

## Transport Coefficients and Cross Sections in Argon and Hydrogen-Argon Mixtures\*

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Using a method described previously the distribution function of electron energies for electrons in argon and mixtures of hydrogen and argon has been obtained by a numerical solution of the Boltzmann equation which takes into account elastic and inelastic collisions. By taking appropriate averages over the distribution function, the electron drift velocity  $w$ , magnetic drift velocity  $w_M$ , and characteristic energy  $\epsilon_K$  are computed. A comparison of calculated and experimental values of these transport coefficients enables us, in the case of pure argon, to extend the previous work of Frost and Phelps and derive the momentum transfer cross section from 0.7 to 25.0 eV. The momentum transfer and inelastic collision cross sections derived in this paper for argon and previously for hydrogen give rise to transport coefficients in mixtures of these two gases which are consistent with most of the available experimental data.

### I. INTRODUCTION

PREVIOUS determinations<sup>1-4</sup> of momentum transfer and inelastic collision cross sections for electrons in inert and molecular gases provide motivation for calculations which verify the validity of these cross sections. The method which is described in this paper provides such a verification in terms of a comparison of experimental and theoretical values of transport coefficients for binary mixtures of hydrogen and argon. Our choice of argon as the inert component and hydrogen as the molecular one was suggested by the observation by a number of investigators<sup>5-7</sup> of a large change in electron drift velocity produced by small concentrations of H<sub>2</sub> in Ar, and by the availability of previous determinations<sup>8</sup> of the momentum transfer cross sections in Ar and of elastic and inelastic cross sections in hydrogen.<sup>1,2</sup> In order to be able to perform calculations for regions of higher electron energy, the determination by Frost and Phelps<sup>3</sup> of the momentum transfer cross section  $Q_m$  in argon for electron energies less than 0.7 eV was extended to an energy of approximately 25 eV. In these calculations we have used the argon inelastic collision cross section determined experimentally by Maier-Leibnitz.<sup>8</sup>

The procedure adopted is essentially the same as that discussed by Frost and Phelps<sup>1</sup> (hereafter called I) and Engelhardt and Phelps<sup>2</sup> (hereafter called II); therefore, we shall give only a brief outline here. The Boltzmann equation [i.e., Eq. (II.2)]<sup>9</sup> is solved numerically for the distribution function  $f$  of electron energies taking into account both momentum transfer and inelastic collisions.

In I and II, cross sections were obtained by comparing experimental and theoretical values of various combinations of transport coefficients—the diffusion coefficient  $D$ , the mobility  $\mu$ , and the Townsend primary ionization coefficient  $\alpha_i$ ; all of these transport coefficients were obtained by taking the appropriate average over the distribution function [see Eqs. (II.6) to (II.12) inclusive]. We have used this technique here with slight modifications in deducing cross sections for pure argon. In the case of mixtures of hydrogen and argon, the previously derived cross sections were checked by comparing experimental and calculated values of the drift velocity  $w = \mu E$  and, in one case, the characteristic energy<sup>2</sup>  $\epsilon_K = eD/\mu$ . In this paper we also compare our calculated results with measurements<sup>7,10</sup> of the “magnetic” drift velocity  $w_M$  for the configuration of crossed electric and small magnetic fields. Following Townsend<sup>10</sup> we define  $w_M$  by

$$w_M = \frac{E}{B} \frac{w_1}{w_T} . \quad (1)$$

Here  $w_1$  is the drift velocity perpendicular to both electric and magnetic fields, and  $w_T$  is the drift velocity transverse to the magnetic field but parallel to the electric field; the ratio  $w_1/w_T$  is the tangent of the angle through which the electrons are deflected.<sup>10</sup> For an energy-independent frequency of momentum transfer collisions, viz.,  $\nu_c = (2\epsilon/m)^{1/2} Q_m = \text{const.}$ , where  $\epsilon$  is the electron energy and  $m$  its mass, it can be shown that  $w_M = w$ . We shall see that in the case of pure argon the condition  $\nu_c = \text{const.}$  is not fulfilled, and that the deviations from this relation are as large as a factor of 4.

### II. MOMENTUM TRANSFER AND INELASTIC CROSS SECTIONS FOR ARGON

In this section we consider the determination of the argon momentum transfer cross section in the energy range from 0.7 to approximately 25 eV; the cross section for energies less than 0.7 eV was obtained from Ref 3.

<sup>10</sup> J. S. E. Townsend, *Electrons in Gases* (Hutchinson's Scientific and Technical Publications, New York, 1947), p. 34.

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<sup>1</sup> L. S. Frost and A. V. Phelps, *Phys. Rev.* **127**, 1621 (1962).

<sup>2</sup> A. G. Engelhardt and A. V. Phelps, *Phys. Rev.* **131**, 2115 (1963). The notation used in this paper is precisely the same as in this reference.

<sup>3</sup> L. S. Frost and A. V. Phelps (private communication, 1962).

<sup>4</sup> C. Ramsauer and R. Kollath, *Ann. Phys. (Leipzig)* **12**, 529 (1932).

