powers in the denominator is $F_1'+f'$. There is no extraneous infrared divergence if the former number is always larger than the latter number for every choice of the subgraph G'.

First, assume $G=G'$. Here the condition is simply

$$4L-f>0.$$

By (B1) and (B2), this is equivalent to

$$W+F/2<2(L+1). \quad \text{(B3)}$$

Note that $L>0$. Next let $G\neq G'$. Since G cannot be made disconnected by removing one internal line, we have

$$F_1'+W_1'>1. \quad \text{(B4)}$$

In this case, the condition is that

$$F_1'+f'<4(L'+F_1'+W_1'-1).$$

Again by (B1) and (B2), this is equivalent to

$$W_2'+F_2'/2<2(L'-1)+3W_1'+\tfrac{5}{2}F_1'. \quad \text{(B5)}$$

By (B4), the right-hand side of (B5) is at least 3.

The simplified Feynman rules can be used to obtain the leading term for a graph G that cannot be made disconnected by removing one internal line provided that (B3) is satisfied for G and that (B5) is satisfied for every connected subgraph $G'\neq G$. In particular, these conditions are always satisfied if G has four external lepton lines and no external W line.

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Monte Carlo Calculations Related to the Analysis of Ultrahigh-Energy Nucleon-Nucleon Interactions*

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The Monte Carlo method is applied to an investigation of angular-distribution parameters currently in use in studies of ultrahigh-energy nuclear interactions. Specifically, center-of-mass system properties of meson showers having a total multiplicity of 16 and corresponding to nucleon-nucleon collisions at 3×10^{12} eV are calculated using the Monte Carlo method and employing input information based largely on experimentally determined average properties of ultrahigh-energy nuclear interactions. The resulting meson showers conserve energy and momentum, the angular distributions on the average possess forward-backward symmetry in the center-of-mass system, and the produced particles of each shower possess only those correlations introduced by energy-momentum conservation. After making an exact transformation of the showers to the laboratory system, some conventional analysis procedures are carried out with the resulting Monte Carlo jets using parameters calculated from the angular distributions of the charged particles. The results give insight into the sensitivity of the parameters to approximations used in the interpretation of the parameters and indicate how well, on the average as well as for individual jets, a parameter $Y(\theta)$ is a measure of the physical quantity y which the parameter is expected to represent. The parameters are studied using the means and standard deviations of their $\log[Y(\theta)/y]$ distributions. The Castagnoli energy is found, rather independently of the details of the Monte Carlo calculations, to be an overestimate of the energy of a jet by an average factor of about 1.8 (antilogarithm of the mean), with a factor 2.3 (antilogarithm of the standard deviation) defining the approximate 68% confidence interval for statistical fluctuations about this average factor in the case of individual jets. Seven other parameters are examined and are found to be generally more sensitive to the details of the Monte Carlo calculations. The factor defining the approximate 68% confidence interval for statistical fluctuations of individual $Y(\theta)/y$ values about the average factor ranges from 1.5 to 3.1 for these other parameters. An application of the $(x-\langle x\rangle)/\sigma$ analysis of the Krakow-Warsaw group indicates that, at least for the jet models considered, fluctuations will not result in spurious two-center effects being indicated by the analysis.

I. INTRODUCTION

In many studies of ultrahigh-energy interactions, experimenters must of necessity deduce most of the properties of the interactions from only the angular distributions of the emitted shower particles. In order to properly evaluate the significance of information so obtained, it is rather important to know the effects of the approximations used as well as the effects of statistical fluctuations. Partly in an attempt to ascertain these effects, we have carried out some common analysis procedures employing the angular distributions of meson showers (jets) which have been calculated by Monte Carlo methods from well-defined input information, much of which is based on experimental data from nuclear interactions at energies around 10^{12} eV. The results of such analysis procedures are compared with the information which the analyses are expected to provide.

The Monte Carlo jets may be utilized in another

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manner. The distributions of a certain parameter may be obtained from the Monte Carlo jets for the purpose of comparison with a corresponding distribution obtained from experimental data on physical jets. Of course, agreement or disagreement of an experimental distribution with the corresponding distribution for the Monte Carlo jets would indicate that some property of physical jets can or cannot be explained, as the case may be, in terms of statistical fluctuations and/or the properties of the Monte Carlo models. We have used our Monte Carlo jets in this spirit to check the possibility of apparent two-center effects arising from the type of analysis used by the Krakow-Warsaw group.

The Monte Carlo models of ultrahigh-energy interactions, if taken seriously, can be used to obtain certain information on interaction properties which are difficult, if not impossible, to determine experimentally. We have not attempted to use the Monte Carlo jets for this purpose.

II. THE MONTE CARLO MODELS

The procedure used to generate the Monte Carlo jets is similar to that which has been used by Lohrmann, Teucher, and Schein.[1] The secondaries of Monte Carlo jets constructed using this method possess no correlations other than those imposed by energy-momentum conservation.[2,3]

For our calculations we have taken the total number of particles produced in each interaction to be 16, since this is probably the approximate number which are produced in nucleon-nucleon collisions at energies $\approx 10^{12}$ eV.[4–6] For a particular Monte Carlo jet, the center-of-mass system emission angle θ^* and azimuthal angle φ for each of the first 15 particles were chosen in a random manner from the angular distributions described below. For all jets the distribution of azimuthal angles was taken to be uniform over the range 0 to 2π. For two samples of jets, θ^* values were determined using the distribution $S(\theta^*)d\Omega^* \propto (1/\sin\theta^*)d\Omega^*$ since this distribution appears to describe the observed average angular distribution of the particles emitted from physical jets.[1,4,7–9] For purposes of comparison, θ^* values for two additional samples of jets were determined using an isotropic angular distribution.

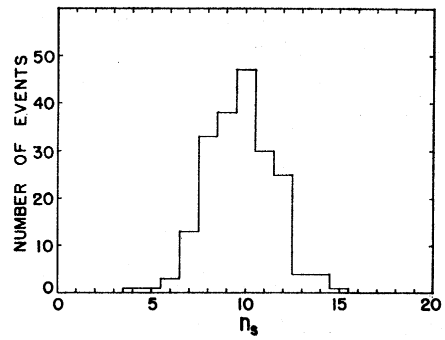

FIG. 1. Typical charged particle multiplicity distribution for a sample of 200 Monte Carlo jets.

Next, the transverse momentum p_t for each of the first 15 particles of each Monte Carlo jet was chosen randomly, and hence independently of θ^*, from an assumed p_t distribution. Two p_t distributions were used. One was the skewed distribution (SPT) given by $S(p_t)dp_t \propto dp_t p_t \exp(-p_t/p_0)$. This distribution with $p_0 = 0.19$ BeV/c (and $\langle p_t \rangle = 2p_0 = 0.38$ BeV/c) was adopted because it appears to be consistent with observations made at cosmic-ray as well as at accelerator energies.[10,11] Mainly for comparison purposes, we have also used a Gaussian p_t distribution (GPT) of the form $G(p_t)dp_t \propto dp_t \exp[-(p_t - \langle p_t \rangle)^2/2\sigma^2]$ with $\langle p_t \rangle = 0.37$ BeV/c and $\sigma = 0.14$ BeV/c. This latter distribution represents well the corrected neutral pion p_t distribution obtained from the early work of the Japanese emulsion chamber group.[12]

As a result of the use of the procedures described above, the first 15 particles of each Monte Carlo jet have uncorrelated values of θ^*, φ, and p_t, as well as uncorrelated values of longitudinal momentum and p_t. This is consistent with observed properties of jets.[7,11,13] Of course, these quantities for all of the particles in a physical jet are slightly correlated because of momentum and energy conservation. In principle, the process of selecting θ^*, φ, and p_t for every particle after the first should not have been entirely random because the available volume in phase space diminishes somewhat

[1] E. Lohrmann, M. W. Teucher, and Marcel Schein, Phys. Rev. **122**, 672 (1961).

[2] No attempt is made here to incorporate into the models the effects of such intermediate particles as ρ, ω, η, f, K^*, etc. If the production of such intermediate particles is significant in physical jets, the number of independent emissions would be lowered, there would be greater restrictions on the available phase space, and in general many of the distributions considered below would tend to be wider.

[3] See L. Van Hove, Nuovo Cimento **28**, 798 (1963) for an interesting paper which is principally concerned with such a model.

[4] R. L. Fricken, Ph.D. thesis, Louisiana State University, Baton Rouge, Louisiana, 1963 (unpublished).

[5] F. Abraham, J. Kidd, M. Koshiba, R. Levi Setti, C. H. Tsao, W. Wolter, C. L. Deney, R. L. Fricken, and R. W. Huggett, Proceedings of the Bristol Conference on Ultra High Energy Nuclear Physics, 1963 (unpublished).

[6] A. G. Barkow, B. Chamany, D. M. Haskin, P. L. Jain, E. Lohrmann, M. W. Teucher, and Marcel Schein, Phys. Rev. **122**, 617 (1961).

