Some Equivalent Approaches to the Self-Consistent Bound State of Strongly Interacting Particles*

R. M. ROCKMORE†
Department of Physics, Brandeis University, Waltham, Massachusetts
(Received 8 May 1963)

It is shown that the method known as the unsubtracted bootstrap, by which one calculates the mass and residue of a self-consistent bound state of strongly interacting particles in the N/D formalism, may be identified with the requirements that the strong-vertex (Z_v) and wave-function (Z_3) renormalization constants of the composite vanish simultaneously. The unsubtracted bootstrap is also shown to be equivalent to the self-consistent bound-state model of Liu by identifying the first vertex equation of that model with $Z_v=0$, and by reducing the second vertex equation to the statement $Z_3=0$ through the application of a Ward identity. The proof of equivalence is confined to S-wave bound states in the lowest order of self-consistency.

I. INTRODUCTION

We have recently been confronted with three alternative approaches to the problem of the self-consistent bound state of strongly interacting particles. These are the techniques of the "bootstrap,"[1] which is a method of calculation embedded in the N/D formalism,[2] the method of vanishing renormalization constants[3] [It is interesting to note that much attention has of late been focused on the vanishing of *only* the wave-function renormalization constant (Z_3) of the composite.[4–7]], and the vertex-equation approach to the self-consistent bound state due to Liu.[8,9]

In all of these methods one determines both coupling constant and mass of the (assumed) composite through the solution of as many simultaneous eigenvalue equations relating these parameters. Were these approaches to yield inequivalent eigenvalue conditions, one might then expect these bound-state parameters to be overdetermined, a situation which would cause us to question seriously our understanding of the bound-state problem in strong-coupling physics.

The purpose of this article is to present a proof of the equivalence of these approaches under the restriction to "unsubtracted bootstraps." A detailed account of such a bootstrap as well as its connection with $Z_3=0$ may be found in Sec. II. In Sec. III we develop a variant of Liu's procedure in the self-consistent bound-state problem and complete our equivalence proof. Throughout, our discussion is confined to S-wave bound states in the lowest order of self-consistency.

II. THE UNSUBTRACTED BOOTSTRAP

Let us consider, as in Ref. 8, a composite-particle model with only three kinds of strongly interacting scalar particles: a stable composite labeled C, of mass M_c, and its two constituents A and B with masses M_a and M_b, respectively. We shall also assume that the composite C is charged, with this charge resulting from the interaction of its charged constituent B with the electromagnetic field. Particle A is taken to be neutral. Now, let $M(s)$ denote the relativistic S-wave elastic scattering amplitude for AB particles. We suppose $M(s)$ satisfies dispersion relations and the elastic unitarity condition[10]

$$\operatorname{Im}M(s)=M^*(s)[q(s)/8\pi s^{1/2}]M(s) \qquad (2.1)$$

for

$$s_{ab}\equiv(M_a+M_b)^2\leq s\,,$$

where s is the square of the total center-of-mass energy and $q(s)=\{[s-(M_a+M_b)^2][s-(M_a-M_b)^2]/4s\}^{1/2}$, the center-of-mass momentum. Following Chew and Mandelstam,[2] one writes

$$(8\pi)^{-1}M(s)=N(s)D^{-1}(s)\,, \qquad (2.2)$$

where[11]

$$N(s)=\frac{1}{16\pi^2 i}\int_\gamma \frac{[M(s')]D(s')ds'}{(s'-s)}\,, \qquad (2.3)$$

and

$$D(s)=1-\frac{1}{\pi}\int_{s_{ab}}^\infty \frac{q(s')N(s')ds'}{s'^{1/2}(s'-s)}\,. \qquad (2.4)$$

$D(s)$ has only the right-hand cut in s coming from unitarity and has a zero corresponding to the "direct" graph of Fig. 1(a) [which gives rise to a pole in $M(s)$ at $s=M_c^2$], while $N(s)$ has only the left-hand cut in s due in lowest order to the "exchange" graph of Fig. 1(b)