<sup>5</sup> J. L. Pack (private communication).

<sup>6</sup> D. Errett, Ph.D. thesis, Purdue University, 1951 (unpublished).

<sup>7</sup> J. S. E. Townsend and V. A. Bailey, *Phil. Mag.* **44**, 1033 (1922).

<sup>8</sup> H. Maier-Leibnitz, *Z. Physik* **95**, 499 (1935).

<sup>9</sup> Eq. (II.2) refers to Eq. (2) of II.

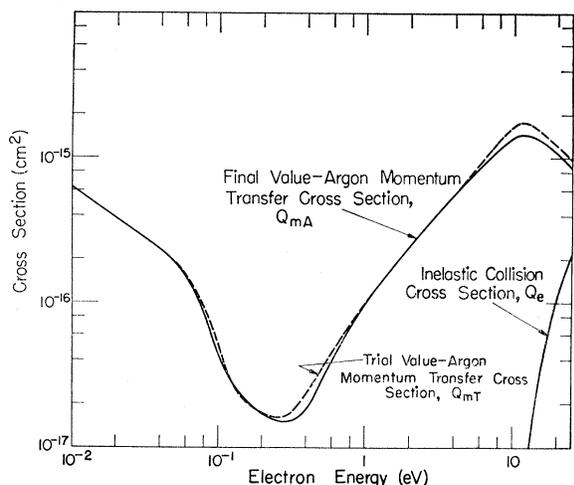


FIG. 1. Momentum transfer and inelastic collision cross sections for electrons in argon as a function of electron energy in eV.  $Q_{mA}$  (solid line) is our final value of the momentum transfer cross section and  $Q_{mT}$  (dashed line) is a trial value.

This cross section is shown in Fig. 1. The dashed momentum transfer cross section,  $Q_{mT}$ , represents the original result obtained by Frost and Phelps<sup>11</sup> by adjusting  $Q_m$  to fit only the experimental drift velocity data.<sup>12</sup> Above an energy of 15 eV this cross section is identical to that reported previously by Barbieri.<sup>13</sup> The solid curve is our final momentum cross section,  $Q_{mA}$ , for argon. The portion of the solid curve below 0.7 eV is the result of later calculations by Frost and Phelps<sup>3</sup> in which the momentum transfer cross section was modified so as to give a better fit to the  $\epsilon_K$  data of Warren and Parker<sup>14</sup> at 77°K. The final  $Q_{mA}$  curve has been compared by Frost and Phelps with the results of electron beam experiments and recent theory, and consequently the discussion will not be repeated here. Here we will present results for both  $Q_{mT}$  and  $Q_{mA}$  in order to show the effect of varying  $Q_m$ . Also shown in Fig. 1 is the total effective inelastic cross section<sup>8</sup> with a threshold at 11.5 eV. This inelastic cross section includes processes whose energy loss is greater than 11.5 eV, e.g., ionization. However, our calculations show that the values of  $w$ ,  $w_M$ , and  $\epsilon_K$  do not change significantly when instead of the single inelastic process shown we use two cross sections having the same total effective cross section, i.e., one for excitation and one for ionization.

Our procedure has been to compare experimental and theoretical values of  $w$  and  $\epsilon_K$  and to make adjustments in  $Q_m$  on this basis. In argon for  $\epsilon_K$  less than approximately 7.0 eV, few electrons have energies great enough to experience inelastic collisions. In this case, one has

<sup>11</sup> L. S. Frost and A. V. Phelps, *Bull. Am. Phys. Soc.* **5**, 371 (1960).

<sup>12</sup> J. L. Pack and A. V. Phelps, *Phys. Rev.* **121**, 798 (1961).

<sup>13</sup> D. Barbieri, *Phys. Rev.* **84**, 653 (1951).

<sup>14</sup> R. W. Warren and J. H. Parker, Jr., *Phys. Rev.* **128**, 2661 (1962).

only one adjustable parameter,  $Q_m(\epsilon)$ , and in principle one can obtain  $Q_m$  by fitting to a plot of either  $w$  or  $\epsilon_K$  versus  $E/N$ . However, as found by Frost and Phelps,<sup>3</sup> values of  $w$  are rather insensitive to the choice of  $Q_m$  when  $w$  varies as slowly with  $E/N$  as in argon for  $10^{-19} < E/N < 2 \times 10^{-17}$  V-cm<sup>2</sup>. Fortunately at the lower values of  $E/N$ , one can make use of  $\epsilon_K$  data to obtain more accurate  $Q_m$  values. Frost and Phelps<sup>3</sup> were unable to obtain a unique  $Q_m$  for energies greater than 0.7 eV because of the dominating influence of the Ramsauer minimum. As is indicated below, we believe one can overcome this difficulty by using mixtures of 10% H<sub>2</sub> and 90% Ar. For  $\epsilon_K < 7.0$  eV we have assumed the inelastic collision cross section of Maier-Leibnitz to be correct and have, therefore, made appropriate adjustments only in  $Q_m$ . We note that because of the decreasing  $\epsilon_K$  at high  $E/N$  (see Figs. 2 and 5) plots of  $v_m/N$  and  $v_u/N$  such as used in Refs. 1 and 2 are not single valued and are of little use in our analysis of pure Ar at high  $\epsilon_K$ .

