Analysis of the 50-MeV Proton-Proton Scattering Data*

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Phase-shift analyses were made of 25 pieces of pp data at 51.8 MeV, including σ, P, D, C_{NN}, and C_{KP}. The higher angular momentum phases were taken to be (i) one-pion exchange only (OPE) and (ii) Amati-Leader-Vitale (ALV) one-, two-, and three-pion exchange. χ^2 was lower than its expected value for both, with ALV slightly better than OPE. Standard deviations were obtained for the phase shifts and for the predicted pion-nucleon coupling constant. The solutions found were discrete, with no other in the neighborhood of the Type No. 1. The relative importance of each of the several kinds of data was examined, with a view to future experiments.

I. INTRODUCTION

RECENT proton-proton scattering experiments in the neighborhood of 50 MeV (incident laboratory energy) have produced a variety of data. It is possible that the data are now sufficient to define the low angular momentum (L) phase shifts to within very narrow limits at that energy. With present experiments this is possible only if the higher L phases are given by a theory or model.

Hoshizaki, Otsuki, Tamagaki, and Watari[1] have made a "modified phase-shift analysis," of the type introduced by Moravcsik,[2] on most of the data considered here. In that method one assumes the higher L contributions to be exactly one-pion exchange (OPE). The lower L phases are determined solely by the condition that the least-square error sum[2] χ^2 should be a minimum. Hoshizaki *et al.* state that such a minimum had not been reached in their analysis.

Another model to be considered is that of Amati, Leader, and Vitale (ALV).[3] ALV have recently calculated nucleon-nucleon phase shifts for $L\geq 2$ via the Cini-Fubini approximation to the Mandelstam representation. Their principle aim was to obtain correctly the total two-pion exchange contribution. Their calculated lower L phases should thus be valid only at moderately low energies (i.e., ALV did not calculate $^1D_2$ beyond 240 MeV). To use the ALV phases, one need only replace the higher L OPE phases in the modified phase analysis searches by the corresponding ALV values.

The two kinds of models are here labeled OPE(N) and ALV(N), where N is the integer specifying the number of low L phases which were free (searched upon). In both cases, the *highest-L* contributions were represented by OPEC amplitudes[2] from which the appropriate lower L contributions had been subtracted. The constants used were $g^2=14.4$ and $\mu=135.1$ MeV.

* Supported in part by the U. S. Atomic Energy Commission.
[1] N. Hoshizaki, S. Otsuki, R. Tamagaki, and W. Watari, Progr. Theoret. Phys. (Kyoto) **29**, 617 (1963).
[2] M. J. Moravcsik, University of California Radiation Lab. Report, UCRL 5317-T 1958 (unpublished); P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **114**, 880 (1959).
[3] D. Amati, E. Leader, and B. Vitale, Phys. Rev. **130**, 750 (1963), and previous publications cited therein.

II. DATA USED AND TREATMENT

The data set used in this analysis is shown in Table I. It contains eighteen relative cross sections, two cross-section normalizations, and one each of absolute cross section, polarization, depolarization, and the spin correlation parameters C_{NN} and C_{KP}. Each normalization pertains to the set of relative values which immediately follows it in the table.

All of the data were treated as though they had been

TABLE I. Data used. N_σ indicates (absolute) normalization for the relative σ's which follow it.

Experimental energy (MeV)	c.m. angle (degrees)	Type	Value	Error	Reference, remarks
50.	70.	D	−0.249	0.075	a
51.5		N_σ	1.000	0.045	b
	16.2	σ	6.7	0.47	c
	17.2	σ	6.4	0.29	c
	18.2	σ	6.4	0.25	c
	20.3	σ	6.5	0.26	c
	22.3	σ	6.6	0.27	c
	24.3	σ	7.0	0.27	c
	26.3	σ	7.1	0.28	c
	30.4	σ	7.7	0.15	c
	35.5	σ	7.7	0.15	c
51.8		N_σ	1.000	0.025	b
	35.5	σ	7.7	0.15	c
	40.5	σ	7.9	0.16	c
	45.5	σ	7.6	0.15	c
	50.6	σ	7.9	0.16	c
	55.6	σ	7.7	0.15	c
	60.7	σ	7.8	0.16	c
	70.7	σ	7.6	0.15	c
	80.8	σ	8.0	0.16	c
	90.8	σ	8.0	0.16	c
51.8	90.	$\sigma_{\text{abs.}}$	8.15	0.13	d interpolated
51.8	45.	$P_{\text{abs.}}$	0.0349	0.0017	e interpolated
52.	90.	C_{KP}	−0.034	0.095	f
52.	90.	C_{NN}	0.13	0.11	f

