Muonic-Decay Branching Ratio of the Lambda Hyperon*

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An estimate of $R_\mu \leqslant 1\times10^{-4}$ has been obtained for the branching ratio $R_\mu=\Gamma(\Lambda\rightarrow p\mu^-\bar{\nu})/[\Gamma(\Lambda\rightarrow p\pi^-)+\Gamma(\Lambda\rightarrow n\pi^0)]$. This result is based upon the observation of two examples of $\Lambda\rightarrow p\mu^-\bar{\nu}$ decay in an effective sample of 19 700 lambdas. The probability that the two events were due to processes other than Λ_μ decay could not be ruled out, so that the experiment gives an upper limit for R_μ. The lambdas were produced by K^- mesons interacting in the Berkeley 30-in. heavy-liquid bubble chamber, filled with a CF_3Br-C_3H_8 mixture.

A TOTAL of 192 000 lambda decays in the charged mode has been examined for evidence of the decay $\Lambda\rightarrow p\mu^-\bar{\nu}$. Two possible examples of the process have been found in an effective sample of 19 700 lambdas. The corresponding upper limit for the branching ratio

$$R_\mu=\frac{\Gamma(\Lambda\rightarrow p\mu^-\bar{\nu})}{\Gamma(\Lambda\rightarrow p\pi^-)+\Gamma(\Lambda\rightarrow n\pi^0)}$$

is $R_\mu \leqslant 1\times10^{-4}$.

The experiment was performed with the Berkeley 30-in. heavy-liquid bubble chamber in a 13-kG magnetic field. The lambdas were produced by interactions in the chamber liquid of 0- to 440-MeV/c K^- mesons. The chamber was filled with a 76–24% mixture by weight of CF_3Br-C_3H_8; a detailed account of the properties of this liquid is contained in Ref. 1.

The fact that 67% of μ^--meson decay after coming to rest in the chamber liquid was utilized in identifying muonic lambda decays. An initial selection was made of lambda decays conforming to the following criteria:

(a) The negative track terminated in the chamber with a $\mu^-\rightarrow e^-\nu\bar{\nu}$ decay, easily recognized in this chamber liquid.

(b) The proton ended in the chamber.

(c) The lambda hyperon was produced in a K^- interaction in the chamber.

(d) The lambda, proton, and meson tracks were at least 3 mm long. A typical event satisfying the selection criteria is shown in Fig 1.

Criterion (a) excludes all $\Lambda\rightarrow p\pi^-$ decays except those in which the pion decayed in flight to a muon, which in turn came to rest and decayed. From the measured pion momentum spectrum of 1000 randomly selected pionic lambda decays, the fraction of pions decaying in flight was calculated to be 0.0189. Thus, criterion (a) separated out a group of lambda decays in which the proportion of Λ_μ events was enhanced by a factor of 53 over that in the original sample.

The remaining criteria ensure that the directions of the lambda, proton, and meson tracks can be measured, and that the proton momentum can be obtained from its range with a mean error of 3%. Multiple scattering in the chamber liquid precluded a reliable determination of the meson momentum by magnetic bending. Neither could the lambda momentum be inferred from the dynamics of the K^- interactions, since these took place predominantly in complex nuclei.

The 992 lambda decays that satisfied the selection criteria divided between $\Lambda_{\pi\mu}$ and Λ_μ decays as follows:

$$N(\Lambda_{\pi\mu})=N_\Lambda F_{\pi\mu}S_{\pi\mu}R_\pi\,, \qquad (1)$$

$$N(\Lambda_\mu)=N_\Lambda F_\mu S_\mu R_\mu\,. \qquad (2)$$

Here N_Λ is the total number of charged and neutral pionic lambda decays in the chamber liquid, $F_{\pi\mu}$ and F_μ are the fractions of $\Lambda\rightarrow p\pi^-$ and $\Lambda\rightarrow p\mu^-\bar{\nu}$ decays that satisfy conditions (a) through (d), and $S_{\pi\mu}$ and S_μ are the respective scanning efficiencies for these events. The branching ratios for charged, pionic lambda decay and for muonic lambda decay are R_π and R_μ, respectively. We take 0.66 for the value of R_π.[2] Rearranging and combining Eqs. (1) and (2), we have

$$R_\mu=\frac{N(\Lambda_\mu)F_{\pi\mu}S_{\pi\mu}R_\pi}{N(\Lambda_{\pi\mu})F_\mu S_\mu}\,.$$