Shown in Fig. 2 is a plot of  $w$ ,  $w_M$  and  $\epsilon_K$  for 77°K. The smooth solid curves represent our calculations using  $Q_{mA}$ , and the smooth dashed curves represent our calculations using  $Q_{mT}$ ; the points<sup>14a</sup> represent the various experimental results of Warren and Parker,<sup>14</sup> Townsend and Bailey,<sup>7,10</sup> Pack and Phelps,<sup>12</sup> Erret,<sup>6</sup> Bowe,<sup>15</sup> Nielsen,<sup>16</sup> Riemann,<sup>17</sup> Herreng,<sup>18</sup> and Caren.<sup>19</sup> The theoretical results of Barbieri<sup>13</sup> and Heylen and Lewis<sup>20</sup> are also shown as smooth curves. The  $\epsilon_K$  data of Warren and Parker from 0.06 to 1.2 eV were taken at liquid-argon temperature, 87°K. Calculations<sup>3</sup> show that from  $\epsilon_K = 0.06$  to 0.2 eV these experimental points should be shifted downward about 10% in order to compare with the 77°K calculations. In the case of the  $w$  data both  $Q_{mA}$  and  $Q_{mT}$  give reasonably good agreement with experiment up to an  $E/N$  of  $10^{-16}$  V-cm<sup>2</sup>. Above  $E/N = 10^{-16}$  V-cm<sup>2</sup> the curve showing our final values of  $w$  falls midway between the results of Erret and those of Riemann, Herreng, and Caren; in this region the  $w$  values found using  $Q_{mT}$  favor Erret's results.

That the final result  $Q_{mA}$  represents an improvement over  $Q_{mT}$  can be concluded from the  $\epsilon_K$  plot where it is seen that  $Q_{mA}$  yields very acceptable agreement with the data of Warren and Parker. On the other hand by using

<sup>14a</sup> [Note added in proof. As the result of an oversight the authors failed to include the experimental measurements of  $w$  by G. Jäger and W. Otto [*Z. Physik* **169**, 517 (1962)] in the region where  $E/N > 1.5 \times 10^{-16}$  V-cm<sup>2</sup>. Their values of  $w$  are somewhat less than ours but with one exception are within 5% of our final curve. Very recent drift velocity data of J. Bambring and H. Raether (private communication, 1963) is slightly lower than that of Jäger and Otto.]

<sup>15</sup> J. C. Bowe, *Phys. Rev.* **117**, 1411 and 1416 (1960).

<sup>16</sup> R. A. Nielsen, *Phys. Rev.* **50**, 950 (1936).

<sup>17</sup> H. Riemann, *Ergeb. Exakt. Naturw.* **22**, 73 (1949).

<sup>18</sup> P. Herreng, *Compt. Rend.* **217**, 75 (1943).

<sup>19</sup> R. P. Caren, Ph.D. thesis, Ohio State University, 1961 (unpublished).

<sup>20</sup> A. E. D. Heylen and T. J. Lewis, *Proc. Roy. Soc. (London)* **A271**, 531 (1963). In Fig. 6 of this reference, curves (b) and (c) quoting the results of Golant (see Ref. 21) are improperly normalized, and should be multiplied by 10.0/11.5 in order that  $\int_0^\infty \epsilon^{1/2} f(\epsilon) d\epsilon = 1$ .