[7] M. Koshiba, Brookhaven National Laboratory Report BNL 772 (T-290), 1962 (unpublished).

[8] B. Edwards, J. Losty, D. H. Perkins, K. Pinkau, and J. Reynolds, Phil. Mag. **3**, 237 (1958).

[9] S. Hasegawa, J. Nishimura, and Y. Nishimura, Nuovo Cimento **6**, 979 (1957).

[10] E. Lohrmann, Nuovo Cimento, Suppl. (to be published).

[11] G. Cocconi, L. J. Koester, and D. H. Perkins, University of California, Lawrence Radiation Laboratory Report UCID-1444, High-Energy Physics Study Seminars No. 28 (Part 2), 1961 (unpublished).

[12] O. Minakawa, Y. Nishimura, M. Tsuzuki, H. Yamanouchi, H. Aizu, H. Hasegawa, Y. Ishii, S. Tokunaga, Y. Fujimoto, S. Hasegawa, J. Nishimura, K. Niu, K. Nishikawa, K. Imaeda, and M. Kazuno, Nuovo Cimento, Suppl. **11**, 125 (1959).

[13] S. J. Lindenbaum and R. M. Sternheimer, Brookhaven National Laboratory Report BNL 772 (T-290), 1962 (unpublished).

as each particle is assigned θ^*, φ, and p_t values. Because of the relatively large available energy which will be assumed and because of the relatively large numbers of particles in each jet, the energy-momentum constraint does not seriously restrict the allowed θ^*, φ, and p_t values until the last few particles of each jet are assigned these quantities. Due to these considerations and the serious practical difficulties in treating the energy-momentum constraint exactly, we have ignored this constraint in the selection of the θ^*, φ, and p_t values for the first 15 particles of each jet, but have let the requirement of the vanishing of the net momentum of the 16 produced particles determine these quantities for the 16th particle. The correlation thus imposed between the momentum of the 16th particle and the momenta of the other particles is abnormally large since this one correlation must compensate for the small momentum correlations that should exist among all the particles. It is reasonable to expect that on the average the correlation imposed in this approximate way will have effects similar to those that would have resulted had all the momenta been properly correlated.

As a result of the treatment of momentum described above, the total momentum of the 16 produced particles of every Monte Carlo jet is zero in the coordinate system in which the jets are constructed. This system will be the center-of-mass system for the collision, under the assumption made in our models that in the center-of-mass system of the collision the net momentum of the surviving collision partners is zero. The surviving collision partners are not considered to be any of the 16 particles assigned θ^*, φ, and p_t values as described above.

In order to be able to calculate center-of-mass system energies E_i^* and in order to have an assignment of which particles are charged and which are neutral, each produced particle was chosen to be a charged pion, neutral pion, charged nonpion, or neutral nonpion with the probabilities for assignment of each type being, respectively, 0.49, 0.24, 0.12, and 0.15. These probabilities correspond to the experimentally-determined average composition of jets.[7] The use of this procedure

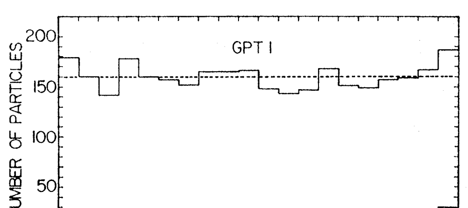

FIG. 2. Typical center-of-mass system angular distribution of particles in a sample of Monte Carlo jets based on the isotropic distribution. Solid line: actual distribution of all particles. Dashed line: isotropic distribution, which was used for the first 15 particles of each jet, normalized to the total number of particles. Cross-hatched area: distribution of the 16th particles only.

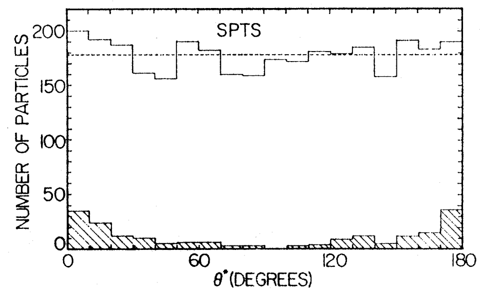

FIG. 3. Typical center-of-mass system angular distribution of particles in a sample of Monte Carlo jets based on the $(1/\sin\theta^*)d\Omega^*$ distribution. Solid line: actual distribution of all particles. Dashed line: distribution of the form $(1/\sin\theta^*)d\Omega^*$, which was used for the first 15 particles of each jet, normalized to the total number of particles. Cross-hatched area: distribution of the 16th particles only.

results in binomial multiplicity distributions for each particle type. A typical distribution of charged particle multiplicity n_s is shown in Fig. 1. The mean value of n_s is 9.8. All particles were assigned masses consistent with the particle-type assignment, with the nonpions being assigned the K-particle mass.

Although the energy that should correspond to the Monte Carlo jets has already been inferred from requiring that the number of produced particles be 16, at this point the specific assumption is made that each Monte Carlo jet results from a collision which in the laboratory system involves a nucleon with energy $E=3\times10^{12}$ eV incident upon a nucleon at rest. The energy W^* available in the center-of-mass system of such a collision is 73 BeV. Each jet, in order to be consistent with energy conservation, must then satisfy the relation $\sum_{\text{all}}E_i^*\leq W^*=73$ BeV (where $\sum_{\text{all}}$ denotes summation over all produced particles) since in our models we make the assumption that the part of the available energy not carried off by the produced particles, $W^*-\sum_{\text{all}}E_i^*$, is the energy carried off by the surviving nucleons. It then turned out that all jets constructed using the isotropic angular distribution satisfied the energy conservation requirement, while 12% of the jets constructed using the $(1/\sin\theta^*)d\Omega^*$ distribution were rejected because they did not satisfy this requirement.

Applying the procedure outlined above and using the various combinations of the two θ^* and the two p_t distributions, we obtained four samples each consisting of 200 events which satisfy the momentum and energy conservation laws. These samples will henceforth be identified by SPTI, SPTS, GPTI, and GPTS. In these designations, the first three letters denote the p_t distribution and the fourth letter denotes the θ^* distribution.[14]

The distributions and tabulations for the four samples are arranged throughout this paper in the order GPTI, SPTI, GPTS, and SPTS since it will be seen that this usually is the order of decreasing "compactness" or

[14] For example, GPTI denotes Gaussian p_t distribution, isotropic angular distribution; SPTS denotes skewed p_t distribution, $(1/\sin\theta^*)d\Omega^*$ angular distribution; etc.

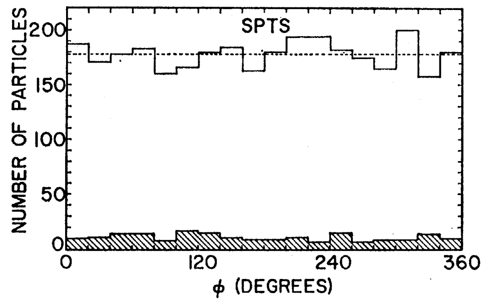

FIG. 4. Typical distribution of azimuthal angles φ of particles in a sample of Monte Carlo jets. Solid line: actual distribution of all particles. Dashed line: random φ distribution, which was used for the first 15 particles of each jet, normalized to the total number of particles. Cross-hatched area: distribution of the 16th particles only.

narrowness of the distribution of a particular quantity characterizing the particles in a given sample. For the reasons given previously, the jets in the SPTS sample have average properties most similar to physical jets.

Figures 2–6 show the essential features of the θ^*, φ, and p_t distributions finally obtained using the above-described procedures. Figures 2 and 3 show typical θ^* distributions obtained for samples for which the isotropic and $(1/\sin\theta^*)d\Omega^*$ angular distributions were used. It is obvious in each figure that the distribution of the 16th particles is more peaked toward 0° and 180° than the distribution used for the first 15 particles, and consequently the over-all angular distributions are slightly more peaked than the respective isotropic or $(1/\sin\theta^*)d\Omega^*$ distributions. However, the effect of the 16th particles on an over-all distribution is rather small. Indeed, the χ^2 probability for statistical fluctuations alone to cause a deviation larger than that obtained is 21% for the GPTS sample and is greater than 44% for the other three samples.

A typical φ distribution is presented in Fig. 4. The φ distribution of the 16th particles is nearly the same distribution that was used for the first 15 particles; hence the over-all distribution deviates little from the latter distribution. For all four samples, the χ^2 probability is greater than 67% for statistical fluctuations to cause deviations larger than those observed.

TABLE I. Average values of p_t, E^*, and K^*.

Quantity[a]	Sample GPTI	SPTI	GPTS	SPTS
$\langle p_t\rangle_{ch}$	0.44	0.47	0.43	0.46
$\langle p_t\rangle_{all}$	0.43	0.46	0.43	0.46
$\langle p_t\rangle_{1-15}$	0.37	0.38	0.37	0.38
$\langle p_t\rangle_{16th}$	1.4	1.6	1.4	1.6
$\langle E^*\rangle_{all}$	0.78	0.83	1.7	1.7
$\langle E^*\rangle_{1-15}$	0.65	0.68	1.3	1.3
$\langle E^*\rangle_{16th}$	2.6	3.0	7.6	7.6
$\langle K^*\rangle$	0.17	0.18	0.36	0.37

[a] All momenta are in BeV/c and all energies are in BeV. A subscript after a closing angular bracket indicates the particles included in the average.