[a] T. C. Griffith, D. C. Imrie, G. J. Lush, and A. J. Metheringham, Phys. Rev. Letters **10**, 444 (1963).
[b] K. Nisimura, J. Sanada, I. Hayashi, S. Kobayashi, D. C. Worth *et al.*, Institute for Nuclear Study, University of Tokyo, Tokyo, Japan, Report No. 45, 1961 (unpublished).
[c] Values from Ref. b above, relativee rrors from K. Nisimura (private communication).
[d] Interpolated from L. H. Johnston and Y. Tsai, Phys. Rev. **115**, 1293 (1959).
[e] Interpolated from C. J. Batty, G. H. Stafford, R. Gilmore, Phys. Letters **2**, 109 (1962). The general slope of $P(45°)$ versus energy was taken from J. N. Palmieri, A. M. Cormack, N. F. Ramsey, and R. Wilson, Ann. Phys. (N. Y.) **5**, 299 (1958).
[f] K. Nisimura, J. Sanada, S. Kobayashi, K. Fukunaga, N. Ryu *et al.*, Institute for Nuclear Study, University of Tokyo, Tokyo, Japan, Report No. 50, 1963 (unpublished); Progr. Theor. Phys. (to be published).

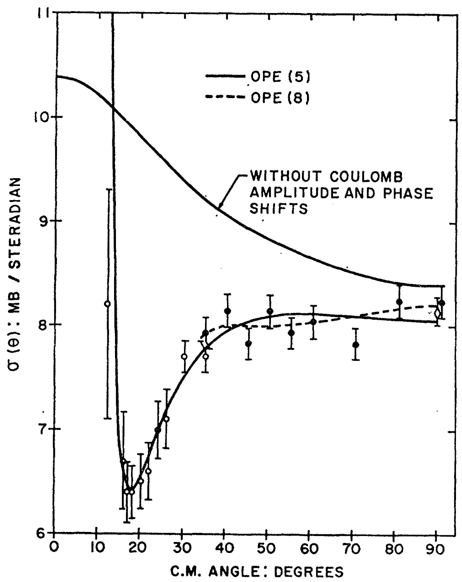

FIG. 1. Cross section at 51.8 MeV as a function of angle. The open circles (○) denote the small-angle telescope Tokyo points, with the OPE(5) associated normalization of 1.00. The darkened circles (●) are the large-angle points, with the OPE(5) associated normalization of 1.03. The diamond (◇) denotes the interpolated Minnesota point.

measured at 51.8 MeV. The only interpolated data were the absolute cross section and polarization values. The latter was obtained by moving the 50-MeV Harwell $P(45°)$ along the general slope of $P(45°)$ measurements plotted against energy. The absolute $\sigma(90°)$ was interpolated from the Minnesota $\sigma(90°)$ versus energy measurements. We note that Hoshizaki *et al.* did not use the absolute cross section and C_{NN} data included here.

The cross-section value at 12.2° c.m. was not used. This point is on the forward Coulomb rise, which is an extremely sharp function of angle (Fig. 1). Since the shape at 12.2° is still strongly influenced by the strong force as well as the Coulomb, the procedure for obtaining the effective angle would be quite complicated and uncertain.[4] We note that all of the analyses to be reported here produced cross-section values at 12.2° higher than the experimental one. Generally, inclusion of the 12.2° point raised χ^2 but did not appreciably change the phase shifts.

