l^{scatt} (mb)	σ_l^{prod} (mb)	σ_l^{scatt} (mb)	σ_l^{prod} (mb)	σ_l^{scatt} (mb)	σ_l^{prod} (mb)
0	0.701	0.703	0.403	0.661	0.409	0.663
1	1.118	1.952	1.898	2.103	1.065	1.932
2	1.309	2.980	0.667	2.395	1.337	2.998
3	1.234	3.694	1.892	4.209	1.226	3.685
4	0.897	3.867	0.590	3.274	0.899	3.870
5	0.548	3.569	0.714	3.985	0.548	3.568
6	0.283	2.933	0.228	2.661	0.283	2.933
7	0.126	2.176	0.144	2.319	0.126	2.176
8	0.048	1.473	0.044	1.405	0.048	1.473
9	0.016	0.915	0.017	0.940	0.016	0.915
10	0.005	0.531	0.005	0.511	0.005	0.531
11	0.001	0.280	0.001	0.280	0.001	0.280
12	0.000	0.155	0.000	0.155	0.000	0.155
$\Sigma\,\sigma_l$	6.29	25.23	6.60	24.90	5.96	25.18
σ_{total}	31.52 (mb)		31.50 (mb)		31.14 (mb)	

[2] B. A. Kulakov, M. F. Lykhachev, A. L. Lyubimov, Yu. A. Matulenko, I. A. Savin, and V. S. Stavinski, in *Proceedings of the 1962 International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 584.

[3] V. Singh and B. M. Udgaonkar, Phys. Rev. **123**, 1487 (1961). V. Cook, B. Cork, W. R. Holley, and M. L. Perl, Phys. Rev. **130**, 762 (1963).

first and second terms in Eq. (6) are taken into account. If the third term with a positive sign is added, then $|1-\eta_0|>1$. Therefore, we choose the negative sign.

$$\begin{aligned}f(\theta)&=i(\exp[\tfrac{1}{2}(A_0+A_1t)]+C\\&\qquad-\exp[\tfrac{1}{2}\{B_0+B_1(u-u_0)\}])\\&=i[\exp(a_0+a_1\cos\theta)+C\\&\qquad-\exp(-b_0-b_1\cos\theta)],\end{aligned}\qquad(6')$$

where

$$a_0=A_0/2-A_1q^2,\qquad a_1=A_1q^2,$$
$$b_0=-B_0/2+B_1q^2,\quad b_1=B_1q^2.$$

For Case (I), the parameters B_0 and B_1 are tentatively estimated with the assumption that $(d\sigma/d\Omega)_{180°}\cong0.001$ mb/sr and $(d\sigma/d\Omega)$ at $|t|=4.75$ $(\text{BeV}/c)^2$ is about 0.003 mb/sr. Then we get

$$B_0\cong-6.91,\quad B_1\cong1.20\ (\text{BeV}/c)^{-2}.\qquad(9)$$

The differential cross sections in the backward direction for Case (I) are shown in Fig. 1 by the dashed line.

We now examine Case (II). As was mentioned before, the scattering amplitude for backward scattering can be approximately expressed by

$$f(\theta)\cong i(C-\exp[\tfrac{1}{2}\{B_0+B_1(u-u_0)\}]).\qquad(10)$$

If there exists a pronounced backward peak, there must be a scattering angle in the region between 90° and 180° at which $d\sigma/d\Omega\cong0$, because

$$C>\exp\{\tfrac{1}{2}[B_0+B_1(u-u_0)]\}\quad\text{at }90°,$$
$$C<\exp\{\tfrac{1}{2}[B_0+B_1(u-u_0)]\}\quad\text{at }180°.$$

