High-Energy Behavior in Production Processes*

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Regge pole-like terms in the high-energy behavior of production amplitudes are evaluated by using the leading asymptotic behavior of sums of Feynman diagrams. The forms obtained depend on the way variables are allowed to tend to infinity.

1. INTRODUCTION

RECENT investigations into the high-energy behavior of scattering amplitudes[1] have been concerned with two-particle to two-particle processes only. However, if Regge poles play an important dynamical role in strong interactions they will also manifest themselves in the high-energy behavior of production amplitudes. Kibble[2] and Ter-Martirosyan[3] have suggested heuristically what form this might take.

The rigorous investigation of this problem is hampered by the fact that the complex singularities of production amplitudes[4] prevent the use of the Froissart-Gribov method of analytic continuation to complex l. In this note we use the methods which have been developed[5–7] to evaluate the leading asymptotic behavior of terms of perturbation theory. They are based on the assumption that the sum of the leading asymptotic terms of a series of diagrams gives the leading term of the sum. These methods will be applied explicitly only to simple ladder-type graphs, although the extension of the theory to more complicated iterated systems which also yield Regge poles[5–8] is quite straightforward. It is now known[9] that perturbation theory also gives other types of high-energy behavior in addition to the Regge-pole behavior but these additional terms will not be considered here. They do not arise for ladder diagrams.

It will be found that a variable-power asymptotic behavior is obtained, although the trajectory function α enters the expression in a more complicated way than for two-particle processes. The most important result is that the asymptotic form depends on how many variables are held fixed and how the remainder are allowed to tend to infinity. The form proposed by Kibble[2] and Ter-Martirosyan[3] does not correspond to any of the limits investigated in this paper.

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2. FIVE-POINT AMPLITUDES

We shall consider bosons all of unit mass interacting through a Yukawa interaction. The diagrams considered are of the type shown in Fig. 1 with the invariants defined as the squares of the sums of pairs of adjacent ingoing momenta in the way indicated. The α_i, β_j, γ_k, γ_l', δ_m, are the Feynman parameters associated with the lines of the diagram. These parameters will also be denoted by the collective symbol ξ_n.

The asymptotic behavior of a physical amplitude will require the addition of a number of terms of this type corresponding to diagrams obtained by permuting the external lines. Only those diagrams in which at least one of the variables s, s_1, s_2 becomes large, and t_1 and t_2 remain fixed, will give significant contributions.

In order to obtain Regge pole-like terms in the asymptotic behavior a sum must be taken over all the different numbers of rungs in the two ladders in Fig. 1. There are a number of different interesting cases corresponding to different types of limit:

(i) $s\to\infty$; s_1, s_2, t_1, t_2 fixed.

The contribution from the diagram Fig. 1 is

$$\pi^2\Gamma(r+1)\frac{g^{2r+1}}{(16\pi^2)^{r-2}}\int_0^1 d\xi\,\frac{[C(\xi)]^{r-1}\delta(\sum\xi-1)}{[D(\xi;s,s_1,\cdots,t_2)]^{r+1}},\qquad(1)$$

where $r=m+n$, and C and D are the Feynman numerator and denominator functions associated with Fig. 1. The coefficient of s in D is

$$g\equiv\alpha_1\cdots\alpha_m\beta_1\cdots\beta_n.\qquad(2)$$

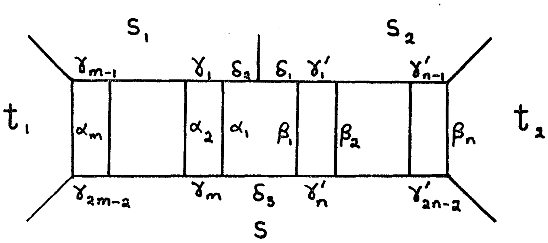

FIG. 1. The type of ladder diagram considered.

