Two-Field Couplings with Especial Reference to Photon-Neutral Meson Interaction†

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The coupling of two particles of identical quantum numbers, apart from their mass, is considered. It is shown that the effects of direct transitions of one particle into another can be treated exactly in terms of six parameters. These can be taken to be the true masses, the mixing parameters, and the partially renormalized coupling constants. This is a generalization of conventional mass renormalization for a single field. The special case of a photon and a neutral vector meson is discussed. Resonance effects may occur if the photon is virtual. However, gauge invariance and the vanishing of the photon mass forbid such effects for real photons, unless a dynamical accident relates the bare photon mass to the effective photon-meson couplings in a particular way.

1. INTRODUCTION

THE discovery of ρ and ω mesons, with the same quantum numbers as the photon, has aroused considerable interest in the general problem of the theory of the interactions of any two particles which are distinguished only by their differing masses.[1]

We discuss this problem here in the context of the Lagrangian formulation. If each of the particles is coupled to other fields, then there is the possibility of their direct conversion into each other.[2] This gives rise to resonance effects, which may be very marked if the masses of the particles are not too different. This direct conversion takes place through diagrams which have the form of off-diagonal self-mass elements. The main point of the first half of this paper is to show that the correct treatment of these effects is a very direct generalization of the standard (Dyson) procedure[3] for the mass renormalization of a single field. This is to extract from the interaction currents the one-particle parts, which are treated exactly, and to base the perturbation expansion only on the residual interaction. In Sec. 2, the usual theory of a single particle is summarized in a manner which generalizes most directly to the two-particle case. In Sec. 3, this generalization is carried out. Since the particles can convert into each other, each propagator has poles at the masses of both particles. The propagators and modified currents which treat the particle conversion and the diagonal self-mass effects exactly, can be expressed in terms of six parameters. These may conveniently be taken to be the true masses of the particles, the partially renormalized coupling constants, and the mixing parameters, which determine the residues of the poles in the propagators. These six constants are the direct generalization of the single true mass in the one-particle case. The true particles are annihilated and created by linear combinations of the fields, which make allowance for the possibility of direct conversion of one particle into the other in external lines.

If the effects of the residual interaction are calculated using perturbation theory, the six parameters mentioned above are related to the parameters of the original bare Lagrangian through expressions which involve divergent integrals. These relations are similar to those which relate the true mass to the bare mass and bare coupling constants in conventional theory. However, if the six constants are treated as finite parameters, taken directly from experiment (as in the conventional case with the observed mass), a finite renormalized perturbation theory can be developed in the usual way. This is shown in the Appendix, in which we also develop a convenient and perspicuous graphical representation.

In Sec. 4 the results of Sec. 3 are extended to cover two particles of spin one. In Sec. 5 we consider the implications of gauge invariance, and the vanishing of the photon mass, on such particle mixing, when one of the particles is a photon.

2. A SINGLE FIELD

Before considering the interaction of two fields with the same quantum numbers, we summarize the standard procedure for a single field. Consider an unrenormalized field $\phi(x)$ with Lagrangian density.[4]

$$L=-\tfrac{1}{2}\phi(x)(-\partial^2+m^2)\phi(x)+j(x)\phi(x)+\cdots, \quad (2.1)$$

where we have written explicitly only those terms which

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[1] I. Yu Kobzarev, L. B. Okun', and I. Ya. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. **41**, 495 (1961) [translation: Soviet Phys.—JETP **14**, 355 (1962)]; J. H. Baroff and T. Fulton, Nuovo Cimento (to be published); P. K. Srivastava, Phys. Rev. **128**, 2906 (1962); G. Feldman, P. T. Matthews and A. Salam, *ibid.* **121**, 302 (1961); R. Jacob and R. G. Sachs, *ibid.* **121**, 350 (1961); R. G. Sachs, Ann. Phys. (N. Y.) **22**, 239 (1963).

[2] We could, of course, have a direct interaction, bilinear in the two fields.

[3] F. J. Dyson, Phys. Rev. **75**, 1736 (1949); That this procedure would have to be generalized in the two-field case was apparently first pointed out by Nambu and Gell-Mann. See M. Gell-Mann in *Proceedings of the Tenth Annual Conference on High Energy Physics, Rochester, 1960* (Interscience Publishers, Inc., New York), p. 792.

[4] The form $\phi\partial^2\phi$ in the Lagrangian is shorthand for $-(\partial_\mu\phi)^2$. We use the space-like metrix, so that $\partial^2=\nabla^2-\partial_t^2$.

depend on $\phi(x)$. The equation of motion is then

$$K_m(\partial^2)\phi(x) \equiv (-\partial^2+m^2)\phi(x) = j(x). \tag{2.2}$$

If the mass of the true particle is M, the expectation value for this equation for a one true-particle state is

$$(-M^2+m^2)\langle 0|\phi(x)|M\rangle = \langle 0|j(x)|M\rangle. \tag{2.3}$$

The object of mass renormalization is to treat exactly the one-particle parts of this current. Assuming that $\phi(x)$ is the only primary field which has a nonvanishing matrix element from the vacuum to the single true-particle state of mass M, we can write

$$\langle 0|j(x)|M\rangle = A\langle 0|\phi(x)|M\rangle, \tag{2.4}$$

then combining (2.3) and (2.4)

$$[K_m(M^2)-A]\langle 0|\phi(x)|M\rangle = 0. \tag{2.5}$$

This is a homogeneous equation[5] for $\langle 0|\phi(x)|M\rangle$ which can be satisfied only if

$$K_m(M^2)-A=0. \tag{2.6}$$

To define a current which has no one-particle expectation value, one can take

$$J(x)=j(x)-A\phi(x). \tag{2.7}$$

This corresponds to rewriting the Lagrangian as

$$L=-\tfrac{1}{2}\phi(K_m(\partial^2)-A)\phi+j\phi-\tfrac{1}{2}\phi A\phi+\cdots. \tag{2.8}$$

The equation of motion is now

$$K_M(\partial^2)\phi(x)\equiv(-\partial^2+M^2)\phi(x)=J(x). \tag{2.9}$$

In the usual interaction representation, which is based on the interaction, $J(x)$, the propagator is Δ, where

$$K_M(-p^2)\Delta(p^2)=1. \tag{2.10}$$

This propagator takes into account exactly the one-particle part of the current $j(x)$.