However, this is not necessarily the case when the contribution from the real part of scattering amplitude is taken into account. The experimental data[1] show that the values of $d\sigma/d\Omega$ at $|t|=4.75$ and $|t|=5.92$ $(\text{BeV}/c)^2$ are of $(0.000_{-0.000}{}^{+0.005})$ and $(0.000_{-0.000}{}^{+0.003})$ mb/sr, respectively. It is said that the data[1] will be consistent with an unpublished calculation of Pomeranchuk[4] which gives $(d\sigma/d\Omega)_{180°}\cong1$ mb/sr. On the basis of these considerations, we tentatively assume that $(d\sigma/d\Omega)$ at $|t|=5.7$ $(\text{BeV}/c)^2$ is nearly equal to zero and $(d\sigma/d\Omega)_{180°}\cong1$ mb/sr. Then

$$B_0\cong0,\quad B_1\cong4.46\ (\text{BeV}/c)^{-2}.\qquad(11)$$

The differential cross section in the backward direction for Case (II) is shown in Fig. 1 by the solid line. It should be noted that there is almost no difference between Cases (I) and (II) so far as the differential cross sections in the region $|t|=(0-4.5)$ $(\text{BeV}/c)^2$ are concerned.

3. PARTIAL-WAVE ANALYSIS AND CROSS SECTIONS DUE TO l WAVE

In this section we state the partial-wave analysis for π^--p scattering at 4.13 BeV/c. From Eqs. (1) and (6′)

$$\begin{aligned}\exp(a_0+a_1\cos\theta)&+C-\exp(-b_0-b_1\cos\theta)\\&=\frac{1}{2k}\sum_l(2l+1)(1-\eta_l)P_l(\cos\theta)\\&=\tfrac{1}{2}\sum_l(2l+1)\xi_lP_l(\cos\theta).\end{aligned}\qquad(12)$$

The ξ_l is given by

$$\begin{aligned}\xi_l&=\int_{-1}^{1}[\exp(a_0+a_1x)+C-\exp(-b_0-b_1x)]P_l(x)dx\\&=\xi_l'+\xi_l''+\xi_l'''.\end{aligned}\qquad(13)$$

$$\xi_l'=\int_{-1}^{1}\exp(a_0+a_1x)P_l(x)dx,\qquad(14)$$

$$\begin{aligned}\xi_l''=\int_{-1}^{1}CP_l(x)dx&=2C\quad\text{for }l=0,\\&=0\quad\text{for }l\neq0,\end{aligned}\qquad(15)$$

$$\xi_l'''=-\int_{-1}^{1}\exp(-b_0-b_1x)P_l(x)dx.\qquad(16)$$

The ξ_l' (or ξ_l''') can easily be estimated by making use of a recurrence formula

$$\xi_{l+1}'=\xi_{l-1}'-[(2l+1)/a_1]\xi_l',\qquad(17)$$

and

$$\begin{aligned}\xi_0'&=[\exp(a_0)/a_1][\exp(a_1)-\exp(-a_1)],\\\xi_1'&=[\exp(a_0)/a_1]\{[\exp(a_1)+\exp(-a_1)]\\&\qquad-(1/a_1)[\exp(a_1)-\exp(-a_1)]\}.\end{aligned}\qquad(18)$$

Values of the η_l's thus obtained are shown in Table I. Then the elastic and inelastic (production) cross sections due to the l wave are estimated, respectively, by

$$\sigma_l^{\text{scatt}}=(\pi/k^2)(2l+1)(1-\eta_l)^2\qquad(19)$$

and

$$\sigma_l^{\text{prod}}=(\pi/k^2)(2l+1)(1-\eta_l^2).\qquad(20)$$

We show in Table II our results for σ_l^{scatt} and σ_l^{prod} in the Cases (I), (II), and (III).

Needless to say, the values of $\sigma^{\text{scatt}}=\sum_l\sigma_l^{\text{scatt}}$ and $\sigma^{\text{prod}}=\sum_l\sigma_l^{\text{prod}}$ agree well with the experimental values because we have adjusted the parameters so that the experimental results for $d\sigma/d\Omega$ may be reproduced and the value of total cross section is determined by value of A_0.

4. CONCLUSIONS

We now should like to summarize the conclusions which can be derived from our analysis.

(1) From the results shown in Table II we can say the following: It is the partial waves with $l=1\sim4$ that play the most important role in elastic scattering. It is the partial waves with $l=1\sim7$ that play the most important role in inelastic scattering.

[4] Y. D. Bayukov, G. A. Leksin, D. A. Suchkov, Y. Y. Shalamov, and V. A. Shebanov, Zh. Eksperim. i Teor Fiz. 41, 52 (1961); [English transl.: Soviet Phys.—JETP 14, 40 (1962)].

