Landau Damping of Ion Acoustic Waves in Highly Ionized Plasmas*

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(Received 29 May 1963; revised manuscript received 19 September 1963)

Ion acoustic waves with frequencies between 5 and 100 kc/sec have been excited in highly ionized cesium and potassium plasmas by modulating the potential of a tungsten grid immersed in the plasma. The waves were detected by another grid, which could be moved along the plasma column. Because the plasma is produced at one end of the column, there is a net flow of plasma between the grids. Phase velocities of 1.3×10^5 cm/sec and 0.9×10^5 cm/sec along and against this drift were measured for cesium and 2.5×10^5 cm/sec and 1.3×10^5 cm/sec for potassium. The damping distance of the waves was found to be independent of ion density in the range between 2×10^{10} cm^{-3} and 3×10^{11} cm^{-3} and equal to 0.55 and 0.25 wavelength along and against the flow in cesium and 0.65 and 0.14 in potassium. A comparison between the results and the collisionless theory shows agreement within about 10%. In particular, it is shown how the present experiment provides a quantitative measurement of the Landau damping of ion acoustic waves.

I. INTRODUCTION

Ion acoustic waves are low-frequency longitudinal plasma density oscillations in which ions and electrons move in phase. They were first predicted in 1929 on the basis of a fluid analysis by Tonks and Langmuir,[1] who found for frequencies well below the ion plasma frequency and for isothermal changes, that the phase velocity v should be given by

$$v^2=\kappa(T_e+T_i)/m_i,$$

where m_i is the ion mass, and T_e and T_i the electron and ion temperatures. These waves differ from ordinary sound waves in that the coupling between the electrons and the ions arises from electric fields resulting from small charge separations. More recently, a number of authors[2,3] have shown on the basis of the collisionless Boltzmann equation that ion waves can exist in a plasma even in the absence of collisions. The collisionless equations also predict an interesting feature of the ion waves; namely, that they should be damped by interaction with ions moving with velocities close to the phase velocity of the wave.[4] This damping, which was first predicted by Landau[5] for high-frequency longitudinal plasma oscillations, is strong even for wavelengths large compared with the Debye distance, if the ion and electron temperatures are comparable.

The first experimental observation of ion acoustic waves was reported by Revans,[6] who found standing waves in an ionized mercury vapor discharge. Recently, Alexeff and Neidigh, and Crawford[7] reported observations of ion waves which occurred naturally in the generation of gas discharges. The frequencies they observed were determined by the size of the plasma.

Propagation and damping of ion waves are most conveniently studied if the waves are generated by an external source. Hatta and Sato[8] excited ion waves in a weakly ionized plasma by means of a grid. Their measurements of the phase velocity and the damping were different from what might be expected in a fully ionized plasma, since their plasma was dominated by ion-atom collisions. Little[9] also succeeded in exciting ion waves with the use of an external coil which modulated the plasma density by varying the magnetic field. The plasma used in his experiments was also weakly ionized ($\sim$1%). No measurements of wave damping have been reported.

It is the purpose of this paper to describe the excitation, propagation, and damping of ion acoustic waves in a low-temperature, highly ionized plasma in which the collisions of ions with atoms are unimportant. Unlike the discharge experiments, the excitation of waves and the generation of the plasma are separated: the steady-state plasma is produced by surface ionization of alkali atoms on a hot tungsten plate; the excitation of waves is achieved by modulating the potential of a grid immersed in the plasma. The damping of the waves excited in this plasma of presumably equal ion and electron temperatures has been found to be so strong that standing oscillations are not produced. A study of wave propagation and damping is thus possible. A brief account of some preliminary experimental results has already been reported.[10]

Section II describes the experimental setup. Section III presents the experimental results. Section IV con-

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] L. Tonks and I. Langmuir, Phys. Rev. **33**, 195 (1929).
[2] J. D. Jackson, J. Nucl. Energy: Pt. C **1**, 171 (1960); I. B. Bernstein, E. A. Frieman, R. M. Kulsrud, and M. N. Rosenbluth, Phys. Fluids **3**, 136 (1960); E. A. Jackson, Phys. Fluids **3**, 786 (1960); I. B. Bernstein and R. M. Kulsrud, Phys. Fluids **3**, 937 (1960).
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[10] A. Y. Wong, N. D'Angelo, and R. W. Motley, Phys. Rev. Letters **9**, 415 (1962).