Typical p_t distributions obtained are shown in Figs. 5 and 6. Mean values of p_t are given in Table I. The p_t distributions for the 16th particle are distinctly different from those used for the first 15 particles. In general, the distribution of p_t values for the 16th particle is much broader and it extends to considerably larger values, causing $\langle p_t\rangle$ for the 16th particle to be greater than that for the other particles by a factor of 3 to 4. Because of the contribution caused by the 16th particle, the p_t distribution for all the particles in a given sample has a small tail ranging up to nearly 5 BeV/c. This tail constitutes, however, only from 3–5% of the area of an over-all distribution. From the way in which the jets were constructed, only the 16th particle might have a correlation of p_t with θ^*. However, it was verified for each sample that the p_t and θ^* values for the 16th particle have no significant correlation.

Thus, under the assumption that in the center-of-mass system the surviving nucleons have zero net momentum and that they carry off that part of the available energy not possessed by the produced par-

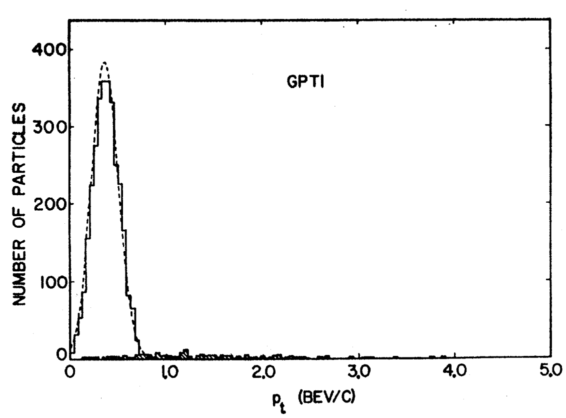

FIG. 5. Typical transverse-momentum distribution of particles in a sample of Monte Carlo jets based on the Gaussian p_t distribution. Solid line: actual distribution of all particles. Dashed line: Gaussian distribution used for the first 15 particles of each jet, normalized to the total number of particles. Cross-hatched area: distribution of the 16th particles only.

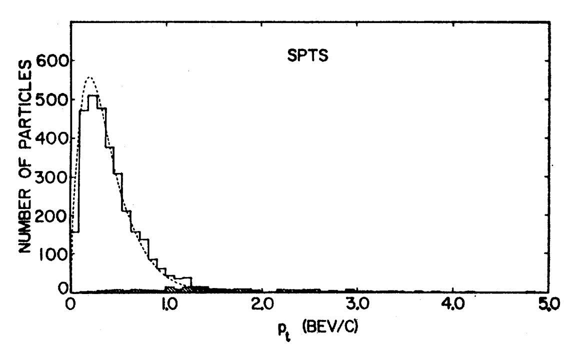

FIG. 6. Typical transverse-momentum distribution of particles in a sample of Monte Carlo jets based on the skewed p_t distribution. Solid line: actual distribution of all particles. Dashed line: skewed distribution used for the first 15 particles of each jet, normalized to the total number of particles. Cross-hatched area: distribution of the 16th particles only.

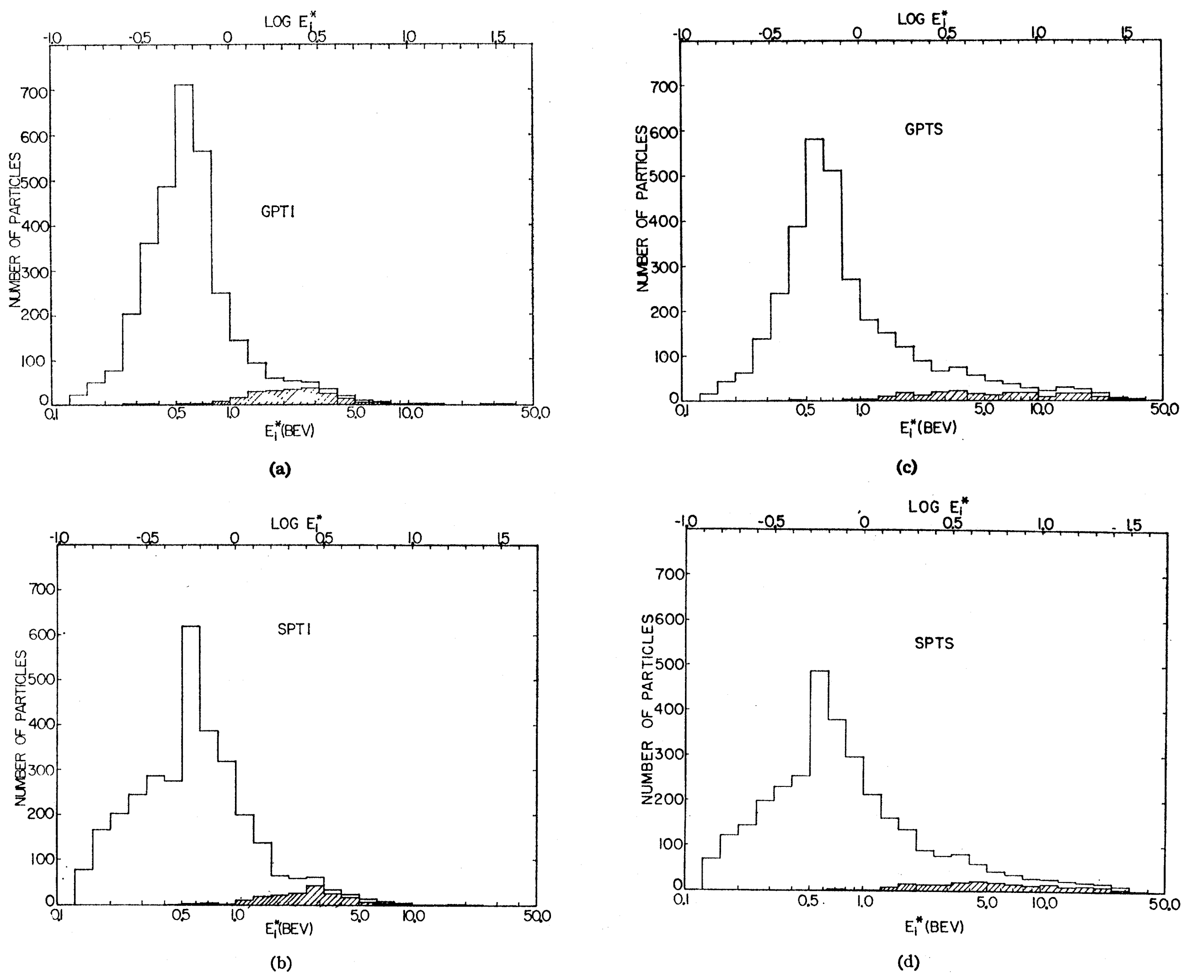

FIG. 7. Center-of-mass system energy distribution for particles in each sample of Monte Carlo jets. Solid line: distribution for all particles. Cross-hatched area: distribution for the 16th particles only.

ticles, we have by the procedures described above constructed in the center-of-mass system jets which satisfy the energy-momentum conservation laws and which have average properties in fair agreement with certain specific distributions.

The center-of-mass system energy distributions obtained for each sample are presented in Fig. 7 and a tabulation of mean values is given in Table I. In all samples the most probable energy is in the range 0.5–0.6 BeV. It is apparent that the energy distribution of the 16th particle is different from that for the rest of the particles, being broader and having a mean value 4 to 6 times larger than that for the other particles.

The inelasticity K of an interaction is defined to be the fraction of the available energy possessed by the produced particles. The center-of-mass system inelasticity for a Monte Carlo jet is then given by $K^* = \sum_{\text{all}} E_i^*/W^*$. Information on the mean K^* values is given in Table I. For our samples, the distribution of K^* is determined primarily by the angular distribution rather than the p_t distribution employed, hence in Figs. 8 and 9 are presented typical K^* distributions for each of the two angular distributions used. The peaks of the distributions occur around $K^*=0.2$, consistent with experimental observations.[7] The mean inelasticity of 0.36 for the jets in the SPTS sample, which best approximates the average properties of physical jets, is in good agreement with average inelasticities which have been experimentally determined by a number of methods.[4,15,16]

Each of the jets whose center-of-mass system properties have just been described was then transformed to the laboratory system using the exact transformation equations for the collision which in the laboratory system involves a nucleon with energy $E=3\times10^{12}$ eV incident upon a nucleon at rest.

For each of the four samples, the distribution of the

[15] B. Peters, *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962), p. 623.

[16] V. V. Guseva, N. A. Dobrotin, N. G. Zelevinskaya, K. A. Kotelnikov, A. M. Lebedev, and S. A. Slavatinsky, J. Phys. Soc. Japan **17**, Suppl. A-III, 375 (1962).