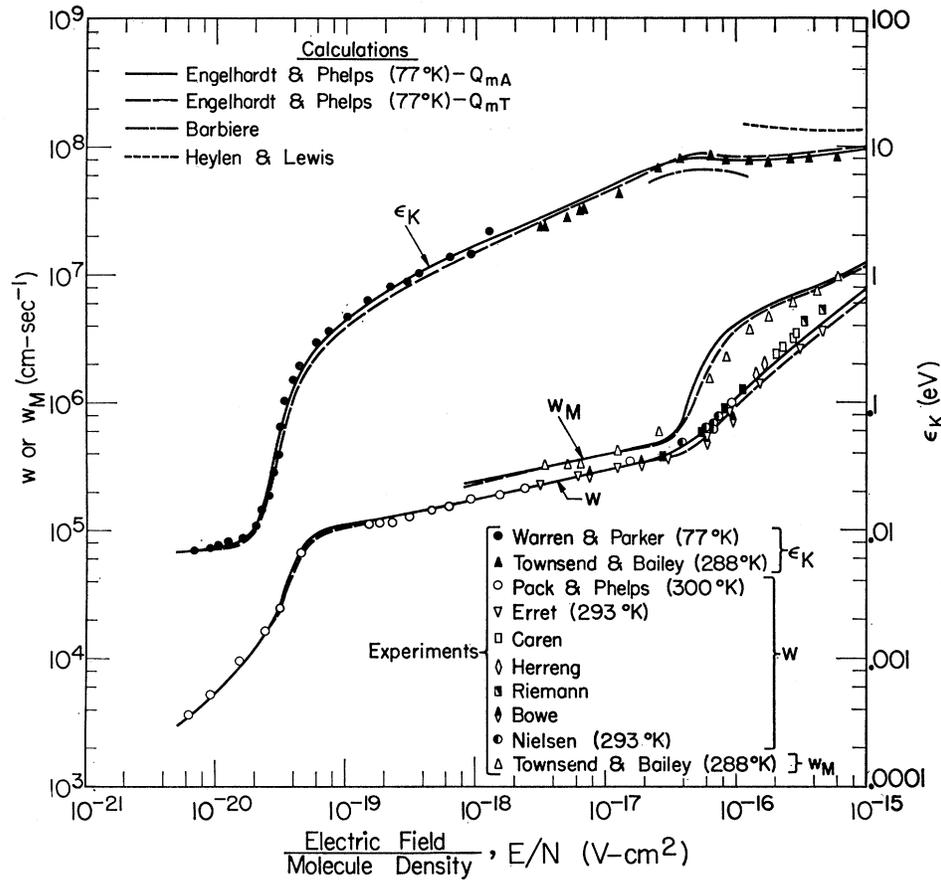


FIG. 2. Drift velocity  $w$ , magnetic drift velocity  $w_M$ , and characteristic energy  $\epsilon_K$  for electrons in argon as a function of  $E/N$ . Below  $\epsilon_K=0.5$  eV the experimental data shown are for gas temperatures  $T_g$  of 77° and 87°K as discussed in the text. Above  $\epsilon_K=0.5$  eV and for  $T_g \leq 300^\circ\text{K}$  the values of  $w$ ,  $w_M$ , and  $\epsilon_K$  are insensitive to  $T_g$ . The solid and dashed curves represent our calculated results using  $Q_{mA}$  and  $Q_{mT}$ , respectively; the points represent the various experimental results.

$Q_{mT}$ , discrepancies as large as 20% are obtained. We have been unable to obtain agreement with the  $\epsilon_K$  values of Townsend and Bailey for  $E/N$  near  $10^{-17}$  V-cm<sup>2</sup> using reasonable  $Q_m$  curves, and so have chosen not to force  $Q_m$  to fit these data. A similar unresolved discrepancy has been found<sup>2</sup> to exist over a portion of the  $\epsilon_K$  data in H<sub>2</sub>. In the case of  $w_M$ , both  $Q_{mA}$  and  $Q_{mT}$  give comparable agreement with the results of Townsend and Bailey. Both cross sections yield  $w_M$  values which change rapidly with  $E/N$  in the vicinity of  $E/N=10^{-16}$  V-cm<sup>2</sup> where inelastic collisions result in an increased relative electron density in the Ramsauer minimum. The large difference between  $w$  and  $w_M$  in this region is an example of the deviation of the "magnetic deflection" factor from unity<sup>1</sup> which is possible when  $\epsilon^{1/2}Q_m(\epsilon)$  varies rapidly with electron energy.

Finally we note that in the highest regime of  $E/N$  studied, the  $\epsilon_K$  values of Heylen and Lewis<sup>20</sup> are approximately 50% greater than ours. In performing their calculations these authors have used a piecewise analytical approximation to the momentum transfer cross section which in principle should not give rise to a distribution function significantly different from our own for  $E/N > 10^{-17}$  V-cm<sup>2</sup>. Essentially the same inelastic cross sections<sup>8</sup> are used in both analysis. However, a perusal of Fig. 3 indicates that there are rather marked differences

between the distribution function of Heylen and Lewis and our own. Since their distribution function has more particles at higher energies, it is to be expected that their

