Vector Meson Decays in Unitary Symmetry*

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The relations among the amplitudes for the decays of a vector meson into two pseudoscalar mesons and a photon and among those for decays into two pseudoscalar mesons and N photons are derived by means of a convenient method based on the symmetry properties of the Lagrangian. These derivations are preceded by the rederivations of the relations among the amplitudes for the decays of a vector meson into two pseudoscalar mesons and among those for the decays into a pseudoscalar meson and N photons.

1. INTRODUCTION

It has become increasingly clear that the strong interactions roughly satisfy the symmetry of the unitary unimodular group SU_3 in three dimensions.[1] In particular, on the basis of the eight-fold way of Gell-Mann[2] and Ne'eman,[3] the various particles and resonances may be included within the multiplets of SU_3. If the strong interactions are invariant under this group of unitary transformations that mix isospin I and strangeness S, then various relations between physical processes involving different I and S can be obtained.

All particles within a unitary multiplet must have the same total angular momentum and parity and the same mass. When the mass-splitting term or SU_3-violating term is turned on, the particles within a given multiplet have different sets of masses that are in remarkably good agreement with a mass formula obtained by Okubo[4] and Gell-Mann.[2]

The purpose of this paper is to obtain, in a simple way, observable consequences of the decays of the vector mesons in the framework of SU_3. Section 2 considers the symmetries of SU_3 and derives relations among the decay amplitudes of vector mesons V into two pseudoscalar mesons P. In Sec. 3, U spin is considered; in Sec. 4 the decay of V into P and N ($N=1, 2, \cdots$) photons is discussed; and in Sec. 5, the decay of V into two P and N photons is discussed.

2. THE DECAY $V\rightarrow P+P$

If the strong interactions are invariant under unitary symmetry, the Lagrangian density between V, P, and the singlet vector meson ω^0 may be written in terms of a 3×3 traceless matrix V representing the eight vector mesons and a similar traceless matrix P representing the eight pseudoscalar mesons. The expression takes the form

$$\mathcal{L}_I=g_0\omega^0\,\mathrm{Tr}(VP)+g_1[\mathrm{Tr}(VPV)+\mathrm{Tr}(VVP)]+g_2[\mathrm{Tr}(VPP)-\mathrm{Tr}(PVP)],\quad(1)$$

where

$$V=\begin{pmatrix}\frac{1}{\sqrt6}\phi^0+\frac{1}{\sqrt2}\rho^0 & \rho^+ & K^{*+}\\ \rho^- & \frac{1}{\sqrt6}\phi^0-\frac{1}{\sqrt2}\rho^0 & K^{*0}\\ K^{*-} & \bar K^{*0} & -\frac{2}{\sqrt6}\phi^0\end{pmatrix}.$$

The interaction VVP is of the D type and that of VPP is of the F type by the requirements of charge-conjugation invariance. We use the units $\hbar=c=1$, and g_0, g_1, and g_2 are coupling constants. Here Tr is the matrix trace over a three-dimensional space that combines isotropic spin I and strangeness S; and P may be obtained from V by the replacement $(\phi^0,\rho,K^*,\bar K^*)\rightarrow(\eta,\pi,K,\bar K)$. The spatial part is suppressed.

The group SU_3 is of rank two. Consequently a set of states that transform into one another under SU_3 form a multiplet which is labeled by two quantum numbers, and there are two independent permutations under which $\mathcal{L}_I$ as well as the free Lagrangian is invariant. One can generate these as well as others by observing that the Lagrangian is invariant under the transformations $V\rightarrow UVU^{-1}$, $P\rightarrow UPU^{-1}$ for any 3×3 matrix U that is unitary and unimodular. Three such matrices are

$$A=\frac{1}{\sqrt2}\begin{pmatrix}1 & -i & 0\\ -i & 1 & 0\\ 0 & 0 & \sqrt2\end{pmatrix},\quad B=\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},\quad C=\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}.$$

