## Angular-Momentum Effects on Neutron Emission by Dy and Tb Compound Nuclei\*

GABRIEL N. SIMONOFF†‡ AND JOHN M. ALEXANDER§ Lawrence Radiation Laboratory, University of California, Berkeley, California (Received 7 June 1963)

We have studied as a function of energy three reactions producing 4.1-h Tb<sup>149</sup> from Tb compound nuclei and nine reactions producing Dy products from Dy compound nuclei. Incident particles were B<sup>10</sup>, B<sup>11</sup>, C<sup>12</sup>, and O<sup>16</sup> of energy 4 to 10.4 MeV per amu. Measurements of the average recoil range give strong evidence that all these reactions proceed by compound-nucleus formation. We report angular distributions of the final heavy products for all these reactions. From angular-distribution data we deduce the average total energies of photons and neutrons for each reaction. In the Tb reactions the average total photon energy is always less than 12 MeV. In the Dy reactions the average total photon energy varies linearly with total available energy from nearly 0 to about 30 MeV. These large differences in total photon energy are attributed to differences in the angular momenta of the initial compound nuclei. The rate of increase of the average kinetic energy of all neutrons (from Dy systems) is approximately proportional to the square root of the excitation energy. The relationships between total photon (or neutron) energy and total available energy seem to be independent of the average angular momentum of all compound nuclei. These relationships vary systematically with the number of emitted neutrons.

#### I. INTRODUCTION

IN this paper, we attempt to gain information on the average energies of neutrons and photons emitted from compound nuclei excited to energies up to approximately 125 MeV. We attempt to separate effects of angular momentum (J) from effects of excitation energy (E) by the comparison of compound nuclei having similar values of Z, A, and E but different values of J. The products  $\mathrm{D}y^{149}$ ,  $\mathrm{D}y^{150}$ , and  $\mathrm{D}y^{151}$  were observed from the compound systems 66Dy154 (formed by C12+Nd142), and Dy156 (formed by two reactions, C12+Nd144 and O<sup>16</sup>+Ce<sup>140</sup>). Also, the product Tb<sup>149g</sup> has been observed from several Tb compound nuclei. Cross-section data imply that the latter reactions proceed from compound systems with  $J \lesssim 7.5\hbar$ , whereas the former reactions involve much higher average angular momenta.2

In this work and in previous studies average range measurements have been used to test the reaction mechanism.3 These measurements give strong evidence that the reactions we study are reactions in which the neutrons are emitted with angular distributions symmetric about 90°.

We report angular-distribution measurements for the products previously mentioned. A relationship between total neutron energy and root-mean-square angle has been derived. This relationship assumes isotropic neutron emission but is not extremely sensitive to this assumption. Using this relationship and our angulardistribution measurements, we obtain average total neturon energies and total photon energies associated with each individual reaction. In the preceding paper4 that presents cross section data we discuss the over-all energy and angular-momentum balance for these reactions.

We conclude that the low-spin Tb compound nuclei that decay to Tb<sup>149g</sup> dissipate less than about 12 MeV in photons. The amount of photon emission from Dy compound nuclei of higher spin is quite different. This qualitative result was previously obtained by Morton, Choppin, and Harvey. Mollenauer has reported observations of photons emitted in complex nuclear reactions.6 His results also indicate that total photon energies increase with increasing J of the compound nucleus. Our results imply that total photon energies up to about 30 MeV are associated with neutron emission from Dy compound nuclei. The average kinetic energy of the emitted neutrons is approximately proportional to  $\sqrt{E}$ . The average total photon energy increases with increasing E or J, or both.

#### II. RECOIL EFFECTS OF THE COMPOUND-NUCLEUS MECHANISM

The basic features of the compound-nucleus mechanism are the following. A projectile and a target nucleus interact to form an excited compound system having a mean life that is long compared with the time required for the projectile to traverse the nuclear diameter. The excited compound nucleus decays by emitting particles and photons until a stable or radioactive final product is formed. The angular distribution of the emitted particles

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<sup>†</sup> On leave from Laboratorie Joliot Curie de Physique Nucléaire Orsay, France.

<sup>‡</sup> Present address: Nouvelle Faculté des Sciences de Bordeaux, Talence (Gironde) France.

<sup>§</sup> Present address: Department of Chemistry, State University of New York at Stony Brook, Stony Brook, New York.

<sup>1</sup> J. M. Alexander and G. N. Simonoff, Phys. Rev. 130, 2383 (1963).

<sup>2</sup> T. D. Thomas, Phys. Rev. 116, 703 (1959).

<sup>3</sup> J. M. Alexander and J. Winghous Phys. Rev. 121, 520 (1964).

<sup>&</sup>lt;sup>8</sup> J. M. Alexander and L. Winsberg, Phys. Rev. **121**, 529 (1961); J. M. Alexander and D. H. Sisson, *ibid*. **128**, 2288 (1962).

<sup>&</sup>lt;sup>4</sup> J. M. Alexander and G. N. Simonoff, preceding paper, Phys. Rev. 132, B93 (1963)

<sup>&</sup>lt;sup>5</sup> J. R. Morton, III, G. R. Choppin, and B. G. Harvey, Phys. Rev. 128, 265 (1962).

<sup>&</sup>lt;sup>6</sup> J. F. Mollenauer, Phys. Rev. 127, 867 (1962).

or photons is symmetric about  $\pi/2$  in the frame of reference of the compound nucleus if the level density of the residual nucleus is large enough to justify the randomphase approximation.<sup>7</sup> In this work we study systems with initial excitation energies of about 50 to 125 MeV, and thus we assume that this approximation is justified.

Let us consider in detail the consequences of this mechanism for two recoil properties of the final products: (a) range-straggling parameter  $\rho$ , and (b) root-meansquare angle (laboratory system),  $\langle \theta_L^2 \rangle^{1/2}$ . Let v denote the velocity given to the compound nucleus by the initial impact of the projectile (this is identical to the velocity of the center of mass). Let V denote the velocity in the c.m. system given to the final product by the evaporation of particles. Let  $\theta$  denote the c.m. angle between v and V and let  $\theta_L$  denote the lab angle between v and v+V. The angular distribution of V is designated by  $W(\theta)$ , and the recoil range is taken as equal to  $k | \mathbf{v} + \hat{\mathbf{V}} |^N$ , where k and N are constants. The projection R of the actual recoil ranges on the beam direction is given by  $R=k|\mathbf{v}+\mathbf{V}|^N\cos\theta_L$ .

If the average quantity  $\langle V^2 \rangle$  is much less than  $v^2$ , and if  $W(\theta)$  is symmetric about  $\pi/2$ , then the average projection of the ranges on the beam direction,  $R_0$ , can be considered to depend only on v, k, and N-and to be independent of V and  $W(\theta)$ . The average range  $R_0$  of the product should be associated with a recoil energy  $E_R$ such that

$$E_R = \frac{E_b A_b A_R}{(A_b + A_T)^2},\tag{1}$$

where mass number is denoted by A, with subscript bindicating the bombarding particle, subscript R the recoil atom or final product, and subscript T the target. The kinetic energy of the projectile in the laboratory system is denoted by  $E_b$ .

The contribution to the measured range straggling from the distribution of  $\mathbf{v} + \mathbf{V}$  is given by

$$\langle (R-R_0)^2 \rangle = \frac{1}{2} \int_0^{\pi} \left[ R(v,V,\theta) - R_0(v,V) \right]^2 W(\theta) \sin\theta d\theta \,. \tag{2}$$

This integral has been evaluated by substituting the appropriate functions of velocity for R and  $R_0$ . For  $V \ll v$ , and for  $W(\theta) = 1$  we have, to order  $(V/v)^3$ ,

$$\langle (R-R_0)^2 \rangle / R_0^2 = N^2 \langle V^2 \rangle / 3v^2; \tag{3}$$

for  $W(\theta) = a + b \cos^2 \theta$ ,

$$\frac{\langle (R-R_0)^2 \rangle}{R_0^2} = \frac{N^2 \langle V^2 \rangle [1 + (3b/5a)]}{3v^2 [1 + (b/3a)]}; \tag{4}$$

and for  $W(\theta)$  proportional to  $1/\sin\theta$ ,

$$\langle (R-R_0)^2 \rangle / R_0^2 = N^2 \langle V^2 \rangle / 2v^2. \tag{5}$$

Detailed calculations by the Monte Carlo method have shown that for  $V^2 \ll v^2$  the range distribution due to evaporation effects can be closely approximated by a Gaussian distribution with straggling parameter denoted by  $\rho_n$ . Thus, we have

$$\rho_n^2 = \langle (R - R_0)^2 \rangle / R_0^2.$$
 (6)