(2) In spite of the fact that a forward peak contains at least $(90-95)\%$ of the total elastic cross section up to 32°, the S matrix for the partial wave with small l value ($l=0, 1, 2, 3$ in our case) is affected remarkably by the existence of a backward peak with a height less than 1/25 times that of the forward peak[5] (see Fig. 1 and Table I). Therefore, it is important to get the detailed information about the backward scattering in order to perform the phase shift analysis. In other words, if the character of backward scattering is not taken into account for the reason that the values of $d\sigma/d\Omega$ in the backward direction are too small to examine in detail, we have many solutions for the set of phase shifts (η_l's). This is seen from our results that the experimental results for the forward scattering can be reproduced by both a solution in Case (I) and a solution in Case (II).

(3) If there exists a pronounced backward peak, there must be a scattering angle in the region (90°–180°) at which the imaginary part of scattering amplitude turns out to be zero.

(4) As is shown in Table I, the value of η_1 for the p wave is nearly equal to zero. This means that the allowable upper limit of backward peak is nearly equal to 1 mb/sr when $d\sigma/d\Omega\cong 0$ at $|t|=5.7$ $(\text{BeV}/c)^2$. Although this value of upper limit should not be taken so seriously, we want to emphasize that there is an allowable limit for the height of backward peak owing to the unitarity of the S matrix.

(5) Let us compare the σ_l^{scatt} (or σ_l^{prod}) for Case (I) with that for Case (II). When l is even, the former is larger than the latter. When l is odd, the latter is larger than the former. This tendency is remarkable in the case where l is small. This behavior can be interpreted as follows: The ξ_l''s with the same sign interfere constructively with each other in the neighborhood of $\cos\theta=1$ and give rise to the forward peak. In order that the ξ_l''''s which are responsible for the backward peak interfere constructively with each other in the neighborhood of $\cos\theta=-1$, the ξ_l''' with even l value must have the opposite sign to the ξ_l''' with odd l value because $P_l(-1)=(-1)^l$. We have taken the minus sign in the third term of Eq. (6). Therefore, the ξ_l''' value associated with odd (even) l becomes positive (negative). This is the reason why we get the results illustrated in Table II. Moreover, the backward peak is mainly due to the pion-nucleon interaction in the region of nucleon core. Therefore, the η_l's with small l values are affected remarkably by the character of the backward scattering.

[5] The experimental result shows that the differential cross section at 0° is nearly equal to 30 mb/sr. In our Case (II), $(d\sigma/d\Omega)_{180°}\cong 1$ mb/sr.

5. DISCUSSION

Previously we analyzed[6] the experimental data for $\pi^- - p$ scattering at 1.4 BeV assuming that the elastic cross section could be expressed by a form $[c/(a-b\cos\theta)]^2$. Then there was an inconsistency such as $\eta_0<0$. This was due to the crude assumption for the form of differential cross section. As is shown in Table I [see the values of η_l for Case (III)], there is no inconsistency in our present analysis. This means that the form (3) for the forward peak is very suitable and that our present results are much more reliable than the previous ones.

In the optical model, the following assumptions have been used very often in order to explain the diffraction peak:

$$\eta_l=a \quad \text{for} \quad 0\leq l\leq L,$$
$$\eta_l=0 \quad \text{for} \quad l>L,$$

where $L=kR$, and R is the radius of the proton in this simple model. Blokhintsev[7] and Perl *et al.*[1] have shown how the same assumptions with the above ones lead to a peak at 180°. However, these assumptions conflict with our results illustrated in Tables I and II.

Finally, we must state the following: Although we have analyzed the experimental data for pion-proton scattering without any estimate for the contribution from the real part of scattering amplitude, it may be necessary to examine its effect in order to discuss the differential cross sections which are much smaller than those in the forward direction, i.e., those in the neighborhood of 90° or in the backward direction.

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[6] D. Ito and S. Minami, Progr. Theoret. Phys. (Kyoto) **14**, 198 (1955).

[7] D. I. Blokhintsev, Nuovo Cimento **23**, 1061 (1962).