The matrix C generates for P the transformation $K^+\leftrightarrow K^0$, $\pi^+\leftrightarrow\pi^-$, $\bar K^0\leftrightarrow K^-$, $\pi^0\leftrightarrow-\pi^0$, and $\eta\leftrightarrow\eta$ and conserves I spin and hypercharge $Y=N+S=S$; the matrix B generates $K^\pm\leftrightarrow\pi^\pm$, $K^0\leftrightarrow\bar K^0$, $\pi^0\leftrightarrow\frac12\pi^0+\frac12\sqrt3\eta$, and $\eta\leftrightarrow\frac12\sqrt3\pi^0-\frac12\eta$ and conserves U spin[5] and charge Q; and the matrix A generates

$$K^+\rightarrow(K^+-iK^0)/\sqrt2,\quad K^-\rightarrow(K^-+i\bar K^0)/\sqrt2,$$
$$\pi^\pm\rightarrow(\pi^++\pi^-\pm i\sqrt2\pi^0)/2,$$
$$K^0\rightarrow(K^0-iK^+)/\sqrt2,\quad \bar K^0\rightarrow(\bar K^0+iK^-)/\sqrt2,$$
$$\pi^0\rightarrow i(\pi^+-\pi^-)/\sqrt2,$$

and $\eta\rightarrow\eta$ and conserves I spin and hypercharge.

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[1] M. Ikeda, S. Ogawa, and Y. Ohnuki, Progr. Theoret. Phys. (Kyoto) 22, 715 (1959); 23, 1073 (1960).

[2] M. Gell-Mann, Phys. Rev. 125, 1067 (1962); California Institute of Technology Report CTSL-20, 1961 (unpublished).

[3] Y. Néeman, Nucl. Phys. 26, 222 (1961).

[4] S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

[5] S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters 10, 361 (1963).

The repetitious use of the permutations generated by A, B, and C together with the fact that $(\phi^0|\eta\eta)=0$ ($\eta\eta$ cannot form a P state) lead to the following relations[6] among the transitions of the type $V\rightarrow P+P$, namely,

$$\begin{aligned}
&\tfrac{1}{2}(\rho^+|\pi^+\pi^0)\\
&=\tfrac{1}{2}(\rho^0|\pi^-\pi^+)=(K^{*+}|K^+\pi^0)=-(K^{*0}|K^0\pi^0)\\
&=\frac{1}{\sqrt{2}}(K^{*0}|K^+\pi^-)=\frac{1}{\sqrt{2}}(K^{*+}|K^0\pi^+)=\frac{i}{\sqrt{3}}(\phi^0|K_1{}^0K_2{}^0)\\
&=\frac{1}{\sqrt{3}}(\phi^0|K^-K^+)=\frac{1}{\sqrt{3}}(\phi^0|\bar{K}^0K^0)=a. \qquad (2)
\end{aligned}$$

The additional expressions related to those above by charge-conjugation invariance are not recorded here.

The physical ϕ and ω vector mesons are expressible in terms of the ϕ^0 (the $I=Y=0$ member of the octet) and ω^0 (the unitary singlet) by[7,8]

$$\omega=\phi^0\sin\theta+\omega^0\cos\theta, \qquad (3)$$

$$\phi=\phi^0\cos\theta-\omega^0\sin\theta. \qquad (4)$$

Since the ω^0 cannot couple to two pseudoscalar mesons, it follows from Eqs. (2) and (4) that

$$(\sin\theta)^{-1}\frac{1}{\sqrt{3}}(\omega|K^-K^+)=(\cos\theta)^{-1}\frac{1}{\sqrt{3}}(\phi|K^-K^+)=\cdots=a. \qquad (5)$$

Summing over the possible modes and using Eqs. (2) and (5), $\theta=38°$, and $\Gamma(\rho^+)=\Gamma(\rho^0)=100$ MeV, we obtain the partial decay widths $\Gamma(K^{*+})=28$ MeV, and $\Gamma(\phi)=2.1$ MeV.[7,8]

3. U SPIN

In order to obtain relations among the transitions $V\rightarrow P+\gamma$ and also among $V\rightarrow P+P+\gamma$, we made use of the U-spin representation because the electromagnetic interactions and SU_3-invariant interactions conserve U spins and the photon transforms like the singlet member ($U=0$) of the unitary octet.[9]