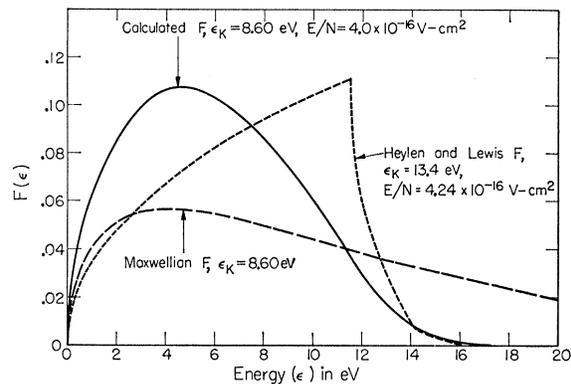


FIG. 3. Energy probability function  $F(\epsilon)$  for electrons in argon as a function of energy  $\epsilon$  in eV. Since  $F(\epsilon) = \epsilon^{1/2}f(\epsilon)$ ,  $F(\epsilon)d\epsilon$  is the probability of an electron having an energy between  $\epsilon$  and  $\epsilon+d\epsilon$ . The results of our calculations are shown as a solid line for  $E/N = 4.0 \times 10^{-16}$  V-cm<sup>2</sup> and  $\epsilon_K = 9.60$  eV. The  $F(\epsilon)$  of Heylen and Lewis is plotted as a dotted line for  $E/N = 4.24 \times 10^{-16}$  V-cm<sup>2</sup> and  $\epsilon_K = 13.4$  eV. The difference of approximately 5% in the two values of  $E/N$  considered by itself leads to only very minor changes in  $F(\epsilon)$ . For illustrative purposes we display for  $\epsilon_K = 9.60$  eV the Maxwellian  $F(\epsilon)$  given by  $F(\epsilon) = 2(\epsilon/\pi\epsilon_K^3)^{1/2} \exp(-\epsilon/\epsilon_K)$ , where  $\epsilon$  and  $\epsilon_K$  are in eV.

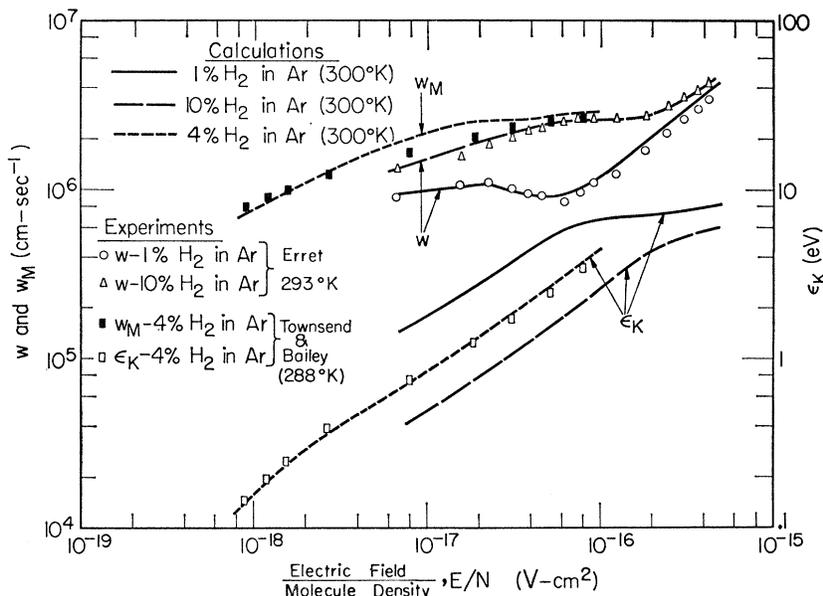


FIG. 4. Drift velocity  $w$ , magnetic drift velocity  $w_M$ , and characteristic energy  $\epsilon_K$  as a function of  $E/N$  for electrons in mixtures of 1, 4, and 10%  $H_2$  in Ar. The solid, dashed, and dotted curves represent our calculated results for 1, 4, and 10%  $H_2$  in Ar, respectively. The various experimental data are shown as points.

value of  $\epsilon_K$  will be substantially greater than our own. Moreover, we have compared our distribution function obtained with  $E/N = 3.0 \times 10^{-16}$  V-cm<sup>2</sup> with that of Golant<sup>21</sup> for  $E/N = 2.94 \times 10^{-16}$  V-cm<sup>2</sup> to discover that the two are almost identical and, incidentally, not very different from our calculated  $F(\epsilon)$  shown in Fig. 3. The agreement with Golant's result should not be too surprising since the elastic and inelastic cross sections used by Golant are quite similar to our own. Consequently we conclude that the method used by Heylen and Lewis to calculate the electron energy distribution function can lead to large errors in the transport coefficients as apparently is the case for Ar.

### III. MIXTURES OF HYDROGEN AND ARGON

#### A. Comparison with Experiment

The calculation of transport coefficients for mixtures of hydrogen and argon stemmed from a desire to check the validity of previously derived momentum transfer and inelastic cross sections for both  $H_2$  and Ar. Since argon has no inelastic processes below 11.5 eV, it is possible for sufficiently low values of  $\epsilon_K$  to scrutinize the inelastic processes of rotation and vibrational excitation in  $H_2$ . The enhanced sensitivity of the transport coefficients to inelastic processes with thresholds between about 0.5 and 10 eV is due to the unusual shape of  $Q_{mA}$  which rises rapidly for  $\epsilon > 0.3$  eV. Consequently the  $f(\epsilon)$  in Ar drops rapidly for  $\epsilon > \sim \epsilon_K$  and leads to a narrower energy spread than in most other gases. Such a behavior is shown in Fig. 3. As a result the effects on transport coefficients of various inelastic processes are more clearly separated. Furthermore, since the onset of vibra-

tional excitation occurs at 0.516 eV, small percentages of  $H_2$  in Ar will result in an even larger relative number of electrons trapped in the vicinity of the Ramsauer minimum for appropriate values of  $E/N$ . In this manner we are able to investigate the validity of the rising part of the vibrational excitation cross section.