The average square of the angle  $\langle \theta_L^2 \rangle$  of the recoil atoms is given by

$$\langle \theta_L^2 \rangle = \frac{1}{2} \int_0^{\pi} \left\{ \tan^{-1} \left( \frac{V \sin \theta}{v + V \cos \theta} \right) \right\} W(\theta) \sin \theta d\theta.$$
 (7)

To order  $(V/v)^3$  for  $W(\theta) = 1$ , we have

$$\langle \theta_L^2 \rangle = 2 \langle V^2 \rangle / 3v^2$$
. (8)

For  $W(\theta) = a + b \cos^2 \theta$ , we have

$$\langle \theta_L^2 \rangle = \frac{2 \langle V^2 \rangle [1 + (b/5a)]}{3v^2 [1 + (b/3a)]}. \tag{9}$$

For  $W(\theta)$  proportional to  $1/\sin\theta$  we have

$$\langle \theta_L^2 \rangle = \langle V^2 \rangle / 2v^2$$
. (10)

The equations given above show relationships between some observable properties and the magnitude of the velocities v and V. The velocity v is, of course, specified by the momentum of the projectile and the mass of the compound nucleus:

$$v^2 = 2A_b E_b / (A_b + A_T)^2$$
. (11)

The value of  $\langle V^2 \rangle$  is determined by the average total kinetic energy  $T_n$  of the emitted particles in the c.m. system and by their angular and energy distributions. The recoil velocity due to emission of photons can be neglected.

Assume that the compound nucleus emits nucleons in random directions then  $W(\theta) = 1$ , and we have  $\langle V^2 \rangle$  $=\sum_{i=1}^{x} V_{i}^{2}$ , where  $V_{i}$  is the additional recoil velocity due to the emission of the ith neutron and x is the total number of emitted neutrons. Then for  $A_R \ge 20$  we obtain

$$\langle V^2 \rangle \approx 8T_n/(A_T + A_b + A_R)^2$$
. (12)

The total energy available in the c.m. system is  $E_{\rm c.m.}+Q$ , therefore, the average total energy emitted as photons  $T_{\gamma}$  is

$$T_{\gamma} = E_{\text{c.m.}} + Q - T_n. \tag{13}$$

<sup>&</sup>lt;sup>7</sup> D. C. Peaslee, Ann. Rev. Nucl. Sci. 5, 99 (1955); T. Ericson, Advances in Physics, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1960), Vol. 9, p. 425.

<sup>8</sup> L. Winsberg and J. M. Alexander, Phys. Rev. 121, 518 (1961).

<sup>&</sup>lt;sup>9</sup> J. M. Alexander, L. Altman, and S. Howry, Lawrence Radiation Laboratory (unpublished calculations).

Thus, from Eqs. (3), (6), (11), and (12) we have

$$\rho_n^2 = \frac{4N^2T_n(A_b + A_T)^2}{3E_bA_b(A_b + A_T + A_R)^2},$$
 (14)

and from Eqs. (8), (11), and (12) we have

$$\langle \theta_L^2 \rangle = \frac{(8T_n)(A_b + A_T)^2}{(3E_b A_b)(A_b + A_T + A_R)^2}.$$
 (15)

In all these relationships the neutron mass is taken as unity.

If the angular distribution of the emitted particles is not isotropic, the development is much more complicated. However, from Eqs. (5) and (10), one can see that even an extreme case of  $W(\theta) \propto 1/\sin\theta$  leads to changes of only about 22% in  $\rho_n$ , and about 13% in  $\langle \theta_L^2 \rangle^{1/2}$ .

### III. EXPERIMENTAL TECHNIQUES AND RESULTS

In our experiments we have made observations of the nuclides 4.1-h Tb<sup>149</sup>, 7.4-min Dy<sup>150</sup>, and 17.9-min Dy<sup>151</sup>. These are the only known alpha-emitting nuclides in the rare-earth region that have convenient decay periods and favorable alpha branching ratios. Therefore, measurement of the alpha radioactivity by ionization chambers allows us to identify these specific products without chemical analysis, thus eliminating chemical-yield errors.

In other work we have observed that cross sections for Tb<sup>149g</sup> from Tb compound nuclei are very small.¹ Also, Dy<sup>150</sup> and Dy<sup>151</sup> cross sections from Dy compound nuclei are very large.⁴ The excitation functions for Dy<sup>149</sup>+Tb<sup>149g</sup> from Dy compound nuclei closely resemble those for Dy<sup>150</sup> and Dy<sup>151</sup>.⁴ We infer that a dominant fraction of the Tb<sup>149g</sup> that is observed from Dy compound nuclei actually comes from radioactive decay of Dy<sup>149</sup> to Tb<sup>149g</sup>. Therefore, we refer to the recoil properties of Tb<sup>149g</sup> produced from Dy compound systems as those of Dy<sup>149</sup>.

### A. Range Measurements

The range measurements were made with thin targets (30 to  $100 \mu g/cm^2$ ), and thin Al catcher foils (approximately 150  $\mu g/cm^2$ ), as described previously.<sup>3,3</sup> On a probability scale,  $F_t$ , the fraction of the total activity that passed through catcher foils of combined thickness t, was plotted against t. These probability plots always indicate that the range distribution can be described as a Gaussian function with two parameters (the average range  $R_0$  and the straggling parameter  $\rho$ ):

$$P(R)dR = \frac{1}{R_0 \rho (2\pi)^{1/2}} \exp \left[ -\left(\frac{R - R_0}{\sqrt{2} R_0 \rho}\right)^2 \right] dR. \quad (16)$$

The results of the range measurements are given in

TABLE I. Range measurements in Al.

Reaction	Bombarding energy E <sub>b</sub> (lab) (MeV)	Observed product	Average range R <sub>0</sub> (mg/cm <sup>2</sup> )	Measured straggling parameter $\rho$	Nuclear reaction straggling parameter $\rho_{\pi^a}$
Ce140 +O16	146.0	Tb149	0.996	0.183	0.09 ±0.035
		$Dy^{150}$	0.991	0.190	$0.102 \pm 0.03$
	140.0	Tb149	0.953	0.186	$0.083 \pm 0.04$
		$Dy^{150}$	0.958	0.197	$0.105 \pm 0.03$
	128,1	$T\dot{b}^{149}$	0.910	0.196	$0.089 \pm 0.033$
		$Dv^{150}$	0.912	0.202	$0.10 \pm 0.03$
	112.4	TĎ <sup>149</sup>	0.803	0.193	≈0
	100.4	Dv151	0.758	0.200	≈ŏ
	100.0	$Dy^{151}$	0.730	0.196	≈ <b>0</b>
	88.2	$D_{\mathbf{y}^{151}}$	0.677	0.199	≈0
Nd144 + C12	120.5	Tb149	0.661	0.245	$0.082 \pm 0.03$
		$Dy^{150}$	0.656	0.248	0.085 ±0.03
	95.0	T b149	0.549	0.224	≈0
	. 0,0	$\widetilde{\mathbf{D}}_{\mathbf{V}^{150}}$	0.551	0.223	≈0
		$D_{y^{151}}$	0.554	0.237	~0 ≈0

<sup>&</sup>lt;sup>a</sup> The value of  $\rho_n$  is given only if it is significantly different from zero.

Table I. The first three columns give the reaction, beam energy, and observed product, respectively. The values of the measured quantities  $R_0$  and  $\rho$  are given in the fourth and fifth columns. The measured straggling parameter is the result of contributions from several sources: (a) finite target thickness  $\rho_w$ , (b) catcher-foil inhomogeneities  $\rho_f$ , (c) inherent straggling in the stopping process  $\rho_s$ , and (d) the nuclear reaction  $\rho_n$ . If all these contributions are treated as Gaussian we have

$$\rho_n^2 = \rho^2 - \rho_w^2 - \rho_f^2 - \rho_s^2. \tag{17}$$

The effects of  $\rho_w$ ,  $\rho_f$ , and  $\rho_s$  have been subtracted as previously described,<sup>3</sup> and we show the values of  $\rho_n$  in the last column.

The values of  $\rho_n$  are not accurate enough to specify  $T_n$  values from Eq. (14). We can only say that the values of  $\rho_n$  are not inconsistent with any conclusions deduced from the angular-distribution results. As shown in Eqs. (14) and (15), the values of  $\rho_n$  and  $\langle \theta_L^2 \rangle^{1/2}$  are both related to  $T_n$ . The major result from the range measurements is the determination of the average range  $R_0$ .

#### B. Angular-Distribution Measurements

The angular-distribution measurements were performed by essentially the same method as developed by Harvey *et al.*<sup>5,10</sup> A thin target layer was exposed to a collimated beam from the Berkeley Hilac. The Nd<sup>142</sup>, Nd<sup>144</sup>, Nd<sup>146</sup>, and Ce<sup>140</sup> targets were prepared from enriched isotopes obtained from the Oak Ridge National Laboratory. The enrichments were 97.4% Nd<sup>142</sup>, 97.3% Nd<sup>144</sup>, 96.2% Nd<sup>146</sup>, and 99.6% Ce<sup>140</sup>. A thick (0.001-in.) Al catcher foil was placed at some distance from the target; and the catcher was cut into rings concentric about the beam.