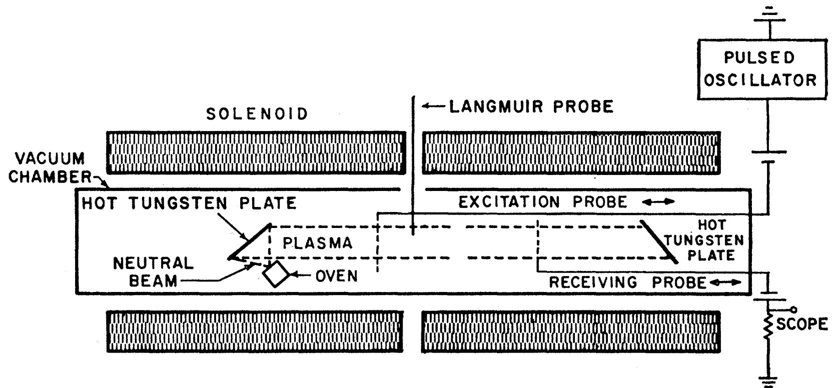

FIG. 1. Diagram of Q-3 machine and the probe arrangement. The distance between the plates is 90 cm.

tains the theory and Sec. V a discussion of the results.

II. EXPERIMENTAL SETUP

The experiment was performed on the Q-3 machine, which is similar in construction to the alkali-ion plasma source described by Rynn and D'Angelo.[11] The plasma is produced, as shown in Fig. 1, by surface ionization of a beam of cesium or potassium atoms on a 3-cm diam tungsten plate heated to $\sim$2500°K. The plasma is terminated at the other end of the column by another hot tungsten plate, and is confined radially by an axial magnetic field which can be increased to 16 000 G steady state. In order to avoid possible ion cyclotron resonance, the magnetic field was adjusted to 12 000 G in cesium and 6000 G in potassium. The ion cyclotron frequency, 140 kc/sec in cesium and 240 kc/sec in potassium, is then greater than the acoustic wave frequency.

The plasma forms a column about 90 cm long and 3 cm in diameter. With the strong magnetic field the main sources of plasma loss are volume recombination and surface recombination of ions striking the grids. Since the plasma is produced at one end of the column, these loss processes induce a drift of plasma away from the producing plate. Ion and electron temperatures are believed to be close to the temperature of the tungsten plates. Ion densities in the 10^{10} cm^{-3} to 10^{12} cm^{-3} range are measured by a Langmuir probe with a cylindrical tip 0.075 cm in length and 0.025 cm in diameter. The walls of the stainless steel vacuum vessel are cooled below 0°C. With a background neutral pressure of 1 to 2×10^{-6} mm Hg the percentage ionization ranges from 40 to 90%.

A tungsten grid 5 cm in diameter made of 0.0025-cm diam wire spaced 0.075 cm apart is immersed in the plasma column perpendicular to the axis, 40 cm from the tungsten plate where the plasma is produced (see Fig. 1). The function of this control grid is to regulate the supply of ions flowing from the plasma-generating hot plate and hence to vary the plasma density. In our experiments the grid is biased at -20 V and a sinusoidal voltage (5 V peak to peak) is applied periodically to the grid for 1 msec. The sinusoidal signal modulates the percentage transmission of the grid by about 10%, presumably by varying the extent of the ion-collecting region around the grid wires. This region extends several Debye distances ($D\cong10^{-3}$ cm) into the plasma, and is a significant fraction of the interwire spacing. If a density perturbation is created, it should propagate down the plasma column. A similar grid, which can be moved longitudinally 30 cm and which is similarly biased, detects the fluctuations in ion density produced by the first grid. A time delay in the reception of the signal, proportional to the distance between the grids, provides evidence that the signal received is the result of wave propagation and is not electrostatic pickup or current conduction between the grids.

[11] N. Rynn and N. D'Angelo, Rev. Sci. Instr. **31**, 1326 (1960).