Figures 4 and 5 exhibit a comparison of experimental and theoretical values of  $w$ ,  $w_M$ , and  $\epsilon_K$  for mixtures of

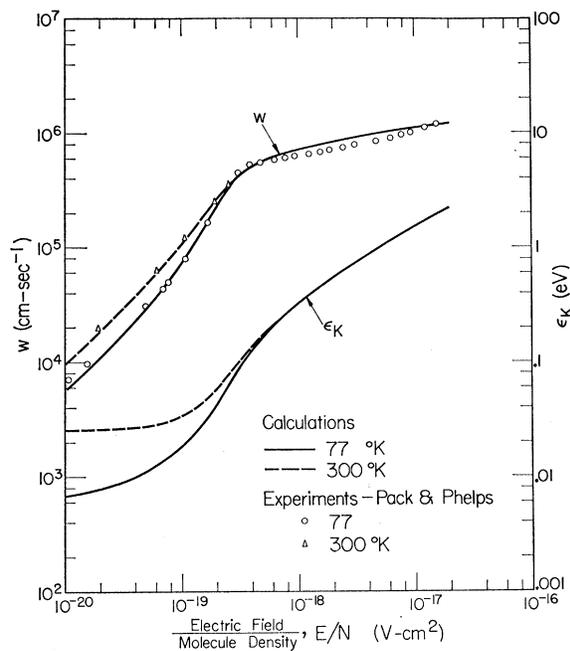


FIG. 5. Drift velocity  $w$  and characteristic energy  $\epsilon_K$  for electrons in a mixture of 1.5%  $H_2$  in Ar as a function  $E/N$ . The solid lines represent our calculations for  $T_0 = 77^\circ K$  and the dashed ones for  $T_0 = 300^\circ K$ . The experimental results of Pack (Ref. 5) are shown as points.

<sup>21</sup> V. E. Golant, Zh. Tekh. Fiz. 27, 756 (1957); 29, 756 (1959) [translation: Soviet Physics—Tech. Phys. 2, 684 (1957); 4, 680 (1959)].

1, 1.5, 4, and 10% H<sub>2</sub> in Ar. The H<sub>2</sub> momentum transfer and inelastic cross sections are the same as those shown in Figs. 1 and 7 of II. The argon momentum transfer and inelastic cross sections used are those shown in Fig. 1. As indicated above, the higher energy portion of  $Q_{mA}$  was obtained by adjusting  $Q_{mA}$  to give a fit between the calculated and measured values of  $w$  in a mixture of 10% H<sub>2</sub> and 90% Ar. The drift velocity in this mixture is sensitive to the value of  $Q_{mA}$  at the higher energies because for this case in the vicinity of the Ramsauer minimum  $0.1 Q_{mH} > 0.9 Q_{mA}$ , where  $Q_{mH}$  is the momentum transfer cross section of H<sub>2</sub>. As a result, considerably fewer electrons are trapped in the Ramsauer minimum and it is easier to investigate the variations of  $Q_{mA}$  for energies greater than about 1 eV than for the case of pure Ar alone. The comparison of experimental and calculated values of  $w$  for 10% H<sub>2</sub> in Ar shown in Fig. 4 indicates that we have been able to obtain satisfactory agreement with the experimental data of Erret. It should be of value to repeat this evaluation of  $Q_{mA}$  using He-Ar mixtures, since the complications of inelastic collisions would not be present.

The comparison in Figs. 4 and 5 of the calculated and experimental values of  $w$ ,  $w_M$ , and  $\epsilon_K$  for various H<sub>2</sub>-Ar mixtures shows generally good agreement and indicates that our assumed cross sections are reasonably consistent with experiments. However, there are several regions of some discrepancy which need to be considered. For example, we note in Fig. 4 that for 1% H<sub>2</sub> in Ar there is a discrepancy of as much as 15% with the values of  $w$  measured by Erret for  $E/N$  values above about  $6 \times 10^{-17}$  V-cm<sup>2</sup>. A comparison of the  $w$  and  $\epsilon_K$  curves for this mixture and for pure Ar shows that this is the same discrepancy in  $w$  values that is present in Erret's data for pure Ar for  $E/N$  above  $3 \times 10^{-17}$  V-cm<sup>2</sup>. However, in pure Ar the discrepancy with respect to other recent data is much smaller or in the opposite direction. This suggests the possibility of error in Erret's data at high  $E/N$ . Since this error does not appear to be present in Erret's pure H<sub>2</sub> or pure N<sub>2</sub> data, we have assumed that the data for 10% H<sub>2</sub>-90% Ar used in the evaluation of  $Q_{mA}$  is correct.