If a transition is made from an initial state $|p,i\rangle$ to a final state $\langle f|$, which includes a particle of four-momentum p, the amplitude is[6]

$$_{\text{out}}\langle f|p,i\rangle_{\text{in}}=\int e^{ipx}\overleftrightarrow{\partial_t}\langle f|\phi_{\text{in}}(x)|i\rangle d^3x \tag{2.11}$$

$$=\int e^{ipx}\overleftrightarrow{\partial_t}\langle f|\phi(x)|i\rangle d^3x$$

$$=\int e^{ipx}K_M(\partial^2)\langle f|\phi(x)|i\rangle d^4x$$

$$=\int e^{ipx}\langle f|J(x)|i\rangle d^4x. \tag{2.12}$$

3. COUPLED SPIN-ZERO FIELDS

We now consider the theory of two stable particles, which apart from their masses, have the same quantum numbers. Suppose that the unrenormalized fields are $\phi_1(x)$ and $\phi_2(x)$ and that the terms in the Lagrangian which depend explicitly on $\phi_i(x)$ are

$$L=-\tfrac{1}{2}\phi_1(x)(-\partial^2+m_1^2)\phi_1(x)-\tfrac{1}{2}\phi_2(x)(-\partial^2+m_2^2)\phi_2(x)+j_1(x)\phi_1(x)+j_2(x)\phi_2(x)+\cdots. \tag{3.1}$$

Then the equations of motion[7] are

$$(-\partial^2+m_i^2)\phi_i(x)=j_i(x),\quad (i=1,2). \tag{3.2}$$

The one true-particle matrix elements of these equations are

$$(-M_k^2+m_i^2)\langle 0|\phi_i(x)|M_k\rangle=\langle 0|j_i(x)|M_k\rangle.\quad i,k=1,2. \tag{3.3}$$

As in the single-particle case, we wish to extract, and treat exactly the one-particle parts of the currents $j_i(x)$. To this end we write

$$\langle 0|j_i(x)|M_k\rangle=\textstyle\sum_j A_{ij}^{(k)}\langle 0|\phi_j(x)|M_k\rangle. \tag{3.4}$$

Define

$$K_{ij}(M_k^2)=(-M_k^2+m_i^2)\delta_{ij}, \tag{3.5}$$

then

$$\textstyle\sum_j[K_{ij}(M_k^2)-A_{ij}^{(k)}]\langle 0|\phi_j(x)|M_k\rangle=0. \tag{3.6}$$

This is a homogeneous equation for the matrix elements, which can only be satisfied[8] if

$$\det[K(M_k^2)-A^{(k)}]=0. \tag{3.7}$$

Explicitly the relation (3.7) between the bare and true masses is

$$[M_k^2-m_1^2+A_{11}(M_k^2)][M_k^2-m_2^2+A_{22}(M_k^2)]-A_{12}^2(M_k^2)=0. \tag{3.8}$$

Since $A^{(k)}$ are symmetric matrices,[8] the elements $A_{ij}^{(k)}$ are a set of six constants. In place of the six constants $A_{ij}^{(k)}$, it is convenient to introduce six new constants G_{ij}, H_{ij} such that $A_{ij}^{(k)}=G_{ij}+M_k^2H_{ij}$.[9]

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[5] Note that $K_m(M^2)=-M^2+m^2\equiv-\delta m^2$. The constant A is $\Pi^*(M^2)$, where $\Pi^*(-p^2)$ is the proper self-energy part, i.e., the mass operator.

[6] In this paper we are only concerned with the mass renormalization of a single field and its generalization to the two-field case. For this reason we omit all Z factors associated with complete field renormalization. For example, the right-hand side of (2.12) should read

$$Z^{-1/2}\int e^{ipx}\overleftrightarrow{\partial_t}\langle f|\phi(x)|i\rangle d^3x.$$

[7] For the sake of simplicity we assume that $j_i(x)$ does not depend on $\phi_i(x)$.

[8] The complete propagator matrix $\Delta'(p^2)$ in momentum space is given by

$$[K(-p^2)-\Pi^*(-p^2)]\Delta'(p^2)=1,$$

where we know from Dyson's analysis that graphically $\Pi^*(-p^2)$ is all proper self-energy parts (i.e., those which cannot be broken into two parts connected by a single line). Since $\Delta'(p^2)$ has poles at $p^2+M_k^2=0$, it follows that (i), $\det[K(M_k^2)-\Pi^*(M_k^2)]=0$. By comparison with Eq. (3.7), (ii), $A^{(k)}=\Pi^*(M_k^2)$. Thus, $A_{ij}^{(k)}$ are constants which can, in principle, be calculated (to any order in perturbation theory, for example) by calculating $\Pi^*(-p^2)$ and solving (i). It is evident from crossing symmetry that $\Pi_{ij}^*(-p^2)=\Pi_{ji}^*(-p^2)$ and the matrices $A^{(k)}$ are symmetric.