A predicted cross-section normalization should be obtained by minimizing χ^2 with respect to N in the equation

$$\chi^2 = \sum_n \left(\frac{p_n - N d_n}{N\epsilon_n}\right)^2.$$

Here p_n is a predicted datum, d_n is its experimental value, ϵ_n is the relative standard error on the experimental datum, and N is the (unknown) predicted normalization. The sum is over the set of relative data to be normalized. Perring[5] in a 140-MeV analysis used a similar equation, but he erroneously omitted the N in the denominator. Perring also included a term to minimize χ^2 with respect to the contributions to it from N itself:

$$[(N-1)/\epsilon_N]^2.$$

That term was automatically taken care of in our search procedure.

Hoshizaki *et al.* did not use the cross section treatment outlined above; instead, they treated as experimental data the quantities[6] $r(\theta)=\sigma(\theta)/\sigma(90°)$. The errors on $\sigma(\theta)$ and $\sigma(90°)$ should have been combined quadratically to obtain the errors on $r(\theta)$. Instead, they simply divided the $\sigma(\theta)$ errors by $\sigma(90°)$ to obtain errors on $r(\theta)$ which were then about $\sqrt{2}$ smaller than they should have been.

III. SEARCH METHOD

The least-squares fitting was carried out using the method reported by Lietzke,[7] which includes a check on whether an extremum was reached in the value of the goodness-of-fit parameter (least-square error sum) χ^2. In all of the analyses reported here, a minimum in χ^2 was actually reached.

Standard deviations for the searched-upon parameters were calculated in the usual fashion[2] from the diagonal elements of the error matrix. For convenience of computation, the error matrix was taken as the inverse of the linearized second derivative matrix. The exact error matrix was also computed for several of the runs and was found to give negligibly different standard deviations.

IV. NUMBER OF SEARCH PARAMETERS

There is an inherent difficulty in the modified phase analysis method, to which little effort has previously been applied. This problem is the uncertainty in how many low L phases should be free (searched-upon), rather than be fixed by the model. If possible, the number of free parameters needed should be determined by applying the F test.[8] The latter yields, for each number of search parameters N, the probability that the last parameter released was sufficiently given by the model value. In practice, however, it often happens that the available theory and data are insufficient to render usefully high F-test probabilities. In that case, one can try the χ^2 test,[8] which indicates the most probable fit. The most probable value of χ^2 is that for which the χ^2 ratio, χ^2/χ^2 expected, is a minimum.

[4] J. N. Palmieri, A. M. Cormack, N. F. Ramsey, and R. Wilson, Ann. Phys. **5**, 299 (1958).

[5] J. K. Perring, Nucl. Phys. **42**, 306 (1963).

[6] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105**, 302 (1957).

[7] M. H. Lietzke, Oak Ridge Report ORNL-3259, April 1962 (unpublished).

[8] P. Cziffra and M. J. Moravcsik, University of California Radiation Laboratory Report, UCRL-8523 Rev., June 1959 (unpublished).

TABLE II. Results of the modified phase analyses. The third column is the ratio of χ^2 to its expected value. The latter is the number of data (25) minus the number of free (searched-upon) phases N. The phase shifts and their standard deviations are nuclear bar, in degrees.

Model (N)	χ^2	Ratio	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$
ALV(5)	15.1	0.76	37.51± .74	18.71±1.99	−6.08± .56	4.46±.46
ALV(6)	9.6	0.51	37.35± .45	15.91± .45	−6.41± .44	5.45±.45
ALV(7)	9.6	0.53	37.62± .91	15.40±2.24	−6.09±1.23	5.52±.53
ALV(8)	9.4	0.56	37.31±1.15	15.86±2.35	−6.08±1.10	5.53±.51
OPE(4)	40.9	1.95	35.89±1.34	21.79±2.52	−6.56± .73	3.68±.48
OPE(5)	16.7	0.84	36.56± .77	17.25±1.97	−7.55± .47	5.06±.41
OPE(6)	13.5	0.71	36.32± .60	15.57±1.78	−7.35± .45	5.72±.47
OPE(7)	11.4	0.63	37.30± .60	14.63±1.89	−4.96±1.24	5.90±.63
OPE(8)	10.7	0.63	37.91± .89	13.84±2.18	−5.10±1.20	5.91±.65
OPE(7)[a]	14.2	0.79	−27.47± .96	5.36±1.88	15.80± .73	4.00±.49