A second region in which we have consistently been unable to obtain as good agreement between calculated and experimental drift velocities as we might expect occurs in pure Ar and in H<sub>2</sub>-Ar mixtures for  $E/N$  such that  $\epsilon_K \sim 0.8$  to 1.0 eV. The  $w$  data of Figs. 4 and 5 show that the maximum discrepancy in this range of  $\epsilon_K$  varies from about 10% in pure Ar and in 10% H<sub>2</sub> in Ar, to about 20% in 1.5% H<sub>2</sub> in Ar. This same discrepancy<sup>22</sup> is seen in the  $w_M$  data of Fig. 4. The presence of this discrepancy in the pure Ar data for  $\epsilon_K$  near 1 eV suggests that we have not found the optimum  $Q_{mA}$  curve for energies below about 1 eV. We have not pursued this

question further because of the relative smallness of the discrepancy, the lack of recent  $\epsilon_K$  values in mixtures, and the repeated inconsistencies between what we consider to be reasonable cross sections and the data of Townsend and Bailey. This state of affairs emphasizes the desirability of accurate measurements of  $w$  or  $w_M$ , and  $\epsilon_K$  for various H<sub>2</sub>-Ar mixtures.

In the case of 4% H<sub>2</sub> in Ar, we note that according to Fig. 4 the cross sections used in our analysis lead to very good agreement between the calculated and experimental values of  $\epsilon_K$ . This agreement is to be compared with the existence of discrepancies of as much as 20% between the calculated and experimental values of  $w_M$ . This effect could be due to the fact that  $w_M$  is more sensitive to variations in  $Q_m$  than is  $\epsilon_K$ , i.e.,  $w$  depends on integrals of  $1/Q_m^2$ , where  $D$  depends on integrals of  $1/Q_m$ .

It is possible to regard the results for various concentrations as a measure of the sensitivity of  $w$ ,  $w_M$ , and  $\epsilon_K$  to changes in the inelastic cross sections for fixed concentration. For example, we observe that for  $E/N = 10^{-17}$  V-cm<sup>2</sup> a decrease of 33% in the effective vibrational cross section caused by changing the mixture composition from 1.5% H<sub>2</sub> in Ar to 1% H<sub>2</sub> in Ar results in a 20% increase in  $\epsilon_K$  and a 13% decrease in  $w$ .

### B. Advantages of Mixture Technique

Here we would like to make use of the results of the preceding calculations to illustrate graphically the manner in which argon serves to improve the separation in energy of various inelastic collision processes. For this purpose it is convenient to employ a combination of transport coefficients which has been shown<sup>1,2</sup> to be a good measure of the cross section for inelastic collisions. Thus we use the energy exchange collision frequency, as defined by the relation

$$\nu_u/N = (ewE/N)/(\epsilon_K - kT_g), \quad (2)$$

where  $k$  is Boltzmann's constant. For mixtures of H<sub>2</sub> in Ar it is most appropriate to think in terms of a hydrogen energy exchange collision frequency,  $[\nu_u/N]_{H_2-Ar}$ , for a mixture of H<sub>2</sub> and Ar which is a function not only of  $\epsilon_K$  but also of the mixture composition, and which is defined by the relation,

$$\left[ \frac{\nu_u}{N} \right]_{H_2-Ar} = \frac{1}{\alpha} \left[ \frac{\nu_u}{N} \right]_T - \frac{1-\alpha}{\alpha} \left[ \frac{\nu_u}{N} \right]_{Ar}. \quad (3)$$

In Eq. (3)  $\alpha$  is the fraction of H<sub>2</sub> in the mixture;  $[\nu_u/N]_T$  is the total energy exchange collision frequency for a mixture of H<sub>2</sub> in Ar, and is obtained by inserting into Eq. (2) the appropriate values as shown in Figs. 4 and 5; and  $[\nu_u/N]_{Ar}$  is the energy exchange collision frequency for pure Ar at the same value of  $\epsilon_K$  as  $[\nu_u/N]_{H_2-Ar}$  and  $[\nu_u/N]_T$ . To be truly meaningful  $[\nu_u/N]_{Ar}$  should be much less than  $[\nu_u/N]_T$ . This condition is fulfilled for the mixtures and values of  $\epsilon_K$  considered.

<sup>22</sup> However, this correlation of discrepancies in  $w_M$  on the basis of  $\epsilon_K$  values is not unique. Fig. 2 and 4 show that for both pure Ar and 4% H<sub>2</sub> in Ar the calculated values of  $w_M$  are higher than the experimental results near  $E/N = 10^{-17}$  V-cm<sup>2</sup>.

