Second-Order Coulomb Excitation via the Giant Dipole Resonance*

JÖRG EICHLER†
California Institute of Technology, Pasadena, California
(Received 14 October 1963)

The contributions to the Coulomb excitation probability due to virtual electric dipole transitions into the giant resonance have been studied. Use has been made of the fact that the second-order transition amplitude can be related to the photonuclear absorption cross section which is known experimentally. Two examples are discussed in some detail: (1) The Coulomb excitation of a first-excited 0^+ level from a 0^+ ground state; (2) second-order corrections to a $0^+ \to 2^+$ transition. In the latter case, the corrections are of the same order of magnitude as those due to the reorientation effect which has been proposed as a means to measure the quadrupole moment of excited states.

I. INTRODUCTION

In the last decade, Coulomb excitation has become a most valuable tool of nuclear spectroscopy.[1] The experimental accuracy has now been increased to such a degree that multiple processes and higher-order corrections can be observed.[2] By multiple excitations it is possible to reach higher energy levels and to study reduced transition probabilities between different states. An example of a second-order correction is the reorientation effect caused by $E2$ transitions between the magnetic substates of the final state in a $0^+ \to 2^+$ excitation. This effect has been proposed[3] as a means for determining the quadrupole moment of excited nuclear states. In general, higher-order effects will be most easily observed for low-lying rotational states because of their large $E2$ matrix elements. In nuclei with less strongly enhanced $E2$-transition probabilities, however, one might expect that contributions from other multipole orders would not be negligible.

It is the purpose of this work to emphasize that second-order $E1$ transitions via the giant dipole resonance may give important contributions to the transition probability in Coulomb excitation. Electric dipole transitions into bound states are usually weak and can be neglected. As will be discussed below, the corrections due to virtual transitions via the giant resonance may: (1) affect the determination of the quadrupole moment by the reorientation effect, and (2) give some information on the giant resonance not attainable by direct absorption studies since the final state reached by Coulomb excitation is different from the initial state. Virtual excitation of the dipole resonance has also been considered in $0^+ \to 0^+$ transitions by two-photon emission,[4,5] but Coulomb excitation seems to be much more promising for a study of the giant resonance.

Section II gives an outline of the calculations and contains a discussion of the quantity η which relates the virtual $E1$ transitions to the photonuclear absorption cross section. Two specific examples will be discussed in Secs. III and IV, namely, $0^+ \to 0^+$ and $0^+ \to 2^+$ transitions.

II. SECOND-ORDER TRANSITION AMPLITUDES

Let us consider the excitation process $i \to n \to f$ where i is the 0^+ ground state of the target nucleus, n is one of the highly excited 1^- states of the giant resonance, and f is the final state with spin $I=0^+$ or $I=2^+$ and projection quantum number M. If the charged projectile causing this transition is a heavy ion with a typical experimentally available energy, it is well justified to treat the motion of the particle in the Coulomb field of the nucleus classically. Under this assumption, the second-order transition amplitude[6] is[1]

$$b_{IM}^{(2)}(E1)=i(-1)^{I/2}\left(\frac{3I+4}{3}\right)^{1/2}\left(\frac{e^2}{\hbar c}\right)^2\frac{Z_1^2}{(v/c)^2a^2}$$
$$\times\sum_{\mu\mu'}\begin{pmatrix}1 & 1 & I\\ \mu & \mu' & M\end{pmatrix}Y_1^{\mu}\left(\frac{\pi}{2},0\right)Y_1^{\mu'}\left(\frac{\pi}{2},0\right)F_{\mu\mu'}, \quad (1)$$

with

$$F_{\mu\mu'}=\sum_n \langle i|z|n\rangle\langle n|z|f\rangle$$
$$\times\mathcal{P}\int_{-\infty}^{\infty} I_{1\mu}(\theta,\xi_{in}+x)I_{1,-\mu'}(\theta,-\xi_{nf}+x)\frac{dx}{x},$$

where $\mathcal{P}$ stands for the principal part of the integral. The projectile is characterized by the charge number Z_1, the velocity v, the deflection angle θ (in the center-of-mass system), and by a, half the distance of closest ap-

* Supported by the U. S. Office of Naval Research.