III. EXPERIMENTAL RESULTS

All of the data reported in this paper were obtained with a plasma source at one end of the column. With this arrangement the plasma drifts from this end to supply losses caused by volume recombination and surface recombination on the grids. We find that the grid excites waves in the directions along and against the flow of plasma. The drift velocity of the plasma can be estimated from the difference between the phase velocities of waves propagating "upstream" and "downstream."

In Fig. 2 are shown some typical experimental results obtained by photographing oscilloscope traces. The time delay between the applied and received signals, which readily distinguishes wave propagation from pickup, is evident. The percentage density fluctuation n_1/n_0 detected by the receiving grid is equal to $e\phi/kT$ within 20%. (ϕ is the fluctuating component of the plasma potential.) This equality is required by the condition that the potential must adjust to keep the ion and electron concentrations almost equal.

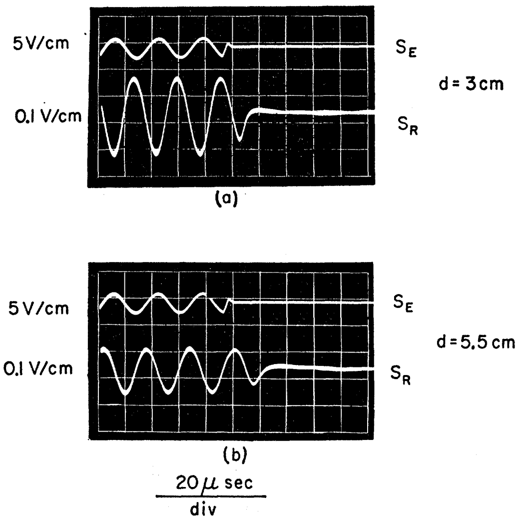

FIG. 2. Oscillograms showing the excitation signals S_E and the received signals S_R in potassium. d is the distance between the grids. The sweep speed is 20 μsec/cm.

1. Phase Velocities

By moving the second grid along the axis of the column, we have measured the phase velocity and the attenuation of the waves both along and against the direction of plasma flow. The phase velocity was calculated from the slope of the phase delay versus the receiving probe position, as shown in Figs. 3 and 4. The important quantity in Figs. 3 and 4 is the incremental time delay with the incremental change in probe position. The absolute magnitude of the phase delay is not a useful quantity, because it is affected by phase shifts introduced by coupling in and out of the plasma. In the figures, the horizontal position of each set of data has been displaced to avoid overlapping of data points.

In general, we were able to measure the phase over more than one wavelength downstream, but only over 0.1–0.2 wavelength upstream. Note that the points fall on a straight line, indicating that the phase velocity is constant, independent of the wave amplitude. The results of the phase velocity measurements as a function of frequency for both cesium and potassium plasmas are shown in Figs. 5 and 6. The notable features of these measurements are: (a) The phase velocity is independent of frequency in the 15–100 kc/sec range. (b) The phase velocity is independent of density in the range used in our experiments ($2\times10^{10}-3\times10^{11}$ ions/cm^{-3}). (c) The average phase velocities in cesium plasma are 1.3×10^5 cm/sec downstream and 0.9×10^5 cm/sec upstream and in potassium plasma are 2.5 $\times10^5$ cm/sec downstream and 1.3×10^5 cm/sec upstream. The phase measurements upstream are more difficult because of stronger wave damping (see Sec. III-2).

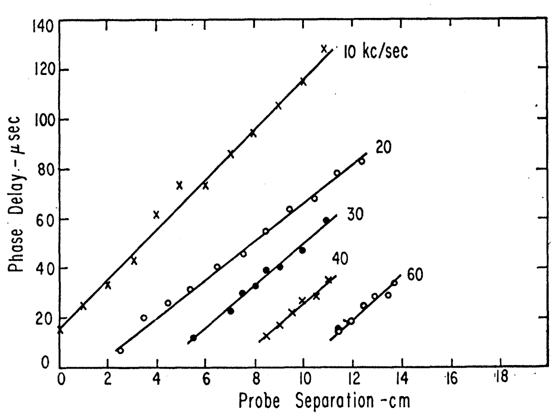

FIG. 3. Phase delay of the downstream ion waves as a function of the separation between the grids in a cesium plasma. The zero of the distance scale has been arbitrarily chosen for each set of data. The density of the plasma is 3×10^{11} cm^{-3}.