The vector mesons form U-spin multiplets as follows: ρ^- and K^{*-}, a doublet; K^{*0}, $M_1{}^*=-(\rho^0-\sqrt{3}\phi^0)/2$, and $\bar{K}^{*0}$, a triplet; $M_0{}^*=-(\sqrt{3}\rho^0+\phi^0)/2$, a singlet; and K^{*+} and ρ^+, a doublet.[10] The matrix representing the vector mesons in U space is

$$V'=\begin{bmatrix}\frac{1}{\sqrt{6}}M_0{}^*+\frac{1}{\sqrt{2}}M\,{}^* & K^{*0} & \rho^-\\[2ex] \bar{K}^{*0} & \frac{1}{\sqrt{6}}M_0{}^*-\frac{1}{\sqrt{2}}M_1{}^* & K^{*-}\\[2ex] \rho^+ & K^{*+} & -\frac{2}{\sqrt{6}}M_0{}^*\end{bmatrix}. \qquad (6)$$

The pseudoscalar mesons form a corresponding multiplet and respresentation.

In the presence of the electromagnetic interactions the symmetry generated by the matrix B is no longer valid because it does not commute with $M_0{}^*$ (which transforms like the photon), but those generated by the matrices A and C are still valid.

For P the matrix C generates the transformation $\pi^\pm\leftrightarrow K^\pm$, $K^0\leftrightarrow\bar{K}^0$, $M_1\leftrightarrow -M_1$, and $M_0\leftrightarrow M_0$; and the matrix A generates $\pi^\pm\rightarrow(\pi^\pm\pm iK^\pm)/\sqrt{2}$, $K^\pm\rightarrow(K^\pm\pm i\pi^\pm)/\sqrt{2}$, $K^0\rightarrow(K^0+\bar{K}^0+i\sqrt{2}M_1)/2$, $\bar{K}^0\rightarrow(K^0+\bar{K}^0-i\sqrt{2}M_1)/2$, $M_1\rightarrow i(K^0-\bar{K}^0)/\sqrt{2}$, and $M_0\leftrightarrow M_0$. Corresponding transformations are generated for V.

One can obtain the Lagrangian density between V, P, and the electromagnetic field A_e by substituting

$$V_0=\begin{bmatrix}\frac{1}{\sqrt{6}}M_0{}^* & 0 & \\[2ex] 0 & \frac{1}{\sqrt{6}}M_0{}^* & \\[2ex] 0 & 0 & -\frac{2}{\sqrt{6}}M_0{}^*\end{bmatrix}\rightarrow\frac{A_e}{\sqrt{6}}\begin{bmatrix}1&0&0\\0&1&0\\0&0&-2\end{bmatrix} \qquad (7)$$

in Eq. (1) described in U space.

[6] Some of these relations have been discussed by J. J. Sakurai, in Proceedings of the International Summer School at Varenna, 1962 (to be published), by Sheldon L. Glashow and Arthur H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963), and by H. J. Lipkin in a Seminar Lecture at Argonne National Laboratory, 1963 (unpublished).

The vector mesons ρ^0, ϕ, and ω and the pseudoscalar mesons π^0 and η are expressible in terms of M_1^*, M_0^*, N_0^*, M_1, and M_0 as

$$\begin{aligned}\rho^0&=-\tfrac{1}{2}\sqrt{3}M_0^*-\tfrac{1}{2}M_1^*,\\ \phi&=-\tfrac{1}{2}M_0^*\cos\theta+\tfrac{1}{2}\sqrt{3}M_1^*\cos\theta+N_0^*\sin\theta,\\ \omega&=-\tfrac{1}{2}M_0^*\sin\theta+\tfrac{1}{2}\sqrt{3}M_1^*\sin\theta-N_0^*\cos\theta,\end{aligned} \tag{8}$$

and

$$\begin{aligned}\pi^0&=-\tfrac{1}{2}\sqrt{3}M_0-\tfrac{1}{2}M_1,\\ \eta&=-\tfrac{1}{2}M_0+\tfrac{1}{2}\sqrt{3}M_1,\end{aligned} \tag{9}$$

and N_0^* is the unitary singlet with $U=0$.