* Work supported in part by the National Science Foundation.
† Present address: Physics Department, Rutgers University, New Brunswick, New Jersey.
[1] F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962).
[2] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960); G. F. Chew, *Dispersion Relations*, edited by G. R. Screaton (Interscience Publishers, Inc., New York, 1961), p. 167.
[3] A. Salam, Phys. Rev. **130**, 1287 (1963).
[4] J. Houard and B. Jouvet, Nuovo Cimento **18**, 466 (1960); M. T. Vaughn, R. Aaron, and R. D. Amado, Phys. Rev. **124**, 1258 (1961).
[5] R. Acharya, Nuovo Cimento **24**, 870 (1962).
[6] J. S. Dowker, Nuovo Cimento **25**, 1135 (1962).
[7] S. Weinberg, Phys. Rev. **130**, 776 (1963).
[8] L. S. Liu, Phys. Rev. **125**, 761 (1962).
[9] S. K. Bose, thesis, Brandeis University, Waltham, Massachusetts, 1963 (unpublished).
[10] Our considerations are limited to the one-channel problem in this note.
[11] $[M(s)]$ is the discontinuity of $M(s)$ in crossing the unphysical cut γ.

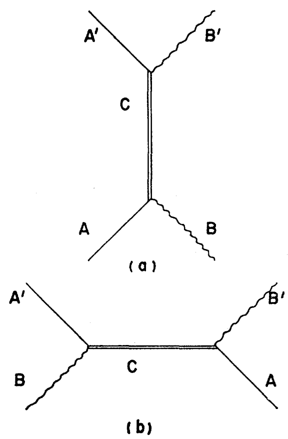


FIG. 1. (a) "Direct" graph in AB scattering with a pole at $s=M_c^2$; this is the output graph in a "bootstrap" calculation. (b) "Exchange" graph in AB scattering with a pole in the crossed channel at $u=M_c^2$; this is the input.

(the Born term with a pole in the crossed channel at $u=M_c^2$).

It is essential to our discussion to assume a *no-subtraction* representation for D, although it is customary[12] to make at least one, so that $D(s)$ takes the usual form

$$\bar{D}(s)=1-\frac{1}{\pi}(s-s_0)\int_{s_{ab}}^{\infty}\frac{q(s')\bar{N}(s')ds'}{s'^{1/2}(s'-s)(s'-s_0)}, \quad (2.5)$$

with the subtraction constant $D(s_0)$ eliminated through the scale transformations

$$\bar{D}(s)=D(s)/D(s_0),\qquad \bar{N}(s)=N(s)/D(s_0). \quad (2.6)$$

From the form of our equation for D [Eq. (2.4)], it is apparent that we assume $D(s)$ tends asymptotically to a constant; hence, by choosing $D(\infty)=1$, we insure that $M(s)$ goes asymptotically like $N(s)$.[13]

If a solution $M(s)$ to the coupled integral equations (2.3) and (2.4) exists, it will have a simple pole at $s=M_c^2$ corresponding to the stable composite C; then, the requirement that such a pole in $M(s)$ correspond to a zero in the D function, will yield the two equations of self-consistency which together determine both the position and residue associated with the composite C:

$$D(M_c^2)=0=1-\frac{1}{\pi}\int_{s_{ab}}^{\infty}\frac{q(s')N(s')ds'}{s'^{1/2}(s'-M_c^2)}, \quad (2.7\text{a})$$

and[14]

$$\frac{1}{\Gamma_0^2}=-\frac{1}{8\pi N(M_c^2)}\frac{dD(M_c^2)}{dM_c^2}=\frac{1}{8\pi^2 N(M_c^2)}\int_{s_{ab}}^{\infty}\frac{q(s')N(s')ds'}{s'^{1/2}(s'-M_c^2)^2}. \quad (2.7\text{b})$$

Equations (2.7a) and (2.7b) epitomize the "unsubtracted bootstrap"[1] in our one-channel model. We note that the second of these equations [Eq. (2.7b)] is unaffected by the presence or absence of a subtraction in D.