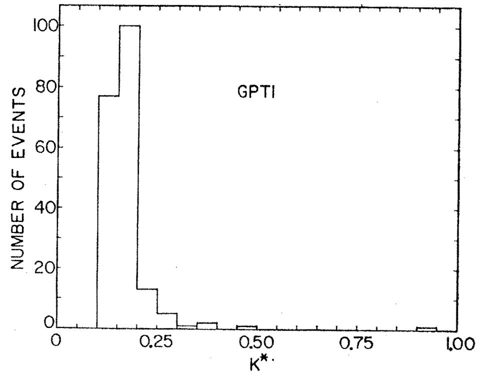

FIG. 8. Typical center-of-mass system inelasticity distribution for a sample of Monte Carlo jets based on the isotropic center-of-mass system angular distribution.

logarithm of the resulting laboratory-system energy ($\log E_i$) can be fitted quite well by a Gaussian curve. Characteristics of these distributions are summarized in Table II and in Figs. 10 and 11.

The resulting distributions of the laboratory system emission angles θ of the produced charged particles are probably best presented and discussed in terms of the parameter $x \equiv \log(\gamma_c \tan\theta)$, where $\gamma_c = (1-\beta_c^2)^{-1/2}$, β_c = velocity (in units of c) of the center-of-mass system with respect to the laboratory system. For all of the Monte Carlo jets $\gamma_c = 40$. Characteristics of the distributions of x are presented in Table II and in Figs. 12 and 13.

If we let β_i^* be the center-of-mass system velocity (in units of c) of a produced charged particle and assume that $\beta_i^* = \beta_c$, then an isotropic center-of-mass system angular distribution results in a distribution of x of the form $f(x)dx = (2.3/2)[dx/\cosh^2(2.3x)]$ with a mean of zero and a standard deviation $\sigma = (12)^{-1/2}\pi \log e = 0.39$. Under the same assumption a $(1/\sin\theta^*)d\Omega^*$ distribution results in a distribution of x given by $g(x)dx = (2.3/\pi) \times [dx/\cosh(2.3x)]$ with a mean of zero and $\sigma = (\pi/2) \times \log e = 0.68$.

The σ value of a particular Monte Carlo x distribution may be the same as, or differ from, 0.39 or 0.68 (whichever value is appropriate) depending upon the relative importance of (a) the tendency of σ to be reduced[17] by the presence in the Monte Carlo jets of

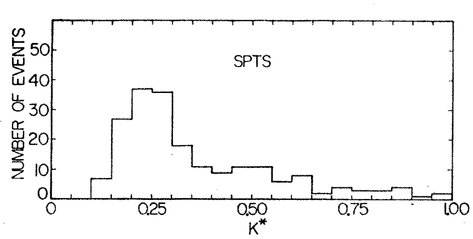

FIG. 9. Typical center-of-mass sytem inelasticity distribution for a sample of Monte Carlo jets based on the $(1/\sin\theta^*)d\Omega^*$ center-of-mass system angular distribution.

[17] K. Kobayakawa, K. Mori, K. Daiyasu, and H. Yokomi, Nuovo Cimento 28, 992 (1963).

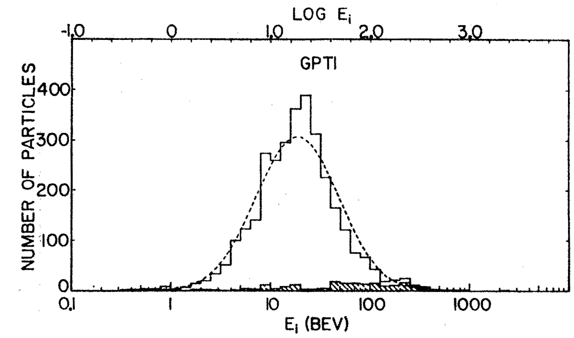

FIG. 10. Typical laboratory-system energy distribution for the particles in a sample of Monte Carlo jets based on the isotropic center-of-mass system angular distribution. Solid line: actual distribution for all the particles in the sample. Dashed line: normalized Gaussian distribution with the same mean and standard deviation as the actual distribution. Cross-hatched area: distribution for the 16th particles only.

particles with $\beta_i^* < \beta_c$, and (b) the tendency of σ to be increased because the θ^* distributions of the Monte Carlo jets are slightly more peaked toward 0° and 180° than the isotropic or $(1/\sin\theta^*)d\Omega^*$ distributions. It appears that effect (b) predominates slightly for the samples based on the isotropic θ^* distribution, while for the samples based on the $(1/\sin\theta^*)d\Omega^*$ distribution effect (a) predominates slightly.

The means of the Monte Carlo x distributions are less than zero because of the occurrence of particles with $\beta_i^* < \beta_c$. These mean values are intimately connected with the discussion to be given below on the Castagnoli formula.

III. DISCUSSION OF RESULTS

Let y be a physical quantity or parameter that characterizes some property of an ultrahigh-energy nuclear interaction, and let $Y(\theta)$ be an estimate of y determined solely from the laboratory-system emission angles θ of the produced charged particles. For a number of parameters we have attempted to use the Monte Carlo

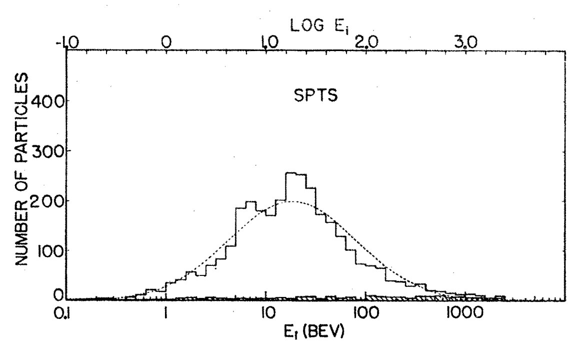

FIG. 11. Typical laboratory-system energy distribution for the particles in a sample of Monte Carlo jets based on the $(1/\sin\theta^*)d\Omega^*$ center-of-mass system angular distribution. Solid line: actual distribution for all the particles in the sample. Dashed line: normalized Gaussian distribution with the same mean and standard deviation as the actual distribution. Cross-hatched area: distribution for the 16th particles only.

TABLE II. Characteristics of the laboratory-system energy and angular distributions.

	Sample							
	GPTI		SPTI		GPTS		SPTS	
Quantity	Mean	σ	Mean	σ	Mean	σ	Mean	σ
$\log_{10}E_t$[a]	1.27	0.42	1.26	0.44	1.28	0.64	1.27	0.64
$\log_{10}(\gamma_c \tan\theta)$	−0.08	0.40	−0.12	0.40	−0.09	0.66	−0.13	0.65
$\log_{10}(\gamma_c'' \tan\theta)$[b]	0.00	0.38	0.00	0.39	0.00	0.64	0.00	0.63

[a] The units of E_t are BeV.
[b] Calculated taking for each event the value of γ_c'' given by the Castagnoli formula, Eq. (1).

jets to determine how well, on the average as well as for individual events, $Y(\theta)$ is a measure of y. This has been done by computing $Y(\theta)$ for each jet from the θ values of the produced charged particles ($\langle n_s\rangle = 9.8$) and then comparing the $Y(\theta)$ values so obtained with the known corresponding y values by finding the distribution of $\log_{10}[Y(\theta)/y]$ and computing for this distribution the mean and the standard deviation σ. Properties of the logarithmic distributions are given here because, for all of the parameters investigated, comparison of the linear and logarithmic distributions of $Y(\theta)/y$ shows that the logarithmic distributions are generally more nearly symmetrical and Gaussian in appearance than the linear distributions. Then, the antilogarithm of the mean 10^m is the average factor by which $Y(\theta)$ overestimates y, and the antilogarithm of σ, 10^σ, is the factor which defines the approximate 68% confidence interval for statistical fluctuations of individual $Y(\theta)/y$ factors about the average factor. The limits of this confidence interval are $(10^{-\sigma})10^m$ and $(10^\sigma)10^m$.

The distribution of $\log[Y(\theta)/y]$ was obtained for each of the four samples of Monte Carlo jets with the expectation that comparison of the results for each sample would show the effects of the approximations under which $Y(\theta)\approx y$, particularly in so far as the approximations are influenced by variation of the θ^* and p_t distributions.