In Sec. V these results will be compared with the

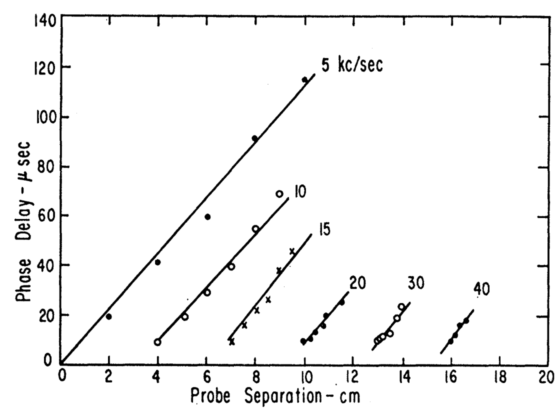

FIG. 4. Phase delay of the upstream ion waves as a function of the separation between the grids in a cesium plasma. The zero of the distance scale has been arbitrarily chosen for each set of data. The density of the plasma is 2×10^{11} cm^{-3}.

predictions of a plane-wave analysis of ion-wave propagation. In order to check that such a plane-wave analysis should be appropriate for our experiment, we have varied the following parameters: (a) the confining magnetic field, between 4 and 14 kG; (b) the plasma diameter, between 1.5 and 3.0 cm.

In varying each of these parameters we were able to detect no systematic variation of phase velocity, within about 10%. In addition, by means of a small 2-mm diam grid which could be moved radially, we have measured both the wave amplitude n_1/n_0 and the phase in the radial direction. Again, over the central 2 cm of plasma column we could detect no change in wave amplitude, within about 10%, or variation in phase, within about 10°.

From these measurements we conclude that a plane-wave analysis should be appropriate. We attribute this absence of plasma boundary effects to the strong confining magnetic field.

2. Attenuation

In addition to the phase velocity we have also measured the wave attenuation, which is quite rapid. Data taken in cesium are shown in Fig. 7. Over a distance of one wavelength the wave amplitude falls exponentially by a factor of 6 downstream and a factor of 50 upstream. In Figs. 8 and 9 is shown the frequency dependence of the wave damping distance δ, where δ is the distance over which the wave amplitude falls by a factor e. The notable features are:

(a) δ is independent of the plasma density between 2×10^{10} ions/cm^3 and 3×10^{11} ions/cm^3.

(b) $1/\delta$ varies linearly with frequency both in cesium and potassium. Therefore, the damping distance δ is a constant fraction of a wavelength. The average damping constants computed from the data of Figs. 5, 6, 8, and 9 in cesium are $\delta/\lambda=0.55$ (downstream) and $\delta/\lambda=0.25$ (upstream). In potassium $\delta/\lambda=0.65$ (downstream) and $\delta/\lambda=0.14$ (upstream). For each of these numbers quoted the mean deviation of the individual measurements included in the averaging is about 10% of the mean. These values of damping constants show no significant trend with ion density or frequency but do exhibit a marked dependence on the direction of wave propagation.

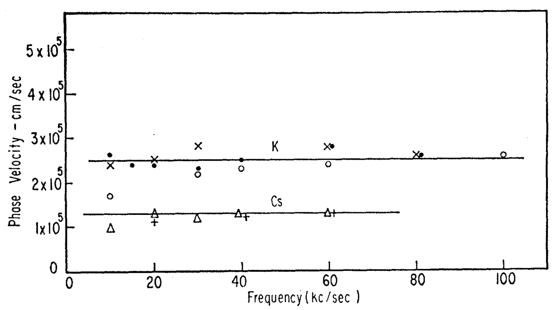

FIG. 5. Phase velocity of the downstream waves in cesium and potassium plasmas as a function of frequency. The different symbols refer to data taken at different ion densities: $\times=1.8\times10^{10}$ cm^{-3}, $\bullet=6\times10^{10}$ cm^{-3}, $\circ=15\times10^{10}$ cm^{-3}, $+=6\times10^{10}$ cm^{-3}, $\triangle=30\times10^{10}$ cm^{-3}.