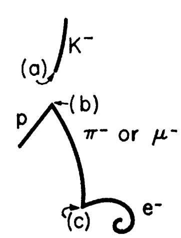

FIG. 1. A typical event satisfying the selection criteria. (a) K^-+nucleus$\rightarrow\Lambda$+fragments, (b) $\Lambda\rightarrow p\pi^-$ or $\Lambda\rightarrow p\mu^-\bar{\nu}$, (c) $\mu^-\rightarrow e^-\nu\bar{\nu}$.

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[1] R. P. Ely, G. Gidal, G. E. Kalmus, L. O. Oswald, W. M. Powell, W. J. Singleton, F. W. Bullock, C. Henderson, D. J. Miller, and F. R. Stannard, Phys. Rev. 131, 868 (1963).

[2] F. W. Crawford, in *Proceedings of the 1962 International Conference on High Energy Physics*, edited by J. Prentki (CERN, Geneva, 1962), p. 832.

Two Monte Carlo calculations, outlined below, gave $F_{\pi\mu}/F_{\mu}=0.0182$; this estimate is independent of the ratio of muon capture to muon decay in the chamber liquid. The ratio $S_{\pi\mu}/S_{\mu}$ was set equal to unity because the scanning criterion for both types of events was the presence of an electron at the end of the negative track from the lambda decay, and therefore the scanning efficiencies for $\Lambda_{\pi\mu}$ and Λ_{μ} decays were expected to be identical. Then we have

$$R_{\mu}=[N(\Lambda_{\mu})/N(\Lambda_{\pi\mu})](0.0182\times0.66). \qquad (3)$$

The quantity $N(\Lambda_{\mu})$ was determined by looking for events in which the length of the negative track was inconsistent with the decay sequence $\Lambda\to p\pi^-$, $\pi^-\to\mu^-\bar{\nu}$ in flight. In this decay sequence the length of the negative-particle track, $L_{\pi}+L_{\mu}$, depends upon P_{π}, the magnitude of the pion momentum in lambda decay, and upon the dynamics of the pion decay. For each $\Lambda_{\pi\mu}$ event, $L_{\pi}+L_{\mu}$ is restricted by kinematics within an interval $L_{\min}$ to $L_{\max}$, where $L_{\min}(L_{\max})$ is the range of the muon when the pion decays at the lambda decay vertex, and the muon is emitted in the pion rest frame antiparallel (parallel) to the pion direction.

The kinematics of $\Lambda\to p\pi^-$ decay are completely specified by two independent variables, the magnitude of the proton momentum and the angle between the proton and the pion. For all events it was assumed that $\Lambda\to p\pi^-$ decay had occurred, and $P_{\pi}\pm\Delta P_{\pi}$ was calculated from the measured values of the proton momentum and ψ, the angle between the charged decay products. The uncertainty in P_{π} arises predominantly from the experimental error on ψ. The limiting lengths of the negative-particle track, $L_{\max}\pm\Delta L_{\max}$ and $L_{\min}\pm\Delta L_{\min}$, were computed from $P_{\pi}\pm\Delta P_{\pi}$. Whenever L, the measured length of the negative particle track [see the track connecting origins (b) and (c) in Fig. 1], lay outside the range $L_{\min}$ to $L_{\max}$, the event was remeasured twice. Also, when L, although within the range $L_{\min}$ to $L_{\max}$, came within two standard deviations of a limiting value, the event was remeasured.