Since most of the results described in this section are based on $Y(\theta)$ values calculated using emission angles of only the *charged* particles produced in each event, the emission angle of the correlated 16th particle has been used in the calculation of $Y(\theta)$ only for events in which the 16th particle is charged. The fraction of events in which this occurs is the same as the average ratio $\langle n_s/N\rangle$, where N is the total number of produced particles per event. If one believes that the momentum correlation which was introduced solely by the 16th particle results in the events having average properties which are similar to those of physical jets with $N=16$ particles, then the average effect of correlations among n_s produced charged particles of a physical jet will probably be obtained on the average by including the 16th particle in a fraction $\langle n_s/N\rangle$ of the total number of Monte Carlo jets used. The average effects on these $\log[Y(\theta)/y]$ distributions caused by the presence or

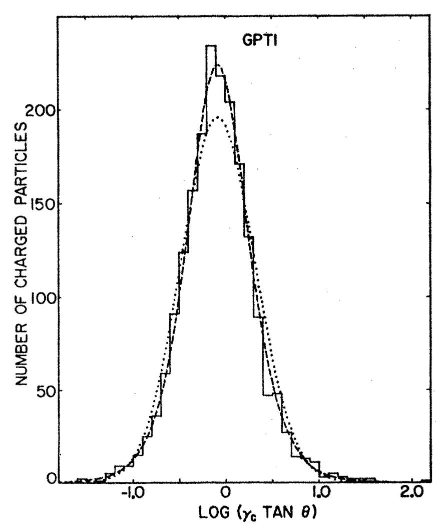

FIG. 12. Typical distribution of $x=\log(\gamma_c \tan\theta)$ for a sample of Monte Carlo jets based on the isotropic center-of-mass system angular distribution. Solid line: actual distribution for all charged particles. Dashed line: normalized curve of the form $(2.3/2) \times[dx/\cosh^2(2.3x)]$ with the same mean as the actual distribution. Dotted line: normalized Gaussian curve with the same mean and standard deviation as the actual distribution.

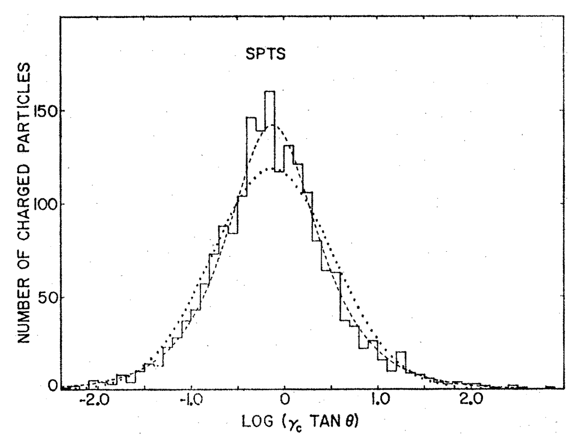

FIG. 13. Typical distribution of $x=\log(\gamma_c \tan\theta)$ for a sample of Monte Carlo jets based on the $(1/\sin\theta^*)d\Omega^*$ center-of-mass system angular distribution. Solid line: actual distribution for all charged particles. Dashed line: normalized curve of the form $(2.3/\pi) \times[dx/\cosh(2.3x)]$ with the same mean as the actual distribution. Dotted line: normalized Gaussian distribution with the same mean and standard deviation as the actual distribution.

TABLE III. Characteristics of the distribution of $\log_{10}[Y(\theta)/y]$.

	Sample							
	GPTI		SPTI		GPTS		SPTS	
Distribution	Mean	σ	Mean	σ	Mean	σ	Mean	σ
$\log(E_c/E)$	0.17	0.20	0.24	0.22	0.17	0.36	0.25	0.36
$\log(E_{ch}/\sum_{ch}E_i)$	0.00	0.15	0.01	0.20	−0.02	0.20	0.01	0.25
$\log[(E_{ch}/\gamma_c)/\sum_{ch}E_i^*]$	−0.10	0.11	−0.12	0.14	−0.14	0.16	−0.16	0.19
$\log(1.65E_{ch}/\sum_{all}E_i)$	−0.04	0.17	−0.02	0.20	−0.08	0.28	−0.06	0.29
$\log(E_{ch}/E)$	−1.05	0.17	−1.01	0.17	−0.81	0.32	−0.79	0.30
$\log(E_c'/E)$	0.01	0.32	0.06	0.34	−0.03	0.48	0.03	0.49
$\log[(E_{ch}/E_c)/K_{ch}^M]$	−0.18	0.13	−0.23	0.15	−0.19	0.25	−0.24	0.24
$\log[(1.65E_{ch}/E_c)/K^M]$	−0.20	0.16	−0.26	0.16	−0.25	0.28	−0.31	0.28

absence of the 16th particle among the charged particles is demonstrated and discussed in the Appendix.

For the last parameter discussed in this section, the analysis was performed in a somewhat different manner from that described above.

The Castagnoli Energy

To estimate the energy of the particle initiating an interaction, a commonly used method is based on the formula of Castagnoli *et al.*[18]

$$\log\gamma_c = -(\textstyle\sum_{ch} \log \tan\theta)/n_s\,, \qquad (1)$$

where $\gamma_c=(1-\beta_c^2)^{-1/2}$ with β_c=velocity (in units of c) of a reference frame c with respect to the laboratory system, and $\sum_{ch}$ denotes summation over the produced charged particles. Now the reference frame c to which Eq. (1) applies will be the center-of-mass system for the interaction provided the following assumptions are satisfied: (a) The produced charged particles are emitted with detailed forward-backward symmetry in the center-of-mass system and (b) each of the produced charged particles has $\beta_i^*=\beta_c$. An estimate of the laboratory system energy E of an incident particle colliding with a nucleon at rest in the laboratory system will at high energies be given by $E_c=2M\gamma_c^2$, where γ_c is given by Eq. (1) and M is the nucleon mass. This estimate is commonly called the Castagnoli energy of the event. When assumptions (a) and (b) are satisfied, one has $E_c=E$.

For each Monte Carlo jet, E_c has been computed and compared with the primary energy $E=3\times10^{12}$ eV. The results are presented in tabular form in Table III. The distribution for the SPTS sample is shown in Fig. 14.

The deviation from zero of the mean of $\log(E_c/E)$ is due entirely to failure of the jets to completely satisfy assumption (b) since the θ^* distributions employed are on the average symmetrical about 90°. The value of $\langle\log(E_c/E)\rangle$ depends little on the correlated 16th track. (See Appendix.) The main dependence of $\langle\log(E_c/E)\rangle$ is on the p_t distribution. The value of $\langle\log(E_c/E)\rangle$ is smaller for the samples having the GPT distribution than for samples having the SPT distribution because there are more numerous serious violations of assumption (b) in the latter cases. (See Fig. 6.) In the case of the SPTS sample E_c overestimates E on the average by a factor of 1.8. Lohrmann *et al.*[1] obtained a factor of 1.7 from similar calculations for jets with $\langle n_s\rangle=7.8$. These authors experimentally obtained a value of 1.3 for p and n jets in emulsions having energies around 250 BeV and heavy prong number $N_h\leq5$. This experimental factor is undoubtedly lower than the value appropriate to nucleon-nucleon collisions, since the effects of having secondary interactions inside the target nucleus and/or heavy effective targets are to systematically lower E_c.

For our samples, the standard deviation σ of the $\log(E_c/E)$ distribution depends on the θ^* distribution rather than on the p_t distribution, the values of σ being greater for the $(1/\sin\theta^*)d\Omega^*$ distribution with its more pronounced forward-backward peaking than for the isotropic distribution. As is demonstrated in the Appendix, the value of σ depends little on the presence or absence of the correlated 16th particle, although it can be seen that the presence of the correlated particle has a slight tendency to make σ smaller.[19] For the SPTS sample, σ was found to be 0.36 whether calculated using

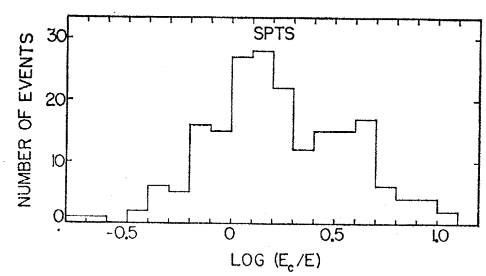

FIG. 14. Distribution of $\log(E_c/E)$ for the SPTS sample of Monte Carlo jets.

[18] C. Castagnoli, G. Cortini, C. Franzinetti, A. Manfredini, and D. Moreno, Nuovo Cimento **10**, 1539 (1953).

[19] See the discussion of the relationship of the correlations to the standard deviation of $\log\gamma_c$ given by D. H. Perkins, *Progress in Elementary Particle and Cosmic Ray Physics* (North-Holland Publishing Company, Amsterdam, 1960), Vol. V, Chap. 4, p. 257.

for each jet only the charged particles ($\langle n_s \rangle = 9.8$) or only the first 10 particles (which are uncorrelated).

We can conclude with fair certainty that for pure nucleon-nucleon collisions with multiplicities around 10 and in which the produced particles are emitted on the average with forward-backward symmetry the σ value for the $\log(E_c/E)$ distribution is about 0.36. From this value one obtains (1/2.3)1.8 and (2.3)1.8 as approximate 68% confidence limits on the fluctuations of individual values of E_c/E about 1.8, the most probable value.[20] With interactions in emulsion which usually involve complex nuclei as targets, it is certainly to be expected that the mean value of E_c/E will be lower and larger fluctuations in this ratio will occur as a result of secondary interactions inside the target nuclei.