† On leave from the Institute for Theoretical Physics, University of Heidelberg, Heidelberg, Germany.

[1] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 (1956); K. Alder and A. Winther, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **31**, No. 1 (1956).

[2] F. S. Stephens, R. M. Diamond, and I. Perlman, Phys. Rev. Letters **3**, 435 (1959); J. de Boer, G. Goldring, and H. Winkler, Proceedings of the Conference on Reactions between Complex Nuclei, Asilomar, California, 1963.

[3] G. Breit, R. L. Gluckstern, and J. E. Russel, Phys. Rev. **103**, 727 (1956); **105**, 1121 (1957).

[4] J. Eichler and G. Jacob, Z. Physik **157**, 286 (1959).

[5] B. Margolis, Nucl. Phys. **28**, 524 (1961).

[6] In general, the amplitude also has a real part, but for the high energies of the intermediate states considered here it can be completely neglected.

proach in a head-on collision.[1] The dipole operator is denoted by z and

$$\xi_{in}=(E_n-E_i)a/\hbar v,\quad \xi_{nf}=(E_f-E_n)a/\hbar v.$$

$I_{1\mu}(\theta,\xi)$ is the classical orbital integral[1] and gives appreciable contributions to the integral of Eq. (1) only if $\xi\approx 0$. For transitions via the giant resonance, $\xi_{in}\approx-\xi_{nf}\gg 1$; hence, $1/x$ does not change much in the region of interest and can be taken out of the integral. This leads to

$$F_{\mu\mu'}=-(\hbar v/a)\rho_{\mu,-\mu'}(\theta,\xi_{if})\sum_n\frac{\langle i|z|n\rangle\langle n|z|f\rangle}{E_n-E_i},\tag{2}$$

where

$$\rho_{\mu\mu'}(\theta,\xi_{if})=\int_{-\infty}^{\infty}I_{1,\mu}(\theta,x)I_{1,\mu'}(\theta,x-\xi_{if})dx\tag{3}$$

no longer contains the energy of the intermediate state. The sum in Eq. (2) is closely related to the (-2) moment of the photonuclear absorption cross section,[7]

$$\sigma_{-2}\equiv\int\frac{\sigma(E)}{E^2}dE=4\pi^2\frac{e^2}{\hbar c}\sum_n\frac{\langle i|z|n\rangle\langle n|z|i\rangle}{E_n-E_i},\tag{4}$$

which is known to be a smooth function[7]

$$\sigma_{-2}\approx 3.5A^{5/3}\,\mu\text{b/MeV}$$

of the mass number A.

we now write

$$\sum_n\frac{\langle i|z|n\rangle\langle n|z|f\rangle}{E_n-E_i}\equiv\eta\sum_n\frac{\langle i|z|n\rangle\langle n|z|i\rangle}{E_n-E_i}\tag{5}$$

keeping η as a parameter which is expected to be smaller than 1. Evaluating the integrals ρ numerically, one can write the second-order transition amplitude, Eq. (1), in terms of σ_{-2} and η.

The ratio η of the two sums could possibly be small for two reasons: (a) The matrix elements $\langle n|z|f\rangle$ could be smaller than $\langle n|z|i\rangle$; (b) there might be cancellations in the left-hand sum of Eq. (5) due to fluctuations in the sign of $\langle n|z|i\rangle/\langle n|z|f\rangle$. If Δ is the correlation length of these fluctuations and Γ the energy interval covered by the sum, then $|\eta|$ should be of the order of Δ/Γ, which may be quite small in general. On the other hand, it is known[8,9] that the dipole state of closed-shell nuclei essentially consists of particle-hole excitations and it is believed[9] that the valence nucleons do not change this interpretation even for nuclei far from closed shells. This means that the giant resonance behaves like a single state and therefore one would not expect the sign of $\langle n|z|i\rangle/\langle n|z|f\rangle$ to fluctuate very rapidly. In other words, if only one shell-model state α contributes to the giant resonance, one can rewrite Eq. (5) as