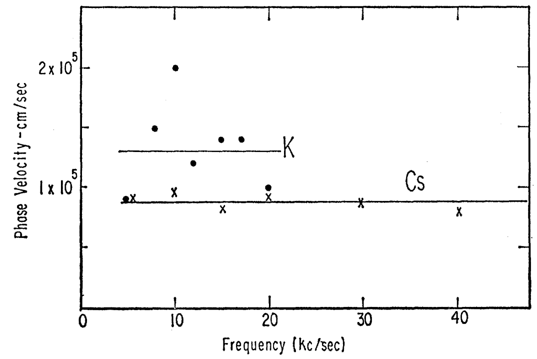

FIG. 6. Phase velocity of the upstream waves in cesium and potassium plasmas as a function of frequency. The density of the plasma is 2×10^{11} cm^{-3}.

IV. THEORY

In our experiments the acoustic wave frequency is much less than the electron-electron collision frequency but greater than the ion-ion collision frequency. We shall therefore follow the analysis of Jensen[12] in treating the electrons as a fluid but shall describe the ions by a collisionless Boltzmann equation in the manner of Fried and Gould.[3] The electrons are characterized by an average electron density n_e, an average velocity v_e, and isotropic pressure p_e. The equation of motion is

$$n_e m_e \frac{dv_e}{dt} = -n_e eE - \frac{\partial}{\partial x} p_e, \tag{1}$$

where $p_e = n_e K T_e$. The ions are described by a velocity distribution function f satisfying the one-dimensional collisionless Boltzmann equation

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + a \frac{\partial f}{\partial v_x} = 0. \tag{2}$$

For longitudinal wave propagation in the direction along x parallel to the external magnetic field, $a = eE/m_i$.

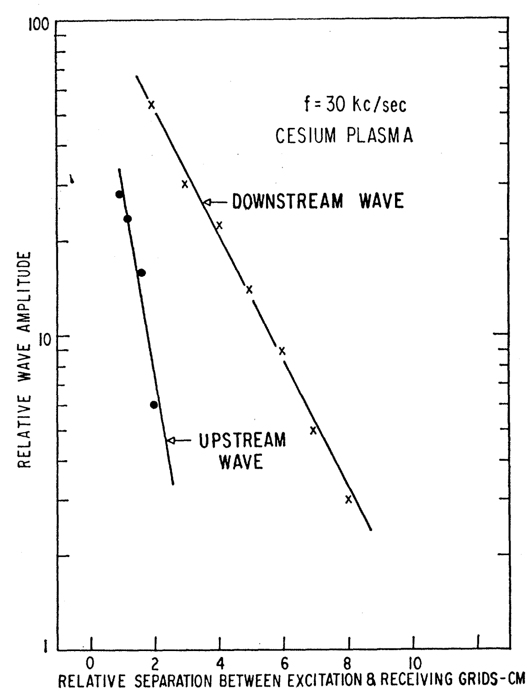

FIG. 7. Ion-wave damping in a cesium plasma.

[12] V. O. Jensen, Risö Report No. 54, Risö, Roskilde, Denmark, 1962 (unpublished). We are also grateful to R. M. Kulsrud, J. M. Dawson, and M. G. Kivelson for pointing out this treatment.

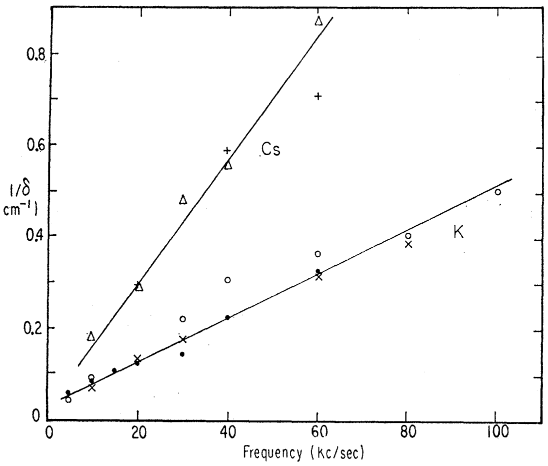

FIG. 8. Reciprocal damping distance, $1/\delta$, as a function of frequency for the downstream waves. The symbols have the same significance as those in Fig. 5.