4. $V\rightarrow P+\gamma$

From the transformations generated by the matrices A and C, one finds to all orders in the electromagnetic interactions and the strong interactions that are SU_3 invariant that

$$\begin{aligned}&(M_0^*|M_0\gamma)\equiv a,\\ &(K^{*0}|K^0\gamma)=(M_1^*|M_1\gamma)=(\bar{K}^{*0}|\bar{K}^0\gamma)\equiv b,\\ &(\rho^+|\pi^+\gamma)=(K^{*+}|K^+\gamma)\equiv c.\end{aligned} \tag{10}$$

Expressing ρ^0, $\phi^0(\pi^0,\eta)$ in terms of M_1^*, $M_0^*(M_1,M_0)$ leads to

$$\begin{aligned}&(\rho^0|\pi^0\gamma)=\tfrac{3}{4}a+\tfrac{1}{4}b,\\ &(\rho^0|\eta\gamma)=(\phi^0|\pi^0\gamma)=\tfrac{1}{4}\sqrt{3}(a-b),\\ &(\phi^0|\eta\gamma)=\tfrac{1}{4}a+\tfrac{3}{4}b.\end{aligned} \tag{11}$$

These relations also hold where there are N ($N=1$, $2, 3\cdots$) number of photons in the final state.

In order to relate a, b, and c, we use the previous transformations generated by the matrices A and C and also that generated by the matrix B ($K^\pm\leftrightarrow K^\mp$, $\pi^+\leftrightarrow\bar{K}^0$, $\pi^-\leftrightarrow K^0$, $M_1\rightarrow\tfrac{1}{2}M_1+\tfrac{1}{2}\sqrt{3}M_0$, and $M_0\rightarrow\tfrac{1}{2}\sqrt{3}M_1-\tfrac{1}{2}M_0$ for P and a corresponding transformation for V) and find to all orders in the SU_3-invariant interactions that

$$(M_0^*|M_0^*M_0)=-(M_1^*|M_0^*M_1)=2(\rho^+|M_0^*\pi^+).$$

Since the electromagnetic current transforms like M_0^* of the octet representation of SU_3, the relation leads to

$$a=-b=2c. \tag{12}$$

The amplitudes $V\rightarrow P+\gamma$ are expressible in terms of $(M_0^*|M_0\gamma)=a$ and $(N_0^*|M_0\gamma)=d$ from Eqs. (8), (9), (11), and (12). They are

$$\begin{aligned}&(K^{*+}|K^+\gamma)=(\rho^+|\pi^+\gamma)=(\rho^0|\pi^0\gamma)\\ &\qquad=\frac{1}{\sqrt{3}}(\rho^0|\eta\gamma)=-\tfrac{1}{2}(K^{*0}|K^0\gamma)=\frac{a}{2},\\ &(\phi|\pi^0\gamma)=\tfrac{1}{2}\sqrt{3}(a\cos\theta-d\sin\theta),\\ &(\phi|\eta\gamma)=-\tfrac{1}{2}(a\cos\theta+d\sin\theta),\\ &(\omega|\pi^0\gamma)=\tfrac{1}{2}\sqrt{3}(a\sin\theta+d\cos\theta),\\ &(\omega|\eta\gamma)=\tfrac{1}{2}(-a\sin\theta+d\sin\theta).\end{aligned} \tag{13}$$

The relations (13) are valid to all orders of the SU_3-invariant interactions and to the lowest order in the electromagnetic interaction.[11]

5. $V\rightarrow P+P'+\gamma$

In this section, we continue to exploit the transformation generated by the matrices C and A, which lead to the relations

$$\begin{aligned}&(K^{*+}|K^+M_0\gamma)=(\rho^+|\pi^+M_0\gamma)\equiv a_1,\\ &(K^{*+}|K^+M_1\gamma)=-(\rho^+|\pi^+M_1\gamma)=-\frac{1}{\sqrt{2}}(K^{*+}|\pi^+K^0\gamma)\\ &\qquad=-\frac{1}{\sqrt{2}}(\rho^+|K^+\bar{K}^0\gamma)\equiv a_2.\end{aligned} \tag{14}$$

These relations together with Eq. (9) lead to

$$\begin{aligned}&(\rho^+|\pi^+\pi^0\gamma)=-\tfrac{1}{2}\sqrt{3}a_1+\tfrac{1}{2}a_2,\\ &(\rho^+|\pi^+\eta\gamma)=-\tfrac{1}{2}a_1-\tfrac{1}{2}\sqrt{3}a_2,\\ &(K^{*+}|K^+\pi^0\gamma)=-\tfrac{1}{2}\sqrt{3}a_1-\tfrac{1}{2}a_2,\\ &(K^{*+}|\pi^+K^0\gamma)=-\sqrt{2}a_2.\end{aligned} \tag{15}$$