In the lowest order of self-consistency, the bootstrap equations (2.7a) and (2.7b) take the form

$$1=\frac{\Gamma_0^2}{32\pi^2}\int_{s_{ab}}^{\infty}\frac{ds'\ln[(M_c^2-2M_a^2-2M_b^2+s')/(M_c^2-2M_a^2-2M_b^2+s'-4q^2(s'))]}{q(s')s'^{1/2}(s'-M_c^2)}, \quad (2.8\text{a})$$

and

$$\begin{aligned}1&=\frac{\Gamma_0^2}{8\pi^2}\int_{s_{ab}}^{\infty}\frac{ds'q(s')}{s'^{1/2}(s'-M_c^2)^2}\\&=\frac{\Gamma_0^2}{8\pi^2}\Bigg\{\frac{M_a^2-M_b^2}{2M_c^4}\ln\frac{M_a}{M_b}-\frac{1}{2}+\frac{1}{2M_c^2}\frac{M_c^2(M_a^2+M_b^2)-(M_a^2-M_b^2)^2}{[M_c^2(2M_a^2+2M_b^2-M_c^2)-(M_a^2-M_b^2)^2]^{1/2}}\\&\qquad\times\Bigg[\tan^{-1}\frac{M_c^2+M_a^2-M_b^2}{[M_c^2(2M_a^2+2M_b^2-M_c^2)-(M_a^2-M_b^2)^2]^{1/2}}\\&\qquad+\tan^{-1}\frac{M_c^2+M_b^2-M_a^2}{[M_c^2(2M_a^2+2M_b^2-M_c^2)-(M_a^2-M_b^2)^2]^{1/2}}\Bigg]\Bigg\}, \quad (2.8\text{b})\end{aligned}$$

where we have substituted for $N(s)$ in Eq. (2.7a), the Born contribution to it from the crossed channel,[15]

$$N_{\text{Born}}(s)=\frac{1}{16\pi}\int_{-1}^{1}d(\cos\vartheta)\frac{\Gamma_0^2}{M_c^2-u}, \quad (2.9)$$

[12] The purpose of this subtraction is to fix D approximately equal to unity in the left-hand cut of interest so that the coupling constant used to calculate the force due to C exchange be the same as that determining the residue of the bound state. Since one usually imposes an additional cutoff on the D integral, the precise role played by such a subtraction is unclear. We do not consider this point any further in this paper except to note that the kinematical structure of our model allows the no-subtraction form for D given in Eq. (2.4).

[13] Thus, at least asymptotically, $N_{\text{Born}}(s)$ is given by the S-wave projection of the "exchange" graph of Fig. 1(b), with $N_{\text{Born}}(s)\to(\Gamma_0^2/8\pi s)\ln(s/\text{const})$ as $s\to\infty$.

[14] Cf. Eq. (5) in M. Nauenberg, Phys. Rev. **124**, 2011 (1961).

[15] $u=2M_a^2+2M_b^2-s+2q^2(s)(1-\cos\vartheta)$.

while we have replaced $N(s)$ in the integrand in Eq. (2.7b) by its value at $s=M_c^2$.

It is worthwhile noting that in the absence of crossing symmetry, we are left with only the second of the bootstrap equations,[6] Eq. (2.8b), so that Zachariasen's[16] world obtains, and we have merely a relation between the mass and residue of the bound state. That relation is easily identified with the statement $Z_3=0$,[5,6] where Z_3 is the wave-function renormalization constant of the composite C. This follows from the fact that

$$Z_3=1+\left.\frac{\partial\Sigma(s)}{\partial s}\right|_{(s=M_c^2)}, \tag{2.10}$$

where $\Sigma(s)$, the self-energy, is familiarly given by[17]

$$\Sigma(s)=i\Gamma_0^2\int\frac{d^4k}{(2\pi)^4}(k^2-M_a^2+i\epsilon)^{-1}\times[(k-p)^2-M_b^2+i\epsilon]^{-1}, \tag{2.11}$$

and has the once-subtracted representation

$$\Sigma(s)-\Sigma(M_c^2)=-\frac{\Gamma_0^2}{8\pi^2}(s-M_c^2)\int_{s_{ab}}^{\infty}\frac{ds'q(s')}{s'^{1/2}(s'-M_c^2)(s'-s-i\epsilon)}. \tag{2.12}$$