We shall linearize (1) and (2) by letting

$$n_e = n_{e0} + n_{e1} \quad \text{and} \quad f(x,v_x,t) = f_0(v_x) + f_1(x,v_x,t).$$

Neglecting the inertial term of the electrons ($m_e/m_i \ll 1$) and assuming no externally applied electric field and isothermal electrons, the linearized equations of (1) and (2) become

$$n_{e0}eE_1 \cong -KT_e(\partial/\partial x)n_{e1}, \tag{3}$$

$$\frac{\partial f_1}{\partial t} + v_x\frac{\partial f_1}{\partial x} + \frac{eE_1}{m_i}\frac{\partial f_0}{\partial v_x} = 0. \tag{4}$$

The self-consistent field E_1 is given by the Poisson's equation

$$\frac{\partial}{\partial x}E_1 = 4\pi e\left[\int f_1 dv_x - n_{e1}\right]. \tag{5}$$

Upon Fourier analyzing in time and Laplace transforming in space each of the first-order quantities n_{e1}, E_1 and f_1 we have

$$n_{e1}(k,\omega) = \frac{n_{e0}e}{(KT)(ik)}E_1(k,\omega) + i\frac{q(\omega)}{k} \tag{6}$$

and

$$f_1(k,\omega) = \frac{1}{i(kv_x-\omega)}\left[\frac{e}{m_i}\frac{\partial f_0}{\partial v_x}E_1(k,\omega) - vg(\omega)\right], \tag{7}$$

where

$$q(\omega) = \int_{-\infty}^{\infty} n_{e1}(0,t)e^{-i\omega t}dt \quad \text{and} \quad g(\omega) = \int_{-\infty}^{\infty} f_1(0,t)e^{-i\omega t}dt.$$

Now $n_{e1}(0,t)$ and $f_1(0,t)$ are the boundary values at $x=0$. Equations (5), (6), and (7) together yield the dispersion relation:

$$k^2 = \frac{4\pi e^2}{m_i}\int_{-\infty}^{\infty}\frac{dv_x k(\partial f_0/\partial v_x)}{(kv_x-\omega)} - k_D^2, \tag{8}$$

where $k_D^2 = 4\pi n_{e0}e^2/KT_e$. We assume for the ions a Maxwellian distribution with a drift velocity V_0:

$$f_0^{+}(v_x) = \frac{n_i}{A\pi^{1/2}}\exp\left[-\frac{(v_x \mp V_0)^2}{A^2}\right],$$

where $A^2 = 2KT_i/m_i$. The drift term in the distribution function takes account of the drift along (−) or against (+) the wave velocity. With these assumptions the integral term in (8) can be evaluated in terms of the "plasma dispersion function[3]"

$$Z(x) = 2i\exp(-x^2)\int_{-\infty}^{ix}\exp(-t^2)dt.$$

The dispersion relation (8) then reduces to

$$\frac{k^2}{k_D^2} - \frac{1}{2}\frac{T_e}{T_i}Z'\left[\frac{1}{A}\left(\frac{\omega}{k} \mp V_0\right)\right] + 1 = 0, \tag{9}$$

where $Z'[x]$ is the derivative of Z with respect to x. The dispersion relation can be solved with the aid of tables of the real and imaginary parts of Z'.[13] For wavelengths long compared to the Debye distance, $k^2/k_D^2 \ll 1$, for equal ion and electron temperatures, and for drift velocities negligible compared with the electron thermal velocity, we obtain an approximate solution corresponding to the least-damped ion oscillation mode

$$\omega/k \mp V_0 = A(1.45 - 0.6i).$$

Assuming the wave number k is complex, the phase velocity ω/k_r is

$$\frac{\omega}{k_r} = \left(1.45 \pm \frac{V_0}{A} + \frac{0.36}{1.45 \pm V_0/A}\right)A. \tag{10}$$

The second term on the right results from the Doppler

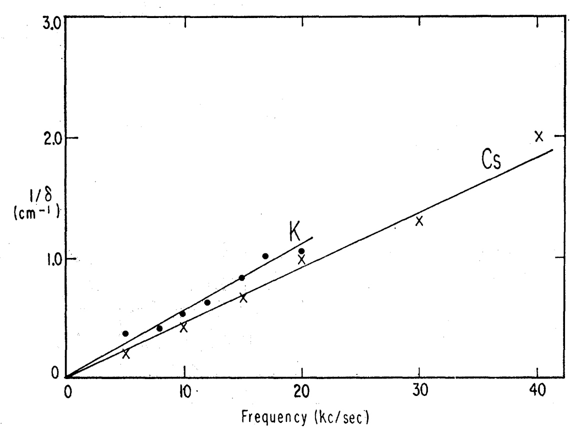

FIG. 9. Reciprocal damping distance $1/\delta$ as a function of frequency for the upstream waves. The density of the plasma is 2×10^{11} cm^{-3}.

