$\gamma+p\rightarrow\eta+p$ Reaction at Low Energy and the Second-Resonant State in the Nucleon*

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The $\gamma+p\rightarrow\eta+p$ reaction near threshold is studied from field-theoretical viewpoint by taking into account the effects of the second-resonant state of the nucleon. It is pointed out that there may be some possibility of strong d-wave η-N interaction at low energy.

1. INTRODUCTION

As simple but important cases of η-meson reactions in one-nucleon systems, we may consider the $\pi+N\rightarrow\eta+N$ and $\gamma+N\rightarrow\eta+N$ reactions. Because the η is a $0^{\pm}$ meson with isotopic spin $I=0$, the η cannot be produced through a strong interaction in a 2π, 3π, $\rho-\pi$, or $\omega-\pi$ system. Therefore, the $\pi+N\rightarrow\eta+N$ reaction might be regarded as one of the most suitable processes to examine the $\eta-N$ interaction. If the target nucleon is a proton, however, the $\eta+N$ cannot be produced in $\pi^{+}-p$ collision owing to the conservation law of isotopic spin, and in $\pi^{-}-p$ collision it will be difficult to observe the particles in the final state $(\pi^{+}\pi^{-}\pi^{0})+n$ because of the existence of two neutral particles. In view of these circumstances we now want to study the reaction $\gamma+p\rightarrow\eta+p$, although there is yet no reliable experimental result for this reaction.

In the estimation of the cross section for $\gamma+p\rightarrow\eta+p$ reaction by means of perturbation theory, the main parts of matrix elements corresponding to the two kinds of Feynman diagrams (cf., Fig. 1) cancel each other as in the case of $\gamma+p\rightarrow\pi^{0}+p$ reaction. Here we should note the following facts: The total energy in the center-of-mass system corresponding to the threshold for the $\gamma+p\rightarrow\eta+p$ reaction is slightly below the mass of the second-resonant state N^{**} in the nucleon, and the isotopic spin of the N^{**} is equal to $\frac{1}{2}$. Thus we can easily suppose that the N^{**} plays an important role in the $\gamma+p\rightarrow\eta+p$ reaction near threshold. If the process $\gamma+p\rightarrow N^{**}\rightarrow\eta+p$ only is taken into account, the following results can be predicted because the N^{**} is the $d_{3/2}$-resonant state in the $\pi-N$ system: (1) The angular distribution for η-meson production has the form something like $(2+3\sin^{2}\theta)$, which is quite differnet from isotropic in spite of the fact that the reaction takes place in the neighborhood of threshold. (2) The cross section $\sigma(\gamma+p\rightarrow\eta+p)$ near threshold will be proportional to $(E-E_{0})^{5/2}$, where E_{0} is the threshold energy.

In Sec. 2, this reaction is studied from field-theoretical viewpoint by taking into account both the process $\gamma+p\rightarrow N^{**}\rightarrow\eta+p$ and the processes corresponding to perturbation, where the N^{**} is described approximately in terms of a Rarita-Schwinger particle[1] with spin $\frac{3}{2}$ and the experimental value for the width of the N^{**} is introduced in our calculation. In Sec. 3, our results with respect to the angular distributions and the energy dependence of total cross section are given. On the basis of our consideration it is pointed out that there may be some possibility of strong d-wave $\eta-N$ interaction at low energy.

2. MATRIX ELEMENTS FOR $\gamma+p\rightarrow\eta+p$ REACTION

In this paper, our study of the $\gamma+p\rightarrow\eta+p$ reaction is restricted to within the incident photon-energy region (740∼830) MeV where only the second-resonant state N^{**} has a large effect on the reaction and the effect of the third-resonant state N^{***} ($I=\frac{1}{2}$, $M=1688$ MeV) is negligible. Because the η is a 0^{-} meson and because the N^{**} is the $d_{3/2}$-resonant state in $\pi-N$ system, the interaction Hamiltonians for the $\eta-N$, $\gamma-N$, $\eta-N-N^{**}$, and $\gamma-N-N^{**}$ systems are given by the following forms[2]:

$$H_{\eta N}=g_{\eta}\bar{\psi}i\gamma_{5}\psi\phi\,, \tag{1}$$

$$H_{\gamma N}=-ie\bar{\psi}\gamma_{\mu}\frac{1+\tau_{3}}{2}\psi A_{\mu}\,, \tag{2}$$

$$H_{\eta NN^{**}}=\frac{G_{\eta}}{\mu}\bar{\Psi}_{\lambda}i\gamma_{5}\psi\frac{\partial\phi}{\partial x_{\lambda}}+\text{c.c.}\,, \tag{3}$$