Model (N)	ϵ_2	$^1D_2$	$^3F_2$	$^3F_3$
ALV(5)	−1.54±.09			
ALV(6)	−2.46±.26	2.21±.23		
ALV(7)	−2.63±.63	2.31±.36	−0.20±.83	
ALV(8)	−2.69±.59	2.31±.35	−0.07±.78	−0.88±.47
OPE(4)				
OPE(5)		1.62±.10		
OPE(6)	−2.71±.31	2.37±.28		
OPE(7)	−3.64±.49	2.73±.18	−0.79±.60	
OPE(8)	−3.40±.49	2.65±.19	1.07±.69	−0.28±.6
OPE(7)[a]	3.05±.26	1.63±.24	0.43±.22	
OPE			0.40	−0.82

[a] Solution of Type No. 2.

To apply the F and/or χ^2 tests, one arranges the phases in such an order that the *slope* of the χ^2 ratio versus N curve is monatonically increasing with N. The final aim is, of course, to have a minimum occur in the χ^2 ratio, and to have monatonically increasing F-test probabilities. One has then to decide upon the termination probability. We note that a zero-slope portion of the χ^2 ratio versus N curve would yield an F-test probability of $\frac{1}{3}$ for the analyses being reported here.

We note that Hoshizaki *et al.* searched on six specific phases, using OPE to represent the higher L phases.

V. RESULTS OF THE MODIFIED PHASE ANALYSIS

The parameters resulting from the modified phase analyses are shown in Table II. The search method used did not guarantee that a minimum in χ^2 had been reached when nine or more phases were searched upon.

For the OPE model, the χ^2 ratio versus N_s curve has possibly reached a minimum at eight search phases, but that is not positive. The χ^2-test probabilities change very slowly after five search parameters, making it a weak test there. The F-test probabilities are too small to be useful, so are not shown. Some selection among the OPE runs is possible if one postulates that the effect of the centrifugal barrier is fairly well represented by recently proposed two-nucleon potential models. The latter are in agreement that, at 50 MeV, the ^3F phases and $^1G_4$ are accurately given by their OPE values. The $L\geq 5$ departures of the potentials from OPE[9] should not be serious at this low an energy, where the high L phases are quite small. The run labeled OPE(5) may be considered, then, to be the prediction of potential-type models.

The ALV model prediction, using all of the phases calculated by ALV, is labeled ALV(5) in Table II. It is a small, but distinct, improvement over the bare OPE contributions used in the corresponding OPE(5) run. Note, however, that releasing $^1D_2$ from its ALV value, run ALV(6), results in a statistically significant improvement in the fit to the data. The resulting $^1D_2$ phase is four standard deviations away from the one calculated by ALV; this is not necessarily serious, since $^1D_2$ is presumably the least accurate of the ALV phases. The χ^2 ratio is a minimum here and the F test yields the comparatively good probability of 0.44, so ALV(6) is probably as good an estimation of the predictions of the model as one can make at present.

VI. PION-NUCLEON COUPLING CONSTANT

If the value of g^2, the pion-nucleon coupling constant, is included with the lower L phases as a searched-upon parameter, one can obtain that value which produces a least-squares fit to the data. In addition, the associated standard deviation can be obtained by including g^2 in the calculation of the error matrix. Such runs were made, with the results for four and five free phases shown in Table III. For six searched-upon phases, g^2 went to a negative value with an exceedingly large standard deviation. The favored model OPE(5) includes the pion-nucleon value within a standard deviation.