The leading asymptotic behavior in s comes from integration over the neighborhood of the zeros of g, i.e., $\alpha_1=\cdots=\beta_n=0$.[5,6] This yields

$$\left\{\pi^2\Gamma(r)\frac{g^{2r+1}}{(16\pi^2)^{r-2}}\int_0^1 d\gamma d\gamma' d\delta \times\frac{[c(\gamma,\gamma',\delta)]^{r-1}\delta(\sum\gamma+\sum\gamma'+\sum\delta-1)}{[d(\gamma,\gamma',\delta;t_1,t_2)]^r}\right\} \times(\ln s)^{r-1}/s\Gamma(r), \quad (3)$$

where c and d are the Feynman numerator and denominator functions of the contracted diagram, Fig. 2. The expression in curly brackets in (3) is just the correct contribution for the Feynman diagram Fig. 2 but evaluated with *two-dimensional* momentum vectors. Thus, (3) may be written as

$$g^2[\alpha'(t_1)]^{m-1}[\alpha'(t_2)]^{n-1}\beta(t_1,t_2)\frac{(\ln s)^{r-1}}{s\Gamma(r)}, \quad (4)$$

where

$$\alpha'(t)=\frac{g^2}{16\pi^2}\int_0^1\frac{d\xi_1 d\xi_2\,\delta(\xi_1+\xi_2-1)}{\xi_1\xi_2 t-1}, \quad (5)$$

and

$$\beta(t_1,t_2)=\pi^2 g^3\int_0^1\frac{d\xi_1 d\xi_2 d\xi_3\,\delta(\xi_1+\xi_2+\xi_3-1)}{[t_1\xi_2\xi_3+t_2\xi_3\xi_1+\xi_1\xi_2-1]^2}. \quad (6)$$

A sum is now taken over leading contributions from all diagrams with $m+n$ equal to a fixed value of r. This yields

$$g^2\beta(t_1,t_2)\frac{(\ln s)^{r-1}}{s\Gamma(r)}\frac{[\alpha'(t_1)]^{r-1}-[\alpha'(t_2)]^{r-1}}{\alpha'(t_1)-\alpha'(t_2)}. \quad (7)$$

A final sum over all values of r yields

$$g^2\beta(t_1,t_2)\frac{s^{\alpha(t_1)}-s^{\alpha(t_2)}}{\alpha(t_1)-\alpha(t_2)}, \quad (8)$$

where

$$\alpha(t)\equiv-1+\alpha'(t) \quad (9)$$

is the same trajectory function as appears in four-point ladder diagrams.[5,6]

As $t_2\to t_1$ (8) becomes

$$g^2\beta(t_1,t_1)s^{\alpha(t_1)}\ln s, \quad (10)$$

corresponding to a double Regge pole.

(ii) s, s_1, $s_2\to\infty$, t_1, t_2 fixed.

We write $s=kS$, $s_1=k_1S$, $s_2=k_2S$, $S\to\infty$. The coefficient of S in D is of the form

$$\alpha_1\cdots\alpha_m\beta_1\cdots\beta_n k+\alpha_1\cdots\alpha_m\Delta_1(\beta,\gamma',\delta_1)k_1 +\beta_1\cdots\beta_n\Delta_2(\alpha,\gamma,\delta_2)k_2, \quad (11)$$

where

$$\Delta_1=\delta_1\Delta_n(\beta,\gamma')+\beta_1\Delta_n'(\beta,\gamma')$$

and

$$\Delta_2=\delta_2\Delta_m(\alpha,\gamma)+\alpha_1\Delta_m'(\alpha,\gamma). \quad (12)$$

$\Delta_n\cdots\Delta_m'$ may be calculated but it is sufficient to notice the following properties:

(a) Δ_1 does not vanish when one of the β's is zero;
(b) when all the β's are zero,

$$\Delta_1=\delta_1\Delta_n(0,\gamma')=\delta_1\prod_{i=1}^{n}(\gamma_i'+\gamma_{i+n-1}')\equiv\delta_1c_1(\gamma');$$

together with similar properties for Δ_2.

To make the coefficients of k, k_1, k_2 vanish it is necessary to set at least two Feynman parameters equal to zero. These pairs may be chosen as follows: (1) one α_i $(i=1,\cdots,m)$ with one β_j $(j=1,\cdots,n)$; (2) δ_1 with β_1; (3) δ_2 with α_1. Since the scaling procedure[7] gives us δ functions with arguments $\alpha_i+\beta_j-1$ we must be careful to make these arguments linearly independent. We take them as follows. First, we have the distinguished pairs (δ_1,β_1), (δ_2,α_1). We may then take $(\alpha_m,\beta_i)(i=1,\cdots,n)$ and, finally, (α_i,β_n) $(i=1,\cdots,m-1)$. These pairs span the space of all possible pairs and are linearly independent.