We note that this identification is consistent with the conjecture that the limit of vanishing wave-function renormalization of an "elementary particle" theory yields a theory in which the particle may be regarded as composite.[4–7] One would then like to equate the "residue equation" (2.7b) to the statement $Z_3=0$ to all orders in the square of the renormalized coupling, Γ_0^2.[18]

III. VERTEX-EQUATION APPROACH TO THE SELF-CONSISTENT BOUND STATE

The vertex-equation approach to the self-consistent bound-state problem taken by Liu[8] follows from the possibility of unsubtracted dispersion relations for both strong and electromagnetic vertex functions and from the observation that the renormalized ABC coupling is conventionally defined by[8]

$$\lim_{s_1\to M_a^2,\,s_2\to M_b^2,\,s_3\to M_c^2}\Gamma(s_1,s_2,s_3)=\Gamma_0. \tag{3.1}$$

These two conditions then lead naturally to the construction of the eigenvalue equations relating Γ_0^2 to M_c^2.

More explicitly, if one considers the vertex function defined by[19]

$$\Gamma(s)=(4A^0B^0)^{1/2}\langle 0|j_c(0)|AB\text{ in}\rangle, \tag{3.2}$$

with[20] $s=(A+B)^2$, one finds, on contracting the A particle in the instate in the usual way,[21]

$$\operatorname{Im}\Gamma(s)=\pi\int\frac{d^3A'd^3B'}{4A'^0B'^0(2\pi)^3}\Gamma(s')(8A'^0B'^0B^0)^{1/2}\times\langle A'B'|j_a(0)|B\rangle\delta(A'+B'-A-B), \tag{3.3}$$

with[22]

$$\Gamma(s)=\frac{1}{\pi}\int_{s_{ab}}^{\infty}ds'\frac{\operatorname{Im}\Gamma(s')}{s'-s-i\epsilon}. \tag{3.4}$$

In lowest order one has

$$(8A'^0B'^0B^0)^{1/2}\langle A'B'|j_a(0)|B\rangle=\frac{\Gamma_0^2}{M_c^2-s}+\frac{\Gamma_0^2}{M_c^2-u}, \tag{3.5}$$

where $u=(A-B')^2$. If one omits the term $\Gamma_0^2/(M_c^2-s)$ in Eq. (3.5) and, in the resulting homogeneous integral equation for $\Gamma(s)$,

$$\Gamma(s)=\frac{\Gamma_0^2}{16\pi^2}\int_{s_{ab}}^{\infty}\frac{ds'q(s')\Gamma(s')}{s'^{1/2}(s'-s-i\epsilon)}\times\int_{-1}^{1}d(\cos\vartheta')\frac{1}{M_c^2-u'}, \tag{3.6}$$

replaces $\Gamma(s')$ by Γ_0 and, further, takes the limit $s\to M_c^2$, then the eigenvalue equation,

$$1=\frac{1}{\pi}\int_{s_{ab}}^{\infty}\frac{ds'q(s')N_{\text{Born}}(s')}{s'^{1/2}(s'-M_c^2)}, \tag{3.7}$$

already familiar as the first bootstrap equation [Eq. (2.7a)], emerges. [If the renormalization term $\Gamma_0^2/(M_c^2-s)$ has been kept, one would have obtained

[16] F. Zachariasen, Phys. Rev. **121**, 1851 (1961).

[17] $p^2=s$.