$$\sum_n\frac{\langle i|z|\alpha\rangle\langle\alpha|n\rangle\langle n|\alpha\rangle\langle\alpha|z|f\rangle}{E_n-E_i}=\eta\sum_n\frac{\langle i|z|\alpha\rangle\langle\alpha|n\rangle\langle n|\alpha\rangle\langle\alpha|z|i\rangle}{E_n-E_i}$$

or

$$\langle\alpha|z|f\rangle=\eta\langle\alpha|z|i\rangle.$$

It would be interesting to test this picture by an experimental determination of η.

Let us consider two experiments in which the effect of the giant dipole resonance may be studied.

III. COULOMB EXCITATION OF A FIRST EXCITED 0^+ LEVEL FROM A 0^+ GROUND STATE

The second order is the lowest order which can contribute to the excitation of the 0^+ level. The final state can be reached either by dipole transitions via the giant resonance or, in competition, by quadrupole transitions via a 2^+ intermediate state m. If we call the corresponding transition amplitudes $b_0^{(2)}(E1)$ and $b_0^{(2)}(E2)$, the second-order transition probability is given by

$$P(2,2)=|b_0^{(2)}(E1)+b_0^{(2)}(E2)|^2.\tag{6}$$

From Eqs. (1) to (5), we obtain

$$b_0^{(2)}(E1)=i1.17\times10^{-4}\frac{A_1^{1/2}A_2^{5/3}E_{\text{MeV}}^{5/2}}{Z_1Z_2^3(1+A_1/A_2)^3}\eta\rho_{11}(\theta,\xi_{if}),\tag{7}$$

where A_1, A_2, Z_1, Z_2 are mass and charge numbers of the incident particle and of the target nucleus, respectively; E_{MeV} is the energy in MeV of the incident particle in the lab system, and ρ is defined[10] by Eq. (3). The amplitude $b_0^{(2)}(E2)$ may be calculated from Ref. 1. It depends, of course, on the enhancement factors γ_{im} and γ_{mf} of the $E2$ transition amplitudes defined by

$$B_{\text{exp}}(E2;a\rightarrow b)\equiv\gamma_{ab}^2\times3\times10^{-5}A_2^{4/3}e^2 10^{-48}\text{ cm}^4.\tag{8}$$

In order to give an idea of the effects to be expected, we present in Table I characteristic amplitudes for the excitation of the 1.75-MeV state of Zr^{90} for $\theta=150°$ (the 2.18-MeV state serving as the 2^+ intermediate level). The transition probability is at the limit of what may

TABLE I. Second-order transition amplitudes for Zr^{90} at 150°.

	$b_0^{(2)}(E1)$	$b_0^{(2)}(E2)$
45-MeV O^{16}	$i8.8\times10^{-3}\eta$	$i(1.4+i0.9)\times10^{-3}\gamma_{im}\gamma_{mf}$
100-MeV Ar^{40}	$i1.8\times10^{-2}\eta$	$i(0.23+i0.12)\times10^{-2}\gamma_{im}\gamma_{mf}$

[7] J. S. Levinger, *Nuclear Photo-Disintegration* (Oxford University Press, New York, 1960).

[8] D. H. Wilkinson, Physica 22, 1039 (1956).

[9] G. E. Brown, L. Castillejo, and J. A. Evans, Nucl. Phys. 22, 1 (1961).

[10] Numerical values of ρ_{11} for backward scattering may be obtained from Fig. 1 and the relation $\rho_{11}(180°,\xi)=[f_{12}(180°,\xi)/f_{11}(180°,\xi)]\cdot I_{22}(180°,\xi)$, where $I_{22}(\theta,\xi)$ is tabulated in Ref. 1.

experimentally be detected at present. The contribution from the 2^+ intermediate state may be larger than that from the 1^- state if the quadrupole transitions are sufficiently enhanced. In principle, it is possible to distinguish between the contributions by using different bombarding particles and by measuring $P(2,2)$ at different scattering angles.