[9] If the theory is symmetric for the exchange $\phi_1\leftrightarrow\phi_2$, $m_1\leftrightarrow m_2$, and the coupling constants $g_1\leftrightarrow g_2$, then $H_{ij}\equiv 0$, and three constants suffice.

To define a current which has no one-particle matrix elements, we can take

$$J_i(x)=j_i(x)-A_{ij}(\partial^2)\phi_j(x)\,, \tag{3.9}$$

where

$$A_{ij}(\partial^2)=G_{ij}+\partial^2 H_{ij}. \tag{3.10}$$

This corresponds to rewriting the Lagrangian as

$$L=-\tfrac{1}{2}\phi_i[K_{ij}(\partial^2)-A_{ij}(\partial^2)]\phi_j +[j_i\phi_i-\tfrac{1}{2}\phi_i A_{ij}(\partial^2)\phi_j]+\cdots. \tag{3.11}$$

The equations of motion in terms of J are

$$[K_{ij}(\partial^2)-A_{ij}(\partial^2)]\phi_j=J_i. \tag{3.12}$$

Thus, from (3.6), (3.7), and (3.8) it follows that

$$\det[K(-p^2)-A(-p^2)]=0\,, \tag{3.13}$$

at the two-point

$$p^2+M_k^2=0. \quad (k=1,2) \tag{3.14}$$

The propagator in the interaction representation based on J_i, which treats exactly the one-particle parts of j_i, is given by

$$[K(-p^2)-A(-p^2)]\Delta(p^2)=1. \tag{3.15}$$

$\Delta(p^2)$ is a matrix propagator [the inverse of $K(-p^2)+A(-p^2)$] each element of which, in general,[10] has poles at both masses, M_k^2.

It is convenient for later purposes to introduce the partially renormalized propagator[11] $\Delta^R(p^2)$. This is to be defined so that the residue of the $\Delta_{ii}{}^R$ element at the mass M_i^2 is unity. Thus, we take

$$\Delta_{ij}{}^R(p^2)\equiv R_i^{-1/2}\Delta_{ij}(p^2)R_j^{-1/2}\,, \tag{3.16}$$

where R_i is the residue of $\Delta_{ii}(p^2)$ at $p^2=-M_i^2$. This leads to

$$\Delta_{ij}{}^R(p^2)=\begin{pmatrix} \dfrac{1}{p^2+M_1^2}+\dfrac{X_2^2}{p^2+M_2^2}, & \dfrac{X_1}{p^2+M_1^2}-\dfrac{X_2}{p^2+M_2^2} \\ \dfrac{X_1}{p^2+M_1^2}-\dfrac{X_2}{p^2+M_2^2}, & \dfrac{X_1^2}{p^2+M_1^2}+\dfrac{1}{p^2+M_2^2} \end{pmatrix}, \tag{3.17}$$

where[12]

$$X_i=\frac{A_{12}(M_i^2)}{[M_1^2-m_2^2+A_{22}(M_1^2)]^{1/2}[-M_2^2+m_1^2-A_{11}(M_2^2)]^{1/2}}. \tag{3.18}$$

In carrying out a perturbation expansion based on the current J_i, the quantity $J_i\Delta_{ij}J_j$ can be regarded as the effective interaction for those matrix elements which involve the particles of mass M_i internally. It is now convenient to introduce the partially renormalized currents[13] $J_i{}^R$ defined such that

$$J_i{}^R\Delta_{ij}{}^R J_j{}^R=J_i\Delta_{ij}J_j. \tag{3.19}$$

Thus,

$$J_i{}^R=R_i^{1/2}J_i. \tag{3.20}$$

Accordingly, the interaction picture, which takes into account exactly the effects of the two single particles on each other, can be expressed in terms of six parameters; either the $A_{ij}{}^{(k)}$, or, the physically more perspicuous parameters, the observed masses M_1^2, M_2^2, the mixing parameters X_1, X_2, and the coupling constants in the partially renormalized currents $J_1{}^R$, $J_2{}^R$.

To complete the presentation we must give the technique for handling external lines. We do this by making use of the asymptotic condition. In order to use this, we must introduce fields ψ_i, which have discrete frequencies corresponding to only the ith single-particle state. In other words, we must introduce the "normal coordinates,"

$$\psi=L\phi\,, \tag{3.21}$$

where L is the matrix that diagonalizes the propagator Δ.

We define the diagonal matrix

$$[K_M(\partial^2)]_{ij}=(-\partial^2+M_i^2)\delta_{ij}. \tag{3.22}$$

Thus, L is given by[14]

$$K(\partial^2)-A(\partial^2)=L^T K_M(\partial^2)L\,, \tag{3.23}$$

[10] We shall see later that when one of the particles is a photon, due to the gauge invariance and the fact that the photon mass is zero, the photon pole occurs only in one element of $\Delta(p^2)$, namely, the photon-photon propagator. (Ref. 1.)

[11] By partial renormalization we mean that part of field renormalization, which is due to the one-particle parts of the interaction. This is the direct generalization of mass renormalization for a single field. We do not discuss conventional coupling constant and field renormalization induced by radiative corrections, coming from the residual interactions J_i. This accounts for the absence of Z factors in (3.29). See Ref. 6.

[12] In the approximation in which A_{12} can be neglected in (3.7), which relates the true and bare masses, we have $X_i\simeq A_{12}(M_i^2)/(M_1^2-M_2^2)$. That the propagator has the form (3.17) is clear, since

$$\langle 0|\phi_1{}^R|M_1\rangle=1,\quad \langle 0|\phi_2{}^R|M_1\rangle=X_1, \text{ etc.,}$$

and

$$\Delta_{ij}{}^R=\langle T(\phi_i{}^R,\phi_j{}^R)\rangle.$$

[13] The partial renormalization, which has been done, has the effect of replacing the unrenormalized coupling constants g_i, appearing in j_i, by the partially renormalized constants $g_i{}^R=R_i^{1/2}g_i$. (See Ref. 11.)