and

$$H_{\gamma NN^{**}}=i\frac{G_{\gamma}}{m}\bar{\Psi}_{\lambda}\gamma_{\mu}\psi F_{\lambda\mu}+\text{c.c.}\,, \tag{4}$$

where Ψ, ψ, ϕ, and $F_{\lambda\mu}=\partial_{\lambda}A_{\mu}-\partial_{\mu}A_{\lambda}$ are the wave functions of the N^{**}, nucleon, η meson, and the field strength of the electromagnetic field, respectively. It should be noted that the interaction Hamiltonian mentioned in Eq. (4) is not necessarily the most general γNN^{**} interaction.[3] As was shown previously,[2] the propagation

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function of the N^{**} is given by

$$S_F(p)_{\lambda\rho}=\frac{1}{p^2+M^2}\Big\{\delta_{\lambda\rho}(i\gamma p-M)+\frac{2}{3M^2}p_\lambda(i\gamma p)p_\rho-\frac{4}{3M}p_\lambda p_\rho -\frac{i}{3M}[p_\lambda(i\gamma p)\gamma_\rho+\gamma_\lambda(i\gamma p)p_\rho]-\frac{i}{M^2}(\tfrac{2}{3}p^2+M^2)[\gamma_\lambda p_\rho+p_\lambda\gamma_\rho+i\gamma_\lambda(i\gamma p+M)\gamma_\rho]\Big\}, \quad (5)$$

where M is the mass of N^{**}. Needless to say, the matrix elements corresponding to Fig. 2(b) are negligible compared with those corresponding to Fig. 2(a). Therefore, we take into account the latter only. By a straightforward calculation, the matrix elements for the reaction $\gamma+p\rightarrow\eta+p$ can be expressed as follows:

$$A_1=ig_\eta e\frac{1}{2^2(2\pi)^2}\delta(I+K-F-q)\phi(q)A_\mu(K)\Big\{\Big[\frac{-m}{2(IK)}+\frac{m}{2(FK)}\Big]\bar{\psi}(F)\gamma_5\frac{\sigma_{\mu\nu}K_\nu}{m}\psi(I)+\Big[\frac{F_\mu}{(FK)}-\frac{I_\mu}{(IK)}\Big]\bar{\psi}(F)i\gamma_5\psi(I)\Big\}, \quad (6)$$

$$A_2=i\Big(\frac{G_\eta}{\mu}\Big)\Big(\frac{G_\gamma}{m}\Big)\frac{1}{2^2(2\pi)^2}\delta(I+K-F-q)\phi(q)A_\mu(K)\frac{1}{(I+K)^2+M^2}\Big\{\Big[\Big(\frac{2m^3}{3M^2}-m-\frac{4m(IK)}{3M^2}\Big)[\mu^2+4m(M-m)] +\frac{m^2\mu^2}{3M}+m(FK)+m(IK)\Big(-2-\frac{m}{M}+\frac{\mu^2}{3M^2}+\frac{2m^2}{3M^2}\Big)-\frac{2m}{M^2}(IK)^2\Big]\bar{\psi}(F)\gamma_5\frac{\sigma_{\mu\nu}K_\nu}{m}\psi(I) +(M-m)\bar{\psi}(F)\gamma_5[\gamma_\mu(qK)-(\gamma K)q_\mu]\psi(I)+\Big[4(M-m)\Big(\frac{2m^2}{3M^2}-1\Big)-2(IK)\Big(\frac{3}{M}-\frac{8m}{3M^2}\Big)-\frac{2m}{3M^2}q(I+K)\Big] \times\bar{\psi}(F)\gamma_5[\gamma_\mu(IK)-(\gamma K)I_\mu]\psi(I)+2[(IK)q_\mu-I_\mu(qK)]\bar{\psi}(F)i\gamma_5\psi(I)\Big\}, \quad (7)$$

where A_1 and A_2 are the matrix elements corresponding to the processes shown in Fig. 1 and Fig. 2(a), respectively. I and F indicate the four momenta of the initial and final protons, and K and q the four momenta of the incident photon and the emitted η meson, respectively. It is needless to say that the expressions for A_1 and A_2 satisfy the condition of gauge invariance.

