PHYSICAL REVIEW VOLUME 132, NUMBER 2 15 OCTOBER 1963

Induced Effects in β Decay*

J. N. HUFFAKER†‡ AND EUGENE GREULING
Physics Department, Duke University, Durham, North Carolina
(Received 20 February 1963; revised manuscript received 17 June 1963)

Formulas for β-decay spectra are calculated for Gamow-Teller allowed and first-forbidden unique selection rules. Included are all possible effects induced by strong interactions, up to first order in momentum transfer. From the measured ratio of B^{12} and N^{12} ft values, and from the assumption that the reduced nuclear matrix elements for these transitions are identical, a relation is deduced between the magnitudes of the two possible induced couplings in the axial vector interaction (the "induced tensor" and "induced pseudoscalar" couplings). The "induced pseudoscalar" coupling must be small in order to produce the observed rate of μ capture in C^{12}, which suggests that the B^{12}–N^{12} ft ratio is due primarily to the "induced tensor" coupling. Positive evidence of an "induced tensor" coupling might be obtained from a careful measurement and analysis of the first-forbidden unique β spectrum of N^{16}.

I. INTRODUCTION

THE term "induced effects" refers to the alterations imposed by strong couplings on the universal vector-axial vector (V-A) weak interaction.[1] We write the effective β-decay interaction Hamiltonian density in the form

$$\mathcal{H}=2^{-1/2}G\bar{\psi}_N H\tau_+\psi_N+\text{H.c.}, \tag{1}$$

where the first term of Eq. (1) gives rise to β^- decay, and its Hermitian conjugate (H.c.) gives rise to β^+ decay. In the absence of strong interactions, the operator H should have the form[1]

$$H_{\text{"bare"}}=\gamma_\mu(1+\gamma_5)L_\mu, \tag{2}$$

where L_μ is the lepton current,

$$L_\mu\equiv(\mathbf{L},L_4)=\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_\nu. \tag{3}$$

We introduce induced effects by writing the effective H appropriate for small momentum transfer as[2]

$$H=[\gamma_\mu(1+\lambda\gamma_5)+i\sigma_{\mu\nu}(A+B\gamma_5)\partial/\partial x_\nu +(C+D\gamma_5)\partial/\partial x_\mu]L_\mu. \tag{4}$$

In Eq. (4), the renormalization of the leading term of the A interaction is represented by λ, and A, B, C, and D are the form factors multiplying the various Lorentz invariants involving first derivatives of L_μ, corresponding to effects first-order in momentum transfer. Time-reversal invariance requires that λ, A, B, C, and D be real.

According to the conserved vector current (CVC) theory,[1] the nucleon current of the V interaction [bracketed terms of Eq. (4) with coefficients 1, A, and C] is a conserved current, proportional to the plus component of the total isotopic spin current. By analogy with electrodynamics, the magnitude of the "weak magnetism" (WM) coupling constant A is given by[3]

$$A\approx(\kappa_p-\kappa_n)/e\approx(3.7/2M), \tag{5}$$

in units $\hbar=c=m=1$, where $\kappa_p-\kappa_n$ is the difference of proton and neutron anomalous magnetic moments, and m and M are masses of electron and nucleon. The correctness of the above value of A has been demonstrated by measurements of β-decay spectra.[4–6] The CVC theory also predicts that the "induced scalar" (IS) coupling constant C must vanish.

The behavior of the β interaction under the transformation G, the product of charge symmetry and charge conjugation, permits the separation of the terms of H into two classes,[2] which transform differently under G. The "induced tensor" (IT) term, with coefficient B, and the IS term are in a separate class from the other four terms. If it is assumed that the β-decay interaction, like the strong interaction, is invariant under G, then one must set $B=C=0$, as is customarily done.[7] The CVC theory requires the V interaction to be G invariant; however, there is no good evidence[2] for applying this invariance principle to the A interaction, since the axial-vector current is not conserved under strong interactions.

The magnitude of the "induced pseudoscalar" (IP) coupling constant D has been estimated with the use of dispersion theory to be[7]

$$D/\lambda\approx-0.04, \tag{6}$$

a magnitude which is practically unobservable in β decay. Measurements[8] of the rate of μ capture in C^{12} have been interpreted[9] as requiring that D have this order of magnitude.