[14] Explicitly, the matrix L is

$$L=(M_1^2-M_2^2)^{-1/2}\times\begin{pmatrix}(-M_2^2+m_1^2-A_{11}(M_2^2))^{1/2} & (-M_2^2+m_2^2-A_{22}(M_2^2))^{1/2} \\ -(M_1^2-m_1^2+A_{11}(M_1^2))^{1/2} & (M_1^2-m_2^2+A_{22}(M_1^2))^{1/2}\end{pmatrix}.$$

or

$$\Delta(p^2)=L^{-1}K_M^{-1}(-p^2)(L^T)^{-1}.$$

The currents $\mathcal{J}_i$, associated with ψ_i are

$$\mathcal{J}=JL^{-1}. \quad (3.24)$$

Equating the residues at the poles in the relation

$$J_i\Delta_{ij}J_j=J_i^R\Delta_{ij}^RJ_j^R=\mathcal{J}_iK_{Mij}^{-1}\mathcal{J}_j, \quad (3.25)$$

we have immediately, using (3.17),

$$\mathcal{J}_1=J_1^R+X_1J_2^R, \quad (3.26)$$

$$\mathcal{J}_2=J_2^R-X_2J_1^R. \quad (3.27)$$

The diagonalized equations of motion are thus

$$K_M\psi=\mathcal{J}. \quad (3.28)$$

The asymptotic condition can now be defined in terms of these fields.

A scattering matrix element involving a type-1 particle, with four-momentum p_1, can be written, as in (2.11). (See also Ref. 6.)

$$\begin{aligned}{}_{\text{out}}\langle f|p_1,i\rangle_{\text{in}}&=\int e^{ip_1x}\overleftrightarrow{\partial}_t\langle f|\psi_1^{\text{in}}(x)|i\rangle d^3x\\&=\int e^{ip_1x}\overleftrightarrow{\partial}_t\langle f|\psi_1(x)|i\rangle d^3x\\&=\int e^{ip_1x}(-\partial^2+M_1^2)\langle f|\psi_1(x)|i\rangle d^4x\\&=\int e^{ip_1x}\langle f|\mathcal{J}_1(x)|i\rangle d^4x\\&=\int e^{ip_1x}\langle f|J_1^R(x)+X_1J_2^R(x)|i\rangle d^4x. \quad (3.29)\end{aligned}$$

This last expression is just what one would expect from "graphology." We see from Eq. (3.18), and Ref. 12, that X_1 is essentially the effective $\phi_1\phi_2$ vertex, divided by the propagator of a particle of mass M_2 evaluated at the mass M_1 (see Appendix).

If the mass difference is small and the coupling of J_2 is much stronger than J_1, the appearance of the second term depending on X_1 may introduce a considerable enhancement of the effective coupling of type-1 particles to the other fields.[15]

Note that the whole calculation could be carried out using the diagonalized fields ψ without changing any of the physical results. In this case the propagator matrix is diagonal, each element having only one pole. However, the particles are now coupled to the $\mathcal{J}_i$ which are linear combinations of the original currents, which depend on the mixing parameters, X_i.

[15] Effects such as these were reported by S. Berman and S. Drell at the MIT-Harvard Conference, 1963 (unpublished). However, see Sec. 5.

4. COUPLED SPIN-ONE FIELDS

We now consider the case of two particles of spin one. We use the formalism developed by the authors.[16] If the fields are $\phi_\mu^{(i)}$ $(i=1,2)$, the equations of motion are

$$\left[-(\partial^2\delta_{\mu\nu}-\partial_\mu\partial_\nu)+m_k^2\left(\delta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\lambda_k^2}\right)\right]\phi_\nu^{(k)}=j_\mu^{(k)}, \quad (4.1)$$

or

$$\left[(-\partial^2+m_k^2)\tau_{\mu\nu}(\partial)+\left(\frac{m_k}{\lambda_k}\right)^2(-\partial^2+\lambda_k^2)\lambda_{\mu\nu}(\partial)\right]\phi_\nu^{(k)}=\tau_{\mu\nu}(\partial)j_\nu^{(k)}+\lambda_{\mu\nu}(\partial)j_\nu^{(k)}, \quad (4.2)$$

where

$$\tau_{\mu\nu}(\partial)=\delta_{\mu\nu}-\partial_\mu\partial_\nu/\partial^2 \quad (4.3)$$

and

$$\lambda_{\mu\nu}(\partial)=\partial_\mu\partial_\nu/\partial^2 \quad (4.4)$$

are the projection operators for the "covariant transverse"—space-like and "covariant longitudinal"—time-like—components of the fields. Thus, m_k are the bare masses of the physical spin-one particles; λ_k are the masses of the time-like (spin-zero) mesons. If the current $j^{(k)}$ is conserved,

$$\lambda_{\mu\nu}(\partial)j_\nu^{(k)}=0, \quad (4.5)$$

and the particles of mass λ^i have no interaction, but serve merely to specify a gauge.[17]

The formalism of Sec. 2 can be taken over and applied directly and independently to $\tau_{\mu\nu}\phi_\nu^{(i)}$ and $\lambda_{\mu\nu}\phi_\nu^{(i)}$. Thus, for example, we can write the A matrix as

$$A_{\mu\nu}(-p^2)=\tau_{\mu\nu}(p)A^{(\tau)}(-p^2)+\lambda_{\mu\nu}(p)A^{(\lambda)}(-p^2). \quad (4.6)$$