The Parameter E_{ch}

Several cosmic ray as well as accelerator experiments have shown that the mean transverse momentum $\langle p_t \rangle$ of particles emitted from nuclear interactions is rather independent of particle energy and emission angle.[7,11,21] If it is assumed that $\langle p_t \rangle$ is independent of emission angle and equal to 0.4 BeV/c, then the parameter E_{ch} defined by $E_{ch} = \langle p_t \rangle \sum_{ch} \csc\theta_i = (0.4 \text{ BeV}) \sum_{ch} \csc\theta_i$ will be closely equal to $\sum_{ch} E_i$, the total laboratory-system energy of the charged particles produced in an interaction.[22]

Values of $\log(E_{ch}/\sum_{ch} E_i)$ have been calculated for the jets in each sample and the results summarized in Table III. The distribution for the SPTS sample is shown in Fig. 15. Within the statistical errors, the means of all four samples are consistent with zero. Hence we can conclude with considerable confidence that the most probable value of $E_{ch}/\sum_{ch} E_i$ will be unity for jets having p_t and θ^* distributions not too different from those used for our calculations. The σ value seems to have a weak dependence on the p_t distribution and the angular distribution used. From the σ value for the SPTS sample we can expect that 1.8 is approximately the 68% confidence limit for fluctuations of the factor by which E_{ch} overestimates or underestimates $\sum_{ch} E_i$ in an individual nucleon-nucleon interaction at an energy of about 3×10^{12} eV.

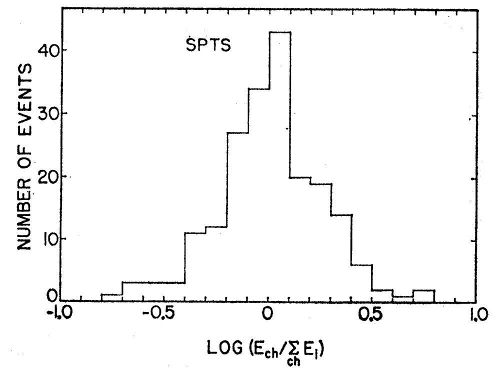

FIG. 15. Distribution of $\log(E_{ch}/\sum_{ch} E_i)$ for the SPTS sample of Monte Carlo jets.

E_{ch}/γ_c as a Measure of $\Sigma_{ch} E_i^*$

For an ultrahigh-energy interaction, E_{ch}/γ_c can be used to estimate the total energy of the produced charged particles in the center-of-mass system of the produced particles by assuming: (a) $E_{ch} = \sum_{ch} E_i$, (b) the Castagnoli γ_c given by Eq. (1) is the correct Lorentz factor for the transformation from the laboratory system to the center-of-mass system of the produced charged particles, and (c) the center-of-mass system of the produced charged particles coincides with the center-of-mass system of all the produced particles.

With the Monte Carlo jets we investigated the properties of this estimator by considering the distribution of $\log[(E_{ch}/\gamma_c)/\sum_{ch} E_i^*]$ for each of the jet samples.[23] Properties of these distributions are given in Table III. Figure 16 shows the distribution for the SPTS sample. It can be seen that rather independently of the θ^* or the p_t distribution, E_{ch}/γ_c underestimates $\sum_{ch} E_i^*$ on the average by a factor of 1.3 to 1.4. This systematic underestimation is almost entirely due to γ_c being systematically overestimated by the use of the Castagnoli formula. The fluctuations in the estimation for an individual event as measured by the σ value are also rather independent of the θ^* and p_t distributions. The factor corresponding to σ ranges from 1.3 to 1.5. Fluctuations of this estimator are smaller than the fluctuations of any of the other estimators considered in this paper.

$1.65 E_{ch}$ as a Measure of $\Sigma_{all} E_i$

In order to obtain an estimate of the energy carried from an interaction by all of the produced particles, both charged and neutral, it is customary to assume (a) that the energy distribution of the neutral produced

[20] Under the assumption that the angles θ are uncorrelated, the dependence of σ on n_s will be given by $\sigma = k/(n_s^{1/2})$ and k will be equal to twice the standard deviation of the log $\tan\theta$ distribution. From the σ value of the $\log(E_c/E)$ distribution for the SPTS sample, one obtains $k = 1.13$. The calculations of Lohrmann *et al.*[1] using a similar Monte Carlo model yield $k = 0.86$. The smaller value probably results from the Monte Carlo jets of that work possessing emission angles θ which are more correlated because (in that work) no adjustment of azimuthal angles was made in order to balance transverse momentum. Neither of these values of k is consistent with the value 1.30 which is expected from the standard deviation of the log $\tan\theta$ distribution of the SPTS sample. This is a result of the emission angles not being independent.

[21] D. H. Perkins, Proceedings of the International Conference on Theoretical Aspects of Very High Energy Phenomena, CERN 61-22, Geneva, 1961 (unpublished), p. 99.

[22] For a discussion of the parameter E_{ch}, see the introductory paper of the group of papers describing the results of the International Cooperative Emulsion Flight, Nuovo Cimento, Suppl. (to be published).

[23] Note that the ratio used in this comparison is equivalently the ratio of an estimator of the average energy of the produced charged particles in the center-of-mass system of the produced particles to the actual value of that energy.

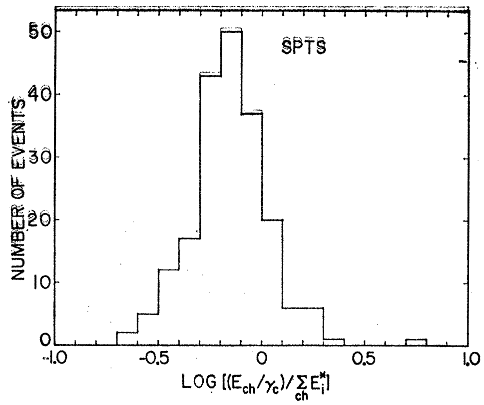

FIG. 16. Distribution of $\log[(E_{ch}/\gamma_c)/\sum_{ch}E_i^*]$ for the SPTS sample of Monte Carlo jets.

particles is the same as that of the charged produced particles, and hence that on the average the relative amount of energy carried by each component is proportional to the average ratio C of the number of particles of each component; and (b) that E_{ch} is a good estimate of the energy of the produced charged particles. Then on the average, $\sum_{all}E_i = CE_{ch}$. For the Monte Carlo jets assumption (a) is approximately satisfied and the appropriate value of C is 1.64.

In Table III are given the properties of the distribution of $\log(1.65E_{ch}/\sum_{all}E_i)$ for each sample of jets. The distribution for the SPTS sample is given in Fig. 17. One finds that the mean of this quantity is indeed near to zero. The σ value for the SPTS case corresponds to a factor of 2.0.

E_{ch} as a Measure of E

Now if one takes $\sum_{ch}E_i \approx E_{ch}$, then K_{ch}, the charged particle inelasticity of an interaction (i.e., the fraction of the available energy which is carried by the produced charged particles), will be given approximately by $K_{ch} \approx E_{ch}/E$. Once a determination of the most probable value of this ratio has been made, E_{ch} can be used as a measure of E.[24] To obtain some information on this

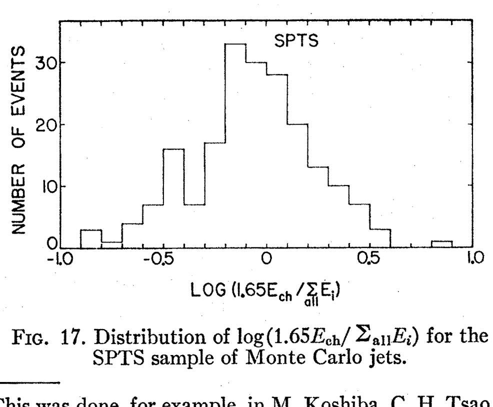

FIG. 17. Distribution of $\log(1.65E_{ch}/\sum_{all}E_i)$ for the SPTS sample of Monte Carlo jets.

method of primary energy estimation, we have found the distribution of $\log(E_{ch}/E)$ for the jets in each sample. The results are summarized in Table III and the distribution for the SPTS sample is given in Fig. 18. In comparing the results for the various samples, the mean and the σ value of a $\log(E_{ch}/E)$ distribution reflect the properties of the inelasticity distribution, being rather insensitive to which p_t distribution is used and smaller for the samples constructed from the isotropic angular distribution. From a comparison of σ values, it can be seen that for each of the samples the fluctuations of E_{ch}/E are approximately 11% smaller than those of E_c/E. Hence, in pure nucleon-nucleon collisions E_{ch} may be only slightly better than E_c as an estimator of E. However, in interactions with complex targets the superiority of E_{ch} is likely to be much more pronounced owing to the relative insensitivity of E_{ch} to effects of secondary interactions inside the target nucleus,[22] whereas E_c is rather sensitive to such effects.