The geometry of the apparatus is shown in Fig. 1. The angular resolution of the beam was defined by two  $\frac{1}{16}$ -in. collimators to approximately 0.5° in some experiments.

<sup>&</sup>lt;sup>10</sup> P. F. Donovan, B. G. Harvey, and W. H. Wade, Phys. Rev. 119, 218, 225 (1960).

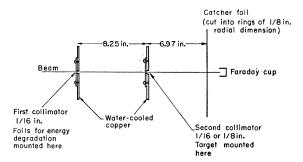


Fig. 1. Schematic diagram of the apparatus used for angular-distribution measurements.

In others the second collimator was  $\frac{1}{8}$  in. in diameter, giving rise to an angular definition of approximately 1°. The effect of the size of the second collimator was measured experimentally (see Table III).

The catcher foil was cut by a stainless steel cutter and a hydraulic press into rings of  $\frac{1}{8}$ -in. radial dimension. Each ring subtended approximately 1°. Two different cutters were used. The dimensions of these cutters were carefully calibrated by weighing several sets of rings cut from sheets of uniform Al foil. The angles defined by each ring are given in Table II.

TABLE II. Angles defined by each cutting edge (deg).a

Ring number	Cutter 1	Cutter 2
4	0	0
1	1.16	1.04
2	2.10	2.04
3	3.12	3.07
4	4.15	4.12
5	5.16	5.15
6	6.16	6.18
7		
8	7.15	7.17
9	8.17	8.21
10	9.19	9.19
11	10.19	10.19
12	11.18	11.16
13	12.13	12.15
14	13.14	13.17
	14.15	14.10
15	15.08	15.08
16	16.02	16.06

<sup>&</sup>lt;sup>a</sup> For each ring the inner and outer angles are given. The outer angle for any ring is the inner angle for the next.

The results of all angular-distribution measurements are given in Table III. The first two columns give the beam energy and target thickness, respectively. As shown in Table II, the two cutters had slightly different dimensions. Therefore, for each experiment we give the cutter, and, for each ring, the fractional cross section per unit angle  $\Delta\sigma/\sigma\Delta\theta$ . The average angle  $\langle\theta_L\rangle$  was calculated by the relationship

$$\langle \theta_L \rangle = \sum_i (\Delta \sigma_i / \sigma) \langle \theta_i \rangle,$$
 (18)

where  $\langle \theta_i \rangle$  is the mean angle of the *i*th ring and  $\Delta \sigma_i / \sigma$  is the fraction of the total activity observed in that ring. The root-mean-square angle was similarly calculated:

$$\langle \theta_L^2 \rangle^{1/2} = \left[ \sum_i (\Delta \sigma_i / \sigma) \langle \theta_i^2 \rangle \right]^{1/2},$$
 (19)

where  $\langle \theta_i^2 \rangle$  is the mean-square angle of the *i*th ring. Values of  $\Delta \sigma_i$  less than 2% of the maximum value of  $\Delta \sigma_i$  were not included in the summations.

The effect of target thickness on the angular distribution of Tb149g was carefully studied for several cases. One series of these experiments is shown in Fig. 2. The values of  $\langle \theta_L \rangle$  and  $\langle \theta_L^2 \rangle^{1/2}$  change significantly but not very rapidly with the target thickness (see Fig. 3). We have used the values of  $d\langle\theta_L\rangle/dW$  and  $d\langle\theta_L^2\rangle^{1/2}/dW$  shown in Fig. 3 to correct these average properties to zero target thickness. The assumption was made that all reactions of the same projectile have the same value of  $d\langle \theta_L \rangle/dW$ and  $d\langle\theta_L^2\rangle^{1/2}/dW$ . This is probably a very good approximation (especially for the C12 and O16 experiments), because the angular distributions and recoil velocities are very similar. The detailed angular distributions in Table III for W=0 were obtained by linearly extrapolating  $\Delta \sigma / \sigma \Delta \theta$  to W = 0 for each ring. This procedure becomes more uncertain, of course, with increasing angle.

The effect of collimator size was carefully studied for two different cases (Nd<sup>144</sup>+122.8-MeV and 77.5-MeV

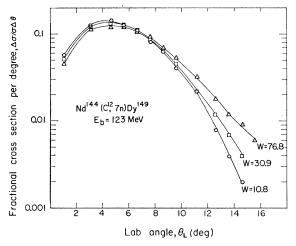


Fig. 2. The effect of target thickness on observed angular distribution. The target thickness W is denoted for each curve in  $\mu g/cm^2$ . These data are for the reaction  $Nd^{144}+123-MeV$   $C^{12} \rightarrow Dy^{149}+7n$ .

Table III. Angular distribution results.

	$\langle\theta_L^2\rangle^{1/2}$	1	5.15	5.35	5.73		6 77	8.64		6.37	7.53	8.07	7.29	8.04		4.83	5.37		5.36	5 57	5.85		5 40	5.64	5 77	21.5	6.58		6.26	6.56	2.66	5.88
	$\langle \theta_L \rangle$		4.51	4.09	5.09		5 07	7.78		5.54	69.9	7.13	6.50	7.13		4.23	4.73		4.68	4 97	5.18		4.82	4 08	5.10	5.43	5.84		5.39	5.63	4.94	5.25
,	18										(0.003)	(0000)		(0.005)																(0.003)		
	17							(0.011)			(0.005)	(0.008)		(0.007)								· ·							(0.004)	(0.004)		
	16						1	22 →		(0.003)	(0.007)	(0.011)	•	(0.011)															(0.005)	(900'0)		(0.003)
	15						0000	← 0.022 →		(0.005)	0.010	0.017		0.013					*****							) <u>.</u> 1	(0.004)		(0.007)	(0.008)	(0.003)	(0,004)
	14	1000	(200.0)	(0.004)	(0.005)		19	45 →		0.008	0.013	0.023	0,008	0.025						0.004	0.004		0.005	0.004	0.004	+ 0.005 →	13 +		0.011	0.014	(0.005)	(0.000)
(deg <sup>-1</sup> )	13	2000	(±00.0)	0.000	0.007		+ 0.016 →	← 0.045 →		0.012	0.028	0.037	0.024	0.036			0.008		(0.000)	0.006	0.009		0.007	0.007	0.008	0.010	← 0.013 →		0.015	0.015	0.010	0.010
Fractional cross section per unit angle Δσ/σΔθ (deg <sup>-1</sup> ) Rino number -	12	000	0000	0.000	0.008		0.026	0.067		0.019	0.037	0.043	0.034	0.044		900'0	0.008		0.013	0.011	0.015		0.010	0.011	0.012	0.018	0.024		0.018	0.021	0.013	0.013
unit ang	11	1499				1499	0.042	0.062	b149g	0.034	0.051	0.057	0.048	0.053	y151	0.010	0.014	y150	0.015	0.017	0.022	y149	0.015	0.021	0.021			y151	0.025	0.027	0.014	
tion per - Ring r	10	Pr <sup>141</sup> (C <sup>12</sup> , 4n) Tb <sup>1499</sup>				Nd <sup>146</sup> (B <sup>10</sup> ,7 <i>n</i> )Tb <sup>149</sup> 9	3 0.064		Nd <sup>146</sup> (B <sup>11</sup> ,8 <i>n</i> )Tb <sup>149</sup> g	2 0.056	1 0.075	1 0.071	3 0.075	5 0.075	22	0.014	0.026	$Nd^{142}(C^{12},4n)Dy^{150}$	97.00	0.030	3 0.036	$Nd^{142}(C^{12},5n)Dy^{149}$	5 0.027	0.030	t 0.032			$Nd^{144}(C^{12},5n)Dy^{151}$	0.038	0.041	0.029	
cross sec	6	Pr <sup>141</sup> (C <sup>12</sup>				I)951PN	9 0.083	8 0.090	Nd146(	8 0.062	2 0.074	6 0.081	0 0.078	4 0.085	)) <sub>771</sub> PN	0.046 0.031	2 0.040	)) <b>271</b> PN	7 0.042	0 0.047	6 0.053	)) <b>27</b> 1PN	5 0.046	5 0.049	9 0.054			)) **:PN	4 0.052	6 0.054	5 0.047	0 0.059
ctional	∞	0 0 055					0,089	8 0.098		1 0.088	9 0.112	1 0.096	0.110	8 0.094			6 0.072		7 0.057	0.070	0.086		5 0.065	2 0.075	0.079		2 0.096		6 0.074	0.076	5 0.075	1 0.080
Fra	7	060			0.111		0.111	0.098		0.101	0.109	0,101	0,111	0.108		0.084	0.096		0.087	0.099	0.110		0.095	0.102	0.110	0.110	0.112		0.096	0.101	0.105	0.101
	9	0.121	0.128	0,133	0.132		0.118	0.096		0.122	0.122	0.108	0.113	0.113		0.115	0.124		0.118	0.130	0.129		0.125	0.132	0.129	0.130	0.124		0.115	0.119	0.117	0.137
	5	0.149	0.151	0.150	0.144		0.122	0.079		0.126	0.104	0.099	0.107	0.101		0,160	0.157		0.148	0,151	0.146		0.153	0.146	0.142	0.137	0.127		0.137	0.127	0.140	0.133
	4	0.169	0.155	0.149	0.147		0.112	0.069		0.114	0.099	0.090	0.100	0.083		0.173	0.159		0.170	0.153	0.138		0.159	0.152	0.147	0.132	0.115		0,140	0.139	0.149	0.129
	3	0.155	0.149	0.132	0.126		0.092	0.049		0.095	0.073	0.069	0.074	0.067		0.159	0.137		0.158	0.134	0.119		0.136	0.132	0.126	0.110	0.092		0.123	0.116	0.135	0.114
	2	0.107					0.067	0.035				0.046	0.048	0.043			0.106		0.101	0.092	0.089		0.097	0.093	0.088		990.0		0.086	0.080	0.100	0.089
	-	0.046	0.044	0.033	0.033		(0.031)	(0.024)		(0.059)	(0.025)	(0.026)	0.025	(0.027)		0.048	0.043	1	0.043	0.033	0.036		0.038	0.038	0.036	← 0.053 →	0.038		0.036	0.039	0.040	0.050
	Cutter	2 <sub>b</sub>	1 <sub>b</sub>	2b	2b		1 <sub>b</sub>	2b		2 <sub>p</sub>	2	T 1		1	7	2р	1ь		1 <sub>b</sub>	2 <sub>b</sub>	1 <sub>b</sub>		2р	4 <b>T</b> p	1 p	1 <sub>b</sub>	1ъ		2	16	7	-
	(µg/cm²)	27.2	30,3	27.2	30.3		25.2	25.2		27.4	27.4	89.0	0	27.4		30.7	30.7		30.7	30.7	30.7		30.7	30.7	30.7	30.7	30.7		77.0	77.0	30.9	30.9
Bombarding energy, $E_b$ (lab)	1	>57.7ª	>59.98	8.79	70.1		75.1	102.4		90.2	103.7	103.7	103.7	112.8	i.	93.0	70.1		70.1	83.4	92.0		83.4	92.0	100.6	111.7	122.8		77.5	77.5	83.4	94.0

