

method of resonance distortion and its relation to other approximate methods of solving collision problems. First of all, the method of resonance distortion is restricted to problems involving near-resonance, since the limiting case of exact resonance is taken as the zeroth-order approximation. This method is most suitable for cases where the coupling between the initial and final states is strong. We can illustrate the nature of this method through a partial-wave analysis. The partial cross sections corresponding to large l for the strong coupling case reduce to the Born partial cross sections, because partial waves of large l are classically equivalent to distant impacts and at large distances, U_{00} , U_{nn} , and U_{0n} are sufficiently small so that the Born approximation is applicable. At lower l the "effective" coupling becomes so large that the use of the Born approximation and the method of distorted waves, which are valid for weak coupling, is not always justifiable. It is in this region of l (called "low- l region") that the method of resonance distortion is useful. For collision-induced transitions with a long-range interaction potential, where the contribution from the

partial cross sections in the "low- l region" constitutes a substantial part of the total cross section (e.g., Table IV), we may expect the method of resonance distortion to yield more accurate results than the usual Born approximation.

For very weak coupling, the results of the method of resonance distortion approach those of the method of distorted waves. This can be seen from Eqs. (108) and (109). When U_{0n} becomes very small, $F_0^{(0)}$ in Eq. (109) is nearly equal to the zeroth-order solution in the method of distorted waves, which is defined by

$$(\nabla^2 + k_0^2 - U_{00})(F_0^{(0)})_{DW} = 0.$$

The difference in F_n as calculated by these two zeroth-order functions should be small compared to F_n itself.

ACKNOWLEDGMENTS

The authors wish to thank Mrs. Barbra Lafon and George Shelton for their assistance in the numerical computations.

Analytical Self-Consistent Field Functions for Cr^+ , Cr and Its Excited States*

MIROSLAV SYNEK

Department of Physics, De Paul University, Chicago, Illinois

(Received 7 October 1963)

Self-consistent field calculations by the expansion method were carried out for Cr^+ , Cr and some of its excited states. The results represent closely the absolute Hartree-Fock solutions. The wave functions were calculated with the requirement to satisfy exactly the cusp condition so that they can be considered to be particularly accurate in the immediate vicinity of the nucleus. The differences in calculated energy levels are compared with experiment.

INTRODUCTION

THE self-consistent field (SCF) expansion method of Roothaan¹⁻³ was applied on a number of cases.⁴⁻⁹ In this paper the application is carried out for

* This work was supported through the grant of the guest privileges at the Department of Physics, University of Chicago, Chicago, Illinois.

¹ C. C. J. Roothaan, *Rev. Mod. Phys.* **23**, 69 (1951).

² C. C. J. Roothaan, *Rev. Mod. Phys.* **32**, 179 (1960).

³ C. C. J. Roothaan and P. S. Bagus, in *Methods in Computational Physics*, edited by B. Alder *et al.* (Pergamon Press, Inc., New York, 1963), Vol. 2.

⁴ C. C. J. Roothaan, L. M. Sachs, and A. W. Weiss, *Rev. Mod. Phys.* **32**, 186 (1960).

⁵ E. Clementi, C. C. J. Roothaan, and M. Yoshimine, *Phys. Rev.* **127**, 1618 (1962).

⁶ C. C. J. Roothaan and P. S. Kelley, *Phys. Rev.* **131**, 1177 (1963).

⁷ M. Synek, *Phys. Rev.* **131**, 1572 (1963); this article, with numerical tabulations and graphs of some wave functions included, is also printed in the Technical Report, Laboratory of

Cr^+ , Cr and some of its excited states. Vector coupling coefficients $J_{\lambda\mu}$ and $K_{\lambda\mu}$ were taken from the tables calculated recently.¹⁰

The computation was done on an IBM 7090 computer with a modification of the previously established program.⁸

RESULTS AND DISCUSSION

The notation and the units used are the usual ones and are identical with those employed in a recent work.⁷

Some of the important results are presented in

Molecular Structure and Spectra, Dept. of Physics, University of Chicago, 1962-3, Pt. 1, p. 229.

⁸ H. D. Cohen, Technical Report, Laboratory of Molecular Structure and Spectra, Dept. of Physics, University of Chicago, 1962-3, Pt. 1, p. 251.

⁹ C. C. J. Roothaan and M. Synek (to be published).

¹⁰ G. L. Malli and C. C. J. Roothaan (to be published).