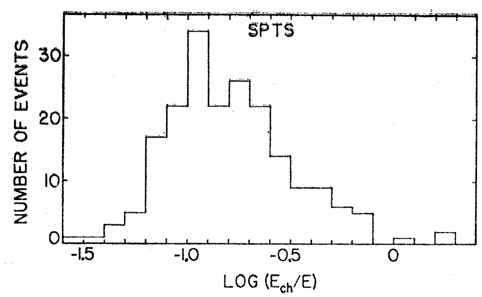

FIG. 18. Distribution of $\log(E_{ch}/E)$ for the SPTS sample of Monte Carlo jets.

The Parameter E_c' as a Measure of E

In large emulsion stacks such as the International Cooperative Emulsion Flight stack it is sometimes possible to make a rather good estimate of the energy E_γ of the electromagnetic cascade resulting from the decay of neutral pions produced in an ultrahigh-energy nuclear interaction. For the estimation of E of such events, Hildebrand and Silberberg[25] have attempted to incorporate this information along with the usually available information on the charged-particle angular distribution into a method for estimating the primary energy of such events. These workers have introduced a "modified Castagnoli energy" E_c' which is given by $E_c' = E_c[E_\gamma/(E_{ch}R)]^{2R/(1+R)}$ where $R \equiv N_{\pi^0}/n_s$.

The parameter E_c' will be equal to the energy E of the particle initiating an interaction if the following assumptions are satisfied: (i) The Castagnoli formula, Eq. (1), gives the correct Lorentz factor for the center-of-mass system of the collision when the mean is taken of the $\log\tan\theta$ values of both the produced charged particles and the produced neutral pions; (ii) the

[24] This was done, for example, in M. Koshiba, C. H. Tsao, C. L. Deney, R. L. Fricken, R. W. Huggett, B. Hildebrand, R. Silberberg, and J. J. Lord, Nuovo Cimento, Suppl. (to be published).

[25] B. Hildebrand and R. Silberberg, Nuovo Cimento, Suppl. (to be published).

Castagnoli formula gives the correct Lorentz factor for the center-of-mass sytem of the produced charged particles (ch system) when, as is customary, the mean is taken over the log $\tan\theta$ values for only the produced charged particles; (iii) the Castagnoli formula gives the correct Lorentz factor for the center-of-mass system of the produced neutral pions (n system) when the mean is taken of the log $\tan\theta$ values of only the produced neutral pions; (iv) $E_{ch}=\sum_{ch}E_i$; (v) $E_\gamma=\sum_{\pi^0}E_i$ where the summation is over the produced neutral pions; (vi) $(\sum_{\pi^0}E_i^n)/(\sum_{ch}E_i^{ch})=(\langle p_t\rangle\sum_{\pi^0}\csc\theta^n)/(\langle p_t\rangle\sum_{ch}\csc\theta^{ch})$ where the superscript on a quantity denotes the coordinate system in which the quantity is evaluated; and (vii) $(\sum_{\pi^0}\csc\theta^n)/(\sum_{ch}\csc\theta^{ch})=R$. In the calculation of E_c', R is assigned a fixed value, namely, the expected average value of N_{π^0}/n_s. Hildebrand and Silberberg put $R=\frac{1}{2}$ for the group of events which they analyzed; the same value was used in the Monte Carlo calculations described below.

It would appear that E_c' will have large fluctuations about E because the many assumptions required for E_c' to equal E involve equalities which are at best statistical. In the calculation of E_c' for each of the Monte Carlo jets, the exact total energy of neutral pions was used for E_γ. Thus assumption (v) is always satisfied in our calculations.

Characteristics of the $\log(E_c'/E)$ distributions obtained from the Monte Carlo calculations are summarized in Table III. One sees that on the average $E_c'\approx E$, independent of the input information used for each sample. Apparently the effects of systematic deviations from some of the equalities involved in assumptions (i)–(vii) are mutually compensating. As is shown in the Appendix, the mean of the $\log(E_c'/E)$ distribution is rather sensitive to the influence of the 16th particle of the jets. This causes us to view with some caution the apparent result that on the average $E_c'\approx E$. The σ values depend essentially on the angular distribution used in our calculations rather than on the p_t distribution. As expected, the fluctuations of individual values of E_c'/E about the mean are quite large. For the SPTS sample the factor corresponding to σ is 3.1, the largest value found for any of the estimators considered in this paper. The distribution of $\log(E_c'/E)_c$ for the SPTS sample is presented in Fig. 19.

E_{ch}/E_c and $1.65E_{ch}/E_c$ as Estimators of Mirror System Inelasticities

It has been pointed out[25,26] that the quantities E_{ch}/E_c and $1.65E_{ch}/E_c$ can be taken as estimators of the mirror system charged particle inelasticity K_{ch}^M and total inelasticity K^M, respectively, where the mirror system is the rest system of the primary particle.

The assumptions which must be satisfied for the equality $E_{ch}/E_c=K_{ch}^M$ to be valid are: (a) $E_{ch}=\sum_{ch}E_i$; (b) the Castagnoli formula, Eq. (1), gives the correct

[26] C. L. Deney, R. L. Fricken, and R. W. Huggett (unpublished).

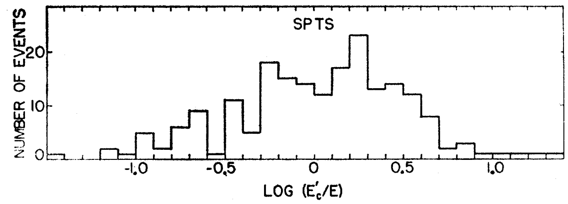

FIG. 19. Distribution of $\log(E_c'/E)$ for the SPTS sample of Monte Carlo jets.

Lorentz factor for the center-of-mass system of the produced charged particles[27]; (c) the velocity β_c (in units of c) of that system with respect to the laboratory system is approximately unity; and (d) the mass M_t of the effective target which actually participates in the collision is equal to M, the nucleon mass.[28] For the equality of $1.65E_{ch}/E_c$ to K^M, assumptions (a) and (b) above are replaced, respectively, by: (a′) $1.65E_{ch}=\sum_{all}E_i$; and (b′) the Castagnoli formula gives the correct Lorentz factor for the center-of-mass system of all the produced particles.

Characteristics of the distributions of $\log[(E_{ch}/E_c)/K_{ch}^M]$ and $\log[(1.65E_{ch}/E_c)/K^M]$ are presented in Table III. Distributions for the SPTS sample are presented in Figs. 20 and 21.

The average factor by which E_{ch}/E_c underestimates K_{ch}^M corresponds to the average factor by which the Castagnoli formula overestimates γ_c. Thus the variation of the mean from sample to sample can be understood in terms of the previous discussion of the Castagnoli energy estimate. The σ values also vary from sample to sample in a manner similar to the σ values for $\log(E_c/E)$; however the former σ values are smaller, corresponding in the case of the SPTS sample to a factor of 1.7.

The amount of the systematic underestimation of K^M by $1.65E_{ch}/E_c$ in each sample can be understood when one considers, as above, the systematic overestimate of γ_c by the Castagnoli formula and also

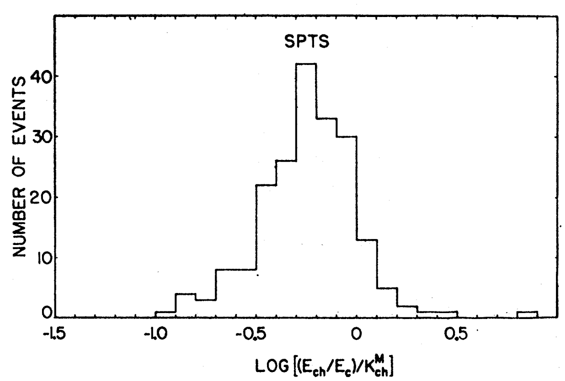

FIG. 20. Distribution of $\log[(E_{ch}/E_c)/K_{ch}^M]$ for the SPTS sample of Monte Carlo jets.

[27] This system need not necessarily coincide with the center-of-mass system of the collision.

[28] If this assumption is not satisfied E_{ch}/E_c (or $1.65E_{ch}/E_c$, discussed below) should be multiplied by M/M_t.

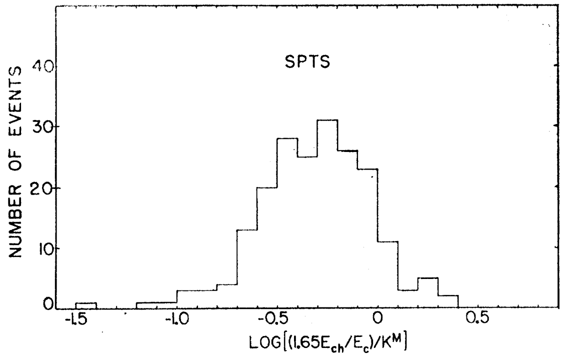

FIG. 21. Distribution of $\log[(1.65E_{ch}/E_c)/K^M]$ for the SPTS sample of Monte Carlo jets.

takes into account the small amount by which the factor 1.65 underestimates $(\sum_{all}E_i)/(\sum_{ch}E_i)$ on the average. The σ values of the $\log[(1.65E_{ch}/E_c)/K^M]$ distributions, as to be expected, are larger than those for the $\log[(E_{ch}/E_c)/K_{ch}{}^M]$ distributions, but only slightly —that for the SPTS sample corresponding to a factor of 1.9.