[18] An analogous "residue equation" was derived some time ago in a discussion of the bound-state problem (for S waves) in potential theory given by R. Blankenbecler, M. L. Goldberger, N. N. Khuri, and S. B. Treiman, Ann. Phys. (N. Y.) **10**, 62 (1960). In their notation ($s=k^2$) one has

$$\frac{1}{\Gamma}=-\frac{1}{\pi}\int_0^{\infty}ds'\frac{s'^{1/2}e^{[p(s')-p(-s_0)]}}{(s'+s_0)^2},$$

where Γ is the residue and $-s_0$, the position of the bound state. The factorization of the D function, $D(s)=(s+s_0)D_r(s)$, enables us to write $|D_r(s)|^{-2}=\exp p(s)$. Since the nonrelativistic vertex $\Gamma(s)$ must have the phase of $D_r^{-1}(s)$ on the right-hand cut, so that $\Gamma(s)=D_r(-s_0)\Gamma(-s_0)/D_r(s)$, the residue equation may also be written as a normalization condition on the bound-state vertex,

$$1=\frac{1}{\pi}\int_0^{\infty}ds'\frac{s'^{1/2}|\Gamma(s')|^2}{(s'+s_0)^2}.$$

[19] We note that with our choice of vertex function (3.2), we recover the *same* vertex function $\Gamma(s)$ in its absorptive part; moreover, $j_c(0)$ projects out only that part of $|AB\text{ in}\rangle$ with angular momentum J, where J is the angular momentum of the composite. (For angular momenta $J\geq 1$, we are led naturally to a Regge-type treatment of the exchange of the composite.)

[20] We use the metric $a\cdot b=a_0b_0-\mathbf{a}\cdot\mathbf{b}$.

[21] $s'=(A'+B')^2$.

[22] $\Gamma(s)$ is analytic in the upper-half s plane.

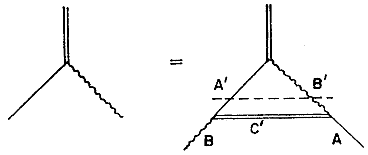


FIG. 2. Diagrammatic representation of the strong vertex equation [Eq. (2.8a)]. The dotted line indicates the two-particle intermediate state appropriate to the dispersion integral for the graph.

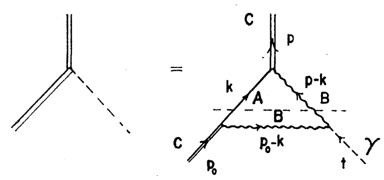


FIG. 3. Diagrammatic representation of the electromagnetic vertex equation [Eq. (3.20)]. The figure is labeled to conform with the text.

instead of Eq. (3.6),

$$\Gamma_0\left[1-\left(\frac{\partial\Sigma(s)}{\partial s}\right)_{(s=M_c^2)}\right]$$
$$=\frac{\Gamma_0^2}{16\pi^2}\int\frac{ds'q(s')\Gamma(s')}{s'^{1/2}(s'-s-i\epsilon)}\int d(\cos\vartheta')\frac{1}{M_c^2-u'};\quad(3.8)$$

however, on recognizing the character of the higher orders of perturbation theory,[23] whence it is apparent that Eq. (3.8) is the lowest order of[24]

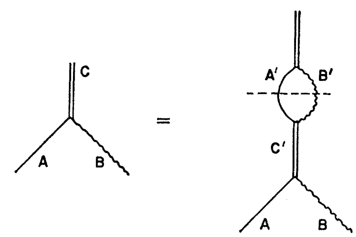
$$\frac{\Gamma_0}{Z_3}=\frac{\Gamma_0^3}{16\pi^2 Z_3}\int\frac{ds'q(s')}{s'^{1/2}(s'-s-i\epsilon)}\int d(\cos\vartheta')\frac{1}{M_c^2-u'},\quad(3.9)$$

we are led rather to (3.7).[25]]

It is interesting to point out here that the unsubtracted bootstrap equation (3.7) may also be identified with the vanishing of the strong vertex renormalization, $Z_v=0$. To see this, one first remarks that to order Γ_0^2, the sum of irreducible vertex parts $L^{(2)}$ may be be written[26,27]

$$L^{(2)}=1+L_1(\Gamma_0)\,,\quad(3.10)$$

FIG. 4. Diagrammatic representation of the "bootstrap" equation (2.8b). It is also the statement $Z_3=0$ in lowest order.