The propagator is

$$\Delta_{\mu\nu}(p^2)=\tau_{\mu\nu}\Delta^{(\tau)}(p^2)+\lambda_{\mu\nu}\Delta^{(\lambda)}(p^2), \quad (4.7)$$

where

$$[K_{ij}^{(\tau)}(-p^2)-A_{ij}^{(\tau)}(-p^2)]\Delta^{(\tau)}(p^2)=1 \quad (4.8)$$

and

$$[K_{ij}^{(\lambda)}(-p^2)-A_{ij}^{(\lambda)}(-p^2)]\Delta^{(\lambda)}(p^2)=1. \quad (4.9)$$

Here

$$K_{ij}^{(\tau)}(\partial^2)=(-\partial^2+m_i^2)\delta_{ij} \quad (4.10)$$

and

$$K_{ij}^{(\lambda)}(\partial^2)=\frac{m_i^2}{\lambda_i^2}(-\partial^2+\lambda_i^2)\delta_{ij}. \quad (4.11)$$

As in Sec. 3, the one-particle part of the general interaction can be treated in terms of twelve phenomenological constants (six each for the space-like and time-like parts) which can be chosen in a manner analogous to that in Sec. 3.

[16] G. Feldman and P. T. Matthews, Phys. Rev. **130**, 1633 (1963).

[17] A vector meson coupled to a nonconserved current is given by taking $\lambda\to\infty$, whereas for a photon λ is arbitrary. This expresses the gauge invariance of photon interactions. (See Ref. 16.)

5. THE PHOTON-VECTOR MESON INTERACTION

We are particularly interested in the case in which one of the particles involved is the photon,

$$\phi_\mu^{(1)}=A_\mu. \tag{5.1}$$

We replace the particle labels, 1, 2 by A, V, respectively.[18] For the moment we allow the photon to have a mass, but the gauge invariance requires that

$$\partial_\mu j_\mu^A=0. \tag{5.2}$$

From (3.4) and (5.2) it follows that[19]

$$A_{AA}^{(\lambda)}(-p^2)=A_{AV}^{(\lambda)}(-p^2)=0. \tag{5.3}$$

The "longitudinal" part of the propagator, as defined in (4.9), is

$$\Delta^{(\lambda)}(p^2)=\begin{bmatrix} \dfrac{\lambda_A^2}{m_A^2(p^2+\lambda_A^2)} & 0 \\ 0 & \dfrac{\lambda_V^2}{m_V^2(p^2+\lambda_V^2)-\lambda_V^2A_{VV}^{(\lambda)}(-p^2)} \end{bmatrix}, \tag{5.4}$$

showing that the time-like photons are not coupled to the rest of the system.[20] The time-like mesons are coupled through the nonconserved part of j_μ^V to the other fields in the system, and thereby undergo a mass renormalization.[21]

The "transverse" part of the propagator is defined by (4.8). This is completely gauge invariant, since the presence of the factor $\tau_{\mu\nu}$ in (4.7) automatically excludes the possibility of any dependence on a particular gauge. Thus, the interaction is described in terms of six phenomenological constants which can be chosen, as in Sec. 3, to be the observed masses M_A^2, M_V^2 of the photons and meson, and the mixing parameters X_A, X_V and the partially renormalized coupling constants.

However, the photon mass is observed to be zero, and this experimental fact must be included in the formalism in addition to the requirements of gauge invariance. This means that $\Delta^{(\tau)}(p^2)$ must have a pole at $p^2=0$, and hence

$$\det[K^{(\tau)}(-p^2)-A^{(\tau)}(-p^2)] \equiv[p^2+m_A^2-A_{AA}^{(\tau)}(-p^2)] \times[p^2+m_V^2-A_{VV}^{(\tau)}(-p^2)]-[A_{AV}^{(\tau)}(-p^2)]^2 \tag{5.5}$$

has a zero at $p^2=0$. Putting

$$A^{(\tau)}(-p^2)=G^{(\tau)}-p^2H^{(\tau)}, \tag{5.6}$$

this requires that

$$(m_A^2-G_{AA}^{(\tau)})(m_V^2-G_{VV}^{(\tau)})-[G_{AV}^{(\tau)}]^2=0. \tag{5.7}$$

It is conceivable that the constants G and the bare masses m_A and m_V are precisely such that this equation is satisfied as a dynamical accident. This would mean that the mass-generating effects of the interaction, of the type proposed by Schwinger[22] are operating, but are exactly cancelled by the nonvanishing bare mass. This could hardly happen as a dynamical fluke and, if it is indeed the case, strongly suggests the operation of some as yet undiscovered general principle of which this is a particular consequence. This general principle is *not* gauge invariance. It seems to us much more probable that the correct solution is the one given by conventional perturbation theory,[23] namely,

$$m_A^2=0,\quad G_{AA}^{(\tau)}=G_{AV2}^{(\tau)}=0. \tag{5.8}$$

This imples that

$$A_{AA}^{(\tau)}(-p^2)=-p^2H_{AA}^{(\tau)}, \tag{5.9}$$

$$A_{AV}^{(\tau)}(-p^2)=-p^2H_{AV}^{(\tau)}, \tag{5.10}$$

and hence, by (3.18), the mixing parameter X_A vanishes,

$$X_A=0. \tag{5.11}$$

The partially renormalized propagator matrix is then

$$\Delta_{AA}^{(\tau)R}(p^2)=\frac{1}{p^2}+\frac{X_V^2}{p^2+M_V^2},\quad \Delta_{AV}^{(\tau)R}(p^2)=\frac{X_V}{p^2+M_V^2},\quad \Delta_{VV}^{(\tau)R}(p^2)=\frac{1}{p^2+M_V^2}, \tag{5.12}$$

so that only the photon-photon propagator has a pole at the photon mass.[10] The mixing parameter X_V is of order e, so the coupling of a *virtual* photon to any

[18] The field $\phi_\mu^{(2)}$ can be taken to represent either a ρ or ω meson. However, in what follows we restrict ourselves to the approximation in which the V particle can be treated as stable.