Two events were found with $L<L_{\min}$, and therefore, the $\Lambda_{\pi\mu}$ interpretation was ruled out for these decays. The muon momentum was calculated from the range of the negative track L for the Λ_{μ} interpretation, and in both cases the momenta of the proton, muon and neutrino in the plane transverse to the lambda trajectory were consistent with the reaction $\Lambda\to p\mu^-\bar{\nu}$.

A Monte Carlo calculation shows that these two events constitute 24% of the Λ_{μ} decays which satisfied the selection criteria. In the calculation, 3000 $\Lambda\to p\mu^-\bar{\nu}$ decays were generated in a random manner. The momenta of the lambdas and their decay points in the chamber were assigned according to the experimentally determined values of these quantities, obtained from the measurement of 1000 randomly selected pionic lambda decays. Since the form of the lambda decay amplitude is not known, the calculation was performed for both a Lorentz-invariant and a non-Lorentz-invariant three-body phase space; the same result was obtained in both cases. In 61.7% of the simulated decays, the tracks of both the proton and the muon were at least 3 mm in length and ended in the bubble chamber. For each of these Λ_{μ} events a corresponding P_{π} was calculated for a $\Lambda\to p\pi$ decay, with the same proton momentum and the same opening angle between the charged decay products. The length of the muon track L_{μ} was compared with the range of values $L_{\min}$ to $L_{\max}$, calculated from P_{π}. In 24% of the events the length of the muon was outside the interval $L_{\min}$ to $L_{\max}$ ($L_{\mu}<L_{\min}$, 20%: $L_{\mu}>L_{\max}$, 4%).

In a second Monte Carlo calculation, 3700 decay sequences of the type $\Lambda\to p\pi^-$, $\pi^-\to\mu^-\bar{\nu}$ in flight were generated in a random manner. In 59.4% of these decays the proton track and the combined length of the poin and muon tracks were at least 3 mm long and ended in the bubble chamber. The value of $F_{\pi\mu}/F_{\mu}$ is therefore $(0.594\times0.0189)/0.617=0.0182$, where 0.0189 is the fraction of pions decaying in flight.

The distribution of $\theta_{\pi\mu}$ (the projection in the film plane of the $\pi-\mu$ decay angle) was also examined to determine if $\Lambda_{\pi\mu}$ decays could be identified by observing the $\pi-\mu$ decay angle. More than 30% of the $\theta_{\pi\mu}$ values were less than 10 deg. Detection and measurement of angles of this magnitude was rendered difficult by multiple scattering in the heavy chamber liquid; consequently, no attempt was made to discriminate between Λ_{μ} and $\Lambda_{\pi\mu}$ decays by this means.

The two examples of Λ_{μ} decay give an estimate of $N(\Lambda_{\mu})\leqslant 2/0.24=8$, and $N(\Lambda_{\pi\mu})=(992-8)=984$. For the present, at least, the estimate of $N(\Lambda_{\mu})$ is considered an upper limit because the Λ_{μ} events could have been simulated by processes other than Λ_{μ} decay. Possible sources of background include:

A. $\Lambda_{\pi\mu}$ decays in which the pion decayed close to the Λ decay vertex. In this case $L_{\min}$ and $L_{\max}$ are computed incorrectly because the measured angle ψ is not the angle between the proton and π^- meson.

B. $\Lambda_{\pi\mu}$ decays in which the pion or proton suffered an unobserved scatter close to the lambda vertex. As in A, the measured ψ is not the pion-proton angle.

C. $\Lambda_{\pi\mu}$ decays in which the proton underwent a charge-exchange interaction. This causes the proton momentum, and hence $L_{\min}$ and $L_{\max}$ to be incorrectly estimated.