We carry out the scalings on (2), (3) first. Under these the coefficient of k is linear and is retained. However, when we carry out the remaining scalings it is of second order and so may be dropped. The final result is thus independent of k. When we have carried out all these integrations, we end up with the final asymptotic form

$$\left\{\pi^2\Gamma(r-1)\frac{g^{2r+1}}{(16\pi^2)^{r-2}}\int_0^1 d\gamma d\gamma' d\delta_3 \times\frac{[c_0(\gamma,\gamma',\delta_3)]^{r-1}\delta(\sum\gamma+\sum\gamma'+\delta_3-1)}{c_1(\gamma')c_2(\gamma)[d_0(\gamma,\gamma',\delta_3;t_1,t_2)]^{r-1}}\right\} \times(\ln S)^r/k_1k_2S^2\Gamma(r+1), \quad (13)$$

where c_0 and d_0 are obtained from c and d by putting $\delta_1=\delta_2=0$. The integral in the curly brackets in (13) reduces to

$$\pi^2g^5[\alpha'(t_1)]^{m-1}[\alpha'(t_2)]^{n-1}. \quad (14)$$

Summing leading terms for all m and n gives

$$\pi^2g^5\left[\frac{\alpha'(t_2)S^{\alpha(t_1)}-\alpha'(t_1)S^{\alpha(t_2)}}{k_1k_2S\alpha'(t_1)\alpha'(t_2)[\alpha(t_1)-\alpha(t_2)]}+\frac{1}{\alpha'(t_1)\alpha'(t_2)S}\right]. \quad (15)$$

(iii) s, $s_1\to\infty$; s_2, t_1, t_2 fixed.

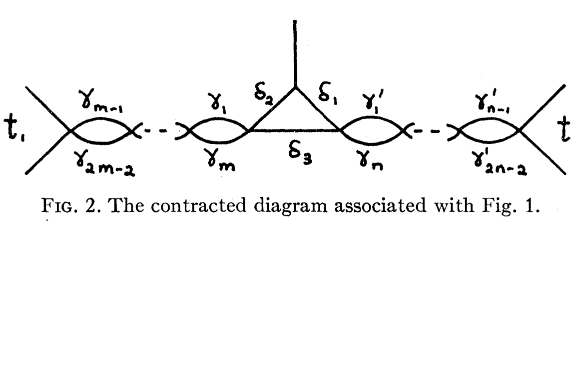

FIG. 2. The contracted diagram associated with Fig. 1.

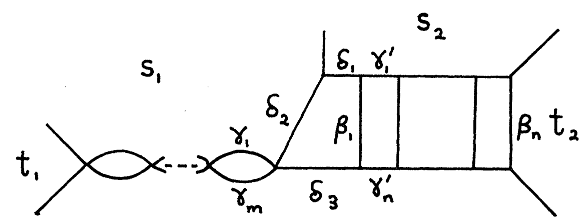

FIG. 3. The semicontracted diagram associated with Fig. 1.

We write

$$s_1 = k_1 s\,, \quad s \to \infty\,.$$

The property (a) of Δ_1 implies that the leading asymptotic behavior is obtained by integrating in the neighborhood of $\alpha_1 = \cdots = \alpha_m = 0$. The β integrations do not enhance the asymptotic behavior. The resulting form is

$$s^{\alpha(t_1)} G(k_1; t_1, t_2, s_2)\,, \tag{16}$$

where the k_1 dependence does not factor out of the expression obtained for G.

(iv) $s_1 \to \infty$, *followed* by $s_2 \to \infty$; s, t_1, t_2 fixed.

The limit $s_1 \to \infty$ is obtained by integrating in the neighborhood of $\alpha_1 = \cdots = \alpha_m = 0$. This yields

$$\pi^2 \Gamma(r) \frac{g^{2r+1}}{(16\pi^2)^{r-2}} \int_0^1 d\beta d\gamma d\gamma' d\delta \times \frac{[C_1(\gamma,\gamma',\delta,\beta)]^{r-1} \delta(\sum \beta + \sum \gamma + \sum \gamma' + \sum \delta - 1)}{\Delta_1(\beta,\gamma',\delta)[D_1(\gamma,\gamma',\delta,\beta; s_2,t_1,t_2)]^r} \times (\ln s_1)^{m-1}/s_1\Gamma(m)\,, \tag{17}$$

where C_1 and D_1 are the Feynman numerator and denominator functions of the semicontracted diagram Fig. 3.