Table III (continued).

(\text{Ag/cm\$}) Cutter 1 2  30.9 1 0.048 0.088  30.9 2 ←0.054 →  30.9 1 ←0.049 →  76.8 1 ←0.044 →  30.9 2 ←0.054 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  10.8 1 ←0.057 →  10.8 1 ←0.057 →  10.8 1 ←0.057 →  30.9 2 ←0.047 →  10.8 1 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 2 ←0.057 →  30.9 3 0.142		0.146 129 + 120 + 0006 0.150 + 0015 0.139 0.139 112 + 121 + 121 + 121 127 + 121 + 121 127 + 121 0.188 0.189	0.149 0 0.147 0 0.133 0 0.115 0 0.115 0 0.115 0 0.115 0 0.115 0 0.116 0 0.117		0.0099 0.0 0.0099 0.0 0.0107 0.0 0.0108 0.0 0.0109 0.0 0.0103 0.0 0.0110 0.0 0.0110 0.0 0.0110 0.0 0.0105 0.0 0.013 0.0	Nd <sup>144</sup> (C) 0.072 0.048 0.082 0.061 0.085 0.065 0.092 0.078 Nd <sup>144</sup> (C) 0.077 0.046 0.077 0.046 0.078 0.051 0.088 0.057 0.099 0.068 0.099 0.068 0.099 0.068 0.099 0.068 0.099 0.068 0.099 0.069	Nd <sup>144</sup> (C <sup>12</sup> ,6n)Dy <sup>150</sup> 0.048 0.037 0, 0.061 0.038 0.066 0.045 C.078 0.058 0.046 0.028 0, 0.051 0.035 0.068 0.048 0, 0.063 0.046 0.063 0.046 0.063 0.046 0.063 0.046 0.064 0.053 0.064 0.063 0.064 0.063 0.064 0.063	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.012 0.012 0.012 0.029 0.011 0.008 0.012 0.013 0.018	(0.008) 0.006 0.008 0.016 (0.006) 0.006 0.008 0.004 0.007 0.003	(0.006) (0.003) (0.003) (0.004) (0.004) (0.005) (0.005) (0.005) (0.005) (0.006) (0.006)	(0.004) (0.004) (0.004) (0.004)	11	(0.003)		(9.4)/12 6.04 5.93 6.33 7.09 5.76 5.80 6.15 6.51 6.01 6.22 6.88
0.048	0 000	0.146 129 → 120 → 096 → 096 → 112 → 0.139 0.139				Ndi44 772 0.00 882 0.00 886 0.00 992 C.0 774 0.00 775 0.00 775 0.00 776 0.00 998 0.00 998 0.00 998 0.00 998 0.00 998 0.00 998 0.00 998 0.00	(C!2,6n)] 48 0.03 49 0.03 49 0.03 49 0.05 40 0.04 40 0.05 40 0.05 40 0.05 40 0.05 40 0.05 40 0.05 40 0.05 40 0.05 40 0.05	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	(0.017) 014 + 024 + 037 + 037 + 0016 + 020 + 020 + 020 + 020 + 020 + 022 + 022 + 032 + 032 + 033 + 038 + 03	(0.012) 0.012 0.012 0.029 0.011 0.013 0.013 0.013 0.018	(0.008) 0.006 0.008 0.016 (0.006) 0.006 0.007 0.007 0.003	(0.006) (0.003) (0.004) (0.004) (0.005) (0.005) 0.006 0.006	(0.004)		(0.003)	5.24 5.22 5.35 6.24 5.03 5.03 5.40 5.73 5.40 5.73 5.40 5.73	6.04 5.93 6.33 7.09 5.76 5.76 6.15 6.01 6.22 6.88
0.048	8 888	0.146 129 + 0.0152 0.0152 0.139 0.139 0.139 112 + 125 + 125 + 121				772 0.0 886 0.0 886 0.0 992 0.0 77 0.0 777 0.0 778 0.0 888 0.0 889 0.0 889 0.0 899 0.0 899 0.0 899 0.0 899 0.0 899 0.0 899 0.0 899 0.0 899 0.0 899 0.0	48 0.03 56 0.04 78 0.05 78 0.05 78 0.05 78 0.05 57 0.03 57 0.03 57 0.04 68 0.04 68 0.04 69 0.05 69 0.05 60 0.03 61 0.03 61 0.03 62 0.03 63 0.04 64 0.05 65 0.03 66 0.05 67 0.03 68 0.04 68 0.05 68 0.0	7 0.027 8	0.017) 014 \to 024 \to 024 \to 037 \to 016 \to 020 \to 020 \to 020 \to 022 \to 022 \to 032 \to 033 \to 032 \to 033 \to 032 \to 033 \t	(0.012) 0.012 0.012 0.029 0.011 0.008 0.013 0.008 0.013 0.008	(0.008) 0.006 0.008 (0.006) (0.008) 0.008 0.004 0.007 0.003	(0.006) (0.003) 0.005 0.012 (0.004) (0.005) 0.006 0.006 0.007	(0.004) (0.003) 0.005 (0.004) (0.004)		(0.003)	5.24 5.22 5.56 6.24 6.24 5.03 5.03 5.73 5.40 6.01	5.93 6.33 7.09 5.76 5.76 6.15 6.01 6.01 6.22 6.88
0.055 ← 0.045 ← 0.054) ← 0.054) ← 0.053 ← 0.053 ← 0.053 ← 0.053 0.062 0.063	000	129 + 096 + 096 + 096 + 096 + 096 + 09132				86 0.0 992 C.0 Nd <sup>144</sup> 00 777 0.0 988 0.0 989 0.0 984 0.0 985 0.0 985 0.0 987 0.0 987 0.0	56 0.04 78 0.05 78 0.05 78 0.05 46 0.02 57 0.03 57 0.03 58 0.04 68 0.04 69 0.05 69 0.05 60 0.05 60 0.05 61 0.03 62 0.03 63 0.04 64 0.05 65 0.03 66 0.05 67 0.03 68 0.05 69 0.05 60	8 8 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9		0.012 0.012 0.029 0.011 0.008 0.013 0.008 0.012 0.018	0.006 0.008 0.016 (0.006) 0.008 0.004 0.007 0.003	(0.003) 0.005 0.012 (0.004) (0.005) 0.006 0.007 0.009	(0.003) 0.005 (0.004) (0.006)		(0.003)	5.22 5.56 6.24 6.24 5.03 5.03 5.73 5.40 6.01	5.93 6.33 7.09 5.76 5.80 6.15 6.51 6.01 6.22 6.88
0.045 ← 0.045 ← 0.054) ← 0.043 ← 0.045 ← 0.053 ← 0.053 ← 0.053 0.063	000	096 + 096 + 096 + 096 + 096 + 096 + 096 + 097 +				Ndiad Ndiad	78 0.04 78 0.05 ((C12,7n)) 74 0.02 51 0.03 57 0.03 58 0.04 63 0.04 64 0.05 65 0.05 67 0.03 68 0.05 69 0.05 69 0.05 60 0.05 60 0.05 60 0.05 61 0.03 62 0.03 63 0.04 64 0.05 65 0.04 66 0.05 67 0.03 68 0.05 69 0.05 60 0.05	SS ← 0.0 Pyles ← 0.018 SS 0.0		0.012 0.029 0.001 0.011 0.003 0.013 0.018 0.007	0.008 0.016 0.006 0.008 0.008 0.004 0.007 0.003	0.005 0.012 (0.004) (0.005) 0.006 0.002 0.004 0.009	(0.003)		(0.003)	5.56 6.24 6.24 5.03 5.05 5.73 5.73 6.01	6.33 7.09 5.76 6.15 6.51 6.22 6.88 5.84
0.054)  (0.054)  (0.054)  (0.054)  (0.062)  (0.063)  (0.063)		0.152 0.139 0.139 112 + 125 + 121 +				Nd <sup>144</sup> Nd <sup>144</sup> Nd <sup>147</sup> 0.0 775 0.0 775 0.0 888 0.0 990 0.0 991 0.0 883 0.0 Ce <sup>440</sup>	78 0.05 ((Cu,7n)); ((Cu,7n)); 446 0.02 551 0.03 557 0.03 68 0.04 63 0.04 65 0.05 60 0.05 60 0.05 61 0.01	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		0.029 0.011 0.008 0.012 0.013 0.018 0.007	0.016 (0.006) (0.008) 0.008 0.004 0.007 0.003	0.012 (0.004) (0.003) (0.005) 0.006 0.002 0.004	(0.004)		(0.003)	6.24 5.03 5.05 5.40 5.73 5.48 6.01	5.76 5.80 6.15 6.01 6.22 6.88 5.84