where[28]

$$L_1(\Gamma_0)=i\Gamma_0^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-M_c^2+i\epsilon)[(k+p_b)^2-M_a^2+i\epsilon][(k-p_a)^2-M_b^2+i\epsilon]},\quad(3.11)$$

$$=\frac{1}{2\pi i}\int\frac{I(p_c'^2)dp_c'^2}{p_c'^2-M_c^2-i\epsilon},\quad(3.12)$$

with[29,30]

$$I(p_c'^2)=-\frac{i\Gamma_0^2}{(2\pi)^2}\int d^4k\,\delta(k^2-M_a^2)\delta[(k-p_c')^2-M_b^2][(k-p_b)^2-M_c^2+i\epsilon]^{-1}\theta[p_c'^2-(M_a+M_b)^2].\quad(3.13)$$

After some straightforward manipulation, one has[31]

$$L_1(\Gamma_0)=\frac{\Gamma_0^2}{16\pi^2}\int_{(M_a+M_b)^2}^{\infty}ds'\frac{\ln\{s'(s'-2M_a^2-2M_b^2+M_c^2)/[M_c^2s'-(M_a^2-M_b^2)^2]\}}{(s'-M_c^2)\{[s'-(M_a+M_b)^2][s'-(M_a-M_b)^2]\}^{1/2}},\quad(3.14)$$

so that to the same order in Γ_0^2, the statement

$$Z_v=1-L_1(\Gamma_0)=0\quad(3.15)$$

[23] See, for example, S. D. Drell and F. Zachariasen, Phys. Rev. **119**, 463 (1960).

[24] We assume, as in R. D. Amado, Phys. Rev. **127**, 261 (1962), Γ_0 to be initially slightly less than the bound-state value, so that Z_3 is small but finite.

[25] Thus, the considerations of Ref. 9, where this renormalization term was kept, are not compatible with our interpretation of this procedure as a bootstrap. [See also R. Blankenbecler and L. F. Cook, Phys. Rev. **119**, 1745 (1960).]

[26] J. Hamilton, *The Theory of Elementary Particles* (Clarendon Press, Oxford, 1959), Chap. 5, Secs. 11 and 12.

[27] S. S. Schweber, *An Introduction to Relativsitic Quantum Field Theory* (Row, Peterson and Comapny, Evanston, Illinois, 1961), Chap. 16.

[28] $p_c=p_a+p_b$, with $p_a^2=M_a^2$ and $p_b^2=M_b^2$.

[29] R. E. Cutkosky, J. Math. Phys. **1**, 429 (1960).

[30] K. Nishijima, Phys. Rev. **126**, 852 (1962).

[31] $s'=p_c'^2$.

is nothing else but Eq. (3.7). We have exhibited this bootstrap equation graphically in Fig. 2.

Liu's second independent vertex equation[32] relating strong-coupling constant and bound-state mass is derived from an (assumed) unsubtracted dispersion relation for the electromagnetic vertex of the composite. We remark that if this relation is *not* identical with the statement $Z_3=0$ and hence with the second bootstrap equation (2.7b), then we should find ourselves confronted with *three* independent relations in the two variables, $\Gamma_0{}^2$ and $M_c{}^2$. As we show below by means of a Ward identity,[33] this is happily *not* the case; Liu's second relation is, indeed, to be identified with $Z_3=0$.

One first notes that

$$\Sigma^{(2)}(p^2)-\Sigma^{(2)}(p_0{}^2)=i\Gamma_0{}^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-M_a{}^2+i\epsilon)}\left\{\frac{1}{[(k-p)^2-M_b{}^2+i\epsilon]}-\frac{1}{[(k-p_0)^2-M_b{}^2+i\epsilon]}\right\},\tag{3.16}$$

$$=-i\Gamma_0{}^2\int\frac{d^4k}{(2\pi)^4}\frac{(p-p_0)\cdot(p+p_0-2k)}{(k^2-M_a{}^2+i\epsilon)[(k-p)^2-M_b{}^2+i\epsilon][(k-p_0)^2-M_b{}^2+i\epsilon]},$$

$$=-(p-p_0)_\mu V_\mu{}^{(2)}(p,p_0),\tag{3.17}$$

where $p_0{}^2=M_c{}^2$. Of course, with regard to dispersion relations, the quantity of interest here[8] is $F(p-p_0)^2$, with