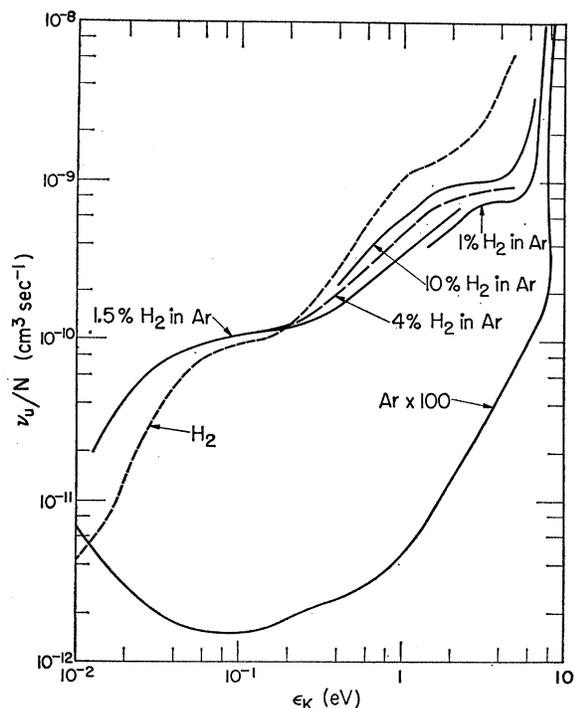


FIG. 6. Calculated energy exchange frequency  $\nu_u/N$  for Ar, H<sub>2</sub>, and 1, 1.5, 4, and 10% H<sub>2</sub> in Ar at  $T_g = 77^\circ\text{K}$  as a function of  $\epsilon_K$  in eV. In the case of the mixtures  $\nu_u/N$  is obtained using Eq. (3). For Ar,  $\nu_u/N$  is shown increased by a factor of 100.

Shown in Fig. 6 are calculated plots of  $[\nu_u/N]_{\text{H}_2\text{-Ar}}$  for H<sub>2</sub>, for 1, 1.5, 4, and 10% H<sub>2</sub>-Ar, and for pure Ar. In the case of Ar we note that  $[\nu_u/N]_{\text{Ar}}$  decreases up to  $\epsilon_K \approx 0.10$  eV because of the Ramsauer minimum. At the point where excitation first begins to assume importance, i.e.,  $\epsilon_K \approx 7.4$  eV,  $[\nu_u/N]_{\text{Ar}}$  starts to increase extremely rapidly, and the curve is even multivalued for a short interval since  $\epsilon_K$  decreases slightly in the region  $6.0 \times 10^{-17} < E/N < 1.0 \times 10^{-16}$  V-cm<sup>2</sup>.

For mixtures, the curves of  $[\nu_u/N]_{\text{H}_2\text{-Ar}}$  indicate clearly the effectiveness of Ar in separating some of the inelastic processes. In particular we wish to emphasize the improvement in the degree to which the effects of vibrational excitation are separated from electronic excitation.<sup>23</sup> As indicated in I and II and shown in Fig. 6,

<sup>23</sup> Although the data of Fig. 6 also seem to indicate that analysis of  $[\nu_u/N]_{\text{H}_2\text{-Ar}}$  should yield more accurate cross sections for rotational excitation, the increased values of the energy exchange collision frequency for  $\epsilon_K < 0.15$  eV are due in part to the rapidly decreasing  $Q_{mA}$  and the resultant broadening of  $f(\epsilon)$ . This is in contrast to the region where vibrational excitation is dominant, and where the increasing  $Q_{mA}$  leads to a narrowing of  $f(\epsilon)$ .

the major contribution of  $\nu_u/N$  for  $\epsilon_K < 0.3$  eV is due to rotational excitation. In pure H<sub>2</sub> the effects of vibrational excitation are seen for  $\epsilon_K$  from 0.3 to about 1.5 eV where electronic excitation begins to be important. Figure 6 shows that especially in the mixtures with low concentrations of H<sub>2</sub>, the vibrational excitation dominates up to an  $\epsilon_K$  of 5 eV. Therefore, if more and better experimental determinations of  $w$  and  $\epsilon_K$  for mixtures were available, it should be possible to make an even more accurate analysis of vibrational excitation than has been previously possible.<sup>1,2,24</sup>

#### IV. SUMMARY AND CONCLUSIONS

In the preceding sections we have discussed the determination of the argon momentum transfer cross section from 0.7 to approximately 25 eV. This cross section has been used in conjunction with previous results for hydrogen to calculate transport coefficients for various H<sub>2</sub>-Ar mixtures. For pure argon satisfactory agreement has been obtained between calculated and experimental values of the drift velocity  $w$ , the "magnetic" drift velocity  $w_M$ , and the characteristic energy  $\epsilon_K$ . On the other hand for mixtures of H<sub>2</sub> and Ar the picture is less satisfactory. In some cases the discrepancy appears to be due to experimental error, whereas in others it may be due to a failure to determine the momentum transfer cross section for argon with sufficient accuracy.

Unfortunately the analysis has been hampered by insufficient data. For example, in the case of mixtures of H<sub>2</sub> in Ar, only for 4% H<sub>2</sub> in Ar is there  $\epsilon_K$  data. Thus, a more complete and satisfactory analysis is predicated upon obtaining additional experimental results for pure Ar and mixtures of H<sub>2</sub> and Ar. As a further inducement for the procurement of such data, it has been demonstrated that improved separation is attained between processes such as vibration and electronic excitation in H<sub>2</sub>.

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<sup>24</sup> M. A. Uman [Phys. Rev. **123**, 399 (1961)] has calculated  $w$  for electrons in H<sub>2</sub>-Ar mixtures using two different approximate theoretical techniques. The agreement he obtains with experiment is less satisfactory than shown in Fig. 4 of this article.