(0.054) (0.054) (0.034) (0.034) (0.04) (0.062) (0.062) (0.063) (0.063)		0.152 0.139 0.139 112 + 125 + 121 + 121 + 121 + 121 + 121 + 127 +				Nd <sup>144</sup> 777 0.0 777 0.0 775 0.0 788 0.0 784 0.0 786 0.0 789 0.0 789 0.0 789 0.0	((C¹2,7n)) 51 0.03 51 0.03 57 0.03 58 0.04 63 0.04 65 0.05 69 0.05 60 0.05 60 0.05 61 0.03 62 0.03 63 0.04 64 0.05 65 0.03 66 0.05 67 0.03 68 0.04 69 0.05 69 0.05 69 0.05 69 0.05 69 0.05 69 0.05 69 0.05 69 0.05 60 0.05	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		0.001 0.008 0.012 0.013 0.008 0.012 0.018	(0.006) 0.006 (0.008) 0.008 0.004 0.007 0.012	(0.004) (0.003) (0.005) 0.006 0.002 0.004 0.009	(0.004)		(0.003)	5.03 5.05 5.40 5.73 5.21 5.09	5.76 5.80 6.15 6.51 6.01 6.22 6.88
(0.054) (0.054) (0.034) (0.034) (0.052) (0.062) (0.063) (0.063) (0.063)		0.152 0.139 0.139 112 + 125 + 121 + 121 + 127 +				777 0.0 775 0.0 888 0.0 890 0.0 884 0.0 886 0.0 991 0.0 933 0.0 Ce <sup>160</sup>	46 0.02 51 0.03 57 0.03 68 0.04 63 0.04 65 0.04 69 0.05 69 0.05 60 0.03 60 0.05 61 0.03	8 0.018 5		0.011 0.008 0.012 0.013 0.008 0.012 0.018	(0.006) 0.006 (0.008) 0.004 0.007 0.012 0.003	(0.004) (0.003) (0.005) 0.006 0.002 0.004 0.009	(0.004)		(0.003)	5.03 5.05 5.40 5.73 5.21 5.09	5.76 5.80 6.15 6.51 6.22 6.88 5.84
0.034 0.034 0.034 0.053 0.062 0.063 0.063	-	0.139 0.139 1125 + 125 + 121 + 1111 + 127 - 0.188				775 0.0 988 0.0 990 0.0 984 0.0 986 0.0 991 0.0 Ce <sup>140</sup>	51 0.03 57 0.03 58 0.04 63 0.04 65 0.04 69 0.05 60 0.05 61 0.03 62 0.03 62 0.03	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	016 + 020 + 0.024	0.008 0.012 0.013 0.008 0.012 0.018	0.006 (0.008) 0.008 0.004 0.007 0.012 0.003	(0.003) (0.005) 0.006 0.002 0.004 0.009	(0.004)		(0.003)	5.05 5.40 5.73 5.21 5.09	5.80 6.15 6.51 6.01 6.22 6.88 5.84
0.034 ← 0.04; ← 0.05; ← 0.05; ← 0.06; 0.062 0.063 0.063	-	0.139 112				0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.0	557 0.03 58 0.04 63 0.04 65 0.05 69 0.05 60 0.03 (O16,5n) I	9 ←0.035 0 ←0.06 6 ←0.3 3 ←0.9 9 ←0.9	020 → 0.024 022 → 022 → 032 → 032 →	0.012 0.013 0.008 0.012 0.018	(0.008) 0.008 0.004 0.007 0.012	(0.005) 0.006 0.002 0.004 0.009	(0.004)		(0.003)	5.40 5.73 5.21 5.48 6.01 5.09	6.15 6.51 6.01 6.22 6.88 5.84
← 0.04; ← 0.05; ← 0.05; ← 0.04; 0.062 0.063 0.063	1142 C	1125 + 1211 + 1211 + 1211 + 1271 + 12				0.0 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00	68 0.04 63 0.04 65 0.04 69 0.05 62 0.03 (O16,5n) I	8 0.035 0 $\leftarrow$ 0.6 6 $\leftarrow$ 0.3 3 $\leftarrow$ 0.9 9 $\leftarrow$ 0.0	0.024 022 → 022 → 032 → 018 →	0.013 0.008 0.012 0.018	0.008 0.004 0.007 0.012 0.003	0.006 0.002 0.004 0.009	(0.004)		(0.003)	5.73 5.21 5.48 6.01 5.09	6.51 6.01 6.22 6.88 5.84
← 0.053 ← 0.04 ← 0.04 ← 0.062 0.062 0.053 0.063	→ → → → → → → → → → → → → → → → → → →	12.5 + 12.1 + 11.1 + 12.7 + 12.7 + 0.188				0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0	63 0.04 65 0.04 69 0.05 62 0.03 (O16,5n)I 14 0.01		022 → 022 → 032 →	0.008 0.012 0.018 0.007	0.004 0.007 0.012 0.003	0.002 0.004 0.009 0.0015	0.006		(0.003)	5.48 6.01 5.09	6.01 6.22 6.88 5.84
← 0.05; ← 0.04¢ ← 0.05; 0.062 0.053 0.045 0.063	→ → → → → → → → → → → → → → → → → → →	1121 → 1111 → 127 → 0.188				0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0	65 0.04 69 0.05 62 0.03 (O16,5x)I 14 0.01		022 → 032 → 018 →	0.012	0.007	0.004	90000		(0.003)	5.48 6.01 5.09	6.22 6.88 5.84
← 0.044 ← 0.053 0.062 0.053 0.045 0.063	1.142 0 1.142 0 1.129 0	1111 → 127 → 0.188				091 0.0 083 0.0 Ce <sup>140</sup> 030 0.0	69 0.05 62 0.03 (O <sup>16</sup> ,5 <i>n</i> )] 14 0.01		032 →	0.003	0.012	0.009	900.0		(0.003)	5.09	5.84
← 0.05; 0.062 0.053 0.045 0.063	→ 0.142 0 0.142 0 0.142 0	0.188 0.189				0.0 Ce <sup>140</sup> 030 0.0	62 0.03 (O16,5 $\pi$ ) I 4 0.01		018 →	0.007	0.003	0.0015		(0.004)		5.09	5.84
0.062 0.053 0.045		0.188				Ce <sup>140</sup> .	$(0^{16}, 5\pi)$ I (016, 50)I (14, 0.01	)y <sup>151</sup>									
0.062 0.053 0.045 0.063		0.188															
0.053		0.189						0 0.006	900.0							3.85	4.40
0.045						0.048 0.019		1 0,004	0.002							4.00	4 53
		0.184	0.159 0	0.120 0.	0.079 0.0	0.043 0.020	20 0.011	1 0.005	0.002	0.001						4.08	4.70
						Ce140	$Ce^{140}(O^{16},6n)Dy^{150}$	Oy150									
	0.134 0.166	0.183	0.167 0	0.120 0.	0.079 0.0	0.024 0.032	32 0.008	8 0.008	0.004							4.03	4.59
2 0.045 (	0.129 0.177	0.184		0.120 0.		0.043 0.020	20 0.011	1 0.005	0.002	0.001						4.08	4.70
0.047	0.114 0.151	0.177	0.166 0	0.131 0.	0.080 0.0	0.054 0.040	40 0.021	1 0.013	0.006							4.44	5.06
2 0.038 (	0.110 0.158	0.177	0.153 0	0.124 0.	0.089 0.0	0.059 0.036	36 0.022	2 0.007	0.003							4.42	4.97
0.040	0.108 0.164	54 0.171	0.160	0.120 0.	0.094 0.0	0.055 0.0	0.033 0.019	0.008	0.003							4.37	4.92
						Ce140	$Ce^{140}(O^{16},7n)Dy^{149}$	Dy149									
2 0.043 (	0.120 0.161	0.172	0.158 0	0.120 0.	0.086 0.0	0.055 0.030	30 0.016	0.007	0.005	0.002	0.0006					4.33	4.91
1 0.043 (	0.107 0.150	0.163	0.153 0	0.126 0.	0.094 0.0	0.064 0.037	37 0.022	2 0.013	0.007	0.005	0.004					4.64	5.28
0.043	0.125 0.165	0.175	0.160 0	0.118 0.	0.083 0.0	0.051 0.027	27 0.014	4 0.006	0.004	0.0004						4.20	4.76
1 0.045 (	0.122 0.154	0.170	0.158 0	0.128 0.	0.092 0.0	0.058 0.034	34 0,013	3 0.010	9000							4.39	4.96
1 0.047	0.114 0.157	7 0.169	0.156 0	0.125 0.	0.093 0.0	0.058 0.035	35 0.020	0.008	0.007							4.43	5.01
1 0.046	0.110 0.153	0.166	0.153 0	0.135 0.	0.091 0.0	0.059 0.037	37 0.019	6 00.00	0.007	0.003						4.50	5.09
	0.113 0.157	7 0.171	0.156 0	0.127 0.	0.088 0.0	0.061 0.033	33 0.018	8 0.008	0.004							4.38	4.95
1 0.039	0.091 0.130	0 0.149	0.146 0	0.135 0.	0.110 0.0	0.077 0.053		0.017	0.009	0.003						4.93	5.55