Tables I-VI.¹¹ The values of cusps are not presented since the cusp conditions are exactly satisfied.

The results computed represent the Hartree-Fock values to about six significant figures for total energies, and to about three decimal places for the radial functions $P_{i\lambda}(r)$. The condition for the required number of nodes of $P_{i\lambda}(r)$ is usually satisfied to four decimal places; as a rule,¹² the occasional deviation did not exceed nine units at the fifth decimal place (in the original computer tabulation). While keeping the same numbers of basis functions, the values of total energies

TABLE I. Comparison with other calculations for total energy $E(\text{au})$.

State \ Source	Other calculations	This work
$\text{Cr}^+, 3d^5, {}^6S$	-1043.1346 ^a	-1043.1379
$\text{Cr}, 3d^4s^2, {}^5D$	-1043.3036 ^b	-1043.3084
$\text{Cr}, 3d^6, {}^5D$	-1043.0909 ^a	-1043.0957

^a See Ref. 14.
^b See Ref. 15.

TABLE II. Optimized exponents ζ of the basis functions for Cr^+ , Cr and its excited states.

Atom and state \ Basis function	Cr^+ $3d^5, {}^6S$	Cr $3d^4s4p, {}^7F$	Cr $3d^4s^2, {}^5D$	Cr $3d^4s, {}^5S$	Cr $3d^4s, {}^5P$	Cr $3d^4s, {}^5G$	Cr $3d^4s, {}^7S$	Cr $3d^4p, {}^7P$	Cr $3d^6, {}^5D$
1s	24.000	24.000	24.000	24.000	24.000	24.000	24.000	24.000	24.000
3s	25.696	25.763	25.671	25.763	25.763	25.763	25.763	25.782	25.518
3s	19.410	21.115	21.668	21.115	21.115	21.115	21.115	20.785	21.037
3s	13.643	13.769	13.874	13.769	13.769	13.769	13.769	13.760	13.859
3s	9.696	9.822	9.726	9.756	9.756	9.756	9.756	9.744	9.860
3s	5.233	4.891	5.082	5.104	4.799	4.865	5.129	5.044	4.958
3s	3.375	3.335	3.442	3.372	3.187	3.237	3.360	3.357	3.239
4s		1.646	1.745	1.592	1.613	1.601	1.602		
4s		0.936	1.014	0.881	0.911	0.912	0.880		
2p	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000	12.000
4p	27.700	27.178	27.178	27.178	27.178	27.178	27.178	27.178	27.454
4p	12.950	13.033	12.967	12.909	12.909	12.909	12.909	12.909	12.963
4p	8.885	9.073	9.006	8.958	8.958	8.958	8.958	8.977	9.133
4p	5.602	5.830	5.506	5.572	5.571	5.588	5.606	5.606	5.694
4p	3.344	3.582	3.318	3.310	3.314	3.317	3.336	3.286	3.314
4p		1.688						1.279	
4p		0.954						0.684	
3d	8.000	8.000	8.000	8.000	8.000	8.000	8.000	8.000	8.000
5d	10.829	10.243	10.243	10.547	10.547	10.547	10.547	10.322	7.893
5d	7.038	6.393	6.393	6.393	6.393	6.393	6.393	6.393	6.950
5d	4.433	3.771	3.653	3.662	3.783	3.715	3.653	3.603	4.505
5d	2.944	3.059	3.021	3.025	2.972	3.004	3.021	2.971	2.446
5d	1.924	2.008	1.950	1.843	1.749	1.759	1.800	1.875	1.255

* Calculated to five decimal places. Values in the table are to be followed by two zeros beyond the third decimal place.

TABLE III. Eigenvectors of coefficients $C_{i\lambda p}$ for Cr^+ , Cr and its excited states.