$$V_\mu{}^{(2)}(p,p_0)=(p+p_0)_\mu F^{(2)}(t^2),\tag{3.18}$$

so that to second order in $\Gamma_0{}^2$,

$$Z_3=1+\left(\frac{\partial\Sigma^{(2)}(s)}{\partial s}\right)_{(s=M_c{}^2)}\tag{3.19}$$

$$=1-[F^{(2)}(t^2)]_{(t^2=0)}=0.\tag{3.20}$$

We remark that the extremely complicated expression for $F^{(2)}(0)$ displayed in Ref. 8 follows from considering the process $C+\bar{C}\to\gamma$ and evaluating the dispersion integral for the appropriate triangle diagram at the value of the photon invariant $t^2=0$. However, we have some latitude here with regard to the choice of dispersion variable and might, for example, have used the process $C+\gamma\to C$ to generate the second eigenvalue equation [we have schematized the resulting eigenvalue equation in Fig. 3]; in this case,[34] $-(p-p_0)_\mu V_\mu{}^{(2)}(p,p_0)$ yields the discontinuity[29]

$$[-(p-p_0)_\mu V_\mu{}^{(2)}(p,p_0)]=i\Gamma_0{}^2\int\frac{d^4k}{(2\pi)^2}\frac{[p^2-M_c{}^2-2k\cdot(p-p_0)]}{[(k-p_0)^2-M_b{}^2+i\epsilon]}\delta(k^2-M_a{}^2)\delta[(k-p)^2-M_b{}^2],\tag{3.21}$$

$$=i\Gamma_0{}^2\int\frac{d^4kd^4q}{(2\pi)^2}\frac{[p^2-M_c{}^2-2k\cdot t]}{[(k-p_0)^2-M_b{}^2+i\epsilon]}\delta(k^2-M_a{}^2)\delta(q^2-M_b{}^2)\delta(p_0+t-k-q),\tag{3.22}$$

corresponding to the channel $C\rightleftarrows A+B$, which, after some further manipulation, may be written as

$$[-(p-p_0)_\mu V_\mu{}^{(2)}(p,p_0)]=-\frac{i\Gamma_0{}^2}{8\pi p^2}\{[p^2-(M_a-M_b)^2][p^2-(M_a+M_b)^2]\}^{1/2},\tag{3.23}$$

so that

$$-(p-p_0)_\mu V_\mu{}^{(2)}(p,p_0)=\frac{1}{2\pi i}\int_{s_{ab}}^{\infty}dp'^2\frac{[-(p'-p_0)_\mu V_\mu{}^{(2)}(p',p_0)]}{p'^2-p^2-i\epsilon},\tag{3.24}$$

$$=-\frac{\Gamma_0{}^2}{16\pi^2}\int_{s_{ab}}^{\infty}\frac{ds'\{[s'-(M_a-M_b)^2][s'-(M_a+M_b)^2]\}^{1/2}}{s'(s'-s-i\epsilon)}.\tag{3.25}$$

---

[32] Equation (5.8) of Ref. 8. The expression for a should read $a=(1/M_\Lambda{}^2)[4M_\pi{}^2M_\Sigma{}^2-(M_\Sigma{}^2-M_\Lambda{}^2+M_\pi{}^2)^2]$.

[33] See Sec. 16e of Ref. 27.

[34] Of course, there is no anomalous threshold (Ref. 8) in this case.

Thus, one finds

$$F^{(2)}(0) = -\frac{\partial\Sigma^{(2)}(M_c^2)}{\partial M_c^2}, \tag{3.26}$$

as expected. Figure 4 exhibits the characteristic structure of the equation $Z_3=0$ in lowest order (for which crossing symmetry is unnecessary). It seems possible to conclude that insofar as the second bootstrap equation [Eqs. (2.7b) and (2.8b)] is established, a no-subtraction dispersion relation treatment of the electromagnetic vertex of the composite is implied.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor M. T. Grisaru for a number of stimulating discussions while this work was being completed. We also wish to thank Professor S. S. Schweber, Dr. H. J. Schnitzer, and D. Leiter for useful comments.