[19] Alternatively,

$$\Pi_{\mu\nu}^{*AA}(-p^2)=\tau_{\mu\nu}\Pi^{*AA}(-p^2),$$
$$\Pi_{\mu\nu}^{*AV}(-p^2)=\tau_{\mu\nu}\Pi^{*AV}(-p^2).$$

[20] We are assuming throughout this section that the theory is invariant under gauge transformations of the field A_μ. As a consequence, $\Delta^{(\lambda)}(p^2)$ is diagonal. The same is true for the field A_μ' for which, by definition, $\Delta^{(\tau)}(p^2)$ is also diagonal. This is to be contrasted with the approach of Baroff and Fulton (Ref. 1) who start with a nondiagonal $\Delta^{(\lambda)}(p^2)$ and transform to gauge-invariant variables.

[21] As pointed out in Ref. 17, we are interested in the limit $\lambda_V\to\infty$.

[22] J. Schwinger, Phys. Rev. **125**, 397 (1962).

[23] We remind the reader that (5.8) states that $\Pi_{\mu\nu}^*(-p^2)=(p^2\delta_{\mu\nu}-p_\mu p_\nu)f(p^2)$, for the AA and AV elements, where $f(p^2)$ is regular at $p^2=0$. The regularity of $f(p^2)$ is *not* a consequence of gauge invariance, but is a plausible inference from the observed vanishing of the photon mass.

Alternatively, the above inference implies that of the two counter terms in the Lagrangian involving direct $A-V$ coupling, $G_{AV}V_\mu\tau_{\mu\nu}A_\nu-\frac{1}{2}H_{AV}(\partial_\mu V_\nu-\partial_\nu V_\mu)(\partial_\mu A_\nu-\partial_\mu A_\mu)$, only the second survives.

strong-interaction complex is enhanced through the propagator $\Delta_{AV}{}^{(\tau)R}$. This represents the effects of the direct transformation of the photon into a V meson, which then interacts strongly with the complex. The best-known example of this is in electron-proton scattering. (See Appendix.)

For the interaction of a real photon we can take over directly the formalism of Sec. 3, and in particular (3.29). Owing to the vanishing of X_A, there is *no enhancement* for external photons, due to direct conversion. The contribution from all such graphs vanishes identically as a consequency of the observed vanishing of the photon mass.[24]

CONCLUSIONS

We have shown how to generalize the conventional procedure of mass renormalization to the case of two interacting particles with the same quantum numbers, but different mass. As in the single-particle case, the procedure is to extract from the interaction currents the one-particle parts and treat them exactly. A perturbation expansion may then be carried out in terms of the residual interaction. Instead of the true mass, which appears in the single-field case, the two fields require six parameters, which may conveniently be taken to be the two true masses, the mixing parameters, and the partially renormalized coupling constants.

In general, the fact that the particles can convert directly into each other gives rise to resonance-type enhancements for both, virtual and external particles. However, if one of the particles is a photon, the vanishing of the photon mass causes the contributions from graphs in which a *real* photon converts directly to a spin-one meson, to vanish identically. On the other hand, for *virtual* photons (as for example in the proton electromagnetic form factors) the resonance effects persist.

The above conclusions can alternatively be expressed in terms of the effective direct $A-V$ interaction. Terms of the form $V_\mu A_\mu$ have sometimes been assumed in the past.[15,22] The objection that this interaction term is not gauge invariant can easily be overcome by modifying it to $V_\mu \tau_{\mu\nu} A_\nu$. However, such a term, barring the dynamical accident discussed after (5.7), gives rise to a nonvanishing photon mass.

It should perhaps be stressed that we have reached these conclusions on the basis of renormalized field theory. Within this framework our results are quite general. Although the validity of field theory is doubted by some for purely strong interactions, it is our belief that the problems of electrodynamics discussed here certainly fall within this framework, and that our result is therefore quite general.

APPENDIX

We develop here a graphical formalism, which clarifies the physical significance of the algebraic manipulations carried out above, and also simplifies the setting up of specific calculations.

We set up a 'one-particle' interaction representation, in which the part of the Lagrangian to be treated exactly is

$$L_0=-\tfrac{1}{2}\phi_i{}^R R_i{}^{1/2}[K_{ij}(\partial^2)-A_{ij}(\partial^2)]R_j{}^{1/2}\phi_j{}^R. \quad \text{(A1)}$$

The interaction is then

$$L_{\text{int}}=\phi_i{}^R j_i{}^R-\tfrac{1}{2}\phi_i{}^R R_i{}^{1/2}A_{ij}(\partial^2)R_j{}^{1/2}\phi_j{}^R. \quad \text{(A2)}$$

Here $\phi_i{}^R$ are the partially renormalized fields,

$$R_i{}^{1/2}\phi_i{}^R=\phi_i.$$

The graphical technique is completely conventional except for the factors corresponding to the two particles associated with the fields ϕ. For these there are three types of propagator which may be represented as illustrated in Table I. We use solid and dotted lines to denote particles of types 1 and 2, respectively. It will be observed that these must join to $J_1{}^R$ and $J_2{}^R$, respectively.

If an external particle of type 1, four-momentum p_1, occurs in a process the procedure is to draw all graphs in which this particle is represented by a $1-1$ line coupled to $J_1{}^R$, and by a $1-2$ line coupled to $J_2{}^R$.