In order to take into account the width Γ of N^{**}, the denominator $(I+K)^2+M^2$ of Eq. (7) must be modified. Of course the N^{**} can decay into $N^*+\pi$, $N+\pi$, $\eta+N$, and $\cdots$. If the width Γ is estimated within the framework of field theory by making use of the damping theory, then Γ is expressed in terms of the coupling constants for the interactions in the $\pi-N^*-N^{**}$, $\pi-N-N^{**}$, $\eta-N-N^{**}$, $\cdots$ systems. Because of the existence of so many unknown parameters, however, we think it suitable to introduce the width phenomenologically in our expression. Our considerations are referred to the center-of-mass system throughout this paper. Then the denominator $(I+K)^2+M^2$ of Eq. (7) has the form

$$(E_r^2-E^{*2})=(E_r+E^*)(E_r-E^*), \quad (8)$$

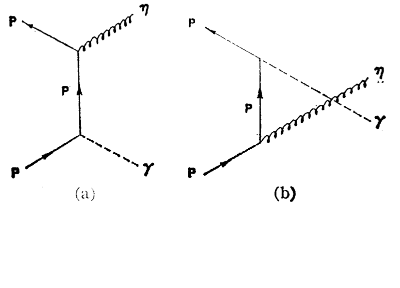


FIG. 1. The Feynman diagrams corresponding to perturbation.

where E_r and E^* stand for the resonance energy $(=M)$ and the total energy of the colliding system, respectively. Comparing the expression in Eq. (8) with the form of the denominator in the Breit-Wigner one-level formula, we modify the former as follows:

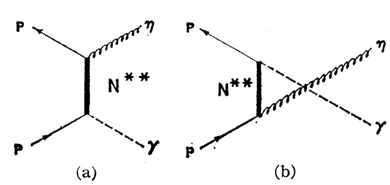
$$(E_r+E^*)[(E_r-E^*)-\tfrac{1}{2}i\Gamma]. \quad (9)$$

As we consider the phenomena in the neighborhood of the second resonance, this approximation may be fairly good.

3. ANGULAR DISTRIBUTIONS AND ENERGY DEPENDENCE OF TOTAL CROSS SECTIONS

Experimental results have shown that the mass of N^{**} is equal to 1510 MeV and its half-width $\frac{1}{2}\Gamma$ is equal to 30 MeV.[4] According to more recent experimental data,[5] however, the value of Γ seems to be about 130 MeV. In this paper we consider the two cases (i) $\frac{1}{2}\Gamma=30$ MeV and (ii) $\frac{1}{2}\Gamma=60$ MeV. By making use

FIG. 2. The Feynman diagrams in the case where the effects of N^{**} are taken into account.



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[5] B. P. Gregory, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 783.

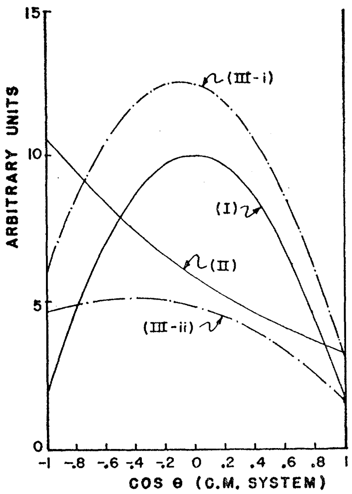

FIG. 3. Angular distributions for the $\gamma+p\rightarrow\eta+p$ reaction just at resonance energy. The following notations are adopted: (I): case of $(g_\eta^2/4\pi)=0$, (II): case of $(G_\eta^2/4\pi)(G_\gamma^2/4\pi)=0$, (III): case of $(G_\eta^2/4\pi)\times(G_\gamma^2/4\pi)=10(g_\eta^2/4\pi)\times(e^2/4\pi)$. (i): case of $\frac{1}{2}\Gamma=30$ MeV, (ii): case of $\frac{1}{2}\Gamma=60$ MeV.

of the matrix elements mentioned in Sec. 2, the cross sections for the $\gamma+p\rightarrow\eta+p$ reaction can be calculated straightforwardly. We show in Figs. 3 and 4 the angular distributions just at resonance energy and the energy dependence of total cross sections for this reaction. Although it is necessary to examine the value of $(G_\eta^2/4\pi)(G_\gamma^2/4\pi)$ relative to $(g_\eta^2/4\pi)(e^2/4\pi)$, only the following cases are considered for simplicity.

$$\begin{aligned}&\text{(I)} && (g_\eta^2/4\pi)=0\,,\\ &\text{(II)} && (G_\eta^2/4\pi)(G_\gamma^2/4\pi)=0\,, \qquad\qquad (10)\\ &\text{(III)} && (G_\eta^2/4\pi)(G_\gamma^2/4\pi)\cong 10(g_\eta^2/4\pi)(e^2/4\pi)\,.\end{aligned}$$

In the case (III), σ_{pert} is of the same order of $\sigma_{N^{**}}$ just at the resonance energy (incident photon energy = 746 MeV), where σ_{pert} and $\sigma_{N^{**}}$ are the cross sections obtained by employing the matrix elements A_1 and A_2, respectively.