VII. OTHER SOLUTIONS

All of the phase shift solutions (to the least-squares fitting problem) so far shown have been of the type labeled No. 1 by Stapp, Ypsilantis, and Metropolis[6] (SYM). It is now generally believed that there are only two probable solutions, Nos. 1 and 2 of SYM. Furthermore, recent analyses[10] at 142 MeV and 210 MeV have discarded Solution No. 2. Nevertheless, it would be valuable to obtain additional confirmation of the uniqueness of Solution No. 1. Searches were made in the vicinity of Solution No. 1 at 51.8 MeV, but no other minimum in χ^2 was found. On the other hand, a

TABLE III. g^2 as a free parameter. N denotes the number of free *phases*. The number of free parameters is $N+1$. The pion-nucleon value is $g^2=14.4$.

Model	χ^2	χ^2 ratio	g^2
OPE(4)	26.4	1.32	6.2±2.1
OPE(5)	16.0	0.84	17.5±3.5

[9] P. Signell and N. R. Yoder, Phys. Rev. **132**, 1707 (1963).
[10] See Ref. 5 for 142 MeV and P. Signell and N. R. Yoder (to be published) [Phys. Rev.] for 210 MeV.

TABLE IV. Comparison of the phases from various models (see text). Nuclear bar, in degrees.

Model	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	ϵ_2	$^1D_2$
YLAM	36.11	13.18	−9.46	6.42	−2.19	2.26
YRB1	38.40	12.15	−8.37	5.73	−1.14	1.83
SBM	42.03	12.40	−9.04	5.10	−1.82	1.54
HJ	38.58	10.73	−8.32	5.44	−1.73	1.79
Yale	36.19	11.95	−9.85	6.13	−2.11	1.95
SW	34.0	14.0	−8.1	6.8	−1.78	2.0
ALV	...	...	...	...	...	1.33
OPE(5)	36.56	17.25	−7.55	5.06	−1.93	1.62
	(±0.77)	(±1.97)	(±0.47)	(±0.41)		(±0.10)

Model	$^3F_2$	$^3F_3$	$^3F_4$	ϵ_4	$^1G_4$	$^3H_4$	$^3H_5$
YLAM	0.23	−0.80	0.07	−0.22			
YRB1	0.14	−0.34	0.21	−0.09	0.14		
SBM	0.34	−0.74	0.08	−0.21	0.15	0.02	−0.07
HJ	0.39	−0.69	0.07	−0.21	0.15	0.02	−0.07
Yale	0.41	−0.85	0.14	−0.25	0.16	0.03	−0.09
SW	0.35	−0.70	0.15				
ALV	0.04	−0.67	0.24	−0.08	0.21	0.04	−0.10
OPE	0.40	−0.82	0.08	−0.22	0.17	0.03	−0.10

number of solutions of the type No. 2 were found. The phases corresponding to one of them are shown in Table II. In view of the variety of data used here, it appears that it will be very difficult (experimentally) to eliminate the other solutions. One is at present forced to rely on the higher energy data combined with model extrapolations to lower energies.

VIII. OTHER MODELS

There have been a number of previous analyses which can yield predictions for the data under consideration. The phase shifts for the various models are shown in Table IV. The Yale[11] energy-dependent phase shifts YLAM and YRB1 were taken from accurate graphs. The phases from the Saylor-Bryan-Marshak[12] (SBM) boundary condition model, the Hamada-Johnston[13] (HJ) potential model, and the Yale[14] potential-with-cutoff model were all computed from the model parameters. The phases predicted by the Scotti-Wong[15] resonant-boson exchange model were available only in the form of the published graphs, so they were known to less accuracy than were the phases for the other models. The Amati-Leader-Vitale[3] (ALV) phases were taken from full-scale graphs supplied by the authors.

TABLE V. Goodness-of-fit of various models to the 51.8-MeV data.