[13] B. D. Fried and S. D. Conte, *The Plasma Dispersion Function* (Academic Press Inc., New York and London, 1961).

TABLE I. Comparison between theoretical and experimental phase velocities for ion-wave propagation in cesium and potassium plasmas.

	Ion-wave phase velocities			
	Cesium		Potassium	
	Theory cm/sec	Experiment cm/sec	Theory cm/sec	Experiment cm/sec
Downstream wave	1.17×10^5	1.3×10^5	2.6×10^5	2.5×10^5
Upstream wave	0.76×10^5	0.9×10^5	1.25×10^5	1.3×10^5
Ion drift velocity	···	0.25×10^5	···	0.9×10^5

shift caused by the ion flow and the third term arises from the strong damping. The damping itself is given by

$$\frac{\delta}{\lambda}=\frac{1}{2\pi}\frac{k_r}{k_i}=0.39\pm0.27\frac{V_0}{A}. \qquad (11)$$

Note that the damping constant δ/λ depends on whether the wave velocity is along (+) or against (−) the ion drift.

V. DISCUSSION OF RESULTS

1. Phase Velocities

The theoretical predictions, that there should be no dispersion, and that the phase velocity should be independent of ion density are borne out by experiment. To compare the absolute magnitudes of the phase velocities we take the plasma temperature to be 2500°K, the plate temperature, and $A=5.6\times10^4$ cm/sec in cesium and $A=1.03\times10^5$ cm/sec in potassium. From the difference in the velocities of waves propagating upstream and downstream we obtain the drift velocities, and then compute from Eq. (10) the phase velocities tabulated in Table I. It should be noted that the drift velocity is not merely half the sum of the velocities upstream and downstream. The full expression of Eq. (10) must be included in the calculation. In cesium the predicted phase velocities are ~12% lower than the experimental values. In potassium the theoretical and experimental phase velocities agree within 5%. The purely statistical errors should be about 5% except for the phase velocity of the upstream waves in potassium, which is subject to 12% error. The agreement between theory and experiment in potassium plasma is within the statistical uncertainty. The small discrepancy (~7% outside statistical error) in cesium is probably caused by a systematic error, which most likely arises in the measurements on the upstream wave. Because of the strong damping of the upstream waves, we were able to measure the wavelength over only ~0.2 wavelength. The attenuation measurements given in the next section indeed suggest that the drift velocity in cesium has been underestimated by about 25%.

2. Wave Damping

The collisionless theory predicts that the ion acoustic waves in plasmas of equal ion and electron temperatures should decay exponentially, that the damping rate for small amplitude waves should not depend on the wave amplitude, that the damping distance should be a constant fraction of a wavelength, and that the damping rate should be independent of the ion density. Each of these predictions is in accord with experiment.

The predicted damping constants δ/λ, which depend on the magnitude of the ion drift, are given in Table II. There is excellent agreement between theory and experiment for wave propagation along and against the ion drift. The 10% difference between theory and experiment in cesium probably originates from too low an estimate of the ion drift velocity, as discussed in the preceding section.

In order to compare theory and experiment without the added complication of the ion drift, we can compute

TABLE II. Comparison between theory and experiment for ion-wave damping in cesium and potassium plasmas. The parameter listed is δ/λ, which is the damping distance normalized to a wavelength.

	Ion-wave damping—δ/λ			
	Cesium		Potassium	
	Theory	Experiment	Theory	Experiment
Downstream wave	0.51	0.55	0.64	0.65
Upstream wave	0.27	0.25	0.14	0.14

the average damping constant δ/λ of both waves. From Eqs. (11)

$$\langle\delta/\lambda\rangle_{\text{av}}=\tfrac{1}{2}[\delta/\lambda\ (\text{upstream})+\delta/\lambda\ (\text{downstream})]=0.39.$$

The experimental value in both cesium and potassium plasmas (from Table II) is 0.40, in excellent agreement with theory.