D. Neutron interactions, $nn\to np\pi^-$, with the subsequent decay in flight of the π^- meson.

E. Radiative lambda decays, $\Lambda\to p\pi^-\gamma$, also with subsequent decay in flight of the π^- meson.

The kinematics of the two Λ_{μ} decay candidates were such as to rule out the possibility that A or C had occurred.

On the basis of two possible cases of Λ_μ decay, R_μ calculated according to Eq. (3) is

$$R_\mu=\left[\left(\frac{2}{0.24}\right)\Big/(984)\right](0.0182\times 0.66)$$
$$=\frac{2}{19\,700}=1\times10^{-4}.$$

The effective sample size of lambda decays in this experiment is seen to be 19 700.

This experiment gives an upper limit of $R_\mu\leqslant 4.5\times10^{-4}$ at the 5% significance level. A lower limit can be set to the branching ratio from the observation of Good and Lind of one unambiguous case of Λ_μ decay in a total of 2500 lambdas,[3] giving $R_\mu\geqslant 0.2\times10^{-4}$ at the 5% significance level. The combined results define a 90% confidence interval for R_μ:

$$0.2\times10^{-4}\leqslant R_\mu\leqslant 4.5\times10^{-4}.$$

[3] M. L. Good and V. G. Lind, Phys. Rev. Letters 9, 518 (1962).

This estimate is consistent with $R_\mu=(1.3\pm0.2)\times10^{-4}$ deduced from the measured β-decay rate of the lambda hyperon,[1] if we assume that the decay processes $\Lambda\to pe^-\bar{\nu}$ and $\Lambda\to p\mu^-\bar{\nu}$ are identical except for the $\mu^-{-}e^-$ mass difference. A theoretical value[4] of $R_\mu=2.4\times10^{-3}$ has been predicted on the basis of the universal $V-A$ weak-interaction theory[5,6] and the hypothesis that the renormalization of the coupling constant due to strong interactions can be ignored in leptonic lambda decays. The measured value of R_μ clearly disagrees with the calculated one; it has already been shown that the Λ_β decay rate is similarly depressed relative to the predicted value.[1]

ACKNOWLEDGMENT

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Bethe-Salpeter Equation for Triplet Amplitude in Intermediate-Vector-Boson Theory of Weak Interactions*

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The Bethe-Salpeter equation for the triplet amplitude in the intermediate-vector-boson theory of weak interactions is shown to have a unique solution in configuration space; the solution has an essential singularity at the light cone, and does not have a Fourier transform. If the neutrino mass is zero, there exists a prescription that "regularizes" the amplitude to zero on the mass shell.

RECENTLY, Feinberg and Pais[1] have conducted interesting studies of a Bethe-Salpeter (B-S) equation that arises in the intermediate-vector-boson theory of weak interactions. In this paper we continue along similar lines; in particular, we show that in general the B-S equation for the triplet part of the amplitude has no solution in momentum space.

The B-S equation under consideration arises from the graphs shown in Fig. 1; for reasons of simplicity, we take $m_e=m_\mu=m_l$, $m_{\nu_e}=m_{\nu_\mu}=m_\nu$, and assume that all masses are finite for the time being for reasons to be explained later. The initial four-momenta of leptons are taken to be zero as in Ref. 1. If the amplitudes for the processes in Figs. 1(a) and (b) are A_1 and A_2, define

$$[(2\pi)^8/4]A_\pm=A_1\pm A_2.$$

This amplitude satisfies (units $\hbar=c=M=1$, with M the boson mass)

$$P_-A_\pm(k)P_+=-4g^2P_-\Delta(k)P_+$$
$$\mp g^2\int d^4k_1P_-\Delta(k-k_1)$$
$$\times\frac{\mathbf{k}_1\otimes\mathbf{k}_1P_-A_\pm(k_1)P_+}{(k_1^2-m_l^2+i\epsilon)(k_1^2-m_\nu^2+i\epsilon)},\quad(1)$$

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