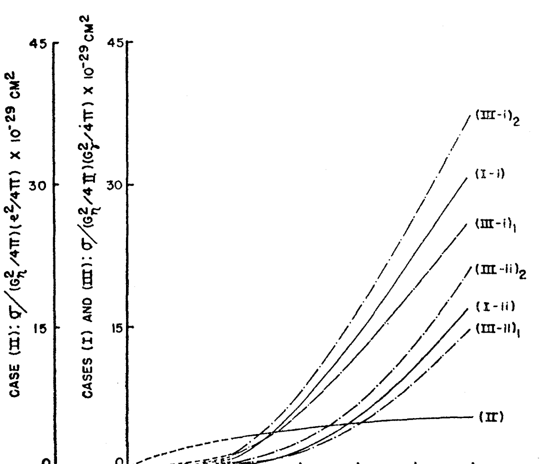

FIG. 4. Excitation function for the $\gamma+p\rightarrow\eta+p$ reaction. The same notations as in Fig. 3 are used. Moreover, case (III) is divided into the following two cases:

$$G_\eta G_\gamma=10^{1/2}g_\eta e \quad\text{and}\quad G_\eta G_\gamma=-10^{1/2}g_\eta e.$$

These cases are characterized by suffices 1 and 2, respectively.

Since the N^{**} is the $d_{3/2}$-resonant state in the $\pi-N$ system, it is produced through the $E1(J=\frac{3}{2})$ or $M2(J=\frac{3}{2})$ interaction, where El and Ml represent electric 2^l-pole and magnetic 2^l-pole, respectively, and J represents the total angular momentum of the $\gamma-N$ or $\eta-N$ system. If the R-matrices for the states $E1(J=\frac{3}{2})$ and $M2(J=\frac{3}{2})$ are denoted by $a_{3/2}$ and $b_{3/2}$, respectively, the differential cross section for the $\gamma+p\rightarrow\eta+p$ reaction due to these states[6] is expressed by

$$\frac{\lambda\!\!\!^{-2}}{4}\left[\frac{5-3\cos^2\theta}{6}a_{3/2}^*a_{3/2}+\frac{3(1+\cos^2\theta)}{10}b_{3/2}^*b_{3/2}-\frac{(1-3\cos^2\theta)}{2\sqrt{5}}(a_{3/2}^*b_{3/2}+a_{3/2}b_{3/2}^*)\right].\quad(11)$$

Based on the experimental results for $\gamma+p\rightarrow\pi+N$ reaction, it is said that the N^{**} is produced through the $E1(J=\frac{3}{2})$ interaction because the experimental data[7] for the angular distribution of this reaction are consistent with a form $(5-3\cos^2\theta)$. As is shown in Fig. 3, the angular distribution in case (I) is expressed approximately by the form $(5-4\cos^2\theta)$. This means that the N^{**} can be excited through the $E1(J=\frac{3}{2})$ and $M2(J=\frac{3}{2})$ states when the interaction Hamiltonian (4) is adopted, although the contributions from the latter state will be much smaller than those from the former.

It must be noted that the cross terms between the matrix elements A_1 and A_2 disappear just at the resonance energy because the denominator of A_2 [cf., Eq. (9)] turns out to be pure imaginary. This makes it easy to compare the contributions from the process corresponding to Fig. 1 with those from the process corresponding to Fig. 2(a), since the angular distribution in case (I) is quite different from that in case (II) as is shown in Fig. 3. We also think it worthwhile to examine experimentally the excitation function of $\sigma(\gamma+p\rightarrow\eta+p)$. If N^{**} plays the most important role in this reaction, then $\sigma(\gamma+p\rightarrow\eta+p)$ rises very steeply with incident energy (cf., Fig. 4).

In addition to the above approaches we may consider an attempt such that the effects of anomalous magnetic moment of the nucleon are taken into account phenomenologically as has been tried by Kaplon[8] and Aizu-Fujimoto-Fukuda[9] in their study for $\gamma+p\rightarrow\pi^0+p$ reaction. We suppose that the effects of the anomalous magnetic moment of the nucleon on the $\gamma+p\rightarrow\eta+p$ reaction are not so remarkable as those of the second-resonant state of the nucleon, although it is necessary to examine them in detail in order to draw this conclusion with confidence.

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