TABLE I. A graphical representation of internal and external particle lines giving the corresponding factors in the matrix element including the interaction currents. The relation between the external line graphs and the corresponding factor is discussed in the text.

Internal lines	
×——× (with ···· above)	$\sim J_1{}^R\left[\dfrac{1}{p^2+M_1{}^2}+\dfrac{X_2{}^2}{p^2+M_2{}^2}\right]J_1{}^R$
○········○ (with —— above)	$\sim J_2{}^R\left[\dfrac{X_1{}^2}{p^2+M_1{}^2}+\dfrac{1}{p^2+M_2{}^2}\right]J_2{}^R$
×—— ······○	$\sim J_1{}^R\left[\dfrac{X_1}{p^2+M_1{}^2}+\dfrac{X_2}{p^2+M_2{}^2}\right]J_2{}^R$
External lines (particle 1)	
——× (with ···· above)	$\sim J_1{}^R\cdots$
+ ······○ (with —— above)	$+\,X_2J_2{}^R\cdots$
(particle 2)	
········○ (with —— above)	$\sim J_2{}^R\cdots$
+ ——× (with ······ above)	$+\,X_2J_1{}^R\cdots$

[24] Thus, subject to the qualifications expressed just above Eq. (5.8) there seems to be no justification for the model of Π^0-decay proposed by M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962). The fallacy in their argument lies explicitly in the form of renormalized field equations assumed by M. Gell-Mann and F. Zachariasen, [Phys. Rev. **124**, 953 (1961), Eq. (4.2)] which are only valid when the two fields A_μ and ϕ_μ are *not* coupled via the mass operator.

These graphs are to be evaluated by including the corresponding propagators, $\Delta_{11}{}^R(p^2)$ and $\Delta_{12}{}^R(p^2)$, coupled appropriately to $J_1{}^R$ and $J_2{}^R$, respectively, multiplying by $p^2+M_1{}^2$, and then evaluating at $p^2+M_1{}^2=0$. (This procedure is correct, but trivial in the one-field case.) This gives rise to graphs with a factor $J_1{}^R$ arising from $\Delta_{11}{}^R(p^2)J_1{}^R$, and graphs with a factor $X_1J_2{}^R$ arising from $\Delta_{12}{}^R(p^2)J_2{}^R$.

As remarked in the text, the two types of terms are just those to be expected on 'physical' grounds. In the first one, particle 1 enters the graphs directly through its current $J_1{}^R$. In the second type, particle 1 converts directly to particle 2, and then enters the graph through $J_2{}^R$. The approximate expression for X_i, given in Ref. 12, is just the factor to be expected, since $A_{12}(M_1{}^2)$ is the effective direct-conversion interaction, and $(M_1{}^2-M_2{}^2)^{-1}$ is the propagator for particle type 2 evaluated at $p^2+M_1{}^2=0$.

The operative parts of the currents, $J_i{}^R$, are the $j_i{}^R$ which arise directly from the original interaction. These currents induce various physical processes, among them being proper self-energy effects, through which one of the particles converts either back to itself, or to the other particle (with no single-particle intermediate state). Corresponding to each graph containing such a part is another graph, which is identical except that the self-energy part is replaced by a direct transition of one particle into another, arising from the counter term A_{ij}. The definition of A_{ij}, (3.4), is such that the two graphs precisely cancel, when

$$p^2+M_k{}^2=0\,,$$

where p is the four-momentum of the line in which the self-energy part occurs. (See Ref. 8.) This is a direct generalization of the cancelling of the leading term in conventional single-field self-energy graphs by the δm^2 counter term. (See Ref. 5.)

The only irreducible proper self-energy part generated from j_i is the lowest order self-energy bubble, $\Pi_{ij}{}^*(-p^2)$. Since this is bilinear in j_i and j_j, the corresponding graph generated from L_{int} is

$$\begin{aligned}R_i{}^{1/2}\Pi_{ij}{}^*(-p^2)R_j{}^{1/2}\\ \equiv R_i{}^{1/2}[(p^2+M_1{}^2)C_{ij}{}^{(1)}+(p^2+M_2{}^2)C_{ij}{}^{(2)}\\ +(p^2+M_1{}^2)(p^2+M_2{}^2)\Pi^c{}_{ij}(-p^2)]R_j{}^{1/2}.\end{aligned}\quad \text{(A3)}$$

The counter term from L_{int} is

$$-R_i{}^{1/2}A_{ij}(-p^2)R_j{}^{1/2}. \quad \text{(A4)}$$

This has been defined to cancel the term in Π^* depending on $C^{(1)}$ and $C^{(2)}$.

Note that for external lines, $J_i{}^R$ can be replaced by $j_i{}^R$, where $j_i{}^R$ couples to the main body of the graph, and does not recombine to form a self-energy part. In view of (A3) and (A4) the contributions from self-energy parts in external lines cancel exactly with the counter terms.

In practice, even in a renormalizable theory, the constants $C^{(i)}$ are infinite, but "physical" quantities can be expressed in terms of the parameters M_i, X_i, and partially renormalized coupling constants. Self-mass effects may be estimated, through (A3) with the use of a cutoff, by solution of (3.13).