IV. CORRECTIONS TO THE COULOMB EXCITATION OF A 2^+ LEVEL

In this case, first-order transitions are possible and occur with the probability $P(1,1)$. The term of next higher order in the perturbation expansion is the interference term $P(1,2)$ between first-order and second-order transitions. Assuming virtual transitions via the giant resonance, we find

$$\frac{P(1,2)}{P(1,1)}=-1.98\times10^{-3}\frac{A_2E_{\text{MeV}}}{Z_2(1+A_1/A_2)}\frac{\eta}{\gamma_{if}}\frac{f_{12}}{f_{11}},\quad(9)$$

where

$$f_{12}=\tfrac{3}{8}I_{2,-2}\,\rho_{-1,1}+\tfrac{1}{4}I_{2,0}\,\rho_{1,1}+\tfrac{3}{8}I_{2,2}\,\rho_{1,-1}$$

and

$$f_{11}=\tfrac{3}{8}I_{2,-2}{}^2+\tfrac{1}{4}I_{2,0}{}^2+\tfrac{3}{8}I_{2,2}{}^2.$$

Here the arguments θ and ξ_{if} in $I_{2\mu}$, $\rho_{\mu\mu'}$, f_{12}, and f_{11} have been suppressed. The ratio f_{12}/f_{11} exhibits a pronounced angular dependence which is given in Fig. 1 for various values of ξ_{if}.

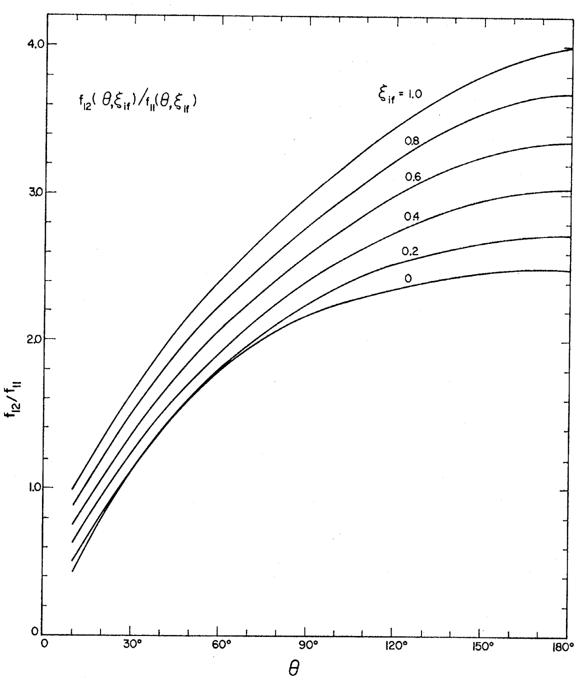

FIG. 1. Angular dependence of second-order Coulomb excitation via the giant dipole resonance. The ratio f_{12}/f_{11} from Eq. (9) is plotted as a function of the scattering angle θ for various values of ξ_{if}.

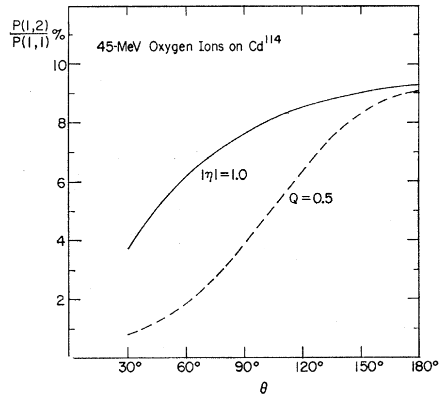

FIG. 2. Angular dependence of second-order effects in the Coulomb excitation of the 556-keV 2^+ state in Cd^{114} by 45-MeV oxygen ions. Solid curve: effect of the giant dipole resonance for $|\eta|=1$ and $\gamma_{if}\cdot\eta$ negative. Broken curve: reorientation effect for $Q=0.5\times10^{-24}$ cm^2.