A Check of Apparent Two-Center Effects

The four samples of Monte Carlo jets were analyzed employing the method introduced by the Krakow-Warsaw cosmic-ray group for investigating double-maximum structure in the angular distributions of jets.[29] In this method of analysis, the differential angular distribution of a jet is presented in terms of the distribution of the quantity $(x-\langle x\rangle)/\sigma$, where $x=\log\tan\theta$ and $\sigma=[\sum_{ch}(x-\langle x\rangle)^2/(n_s-1)]^{1/2}$. The intervals of $(x-\langle x\rangle)/\sigma$ used for plotting the distribution are chosen to correspond to equal areas under a Gaussian curve.

Figure 22 shows for the GPTI and SPTS samples of jets the composite distribution of $(x-\langle x\rangle)/\sigma$ for all jets, as well as for only those having $\sigma>0.6$. It is apparent that, for the Monte Carlo jets, a systematic selection of events having statistical fluctuations towards a bimodal angular distribution is not introduced by the selection criterion $\sigma>0.6$ which is sometimes used by the Krakow-Warsaw group for selecting an enriched sample of two-center-type events. This confirms the conclusions of somewhat similar statistical studies made by Gierula *et al.*[29,30]

IV. CONCLUSIONS

In Table IV are summarized some results obtained from the SPTS jet sample, which of the four samples considered should have properties corresponding the most closely to those of physical jets. The first column of the table presents the average correction factor by which $Y(\theta)$ should be multiplied to obtain y. The second column gives factors which are the antilogarithms of σ of the $\log[Y(\theta)/y]$ distributions. These factors define the approximate 68% confidence interval for fluctuations of $Y(\theta)/y$ from the mean in individual jets.

TABLE IV. Average correction factors and a measure of their fluctuations as determined from the SPTS sample of Monte Carlo jets.

Application of average correction factors: $y=\langle y/Y(\theta)\rangle Y(\theta)$	Factor defining 68% confidence interval
$E=0.56E_c$	2.3
$E=6.2E_{ch}$	2.0
$E=0.93E_c'$	3.1
$\sum_{ch}E_i=1.0E_{ch}$	1.8
$\sum_{all}E_i=1.1(1.65E_{ch})$	2.0
$\sum_{ch}E_i{}^*=1.4(E_{ch}/\gamma_c)$	1.5
$K_{ch}{}^M=1.7(E_{ch}/E_c)$	1.7
$K^M=2.0(1.65E_{ch}/E_c)$	1.9

The most reliance can be placed in the results relating to the Castagnoli energy E_c since it has been shown that these results are rather independent of the details of the Monte Carlo calculation. Results for the other parameters were found to be somewhat more sensitive. It can be seen from Table IV, however, that for all parameters, the factors corresponding to σ of the $\log[Y(\theta)/y]$ distributions are not greatly different from 2.

The application of the $(x-\langle x\rangle)/\sigma$ analysis of the Krakow-Warsaw group indicates, at least for the interaction models considered, that fluctuations will not cause spurious two-center effects to be indicated by such an analysis.

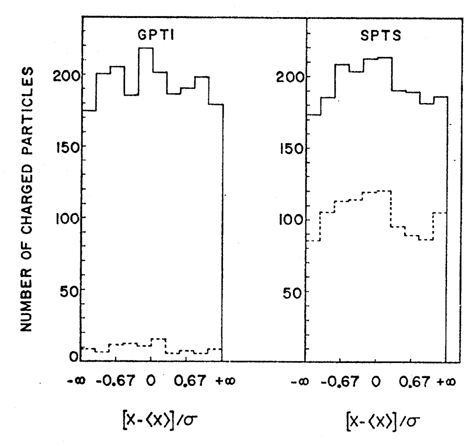

FIG. 22. Typical distributions of $(x-\langle x\rangle)/\sigma$ for samples of jets based on two different center-of-mass system angular distributions. The Gaussian distribution in these coordinates would be represented by a horizontal line. Solid line: distribution for the charged particles of all the events in each sample. Dashed line: distribution for the charged particles of those events in each sample having $\sigma>0.6$. The plot to the left is for a sample of Monte Carlo jets based on the isotropic center-of-mass system angular distribution. The plot to the right is for a sample of Monte Carlo jets based on the $(1/\sin\theta^*)d\Omega^*$ center-of-mass system angular distribution.

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TABLE V. Effect of the 16th particle on characteristics of the distribution of $\log_{10}[Y(\theta)/y]$.

Distribution	16th particle included	GPTI Mean	GPTI σ	SPTI Mean	SPTI σ	GPTS Mean	GPTS σ	SPTS Mean	SPTS σ
$\log(E_c/E)$	yes	0.16	0.20	0.23	0.22	0.15	0.34	0.23	0.35
	no	0.18	0.22	0.26	0.22	0.20	0.38	0.26	0.36
$\log(E_{ch}/\Sigma_{ch}E_i)$	yes	−0.05	0.16	−0.04	0.20	−0.08	0.21	−0.06	0.26
	no	0.07	0.09	0.11	0.15	0.09	0.13	0.12	0.19
$\log[(E_{ch}/\gamma_c)/\Sigma_{ch}E_i^*]$	yes	−0.14	0.11	−0.16	0.14	−0.18	0.16	−0.21	0.18
	no	−0.04	0.08	−0.05	0.12	−0.06	0.15	−0.08	0.18
$\log(1.65E_{ch}/\Sigma_{all}E_i)$	yes	−0.02	0.18	−0.01	0.21	−0.06	0.26	−0.04	0.28
	no	−0.06	0.15	−0.04	0.17	−0.12	0.30	−0.10	0.30
$\log(E_{ch}/E)$	yes	−1.03	0.17	−0.99	0.18	−0.77	0.31	−0.74	0.31
	no	−1.09	0.14	−1.05	0.14	−0.87	0.32	−0.86	0.27
$\log(E_c'/E)$	yes	−0.06	0.30	−0.02	0.32	−0.16	0.43	−0.11	0.45
	no	0.14	0.30	0.20	0.32	0.18	0.47	0.25	0.47
$\log[(E_{ch}/E_c)/K_{ch}^M]$	yes	−0.21	0.14	−0.26	0.16	−0.23	0.25	−0.28	0.24
	no	−0.14	0.10	−0.18	0.13	−0.13	0.24	−0.18	0.24
$\log[(1.65E_{ch}/E_c)/K^M]$	yes	−0.18	0.16	−0.24	0.16	−0.21	0.28	−0.27	0.27
	no	−0.24	0.16	−0.30	0.16	−0.32	0.28	−0.37	0.27

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APPENDIX: EFFECT OF THE 16th PARTICLE ON CHARACTERISTICS OF THE DISTRIBUTION OF $\log_{10}Y[(\theta)/y]$

For a given Monte Carlo jet, $Y(\theta)$ values are calculated from the θ values of only the produced charged particles of each jet, and the y values usually depend on properties of only the produced charged particles. Thus, it is possible to ascertain the influence of the 16th particle of the Monte Carlo jets on the characteristics of a $\log_{10}[Y(\theta)/y]$ distribution by comparing for a given sample of jets the distribution obtained for those jets for which the 16th particle is charged (numbering about 120 in a given sample) to the distribution obtained for those jets for which the 16th particle is neutral (numbering about 80 in a given sample). For the former distribution the $Y(\theta)$ values, and in the case of most parameters also the y values, will have been calculated including the quantities associated with the 16th particles. However, for the latter distribution the $Y(\theta)$ and y values will have been calculated excluding quantities associated with the 16th particles.

Table V presents the comparison described above. Quite generally, the σ values appear to be very insensitive to the effect of the 16th particle. The σ value of the $\log(E_{ch}/\sum_{ch}E_i)$ distribution appears to be influenced the most, but even in this case the factor corresponding to σ becomes larger by only about 20% when the quantities associated with the 16th particles are used in the calculations. There is a slight tendency for the presence of the 16th particle to decrease the value of σ for the $\log(E_c/E)$ and $\log(E_c'/E)$ distributions, whereas for the rest of the distributions there seems to be a slight tendency for σ to be increased.

The means of most of the distributions are also rather insensitive to the influence of the 16th particle. The mean of the $\log(E_c/E)$ distribution is the least sensitive, the effect shown in the table being comparable to the statistical uncertainty of the mean obtained. Most of the factors corresponding to the means differ by only about 26% in this comparison. However, the distribution of $\log(E_c'/E)$ seems to be rather sensitive to the effect of the 16th particle, with the factor corresponding to the mean of the distribution changing by about a factor of 2 when the 16th particle is included in the calculations. For 6 out of the 8 quantities whose distributions are investigated, the presence of the 16th particle tends to bring the mean of the distribution closer to zero, and for 5 of the quantities, the presence of the 16th particle tends to reduce the mean.