From charge conjugation invariance, Eq. (15) also holds for the negatively charged particles. Similarly, by use of $\rho^0=-\tfrac{1}{2}\sqrt{3}M_0^*-\tfrac{1}{2}M_1^*$ and $\phi^0=-\tfrac{1}{2}M_0^*+\tfrac{1}{2}\sqrt{3}M_1^*$, we get

$$\begin{aligned}&(M_0^*|\pi^-\pi^+\gamma)=(M_0^*|K^-K^+\gamma)\equiv b_1,\\ &(M_1^*|\pi^-\pi^+\gamma)=-(M_1^*|K^-K^+\gamma)\\ &\qquad=(1/\sqrt{2})(K^{*0}|\pi^-K^+\gamma)\equiv b_2,\end{aligned} \tag{16}$$

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[7] S. L. Glashow, Phys. Rev. Letters **11**, 48 (1963).

[8] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962); also unpublished; S. Okubo (to be published).

[9] C. A. Levinson, H. J. Lipkin, and S. Meshkov (to be published); A. J. Macfarlane and E. C. G. Sudarshan (to be published).

[10] The present choice of phase relating M_1^* and M_0^* to ρ^0 and ϕ^0 leaves the matrix elements invariant under various transformations. This phase differs from Ref. 5.

[11] These relations have been obtained in a different way by Glashow, Ref. 7, and by S. Okubo, Phys. Letters **4**, 14 (1963). The relations that follow from charge-conjugation invariance are not recorded in the present paper.

and

$$
\begin{aligned}
(\rho^0|\pi^-\pi^+\gamma) &= -\tfrac{1}{2}\sqrt{3}b_1-\tfrac{1}{2}b_2,\\
(\phi^0|\pi^-\pi^+\gamma) &= -\tfrac{1}{2}b_1+\tfrac{1}{2}\sqrt{3}b_2,\\
(\phi^0|K^-K^+\gamma) &= -\tfrac{1}{2}b_1-\tfrac{1}{2}\sqrt{3}b_2,\\
(K^{*0}|\pi^-K^+\gamma) &= \sqrt{2}b_2.
\end{aligned}
\tag{17}
$$

Again, we obtain using charge conjugation invariance and G parity,

$$
\begin{aligned}
&(M_0{}^*|M_0M_0\gamma)\equiv c_0,\\
&(M_0{}^*|M_1M_1\gamma)=\tfrac{1}{2}(M_0{}^*|(\bar{K}^0K^0+K^0\bar{K}^0)\gamma)\\
&\qquad=\tfrac{1}{2}(M_0{}^*|(K_1{}^0K_1{}^0+K_2{}^0K_2{}^0)\gamma)\equiv c_1,\\
&(M_1{}^*|M_0M_1\gamma)_s=(K^{*0}|M_0K^0\gamma)_s\equiv c_s,\\
&(M_1{}^*|M_0M_1\gamma)_a=(K^{*0}|M_0K^0\gamma)_a\equiv c_a,\\
&\tfrac{1}{2}(M_1{}^*|(\bar{K}^0K^0-K^0\bar{K}^0)\gamma)\\
&\qquad=\frac{i}{2}(M_1{}^*|(K_1{}^0K_2{}^0-K_2{}^0K_1{}^0)\gamma)\\
&\qquad\qquad=(K^{*0}|K^0M_1\gamma)_a=0,\\
&\tfrac{1}{2}(M_1{}^*|(K_1{}^0K_1{}^0+K_2{}^0K_2{}^0)\gamma)\\
&\qquad=\frac{i}{2}(M_0{}^*|(K_1{}^0K_2{}^0-K_2{}^0K_1{}^0)\gamma)\\
&\qquad\qquad=(K^{*0}|M_1K^0\gamma)_s=0,
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
(\phi^0|\pi^0\pi^0\gamma) &= \tfrac{1}{8}(-3c_0-c_1+6c_s),\\
(\phi^0|\pi^0\eta\gamma) &= \tfrac{1}{8}\sqrt{3}(-c_0+c_1-2c_s-4c_a),\\
(\rho^0|\pi^0\pi^0\gamma) &= \tfrac{1}{8}\sqrt{3}(-3c_0-c_1-2c_s),\\
(\rho^0|\pi^0\eta\gamma) &= \tfrac{1}{8}(-3c_0+3c_1+2c_s+4c_a),\\
(i/2)(\phi^0|(K_1{}^0K_2{}^0-K_2{}^0K_1{}^0)\gamma) &= 0,\\
\tfrac{1}{2}(\phi^0|(K_1{}^0K_1{}^0+K_2{}^0K_2{}^0)\gamma) &= -\tfrac{1}{2}c_1,\\
(K^{*0}|\pi^0K^0\gamma)_a &= -\tfrac{1}{2}\sqrt{3}c_a,\\
(K^{*0}|\pi^0K^0\gamma)_s &= -\tfrac{1}{2}\sqrt{3}c_s.
\end{aligned}
\tag{19}
$$