A simple example of the above formalism is the electromagnetic form factors of the proton, as deduced from electron-proton scattering. In this case, particle 1 is the photon, and particle 2 a V particle (ρ or ω meson). The appropriate diagrams are shown in Fig. 1. The coupling of the photon and meson to the electron and proton can be expressed in terms of four currents. In an obvious notation the orders of magnitude of these currents are

$$\begin{aligned}j_{A\mu}{}^P\sim e,\quad j_{V\mu}{}^P\sim g_V(\simeq 1),\\ j_{A\mu}{}^e\sim e,\quad j_{V\mu}{}^e\sim e^6.\end{aligned}\quad \text{(A5)}$$

The mixing parameter X_V is of order e. Thus,

$$\begin{aligned}M=j_{A\mu}{}^e\left\{\frac{1}{p^2}+\frac{X_V{}^2}{p^2+M_V{}^2}\right\}j_{A\mu}{}^P+j_{A\mu}{}^e\frac{X_V}{p^2+M_V{}^2}j_{V\mu}{}^P\\ +j_{V\mu}{}^e\frac{X_V}{p^2+M_V{}^2}j_{A\mu}{}^P+j_{V\mu}{}^e\frac{1}{p^2+M_V{}^2}j_{V\mu}{}^P.\end{aligned}\quad \text{(A6)}$$

We have already replaced $\tau_{\mu\nu}$ by $\delta_{\mu\nu}$ since the electromagnetic current is conserved. To order e^2 we have, by (A5),

$$M\simeq j_{A\mu}{}^e\left[\frac{1}{p^2}j_{A\mu}{}^P+\frac{X_V}{p^2+M_V{}^2}j_{V\mu}{}^P\right]\equiv j_{A\mu}{}^e\frac{1}{p^2}F_\mu(p^2)\,, \quad \text{(A7)}$$

where $F_\mu(p^2)$ is the required form factor. Thus,

$$F_\mu(p^2)=j_{A\mu}{}^P+\frac{p^2X_V}{p^2+M_V{}^2}j_{V\mu}{}^P. \quad \text{(A8)}$$

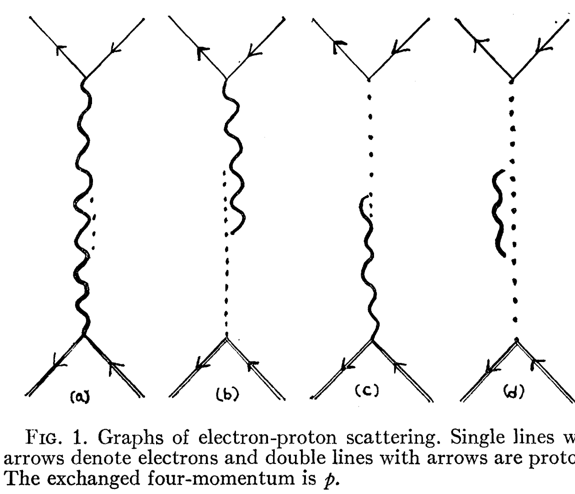

FIG. 1. Graphs of electron-proton scattering. Single lines with arrows denote electrons and double lines with arrows are protons. The exchanged four-momentum is p.

The 'Dirac' form factor F_1 is the coefficient of γ_μ. Since

$$j_{A\mu}{}^P = e\gamma_\mu, \quad (A9)$$

and taking

$$j_{V\mu}{}^P = g_V\gamma_\mu, \quad (A10)$$

$$F_1(p^2) = e + \frac{p^2 X_V g_V}{p^2 + M_V{}^2}, \quad (A11)$$

which is just the Clementel-Villi form obtained from subtracted dispersion relations.[25]

In this two-field theory, perturbation calculations could alternatively have been developed in terms of the diagonalized fields ψ_i, defined in Eq. (3.21). This has the advantage that the propagator matrix is diagonal. However, all the complication is transferred to the interaction. The mixing parameters X_i now appear in the currents $\mathcal{J}_i$ and the extraction of the finite parts of the mass operator is considerably less transparent.

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Photoproduction of π^+ Mesons from Hydrogen*

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The differential cross section for π^+ photoproduction has been determined at 19 points, at center-of-mass angles from 30 to 150 deg, and at photon energies from 162 to 225 MeV. The data are concentrated near 180 MeV, where a full angular distribution has been determined. The relative values of the cross sections are accurate to 5% or better, and the absolute normalization is accurate to 4%. The experiment provides data of improved accuracy which are in general consistent with previous results. The extrapolation to threshold gives a value for $(k^*/p^*)(d\sigma/d\Omega)^*$ at threshold of 16.1 ± 0.7 μb/sr, where k^*, p^*, and $(d\sigma/d\Omega)^*$ are the photon energy, pion momentum, and differential cross section, all in the center-of-mass system.

INTRODUCTION

THE process $\gamma+p\rightarrow\pi^++n$ has been studied for a long time, and the reaction provides, particularly at low energy, one of the simplest testing grounds for our knowledge of pion physics. The advent of dispersion theory has resulted in new theoretical calculations,[1,2] with the most recent ones including the effect of the π-π interaction.[3,4] The measurements reported here were undertaken to determine differential cross sections with improved accuracy, in the region moderately close to threshold.

The pions were detected with a magnet spectrometer and counter telescope, and the arrangement was appropriate to pions with laboratory momenta from about 55 to 102 MeV/c, at laboratory angles between about 30° and 130°. A complete angular distribution could be measured for a laboratory gamma-ray energy of 180 MeV, while at other energies cross sections were determined for angles where the pion momentum lay within the experimental range.

When these measurements were begun, the work of Beneventano *et al.*[5,6] constituted the most accurate and comprehensive study in this energy interval. More recently, Adamovich *et al.*[7] have performed an experiment using emulsion techniques, with accuracy comparable to this one.

APPARATUS

The intensity of the electron beam of the Stanford Mark III linear accelerator was measured with a secondary emission monitor (SEM)[8] consisting of three foils of 0.0003-in. aluminum, enclosed in a separate vacuum chamber with 0.003-in. dural windows. The SEM was automatically oscillated both horizontally and vertically in order to average over a foil area about 1.5-in. square. This monitor was calibrated at regular intervals against a Faraday cup of efficiency

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