For $|\eta|$ close to one, the second-order correction is quite appreciable. This may be seen in the case of Cd^{114} bombarded by 45-MeV O^{16} ions (Fig. 2). We have used the value[11] $|\gamma_{if}|=5.74$ and $|\eta|=1$ and have arbitrarily chosen a negative sign for the product $\gamma_{if}\eta$. The second-order correction due to the giant resonance is of particular interest since it has been proposed[3] to measure the quadrupole momemt of excited states by a similar second-order correction, namely the reorientation effect (higher-order transition between magnetic substates) in Coulomb excitation. For comparison, we have therefore calculated $P(1,2)/P(1,1)$ arising from the reorientation effect for 45-MeV O^{16} ions on Cd^{114}. The result has been plotted in Fig. 2, assuming a quadrupole moment of $Q=0.5\times10^{-24}$ cm^2 (the effect is linear in Q). The Coulomb excitation

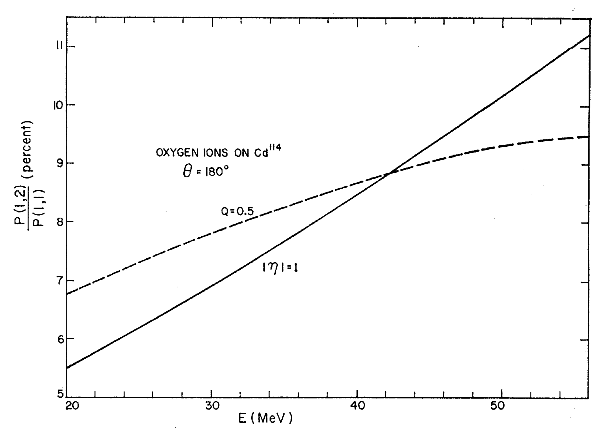

FIG. 3. Energy dependence of second-order effects in the Coulomb excitation of the 556-keV 2^+ state in Cd^{114} by oxygen ions ($\theta=180°$). Solid curve: effect of the giant dipole resonance for $|\eta|=1$ and $\gamma_{if}\cdot\eta$ negative. Broken curve: reorientation effect for $Q=0.5\times10^{-24}$ cm^2.

[11] Nuclear Data Sheets, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C.).

parameters in this example are such[12] that third-order reorientation effects are small.

In Fig. 3 we have plotted $P(1,2)/P(1,1)$ as a function of the bombarding energy E for the fixed angle $\theta=180°$ using again the example of Cd^{114} with $Q=0.5\times10^{-24}$ cm^2 and $|\eta|=1$. The assumed quadrupole moment is probably quite realistic, so that both second-order effects may have the same size. The difference in the energy dependence (Fig. 3) or the difference in the angular behavior (Fig. 2) might serve to distinguish between the two effects.

[12] D. L. Lin and J. F. Masso, Proceedings of the Conference on Reactions between Complex Nuclei, Asilomar, California, 1963 (unpublished).

Summarizing, we may say that any attempt to determine a quadrupole moment by the reorientation effect must take into account virtual transitions via the giant dipole resonance. This requires a higher experimental accuracy, but, on the other hand, a determination of the structure parameter η is an interesting problem in itself.

ACKNOWLEDGMENTS

The author would like to thank Dr. J. de Boer for valuable discussions which first drew his interest to Coulomb excitation. He furthermore wants to acknowledge the kind hospitality of the California Institute of Technology.