The subscripts $s(a)$ on the decay amplitudes in Eqs. (18) and (19) indicate that the two-pseudoscalar-meson system is symmetric (antisymmetric) in U spin or I spin.

Relations (14) to (19) are valid to all orders in the electromagnetic interactions and all orders in the SU_3-invariant interactions. They are derived from the U-spin formalism and the fact that the Lagrangian is expressible in terms of the trace of the product of 3×3 traceless matrices. The relations are also valid when the photon in the final state is replaced by N photons. Note that the specific form of the Lagrangian was not required.

The decay amplitudes are expressed in terms of eight amplitudes a_1, a_2, b_1, b_2, c_0, c_1, c_s, and c_a that are linearly dependent. We now proceed to eliminate the redundant ones. Equation (18) was obtained in the U representation. Let us now transform the matrix elements back to the I representation by the unitary transformation

$$
\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}
\begin{pmatrix}
\frac{1}{\sqrt{6}}M_0{}^*+\frac{1}{\sqrt{2}}M_1{}^* & K^{0*} & \rho^-\\
\bar{K}^{*0} & \frac{1}{\sqrt{6}}M_0{}^*-\frac{1}{\sqrt{2}}M_1{}^* & K^{*-}\\
\rho^+ & K^{*+} & -\frac{2}{\sqrt{6}}M_0{}^*
\end{pmatrix}
\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}
=\begin{pmatrix}
\frac{1}{\sqrt{6}}\phi^0+\frac{1}{\sqrt{2}}\rho^0 & \rho^+ & K^{*+}\\
\rho^- & \frac{1}{\sqrt{6}}\phi^0-\frac{1}{\sqrt{2}}\rho^0 & K^{*0}\\
K^{*-} & \bar{K}^{*0} & -\frac{2}{\sqrt{6}}\phi^0
\end{pmatrix}.
\tag{20}
$$

Since the photon transforms like $M_0{}^*$ it transforms into the scalar part of the electromagnetic current in the I representation

$$M_0{}^*=-\tfrac{1}{2}I_s-\tfrac{1}{2}\sqrt{3}I_v\rightarrow-\tfrac{1}{2}I_s,$$

where I_s and I_v are the scalar and vector parts, respectively.

From the transformation indicated by Eq. (20) for V and a similar one for P, the second and third relation of Eq. (18) transform into

$$(\phi^0|\pi^0\pi^0\gamma)=(\phi^0|\pi^-\pi^+\gamma)=-\tfrac{1}{2}c_1, \qquad (21)$$
$$(\rho^0|\eta\pi^0\gamma)_s=(\rho^+|\eta\pi^+\gamma)_s.$$

We have used the fact that only the I_s part contributes to the amplitude $(\phi^0|\pi\pi\gamma)$ due to G parity. Combining Eqs. (17), (19), and (21) we get for the case in which the two pseudoscalar mesons are in a symmetric state in I spin (denoted by a superscript s)

$$\begin{aligned} c_0&=2a_1{}^s+2\sqrt{3}a_2{}^s+b_1-\sqrt{3}b_2,\\ c_1&=b_1-\sqrt{3}b_2,\\ c_s&=a_1{}^s+\sqrt{3}a_2{}^s. \end{aligned} \qquad (22)$$