Atom and state \ Basis function	Cr^+ $3d^5, {}^6S$	Cr $3d^4s4p, {}^7F$	Cr $3d^4s^2, {}^5D$	Cr $3d^4s, {}^5S$	Cr $3d^4s, {}^5P$	Cr $3d^4s, {}^5G$	Cr $3d^4s, {}^7S$	Cr $3d^4p, {}^7P$	Cr $3d^6, {}^5D$
1s	1s	1s	1s	1s	1s	1s	1s	1s	1s
1s	0.97998	0.97996	0.97996	0.97997	0.97997	0.97997	0.97997	0.97997	0.97999
3s	0.02557	0.02470	0.02523	0.02473	0.02466	0.02468	0.02473	0.02467	0.02648
3s	0.00332	0.00364	0.00273	0.00356	0.00369	0.00366	0.00356	0.00382	0.00141
3s	0.00355	0.00441	0.00490	0.00444	0.00428	0.00431	0.00444	0.00406	0.00491
3s	-0.00150	-0.00179	-0.00200	-0.00183	-0.00167	-0.00171	-0.00183	-0.00160	-0.00189
3s	0.00036	0.00049	0.00063	0.00051	0.00043	0.00045	0.00050	0.00038	0.00041
3s	-0.00016	-0.00028	-0.00034	-0.00026	-0.00025	-0.00025	-0.00026	-0.00018	-0.00019
4s		0.00005	0.00007	0.00005	0.00005	0.00005	0.00005		
4s		0.00002	-0.00003	-0.00002	-0.00002	-0.00002	-0.00002		

¹¹ The numerical tabulations are presented in Tables VII-XV. As a rule (see Ref. 12) the tabulated values are rounded to four decimal places. The tables are deposited as Document number 7766 with the ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington, D. C. A copy may be secured by citing the Document number and by remitting \$1.25 for photoprints, or \$1.25 for 35-mm microfilm. Advance payment is required. Make checks or money orders payable to: Chief, Photoduplication Service, Library of Congress.

¹² The only exception occurs for the state $3d^4s4p, {}^7F$.

TABLE III (continued).