 $^{\rm A}$  The energy-degrading foils were damaged by the beam.  $^{\rm b}$  The second collimator was  $\frac{1}{7}$  in.  $^{\rm e}$  Values in parentheses were obtained by graphical extrapolation.

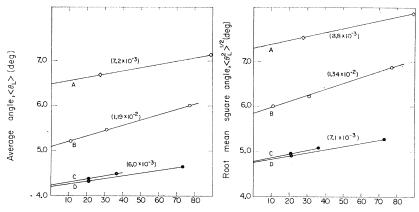


Fig. 3. The dependence of (a) the average angle  $\langle \theta_L \rangle$  and (b) the rootmean-square angle  $\langle \theta_L \rangle^{1/2}$  on target thickness W. Curves A are for the reaction Nd<sup>146</sup>+104-MeV B<sup>11</sup>  $\rightarrow$  Tb<sup>149 $\sigma$ </sup>+8n; B for Nd<sup>144</sup>+123-MeV C<sup>12</sup>  $\rightarrow$  Dy<sup>149</sup>+7n; C for Ce<sup>140</sup>+139-MeV O<sup>16</sup>  $\rightarrow$  Dy<sup>149</sup>+7n; D for Ce<sup>140</sup>+111-MeV O<sup>16</sup>  $\rightarrow$  Dy<sup>149</sup>+7n. The numbers in parentheses denote the slopes of the lines in deg/( $\mu$ g/cm<sup>2</sup>).

Target thickness,  $W(\mu g/cm^2)$ 

C¹²). The angular distribution was measured with two  $\frac{1}{16}$ -in. collimators (angular definition  $\approx 0.5^\circ$ ) and in a separate experiment with the second collimator  $\frac{1}{8}$  in. (angular definition  $\approx 1^\circ$ ). The average angles  $\langle \theta_L \rangle$  and  $\langle \theta_L^2 \rangle^{1/2}$  were enlarged by 0.25 and 0.30°, respectively, by the poorer angular definition of the beam. We assume that no correction is necessary for experiments with two  $\frac{1}{16}$ -in. collimators, and for the other experiments we correct the average angles by the above values. The corrected values of the average angles are given in Table IV along with average energies that are discussed later.

### IV. DISCUSSION

### A. Ranges

In preceding papers we have presented an internalconsistency argument for using average range values to test the validity of the compound-nucleus model.<sup>3</sup> The lack of independent range-energy data for heavy-recoil atoms necessitates this consistency test. First, assume that the compound-nucleus mechanism is valid. Thus, Eq. (1) should give the recoil energy  $E_R$ , appropriate to the average range  $R_0$ . Then the values of  $R_0$  are plotted

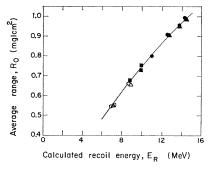


Fig. 4. Average range  $R_0$  in Al versus the calculated recoil energy  $E_R$ . Symbols are as follows:  $\mathrm{Dy^{151}} \ \Box$ ;  $\mathrm{Dy^{150}} \ \triangle$ ;  $\mathrm{Dy^{149}} \ \odot$ . Open symbols are for the reactions  $\mathrm{C^{12}} + \mathrm{Nd^{144}}$ ; closed, for  $\mathrm{O^{16}} + \mathrm{Ce^{140}}$ . The smooth curve is from Ref. 3.

versus  $E_R$ , as in Fig. 4. From this figure we see that one smooth curve fits all the measurements. Furthermore, this curve, which involves data for  $Dy^{150}$  and  $Dy^{151}$ , is identical with that for  $Tb^{149}$  range measurements from many other reactions.<sup>3</sup> This test implies that Eq. (1) gives a correct description of the recoil energy or, in other words, that the projectile transfers all its momentum to the compound system. We conclude that the most likely mechanism for all these reactions is compound-nucleus formation, followed by emission of particles with forward-backward symmetry. All further discussion is based on this conclusion.

#### B. Angular Distributions

From the average recoil-range measurements we have concluded that, in all the reactions studied here, the angular distribution of the emitted neutrons is essentially symmetric about the  $\pi/2$  plane in the center-of-mass system. We use measurements of the angular distribution of the final products to calculate the average kinetic energies of the neutrons and also average total photon energies.

The angular distribution of the final products depends on the energy and angular distributions of the emitted neutrons (see Sec. II). If the neutrons are emitted only as s waves, then their emission is isotropic. However, if neutrons are emitted with nonzero l values, then forward-backward peaking is expected. The classical limit to this forward-backward preference is given by an angular distribution of the form  $W(\theta) \simeq 1/\sin\theta$ . Experimental studies of heavy-ion reactions have shown that alpha particles and fission fragments are emitted with angular distributions approaching this limit; neutrons and protons are emitted with much less forward-backward peaking. Ericson's formulation of this problem

<sup>&</sup>lt;sup>11</sup> W. J. Knox, A. R. Quinton, and C. E. Anderson, Phys. Rev. 120, 2120 (1960); H. C. Britt and A. R. Quinton, *ibid.* 120, 1768 (1960); V. E. Viola, Jr., T. D. Thomas, and G. T. Seaborg, University of California Radiation Laboratory Report UCRL-10248, 1962 (unpublished); H. W. Broek, Phys. Rev. 124, 233 (1961).

TABLE IV. Average angles and energies.