The amplitude b_1 and b_2 only exist when the two pseudoscalar mesons are in a symmetric state. For the antisymmetric case (denoted by a superscript a) one finds

$$c_a=-a_1{}^a-\sqrt{3}a_2{}^a. \qquad (23)$$

Putting together Eqs. (15), (17), (19), (22), and (23), we finally obtain for both cases

$$\begin{aligned} &(\rho^+|\pi^+\pi^0\gamma)=-\tfrac{1}{2}\sqrt{3}a_1+\tfrac{1}{2}a_2,\\ &\sqrt{3}(\rho^+|\pi^+\eta\gamma)=\sqrt{3}(\rho^0|\pi^0\eta\gamma)=(K^{*0}|K^0\pi^0\gamma)\\ &\qquad=(\phi^0|\eta\pi^0\gamma)=-\tfrac{1}{2}\sqrt{3}a_1-\tfrac{3}{2}a_2,\\ &(K^{*+}|K^+\pi^0\gamma)=-\tfrac{1}{2}\sqrt{3}a_1-\tfrac{1}{2}a_2,\\ &(K^{*+}|\pi^+K^0\gamma)=-\sqrt{2}a_2,\\ &(\rho^0|\pi^-\pi^+\gamma)=-\tfrac{1}{2}\sqrt{3}b_1-\tfrac{1}{2}b_2,\\ &(\phi^0|\pi^-\pi^+\gamma)=(\phi^0|\pi^0\pi^0\gamma)=\tfrac{1}{2}(\phi^0|(K_1{}^0K_1{}^0+K_2{}^0K_2{}^0)\gamma)\\ &\qquad=(\phi^0|K_1{}^0K_1{}^0\gamma)=(\phi^0|K_2{}^0K_2{}^0\gamma)\\ &\qquad=-\tfrac{1}{2}b_1+\tfrac{1}{2}\sqrt{3}b_2, \qquad (24)\\ &(\phi^0|K^-K^+\gamma)=-\tfrac{1}{2}b_1-\tfrac{1}{2}\sqrt{3}b_2,\\ &(K^{*0}|\pi^-K^+\gamma)=\sqrt{2}b_2,\\ &(\rho^0|\pi^0\pi^0\gamma)=-\sqrt{3}a_1{}^s-3a_2{}^s-\tfrac{1}{2}\sqrt{3}b_1+\tfrac{3}{2}b_2.\\ &(\phi^0|K_1{}^0K_2{}^0\gamma)=0. \end{aligned}$$

The previous results are independent of the form of the Lagrangian and are based on the fact that the Lagrangian is expressible in terms of the trace of the product of 3×3 traceless matrices.

The triple product of **8** representation of SU_3 contains the **8** representation eight times so there are in general eight independent amplitudes.[12] Time reversal invariance and G parity reduce the number to five amplitudes a_1, a_2, $a_1{}^s+\sqrt{3}a_2{}^s$, b_1, and b_2. Equation (24) is valid to all orders in the SU_3 invariant strong interactions and lowest order in the electromagnetic interaction. The decay amplitudes of the physical particles ϕ and ω can be obtained from Eqs. (4), (5), and (24).

In the "pole approximation," in which one regards the dominant mechanisms[13] of $V\to P+P+\gamma$ as being (1) $V\to P+V'\to P+P'+\gamma$ and (2) $V\to P+P'$ accompanied by $V\to V'+\gamma$ or $P\to P''+\gamma$ or $P'\to P''+\gamma$, one finds that the decays $V\to P+P+\gamma$ are expressible in terms of four amplitudes that are linear combinations of a_1, a_2, b_1, and b_2 so that information on these amplitudes can be obtained.

The various relations given in Eq. (24) are related to partial decay widths by suitable kinematic factors. The relations between the partial decay widths indicated in Eq. (24) can be compared with experiment in order to check the validity of SU_3 symmetry in vector-meson decays.

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[12] Let V and P belong to the **8** representation. Then the product representation (for VV or PP) is **27**, **8s**, **1**, **10**, $\overline{\mathbf{10}}$, and **8a** where **8s** is even and **8a** is odd under R invariance[2]. The channel amplitudes for $VV\to PP$ are then A_{27}, A_{8s}, A_1, A_{8a}, A_{10}, $A_{\overline{10}}$, A_{8as}, and A_{8sa}. Time-reversal invariance requires $A_{8sa}=A_{8as}$. Equation (21) requires $A_{10}=A_{\overline{10}}$ and $A_{8as}=0$ which is satisfied when R invariance is satisfied. In other words, for the case $V+V\to P+P$, charge conjugation (or G parity) leads to R invariance[2], independent of the form of the interaction.

[13] For example, see Paul Singer, Phys. Rev. **130**, 2441 (1963).