Atom and state Basis function	Cr ⁺	Cr	Cr	Cr	Cr	Cr	Cr	Cr	Cr
	3d ⁵ , ⁶ S	3d ⁴ 4s4p, ⁷ F	3d ⁴ 4s ² , ⁵ D	3d ⁵ 4s, ⁵ S	3d ⁵ 4s, ⁵ P	3d ⁵ 4s, ⁵ G	3d ⁵ 4s, ⁷ S	3d ⁵ 4p, ⁷ P	3d ⁶ , ⁵ D
	2s	2s	2s	2s	2s	2s	2s	2s	2s
1s	-0.29373	-0.29358	-0.29364	-0.29362	-0.29363	-0.29363	-0.29364	-0.29364	-0.29364
3s	-0.00091	-0.01743	-0.01942	-0.01565	-0.01588	-0.01582	-0.01560	-0.01177	-0.01639
3s	0.10826	0.10399	0.09324	0.09967	0.10019	0.10005	0.09957	0.09869	0.10115
3s	0.52078	0.52396	0.54316	0.53563	0.53470	0.53494	0.53580	0.53325	0.51317
3s	0.44741	0.46139	0.45820	0.45364	0.45500	0.45469	0.45354	0.45391	0.47416
3s	0.00625	0.01308	0.00694	0.00983	0.00989	0.00986	0.00969	0.00882	0.01185
3s	0.00132	-0.00252	0.00078	-0.00026	-0.00121	-0.00101	-0.00021	0.00002	-0.00115
4s		0.00060	-0.00013	0.00003	0.00031	0.00027	0.00021		
4s		0.00011	0.00006	-0.00019	-0.00004	-0.00003	0.00006		
	3s	3s	3s	3s	3s	3s	3s	3s	3s
1s	0.10596	0.10713	0.10714	0.10604	0.10646	0.10642	0.10622	0.10610	0.10596
3s	0.00448	0.00626	0.01008	0.00874	0.00430	0.00548	0.01039	0.00501	0.00866
3s	-0.05851	-0.04054	-0.04250	-0.04600	-0.03559	-0.03833	-0.04986	-0.04069	-0.04572
3s	-0.16396	-0.20250	-0.20042	-0.19068	-0.21303	-0.20749	-0.18393	-0.19912	-0.18312
3s	-0.35633	-0.31096	-0.32959	-0.32993	-0.29655	-0.30482	-0.34067	-0.31717	-0.32907
3s	0.61141	0.69610	0.62296	0.62120	0.75169	0.72111	0.63050	0.63162	0.69135
3s	0.59404	0.47469	0.56120	0.56523	0.41863	0.45307	0.56279	0.54992	0.49067
4s		0.00142	0.00274	0.00405	-0.00029	0.00059	0.00124		
4s		0.00036	-0.00045	-0.00024	0.00030	0.00009	0.00004		
	4s	4s	4s	4s	4s	4s	4s	4s	4s
1s		-0.02630	-0.02372	-0.01648	-0.02179	-0.02169	-0.02220		
3s		-0.00538	-0.00519	-0.00284	-0.00372	-0.00458	-0.00592		
3s		0.01895	0.01548	0.01063	0.01387	0.01583	0.01919		
3s		0.03314	0.03437	0.02322	0.03188	0.02801	0.02217		
3s		0.10485	0.09337	0.06389	0.08126	0.08607	0.09916		
3s		-0.25596	-0.19616	-0.13275	-0.22081	-0.21732	-0.19912		
3s		-0.05693	-0.09634	-0.06549	-0.03300	-0.03684	-0.07238		
4s		0.74737	0.53841	0.38375	0.56579	0.56795	0.61536		
4s		0.34613	0.56328	0.70890	0.53342	0.52825	0.49368		
	2p	2p	2p	2p	2p	2p	2p	2p	2p
2p	0.72387	0.72388	0.72377	0.72384	0.72383	0.72383	0.72383	0.72386	0.72396
4p	0.00434	0.00432	0.00452	0.00480	0.00479	0.00480	0.00480	0.00484	0.00470
4p	0.22953	0.21977	0.22570	0.22971	0.22980	0.22978	0.22972	0.22918	0.22288
4p	0.12813	0.13712	0.12980	0.12525	0.12520	0.12525	0.12534	0.12552	0.12979
4p	-0.00728	-0.00702	-0.00478	-0.00435	-0.00444	-0.00446	-0.00448	-0.00416	-0.00161
4p	0.00273	0.00336	0.00210	0.00200	0.00200	0.00199	0.00201	0.00211	0.00144
4p		-0.00078						-0.00033	
4p		0.00037						0.00018	
	3p	3p	3p	3p	3p	3p	3p	3p	3p
2p	-0.25297	-0.25653	-0.25633	-0.25302	-0.25400	-0.25389	-0.25343	-0.25313	-0.25254
4p	-0.00077	-0.00090	-0.00051	-0.00078	-0.00079	-0.00087	-0.00083	-0.00114	-0.00109
4p	-0.07296	-0.06931	-0.07651	-0.07445	-0.07460	-0.07377	-0.07394	-0.07061	-0.06911
4p	0.07096	0.04597	0.07977	0.07309	0.07301	0.06994	0.06983	0.06023	0.04923
4p	0.60242	0.56620	0.63795	0.61394	0.61913	0.61800	0.60855	0.62698	0.61382
4p	0.44463	0.48389	0.40061	0.43358	0.42785	0.43145	0.44089	0.43017	0.45461
4p		0.01969						0.00315	
4p		-0.00309						0.00034	
	4p	4p	4p	4p	4p	4p	4p	4p	4p
2p		0.05196						0.03446	
4p		0.00013						0.00032	
4p		0.01427						0.00770	
4p		-0.01201						-0.00425	
4p		-0.12820						-0.10582	
4p		-0.09909						-0.02989	
4p		0.34420						0.41273	
4p		0.74890						0.68564	
	3d	3d	3d	3d	3d	3d	3d	3d	3d
3d	0.15360	0.16224	0.16364	0.15064	0.14777	0.14821	0.14912	0.15174	0.14436
5d	0.03178	0.06190	0.05956	0.05238	0.05278	0.05192	0.05167	0.05397	0.16649
5d	0.28184	0.38530	0.39558	0.37786	0.36559	0.36973	0.37485	0.37286	0.06577
5d	0.36364	0.39340	0.43949	0.37656	0.31463	0.33393	0.37440	0.44134	0.41303
5d	0.26447	0.08687	0.04863	0.12272	0.19863	0.17093	0.12070	0.04231	0.38617
5d	0.20465	0.19501	0.17622	0.22957	0.24785	0.25151	0.24914	0.24809	0.20894

could be made more accurate, mainly, if the cusp condition were sacrificed.

The results are compared with other calculations wherever available.

A numerical SCF calculation for the state $3d^6, ^5D$ of Cr was published by Porter.¹³ However, his calculation was done without exchange and, furthermore, it was done only for the purpose of an approximate orientation. Hence, his numerical wave functions (after normalization) cannot be compared with ours.

Analytical (i.e., expansion method) SCF calculations on the ground state of Cr^+ , $3d^5, ^6S$, and on the two states of Cr, $3d^6, ^5D$ and $3d^4s^2, ^5D$, were carried out by Watson.^{14,15} The comparison of the results for the total energies is given in Table I.