Bombarding energy (lab) $E_b$ (MeV)	Corrected average angle $\langle \theta_L \rangle$ (deg)	Corrected root-mean-square angle $\langle \theta_{L^2} \rangle^{1/2}$ (deg)	Total available energy, $E_{\mathfrak{o}.\mathfrak{m}}$ . $+Q$ (MeV)	Average total neutron energy, Tn (MeV)	Average total photon energy, $T_{\gamma}$ (MeV)
		Pr141 (C12	4n)Th149g		
>57.7 >59.9 67.8 70.1	3.94 4.08 4.40 4.48	4.49 4.64 4.98 5.02	>6.2 >7.9 15.2 17.3	<6.2 <6.8 9.0 9.4	>0.0 >1.1 6.2 7.9
		Nd146 (B10	$(7n) \text{Tb}^{149g}$		
75.1 102.4	5.54 7.35	6.25 8,12	15.7 41.2	12.8 29.5	2.9 11.7
		NA146/1211	,8n)Tb149g		
90.2 103.7 112.8	5.10 6.50 6.94	5.83 7.29 7.80	17.8 30.3 38.8	14.6 26.3 32.8	3.2 4.0 6.0
		Nd142(C15	$^{2}$ ,3 $^{2}$ ) $^{2}$ Dy <sup>151</sup>		
55.6 7 <b>0.</b> 1	3.61 4.11	4.11 4.66	9.3 22.6	5.0 8.2	4.3 14.4
		Nd142(C1	2,4n)Dy <sup>150</sup>		
70.1 83.4 92.0	4.06 4.30 4.56	4.65 4.86 5.14	14.7 27.0 34.9	8.1 10.5 13.0	6.6 16.5 21.9
		Nd142(C1	$^{2},5n)$ Dy $^{149}$		
83.4 92.0 100.6 111.7 122.8	4,20 4,36 4,48 4,81 5,22	4.78 4.93 5.06 5.44 5.87	16.8 24.7 32.7 42.9 53.1	10.1 11.9 13.7 17.5 22.5	6.7 12.8 19.0 25.4 30.6
		Nd144(C1	$^{2}$ ,5 $n$ )Dy <sup>151</sup>		
77.5 83.4 94.0	4.48 4.57 4.88	5.23 5.25 5.47	15.2 20.7 30.5	11.1 12.2 14.9	4.1 8.5 15.6
		Nd144(C1	2,6n)Dy150		
94.0 99.7 111.6 122.8	4.87 4.85 5.19 5.32	5.64 5.52 5.92 6.06	22.6 27.8 38.8 49.2	15.8 16.0 20.6 23.8	6.8 11.8 18.2 25.4
		Nd144 (C1	$^{2},7n)$ Dy $^{149}$		
94.0 99.7 111.6 122.8	4.66 4.68 5.03 5.09	5.35 5.39 5.74 5.84	12.4 17.6 28.6 39.0	14.1 15.2 19.3 21.9	-1.7 2.4 9.3 17.1
		Ce140 (O16	(5n)Dy <sup>15!</sup>		
89.7 101.0 111.0	3.63 3.78 3.95	4.16 4.27 4.55	15.4 25.5 34.5	11.0 13.0 16.1	4.4 12.5 18.4
		Ce140 (O1	$^{6}$ ,6 $n$ ) $\mathrm{Dy^{150}}$		
101.0 111.0 121.1 130.4 139.2	3.81 3.95 4.22 4.20 4.24	4.33 4.55 4.80 4.71 4.77	17.6 26.6 35.7 44.0 51.9	13.3 16.1 19.6 20.3 22.3	4.3 10.5 16.1 23.7 29.6
		Ce140 (O16	$^{6},7n)\mathrm{Dy^{149}}$		
111.0 121.1 130.4 139.2 163.0	4.20 4.17 4.21 4.26 4.71	4.76 4.70 4.75 4.82 5.29	16.4 25.5 33.8 41.7 63.1	17.8 19.0 20.9 23.0 32.3	-1.4 6.5 12.9 18.7 30.8

leads us to expect that most of the neutrons are emitted with nearly isotropic angular distributions. As shown in Sec. II, the value of  $\langle \theta_L^2 \rangle^{1/2}$  is not very sensitive to small anisotropies in neutron emission.

Let us assume initially that all neutrons are emitted isotropically. From Eqs. (13) and (15) we can calculate the average energy emitted as photons, and the average kinetic energy of the neutrons for each reaction. The results of these calculations are given in Table IV. First