Let us consider the following collisional processes which might affect the ion-wave damping: (1) ion-electron collisions, (2) ion-atom collisions, and (3) ion-ion collisions. Of these the last appears to be the most important.

Ion-electron collisions are relatively ineffective in damping ion waves, because of the large ratio between ion and electron masses, and because the electrons and ions tend to oscillate in phase. On the basis of the plasma fluid equations,[14] in which one takes account of electron-ion collisions by means of a resistive term in the momentum equations, one can show that the resistive damping gives

$$\delta/\lambda=4\omega_{pi}^2/\eta\omega^3, \quad \text{(Resistivity)}$$

[14] L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1962), 2nd ed., p. 23.

where ω_{pi} is the ion plasma frequency and η the resistivity in seconds. For our experimental conditions we calculate $\delta/\lambda \sim 10^{11}$ for a frequency of 100 kc/sec, a completely negligible damping.

Ion waves can also be attenuated by the disruption of the organized motion by ion-atom collisions. As done by Hatta and Sato,[8] one can compute this rate by putting a collisional term in the momentum equations. The result for small damping, $\nu_a^2 \ll 4\omega^2$, is

$$\frac{\delta}{\lambda} \cong \frac{\sqrt{2}}{\pi} \frac{\omega}{\nu_a}, \quad \text{(ion-atom collisions)}$$

where ν_a is the ion-atom collision frequency. There are two types of collisions to be considered: charge-exchange collisions and collisions of ions with impurity atoms. For the latter with a background pressure of 2×10^{-6} mm Hg and a collision cross section of $\sim 10^{-14}$ cm^2,[15] one expects $\delta/\lambda = 1000$. For the former we take the partial pressure of cesium vapor, determined by the wall temperature (0°C) as 10^{-7} mm Hg. For a collision cross section of 5×10^{-14} cm^2,[16] we find for collisions between cesium atoms and ions, $\delta/\lambda = 1000$. Therefore, neither type of atomic collision should be important in attenuating the ion waves.

The effect of ion-ion collisions (viscosity) should be to reduce the collisionless damping by disrupting the phasing between the resonant ions and the wave. To our knowledge no general theory which takes account of the actual velocity distribution of the ions exists. However, it is possible to include ion-ion collisions in the fluid approximation by inserting a viscous term in the momentum equations. For a neutral gas this was done by Stokes, who neglected the effect of heat conduction which is of the same order of importance as viscosity. As shown by Lamb,[17] Stokes' treatment gives for small damping

$$\delta/\lambda = \beta(\nu_i/\omega),$$

where ν_i is the (like particle) collision frequency and β a numerical coefficient, which for our case is a fraction of unity. A recent application of this analysis to plasma waves by Kuckes[18] gives $\beta = 0.43$, where ν_i is taken to be the ion-ion collision rate defined by Marshall.[19] Kuckes assumes that the electrons are isothermal and takes into account both ion viscosity and heat conduction. The equation above is valid only for large collision frequencies, $\nu_i/\omega > 1$, and therefore does not directly apply to our results. In cesium we have varied the parameter ν_i/ω over a factor of 30 from 0.027 to 0.83 without observing any systematic effect on the damping rate. Therefore, we believe that the collisionless equations are valid for the range of parameters explored in this work.

This is, to our knowledge, the first experiment in which a quantitative measurement of the Landau damping of waves has been reported. The results of several previous experiments with electron beams have been interpreted as qualitative evidence for collisionless damping of longitudinal *electron* oscillations. Kofoid[20] has found that only plasma oscillations of wavelength greater than about one Debye distance can be excited in a plasma by electron beams. Caulton, Hershenov, and Pasche[21] have observed the spatial damping of plasma oscillations in electron beams and have considered this attentuation as qualitative evidence of Landau damping.

ACKNOWLEDGMENTS

We thank the members of the Engineering and Development Division of the Princeton Plasma Physics Laboratory for the design and fabrication of the Q-3 plasma source, and C. Harrison, J. Frangipani, and K. Schuler for assistance in the experiments. One of us (A. Y. W.) would like to thank Professor D. Pines and Professor T. H. Stix for initial discussions and criticism.

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