Model	χ^2	χ^2/χ^2[OPE(5)]	Remarks
YLAM	71	4.2	P much too high
YRB1	66	3.9	$\sigma(45°)$ much too high, $\sigma(90°)$ much too low.
SBM	199	11.9	P much too low, $\sigma_{\text{abs.}}$ much too high.
HJ	57	3.4	P much too low
Yale	47	2.8	P much too high
SW	158	9.5	P much too high
ALV(5)	15	0.9	
OPE(5)	17	1.0	

The values of χ^2 for the various models are shown in Table V. The rather high values of χ^2 are consistent with the phases shown in Table IV. There, one sees that the model phases are consistently low for the $^3P_0$, high for the $^3P_1$, etc., Noyes[16] has pointed to the danger of extending phenomenological energy-dependent analyses into energy regions where the data are (at the time) insufficient to specify discrete solutions. This remark applies to all of the above models, unless one views the scalar-boson-exchange mechanism of Scotti and Wong as having a theoretical basis.

A probability of 0.04 corresponds to a χ^2 of about 40 with 25 degrees of freedom.[8] Smaller probabilities, corresponding to the larger χ^2 values in Table V can not be meaningfully evaluated without a larger number of data. The $^1S_0$ is usually a difficult phase to fit with a model. The extent to which the large χ^2 values of the models were due to the $^1S_0$ was examined by making the $^1S_0$ a free (searched-upon) parameter. The resulting values of χ^2 were virtually unchanged except for two models: SBM and SW. The SBM χ^2 went down to 41, making it the best fit of any model. The polarization was still too small. The SW χ^2 went down to 71, with the polarization still much too high.

TABLE VI. Fractional *increase* in the phase shift standard deviations, for OPE(5), resulting from the removal of various data.

Data removed	$^1S_0$	$^1D_2$	$^3P_0$	$^3P_1$	$^3P_2$
σ ($\theta\geq 35°$)	−0.27	0.30	−0.36	−0.28	−0.37
N_σ (small angles)	0.04	0.10	0.04	0.04	0.04
σ ($\theta\leq 35°$)	0.52	0.80	0.49	2.24	0.15
N_σ's, $\sigma_{\text{abs.}}$	4.16	0.20	0.46	0.36	0.00
P	0.40	0.20	0.03	0.64	0.63
D	0.32	0.00	0.12	0.02	0.05
C_{NN}	0.04	0.00	0.11	0.00	0.17
C_{NN},D	0.71	0.00	0.58	0.43	0.59
C_{KP}	0.08	0.00	0.40	0.17	0.41

IX. FURTHER EXPERIMENTS

If future experimental work is to be done at energies near to 50 MeV, it will be useful to have some advance indication of the relative usefulness of the different

[11] G. Breit, M. H. Hull, Jr., K. E. Lassila, and K. D. Pyatt, Jr., Phys. Rev. **120**, 2227 (1960).
[12] D. P. Saylor, R. A. Bryan, and R. E. Marshak, Phys. Rev. Letters **5**, 266 (1960).
[13] T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).
[14] K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).
[15] A. Scotti and D. Y. Wong, Phys. Rev. Letters **10**, 142 (1963).
[16] H. P. Noyes (private communication).

TABLE VII. Fractional *decrease* in the phase shift standard deviations, for OPE(5), resulting from the halving of the standard deviations of several experimental data.

Datum	$^1S_0$	$^1D_2$	$^3P_0$	$^3P_1$	$^3P_2$
P	0.01	0.00	0.00	0.02	0.02
C_{KP}	0.12	0.00	0.34	0.13	0.29
D	0.32	0.00	0.27	0.02	0.22

kinds of experiments. Iwadare,[17] for instance, suggested that correlation experiments would be more fruitful than triple scattering. It would also be interesting to know what experimental work would be more likely to further restrict the phase shifts at 52 MeV.