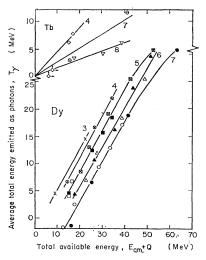


Fig. 5. Total photon energy versus total available energy. The upper curves are for Tb compound nuclei and the product Tb<sup>149</sup>c. The lower curves are for Dy compound nuclei and products Dy<sup>149</sup>, Dy<sup>140</sup>, and Dy<sup>151</sup>. The number of emitted neutrons is indicated for each curve. Symbols are as follows:

```
\begin{array}{lll} \Pr^{141}(C^{12},4n) \operatorname{Tb}^{149g} \diamond; & \operatorname{Nd}^{146}(B^{10},7n) \operatorname{Tb}^{149g} \ominus; \\ \operatorname{Nd}^{146}(B^{11},8n) \operatorname{Tb}^{149g} \lor; & \operatorname{Nd}^{142}(C^{12},3n) \operatorname{Dy}^{151} \times; \\ \operatorname{Nd}^{142}(C^{12},4n) \operatorname{Dy}^{150} \otimes; & \operatorname{Nd}^{142}(C^{12},5n) \operatorname{Dy}^{149} \boxtimes; \\ \operatorname{Nd}^{144}(C^{12},5n) \operatorname{Dy}^{151} \Box; & \operatorname{Nd}^{144}(C^{12},6n) \operatorname{Dy}^{150} \triangle; \\ \operatorname{Nd}^{144}(C^{12},7n) \operatorname{Dy}^{149} \circlearrowleft; & \operatorname{Ce}^{140}(O^{16},5n) \operatorname{Dy}^{151} \blacksquare; \\ \operatorname{Ce}^{140}(O^{16},6n) \operatorname{Dy}^{150} \blacktriangle; & \operatorname{Ce}^{140}(O^{16},7n) \operatorname{Dy}^{149} \clubsuit. \end{array}
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we give the bombarding energy; then the average angles  $\langle \theta_L \rangle$  and  $\langle \theta_L^2 \rangle^{1/2}$  corrected for target thickness and angular definition of the beam. In the last three columns we give the total available energy (Seeger's mass formula was used<sup>12</sup>), the average total kinetic energy of the neutrons, and average total photon energy. We estimate that the values of  $T_n$  have a standard error from experimental sources of not more than about 10%.

If the neutrons are not emitted isotropically, the true energies will differ from those given in Table IV. The maximum alteration due to this effect can be estimated from Eq. (10), which indicates that  $\langle \theta_L^2 \rangle$  for isotropic neutron emission is approximately 33% greater than for  $W(\theta) \propto 1/\sin\theta$ . Thus, if all the neutrons are emitted with this extremely anisotropic angular distribution, then the neutron kinetic energies should be increased by about 33% [see Eq. (15)]. Also, the total photon energies should be correspondingly decreased [see Eq. (13)]. In this paper we proceed with the discussion based on the approximation of isotropy. For this reason the neutron energies in Table IV are probably somewhat too small, and the photon energies are too large. Note that these errors are systematic. Therefore, they probably have only a small effect on the dependence of  $T_n$ and  $T_{\gamma}$  on reaction type and bombarding energy. Precise measurements of range straggling due to the velocity distribution would give a test of this approximation. [Compare Eqs. (4) and (9).]

<sup>&</sup>lt;sup>12</sup> P. A. Seeger, Nucl. Phys. 25, 1 (1961).

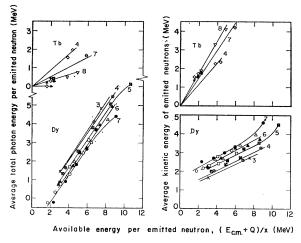


Fig. 6. Average total energy of photons (a) and average energy of neutrons (b) versus available energy per emitted neutron  $(E_{\text{c.m.}}+Q)/x$  for reactions in which x neutrons are emitted. Symbols are as in Fig. 5.

In Fig. 5 we plot the average total photon energy  $T_{\gamma}$  against the total available energy.<sup>13</sup> There is a striking difference between the (HI,xn) reactions (HI means heavy-ion induced) leading to Dy<sup>149</sup>, Dy<sup>150</sup>, Dy<sup>151</sup>, and those leading to Tb<sup>149g</sup>. Increasing the available energy leads to a rather slowly increasing photon energy for Tb<sup>149g</sup> reactions. But for Dy reactions most of the available energy greater than about 10 or 15 MeV is dissipated by photon emission.

There is a small internal inconsistency in the  $T_{\gamma}$  values that we have calculated. These values become negative for two cases; this effect is on the border line of our experimental errors. Also, this result depends on the masses used to calculate Q values. We have Seeger's mass formula for both the target- and heavy-product nuclei. If the angular distribution of the neutrons is peaked forward and backward, this inconsistency is even more pronounced.

As discussed in another paper, the reactions leading to  $\text{Tb}^{149g}$  probably involve only systems of low-angular momentum ( $\lesssim 7.5 \hbar$ ). The results of this study imply that for these Tb compound nuclei of low-spin photon emission does not compete favorably with neutron emission. The reactions leading to  $\text{Dy}^{149}$ ,  $\text{Dy}^{150}$ , and  $\text{Dy}^{151}$  have very high cross sections<sup>4</sup>; thus, the observed

products must be formed from compound nuclei that have angular momentum distributions typical of most compound systems. Presumably, this primary angular-momentum distribution gives rise to a large number of compound nuclei of high spin.<sup>2</sup> As the excited nuclei decay, the angular momentum must be removed by particle and photon emission. Angular momentum barriers increase the lifetime for neutron emission, and, thus, photon emission becomes a competitive process. Grover has described the features of this competition.<sup>14</sup>

Another way of presenting our experimental results is to plot the average energies per emitted neutron versus the available energy per neutron  $(E_{\text{c.m.}}+Q)/x$ . These plots are shown in Fig. 6. Plots of cross section versus available energy per neutron lead to very similar results for these and other similar reactions. The reactions  $(\text{HI},xn)\text{Dy}^{149}$ ,  $\text{Dy}^{150}$ ,  $\text{Dy}^{151}$  all peak at about 5.9 MeV per neutron.<sup>4</sup> The reactions  $(\text{HI},xn)\text{Tb}^{149g}$  all peak at 3 to 4 MeV per neutron.<sup>1</sup>

The Tb<sup>149 $\sigma$ </sup> reactions give values of  $T_n$  and  $T_{\gamma}$  that are expected from evaporation theory without angular-momentum effects. Increasing available energy goes mainly into kinetic energy of the neutrons. For Dy reactions the average kinetic energy of the neutrons increases rather slowly with available energy. For the smaller available energies almost no energy goes to photons. For the higher available energies the photon and neutron energies are comparable.

It has frequently been assumed that the classical rotational energy of a compound nucleus is not available for nuclear evaporation. Thus, this rotational energy is expected to be dissipated by additional photon emission. Such an effect is not apparent from the angular distribution results. The reactions of  $C^{12}$  with  $Nd^{144}$  and of  $O^{16}$  with  $Ce^{140}$  both form  $Dy^{156}$  compound nuclei. Over the energy region of our studies, the average squares of the angular momenta differ by about 25% for a given value of the excitation energy. And yet, in Figs. 5 and 6, the values of  $T_n$  and  $T_\gamma$  are usually indistinguishable. (A possible exception is for  $Dy^{149}$  production at energies near threshold.) The relationship between average total photon energy and angular momentum is discussed further in the preceding paper.

These values of average neutron and photon energies are associated with specific reactions involving neutron emission. Mollenauer's observations<sup>6</sup> of photons are, on the other hand, not associated with such specific reactions. By reference to the excitation functions, we can extract information about average energies of all neutron-emitting reactions. Excitation functions for all the  $(HI,xn)Dy^{149}$ ,  $Dy^{150}$ ,  $Dy^{151}$  reactions peak at about 5.9 MeV per emitted neutron.<sup>4</sup> Thus, if we compare  $T_n$  and  $T_\gamma$  values at 5.9 MeV per neutron, we get a measure of the variation of these quantities with number (x) of

<sup>&</sup>lt;sup>13</sup> The total photon energies that we have deduced may be compared with the results of Morton et~al. (Ref. 5; see Table I in particular.) The comparisons cannot be made quantitatively because of differences in the experimental conditions and the method of analysis. We cannot calculate values of the root-mean-square angle from the data of Morton et~al. because of their rather low-angular resolution. Their analysis involved a Monte Carlo calculation of the angular distribution with an adjustable parameter denoted by the symbol  $E_{\gamma}$ . In the Monte Carlo calculation the quantity  $E_{\gamma}$  was added to the Q value for the reaction. Thus,  $E_{\gamma}$  represents a part of the average total photon energy  $(T_{\gamma})$ . The values of  $E_{\gamma}$  from Ref. 5 can be compared with the values of  $T_{\gamma}$  in Fig. 5 by the relationship  $E_{\gamma}+5\approx T_{\gamma}$  (except for energies near threshold). The consistency of this comparison of the two studies is very gratifying.

 <sup>&</sup>lt;sup>14</sup> J. R. Grover, Phys. Rev. **127**, 2142 (1962); **123**, 267 (1961).
 <sup>15</sup> G. A. Pik-Pichak, Zh. Eksperim. i Teor. Fiz. **38**, 768 (1960) [translation: Soviet Phys.—JETP **11**, 557 (1960)].

neutrons or excitation energy (E). The values of the average neutron energy (at 5.9 MeV per neutron) in Fig. 6 are proportional to  $(E)^{0.4\pm0.15}$ . This relationship reflects the increase in nuclear temperature with excitation energy. The excitation functions give information related to the energy and angular momentum of the first neutron emitted in the evaporation chain. A more detailed comparison of the results of this study with excitation function measurements is given in the following paper.4

#### C. Conclusions

To summarize this study we may list the following conclusions: (a) The reactions involving neutron emission that lead to Dy<sup>149</sup>, Dy<sup>150</sup>, and Dy<sup>151</sup> proceed by compound-nucleus formation. (b) The energetics of the decay of Dy156 (excited to 65 to 125 MeV) to Dy149,  $\mathrm{Dy^{150}}$ , and  $\mathrm{Dy^{151}}$  are almost the same for  $\mathrm{C^{12}+Nd^{144}}$  and for O16+Ce140 in spite of a difference of about 25% in  $\langle J^2 \rangle$ . (c) Compound nuclei of low spin (as measured by reactions forming Tb149g) have very different decay

properties from those of high spin (as measured by reactions forming Dy<sup>149</sup>, Dy<sup>150</sup>, and Dy<sup>151</sup>). (d) The lowspin compound systems dissipate less than about 12 MeV in photons; the remaining energy appears as kinetic energy of the emitted neutrons. (e) The compound systems of higher spin dissipate, on the average, about one-half their available excitation energy by photon emission. (f) For a given reaction, the average total photon energy  $(T_{\gamma})$  increases almost linearly with the available energy, and extends to  $T_{\gamma}$  values of approximately 30 MeV for available energies of 50 to 60 MeV. (g) The average kinetic energy of the neutrons increases approximately as the square root of the excitation energy.

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# Inelastic Scattering of 10.2-MeV Protons by N14†

P. F. Donovan

Bell Telephone Laboratories, Inc., Murray Hill, New Jersey, and Brookhaven National Laboratory, Upton, New York

J. F. MOLLENAUER AND E. K. WARBURTON Brookhaven National Laboratory, Upton, New York (Received 8 August 1963)

The inelastic scattering of protons from N<sup>14</sup> was studied at an incident proton energy of 10.2 MeV. Proton groups were observed corresponding to all the well-established N<sup>14</sup> states below 8.0 MeV. No evidence was obtained for the levels at 7.60, 7.40, 6.60, and 6.05 MeV which were previously reported in this reaction. Angular distributions and total cross sections were measured for inelastic scattering to the N14 states between 3.95 and 7.03 MeV. The relative cross sections are found to be in rather good agreement with shellmodel predictions.

### I. INTRODUCTION

HE present investigation of the inelastic scattering of protons from N14 was undertaken for two reasons. First, previous work on this reaction was done at  $E_p = 9.5$  MeV by Burge and Prowse<sup>1</sup> using photographic emulsions to detect the scattered protons. These authors reported levels in  $N^{14}$  at 7.60 $\pm$ 0.02, and  $7.40\pm0.02$  MeV, and probable levels at  $6.60\pm0.04$  and 5.95 MeV, in addition to the well-known levels2 below 7-MeV excitation in N14. Later, Hossian and Kamal3 reported results from reading of emulsions which were a part of the same series of exposures used by Burge and Prowse.1 Hossian and Kamal reported levels in  $N^{14}$  at  $6.05\pm0.02$  and  $6.75\pm0.03$  MeV in addition to the well-known levels. One purpose of the present work, then, was to study the proton spectrum from  $N^{14}(p,p')N^{14}$  at a proton energy close to that of the previous work as a check on the existence of N14 levels near 7.6, 7.4, 6.7, and 6.0 MeV.

The second reason for undertaking this study was to obtain relative cross sections for excitation of the N14

<sup>†</sup> Work performed in part under the auspices of the U.S. Atomic

Energy Commission.

<sup>1</sup> E. J. Burge and P. J. Prowse, Phil. Mag. 1, 912 (1956).

<sup>2</sup> F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. 11, 1 (1959).

<sup>&</sup>lt;sup>3</sup> A. Hossian and A. N. Kamal, Indian J. Phys. 31, 553 (1957).