The coefficient of s_2 in D_1 is

$$\beta_1 \cdots \beta_n \delta_2 c_2(\gamma)\,. \tag{18}$$

Thus, the leading asymptotic behavior will be given by integrating in the neighborhood of $\beta_1 = \cdots = \beta_n = \delta_2 = 0$. However, in evaluating this behavior a new feature is encountered. The property (b) of Δ_1, together with the presence of Δ_1 in the denominator of the integral in (17), means that the δ_1 integration also affects the asymptotic behavior of (17). The effect is evaluated in the Appendix.

Application of (A4) to (17) gives

$$\left\{ \pi^2 \Gamma(r-1) \frac{g^{2r+1}}{(16\pi^2)^{r-2}} \int_0^1 d\gamma d\gamma' d\delta_3 \times \frac{[c_0(\gamma,\gamma',\delta_3)]^{r-1} \delta(\sum \gamma + \sum \gamma' + \delta_3 - 1)}{c_1(\gamma')c_2(\gamma)[d_0(\gamma,\gamma',\delta_3; t_1,t_2)]^{r-1}} \right\} \times \frac{(\ln s_1)^{m-1}}{s_1 \Gamma(m)} \cdot \frac{(\ln s_2)^{n+1}}{s_2 \Gamma(n+2)}\,. \tag{19}$$

The integral in the curly brackets reduces to (14). Summing over m and n gives

$$\pi^2 g^5 s_1{}^{\alpha(t_1)} \left[\frac{s_2{}^{\alpha(t_2)}}{\alpha'^2(t_2)} - \frac{1 + \alpha'(t_2) \ln s_2}{s_2 \alpha'^2(t_2)} \right]. \tag{20}$$

The form (20) is independent of s. Note that the form depends upon the order in which s_1 and s_2 tend to infinity.

3. HIGHER AMPLITUDES

The types of limit existing for six-point and higher amplitudes are considerably complicated by the existence of nonlinear Gram determinant relations between the invariants. We shall be content to illustrate the type of asymptotic form obtained by an example corresponding to Fig. 4. The limit considered will be one in which $t_1 \equiv (p_2+p_3)^2$, $t_2 \equiv (p_2+p_3+p_4)^2$, $t_3 \equiv (p_1+p_6)^2$ remain finite and

$$\begin{aligned} (p_3+p_4)^2 &= \lambda_{34} S\,, \\ (p_4+p_5)^2 &= \lambda_{45} S\,, \\ (p_5+p_6)^2 &= \lambda_{56} S\,, \quad S \to \infty\,. \end{aligned} \tag{21}$$

Other invariants, such as $(p_1+p_2)^2$, may also tend to infinity, but in the approximation of taking leading asymptotic behavior this does not affect the form of the answer [in the same way that Eq. (4) is independent of k]. The calculation is identical to that described in detail in Sec. 2(ii) except that the d lines now consist of three disjoint lines. The resulting form is

$$\pi^4 g^8 / 2\lambda_{34}\lambda_{45}\lambda_{56} S^2 \times \left[\frac{\sum_{\text{cycl.}} \alpha'(t_2)\alpha'(t_3)[\alpha(t_2) - \alpha(t_3)] S^{\alpha(t_1)}}{\prod \alpha'(t_1)[\alpha(t_2) - \alpha(t_3)]} + \frac{1}{\alpha'(t_1)\alpha'(t_2)\alpha'(t_3)} \right]. \tag{22}$$

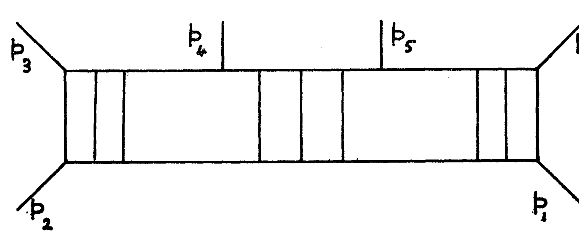

FIG. 4. The type of six-point ladder considered.