In an attempt to obtain such information, various parts of the data set were removed and the OPE(5) analysis repeated. The results are shown in Table VI. The removal of the large-angle telescope cross section points, the first line in Table VI, resulted in a precipitous drop in χ^2 to 0.3 of its expected value and an accompanying *decrease* in the phase shift standard deviations. The only phase shift which changed appreciably, the $^3P_0$, went up only 0.5°. The behavior described above is what one might expect from the poor fit to the large-angle points shown in Fig. 1. The ALV(6) prediction is similar to the better fitting OPE(8) curve in Fig. 1, indicating that the effect is less pronounced for the ALV model. This also shows the *source* of the aforementioned slight superiority of ALV to the usual modified phase analyses. A striking aspect of Table VI is the way some kinds of data have an exceedingly strong effect on one phase shift, and the way several of the phase shifts respond only to a few kinds of measurements. The nonlinear behavior shown in the next-to-last line of Table VI (compare to the two lines above it) is probably due to off-diagonal elements in the error matrix.

One can also examine the effect of enhancing one datum over the others by halving its experimental standard deviation. This may give some indication of the effect, on the phase-shift standard deviations, of more precise measurements of those quantities. Table VII displays such results for C_{KP}, D, and P. The main result, not unexpected from Table VI, is an indication that more precise measurements of C_{KP} and D would be of more value than P.

The above indicates that the most useful experiments to perform first, at other energies, would be σ, P, C_{KP}, and C_{NN}; unless D is measured to high accuracy. Note that very accurate relative cross sections have already been obtained[18] at 9.69, 18.2, 25.63, 39.4, and 68.3 MeV. One should also note that there seem to be discrepancies in $P(45°)$ between various experimental groups,[19] outside what one would expect statistically.

[17] J. Iwadare, Proc. Phys. Soc. (London) **78**, 1188 (1961).

X. SUMMARY OF PRINCIPAL RESULTS

The Amati-Leader-Vitale (ALV) multi-pion exchange phases are found to give a slightly better fit than modified phase analyses (OPE) to the 51.8 MeV *pp* data; the difference is in the relatively poorly fit large-angle cross sections.

The ratio of χ^2 to its expected value drops sharply as the number of search-phases N is increased, until $N=6$ for ALV and $N=5$ for OPE. The ALV phases are exceedingly stable to increasing N, corresponding to the only slightly decreasing χ^2. For OPE, on the other hand the higher angular momentum phases (of those being searched upon) are rather unstable to increasing N, drifting in some cases a number of standard deviations from their OPE values. Since χ^2 continues decreasing as N is increased, the better fits for larger N are statistically an improvement. Physically, however, one suspects either the large-angle cross section data, the adequacy of OPE, or both.

The limits on the interesting $^3P_0$ phase shift[1] are 12.7° and 19.2° using ALV(6) and OPE(5). Thus, the most interesting result of Hoshizaki *et al.*, a larger-than-expected $^3P_0$, is confirmed.

A tentative prediction of the pion-nucleon coupling constant g^2 $(=14.4)$ is 17.5 ± 3.5, which is at least a moderate success.

The data are sufficient to produce discrete solutions, but allow many of the type commonly called Solution No. 2. Hoshizaki *et al.*'s finding that there is no other solution in the neighborhood of type No. 1 is confirmed.

A number of current two-nucleon models were examined; none fitted the data very well statistically. This is either due to there being no attempt to fit this (not then available) data, or due to an incompatability of the 52 MeV with the data at nearby energies. Another alternative would be inadequacy of the models themselves. This question may be answered if the model parameters are readjusted with the inclusion of the present data.

[18] For 9.69 MeV, L. H. Johnston and D. E. Young, Phys. Rev. **116**, 989 (1959); 18.2 MeV, J. L. Yntema and M. G. White, Phys. Rev. **95**, 1226 (1954); 25.63 MeV, T. H. Jeong, L. H. Johnston, D. E. Young, and C. N. Waddell, Phys. Rev. **118**, 1080 (1960); 39.4 MeV, L. H. Johnston and D. A. Swenson, Phys. Rev. **111**, 212 (1958); 68.3 MeV, D. E. Young and L. H. Johnston, Phys. Rev. **119**, 313 (1960).

[19] See Ref. e, Table I.