TABLE IV. Calculated energies and experimental energies.^a

Atom or ion	State	Calculated total energy	Energies relative to ground state		Estimate of correlation energy difference
			Calculated	Experimental	
Cr^+	$3d^5, ^6S$	-1043.1379	0.2160	0.2486	-0.0326
Cr	$3d^4s^4p, ^7F$	-1043.2744	0.0795	0.1152	-0.0357
	$3d^4s^2, ^5D$	-1043.3084	0.0455	0.0364	0.0091
	$3d^5s, ^6S$	-1043.3154	0.0385	0.0346	0.0039
	$3d^5s, ^5P$	-1043.2132	0.1407	0.0996	0.0411
	$3d^5s, ^5G$	-1043.2305	0.1234	0.0935	0.0299
	$3d^5s, ^7S$	-1043.3539	0	0	0
	$3d^5s^2, ^7P$	-1043.2583	0.0956	0.1066	-0.0110
	$3d^6, ^5D$	-1043.0957	0.2582	0.1620	0.0962

^a See Ref. 16.

TABLE V. Orbital energies ϵ .

Atom and state	Cr^+ $3d^5, ^6S$	Cr $3d^4s^4p, ^7F$	Cr $3d^4s^2, ^5D$	Cr $3d^5s, ^6S$	Cr $3d^5s, ^5P$	Cr $3d^5s, ^5G$	Cr $3d^5s, ^7S$	Cr $3d^5s^2, ^7P$	Cr $3d^6, ^5D$
1s	-220.65076	-220.62782	-220.59149	-220.39096	-220.42135	-220.41613	-220.38375	-220.44881	-220.30459
2s	-26.46663	-26.47595	-26.43855	-26.20988	-26.24773	-26.24204	-26.20758	-26.26771	-26.11747
3s	-3.54816	-3.53224	-3.49864	-3.29183	-3.31511	-3.31077	-3.28394	-3.34649	-3.18330
4s		-0.33145	-0.23933	-0.17818	-0.21751	-0.21679	-0.22079		
2p	-22.39883	-22.40307	-22.36467	-22.14117	-22.17770	-22.17209	-22.13781	-22.19968	-22.05020
3p	-2.31925	-2.28422	-2.24831	-2.06079	-2.07916	-2.07514	-2.04994	-2.11635	-1.95714
4p		-0.18370						-0.12148	
3d	-0.64713	-0.60656	-0.56849	-0.38500	-0.33606	-0.34070	-0.37265	-0.44176	-0.20060

TABLE VI. Virial theorem. (Values of E_p/E_k)

Atom or ion	State	E_p/E_k
Cr^+	$3d^5, ^6S$	-1.9999691
Cr	$3d^4s^4p, ^7F$	-2.0000015
	$3d^4s^2, ^5D$	-2.0000009
	$3d^5s, ^6S$	-1.9999815
	$3d^5s, ^5P$	-1.9999836
	$3d^5s, ^5G$	-1.9999807
	$3d^5s, ^7S$	-1.9999770
	$3d^5s^2, ^7P$	-1.9999872
	$3d^6, ^5D$	-1.9999509

Since total experimental energies are not available, the energy differences between the excited states and the ground states are used for the comparison with

¹³ A. Porter, Mem. and Proc. Manchester Lit. Phil. Soc. **79**, 75 (1934).

¹⁴ R. E. Watson, Tech. Rept. No. 12, June 1959, Solid-State and Molecular Theory Group, Massachusetts Institute of Technology, Cambridge, Massachusetts (unpublished).

¹⁵ R. E. Watson, Phys. Rev. **119**, 1934 (1960).

experiment in Table IV. Since the spin-orbit interaction is neglected in the calculation, the experimental energies¹⁶ are averaged over the multiplet components. An estimate of the correlation energy differences between the excited states and the ground state is given by the last column in Table IV. It shows that the correlation energy depends significantly on the state, not only on the configuration.

ACKNOWLEDGMENTS

The author is indebted to Professor Clemens C. J. Roothaan of the University of Chicago, whose developments in the SCF expansion method enabled this application.

The author also appreciates the technical assistance of George Stungis and Max Striegl of the De Paul University, and the staff of the Computation Center at the University of Chicago.

¹⁶ C. E. Moore, U. S. Natl. Bur. Std. Circular No. 467 (U. S. Government Printing Office, Washington, D. C., 1949).