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APPENDIX

The new feature encountered in evaluating the asymptotic behavior of (17) is due to the fact that putting β_1 equal to zero in the factor Δ_1 in the denominator gives a δ_1 integration which diverges at $\delta_1=0$. In order to evaluate correctly the asymptotic form of (17), it is necessary therefore to integrate in the neighborhood of $\beta_1=\cdots=\beta_n=\delta_2=0$ *and* $\delta_1=0$. In order to evaluate the leading asymptotic behavior it is only necessary to consider the linear terms in Δ_1. The structure of Δ_1 is such that these terms only involve δ_1 and β_1. Thus, the leading behavior can be obtained by evaluating

$$\int_0^\epsilon \frac{d\beta d\delta_1 d\delta_2}{[c_1\delta_1+B\beta_1][c_2\beta_1\cdots\beta_n\delta_2 S+d]^r}. \qquad \text{(A1)}$$

The δ_1 integration is performed first to give

$$-\frac{1}{c_1}\int_0^\epsilon \frac{d\beta d\delta_2[\ln B\beta_1-\ln(c_1\epsilon+B\beta_1)]}{[c_2\beta_1\cdots\beta_n\delta_2 S+d]^r}. \qquad \text{(A2)}$$

The second term in the numerator of (A2) is bounded when $\beta_1=0$ so does not contribute to the leading asymptotic behavior. It will, therefore, be omitted. The β_1 integration is now performed and yields

$$\frac{1}{(r-1)c_1c_2d^{r-1}}\int_0^\epsilon \frac{d\beta_2\cdots d\beta_n d\delta_2}{\beta_2\cdots\beta_n\delta_2 S} \times[\ln(\epsilon\beta_2\cdots\beta_n\delta_2 S+1)+O(1)] \qquad \text{(A3)}$$

$$\sim\frac{1}{(r-1)d^{r-1}}\cdot\frac{1}{c_1c_2}\cdot\frac{(\ln S)^{n+1}}{S\Gamma(n+2)},\quad S\to\infty. \qquad \text{(A4)}$$

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Low-Energy $\bar{K}$–d Scattering*

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The magnitude of recoil and binding effects in the multiple-scattering corrections to the impulse approximation in low-energy $\bar{K}$-d scattering is examined by the introduction of a model which makes tractable the numerical evaluation of the double-scattering terms. The finite mass of the nucleons and the n-p interaction in continuum states are both taken into account. It is concluded that estimates of multiple-scattering corrections which ignore these effects are not reliable. The model is used to compute the sum of the cross sections for $K^-+d\to K^-+d$, $K^-+d\to K^-+p+n$. Comparison with the rather limited data available in the region 100 to 200 MeV/c favors the so-called solution II found by Humphrey and Ross in their analysis of $\bar{K}$-p data based on the Dalitz scattering lengths. A pseudopotential or optical-model-like approach to meson-deuteron scattering, which may be useful in other problems, is also described.

1. INTRODUCTION

SOME years ago, Dalitz[1] introduced two complex scattering lengths, which seem adequate for the phenomenological description of low-energy $\bar{K}$,N scattering and absorption processes. Considerable ambiguity in the values of A_0 and A_1, the $I=0$ and $I=1$ scattering lengths, respectively, was allowed by the data, and a number of attempts were made to reduce the ambiguity by a comparison of the rather limited data on K^--d reactions with theoretical predictions.[2] The present work was begun in an attempt to estimate the validity of previous calculations and to improve them, if possible.

More recently, the work of Ross and Humphrey[3] narrowed the ambiguity to a choice of two solutions, so-called solutions I and II, corresponding, respectively, to

$$\text{I}:\quad A_0=-0.22+2.74i\text{ F},\quad A_1=0.02+0.38i\text{ F},$$
$$\text{II}:\quad A_0=-0.59+0.96i\text{ F},\quad A_1=1.2+0.56i\text{ F}.$$

Akiba and Capps[4] then showed that only solution II is consistent with the data of Tripp *et al.*[5] obtained in the reaction $K^-+p\to\Sigma+\pi$ at 400 MeV/c.

We may, thus, turn the problem around and ask to what extent an analysis of K^--d scattering processes supports this choice, or better, to what extent one may correctly predict K^--d scattering and reaction cross sections, using this choice of the phenomenological scattering lengths.

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