PHYSICAL REVIEW VOLUME 133, NUMBER 5B 9 MARCH 1964

Recoil Study of the $Zn^{68}(p,2p)Cu^{67}$ Reaction*

DAVID L. MORRISON† AND ALBERT A. CARETTO, JR.
Department of Chemistry, Carnegie Institute of Technology, Pittsburgh, Pennsylvania
(Received 21 October 1963)

The excitation function and the product recoil behavior of the $Zn^{68}(p,2p)Cu^{67}$ reaction was studied using incident protons of energy between 80 and 430 MeV. The thick-target thick-catcher technique was used in which effective recoil ranges were measured in the forward, backward, and transverse directions. The data were interpreted in terms of the knock-out mechanism. The data were also fitted to a recoil velocity distribution written in terms of a power series in the cosine of the scattering angle. Ranges calculated by this treatment are consistent with the interpretation that the reaction proceeds mainly by the knock-out mechanism. Reasonable agreement was obtained between the recoil kinetic energy, calculated on the basis of the assumed recoil velocity distribution, and that which would be obtained from an abrupt removal of a proton from the top of the nuclear well in Zn^{68}.

INTRODUCTION

THAT class of high-energy nuclear reactions known as the "simple reactions" are thought to involve the interaction of the incident particle with the target nucleus via nucleon-nucleon collisions within the nucleus. For $(p,2p)$ and (p,pn) reactions, only one collision of the incident proton with the appropriate nucleon is required. In principle, if it can be assumed that the interaction involved in these simple reactions involves only the collision of the incident proton with a target nucleon, no other effects manifesting themselves, then an observation of the momentum distribution of the products should reflect the momentum distribution of the struck nucleons prior to the collision. Several groups of experimenters have measured the angular and energy distribution of the protons emitted in $(p,2p)$ reactions,[1–3] but no recoil distribution of the product nucleus has been measured at incident energies above 100 MeV. Most studies of $(p,2$ nucleon) reactions have been confined to measurements of the cross section for the reaction as a function of the bombarding protons.[4–9] Only a few product recoil studies of (p,pn) reactions have been reported,[10–13] and of these only the $C^{12}(p,pn)C^{11}$ reaction, as studied by Singh and Alexander, and the $Cu^{65}(p,pn)Cu^{64}$ reaction, as studied by Merz and Caretto, were investigated in sufficient detail to examine the assumptions underlying the concepts of simple high-energy reactions. As an extension of these studies, preliminary recoil studies have been made of the $Zn^{68}(p,2p)Cu^{67}$ reaction since (i)

* Research performed under contract with U. S. Atomic Energy Commission.

† Presented in partial fulfillment of the Ph.D. degree in the Department of Chemistry, Carnegie Institute of Technology. Present address: Batelle Memorial Institute, Columbus, Ohio.

[1] H. Tyren, P. Hillman, and Th. A. J. Maris, Nucl. Phys. **7**, 10 (1958).

[2] Th. A. J. Maris, P. Hillman, and H. Tyren, Nucl. Phys. **7**, 1 (1958).

[3] T. J. Gooding and H. G. Pugh, Nucl. Phys. **18**, 46 (1960).

[4] S. Markowitz, F. S. Rowland, and G. Friedlander, Phys. Rev. **112**, 1295 (1958).

[5] H. P. Yule and A. Turkevich, Phys. Rev. **118**, 1591 (1960).

[6] N. T. Porile, Phys. Rev. **125**, 1379 (1962).

[7] P. P. Strohal and A. A. Caretto, Jr., Phys. Rev. **121**, 1815 (1961).

[8] N. T. Porile and S. Tanaka, Phys. Rev. **130**, 1541 (1963).

[9] L. P. Remsberg and J. M. Miller, Phys. Rev. **130**, 2069 (1963).

[10] N. M. Hintz, Phys. Rev. **86**, 1042 (1952).

[11] E. R. Merz and A. A. Caretto, Jr., Phys. Rev. **126**, 1173 (1962).

[12] S. Singh and J. M. Alexander, Phys. Rev. **128**, 711 (1962).

[13] N. I. Borisova, M. Ya. Kuznetsova, L. N. Kurchatova, V. N. Mekhedov, and L. V. Chistyakov, Zh. Eksperim. i Teor. Fiz. **37**, 366 (1959) [English transl.: Soviet Phys.—JETP **10**, 261 (1960)].