We have calculated formulas for Gamow-Teller (GT)

* Supported by the National Science Foundation and the U. S. Army Research Office, Durham, North Carolina.
† Based in part on a thesis submitted to the Graduate School of Arts and Sciences of Duke University in partial fulfillment of the requirements of the Ph.D. degree (1963).
‡ Present address: Physics Department, University of Alabama, University, Alabama.
1 R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
2 S. Weinberg, Phys. Rev. **112**, 1375 (1958).
3 M. Gell-Mann, Phys. Rev. **111**, 362 (1958).
4 T. Mayer-Kuckuk and F. C. Michel, Phys. Rev. **127**, 545 (1962).
5 Y. K. Lee, L. W. Mo, and C. S. Wu, Phys. Rev. Letters **10**, 253 (1963).
6 N. W. Glass and R. W. Peterson, Phys. Rev. **130**, 299 (1963).
7 M. Goldberger and S. Treiman, Phys. Rev. **111**, 354 (1958).
8 G. T. Reynolds, D. B. Scarl, R. A. Swanson, J. R. Waters, and R. A. Zdanis, Phys. Rev. **129**, 1790 (1963).
9 M. Morita and A. Fujii, Phys. Rev. **118**, 606 (1960). This paper contains references to older data and analyses.

allowed and first-forbidden unique β-decay spectra, using the effective interaction given by Eq. (4). In order to handle derivatives of lepton functions correctly, we employed the spherical tensor formalism.[10,11] Previously published formulas omit terms which may be significant in isolating induced effects.[12–15]

Weinberg[2] suggested that a finite IT coupling might cause the ft values of B^{12} and N^{12} to differ by a few percent. Recent measurements of the lifetimes of these transitions indicate that the ft values differ by $(14\pm2.5)\%$[16] or $(16\pm3)\%$.[17] Assuming that the nuclear matrix elements for these decays are identical, we deduced from our formulas a relation between the strengths of IT and IP couplings required to produce the observed ft ratio.

Next, we amended the μ-capture analysis of Morita and Fujii[9] to include IT coupling. We found that μ capture is relatively insensitive to IT coupling. Consequently, the measured rate[8] of μ capture in C^{12} restricts IP coupling to an amount slightly larger than the Goldberger-Treiman estimate [Eq. (6)].

The conclusion to be deduced from our analyses of the B^{12}–N^{12} ft ratio and of μ capture in C^{12} is that there is a significant amount of IT coupling in the β interaction. Our calculation of the effect of IT coupling on the shape of the first-forbidden unique β spectrum of N^{16} indicates that a measurement of this spectrum with attainable accuracy[5] could provide evidence for the violation of G invariance in the weak interaction.

II. CALCULATION OF SPECTRA

Writing Eq. (4) in terms of even and odd nuclear operators (using the standard representation $\boldsymbol{\gamma}=-i\beta\boldsymbol{\alpha}$ and $\gamma_4=\beta$), and retaining terms which might contribute significantly to Gamow-Teller (GT) β^- spectra, we obtain

$$H_{\rm GT}=i\lambda\beta\boldsymbol{\sigma}\cdot\mathbf{L}-i\beta\boldsymbol{\alpha}\cdot\mathbf{L}+\lambda\beta\gamma_5L_4-iA\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\times\mathbf{L}$$
$$-iB\boldsymbol{\sigma}\cdot(\partial\mathbf{L}/\partial x_4-\boldsymbol{\nabla}L_4)+D\gamma_5(\boldsymbol{\nabla}\cdot\mathbf{L}+\partial L_4/\partial x_4).\quad(7)$$

The two significant features of the method we employ[11] are that a Foldy-Wouthuysen transformation[18] (FWT) is used to replace odd nuclear operators and that L_μ and its derivatives are treated with techniques of Racah algebra.

Use of the FWT can be illustrated with the treatment of the term $-i\beta\boldsymbol{\alpha}\cdot\mathbf{L}$ of Eq. (7). To first order in $1/M$ this term is replaced by the following two terms appropriate to GT selection rules:

$$-i\beta\boldsymbol{\alpha}\cdot\mathbf{L}\approx-(i/M)\mathbf{L}\cdot\mathbf{p}-(i/2M)\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\times\mathbf{L}.$$

In GT allowed transitions, these terms yield matrix elements corresponding to orbital and spin transition magnetic moments for "bare" nucleons. Recalling that the term proportional to A corresponds to the anomalous magnetic moment [see Eq. (5)], we have Gell-Mann's analogy[3] in detail. There is one peculiarity about the β-decay analogs: because of the Coulomb field of the nucleus, the spectral shape of the "orbital" term is different from the shape of the "spin" term.[19]

After applying the FWT to all odd operators of Eq. (7), we further simplify $H_{\rm GT}$ by use of the nonrelativistic approximation $\beta\approx1$ and the identity

$$\partial L_\mu/\partial x_4\equiv W_0L_\mu,\quad(8)$$

where W_0 is the maximum β energy. The resulting Hamiltonian is

$$H_{\rm GT}=i\lambda\boldsymbol{\sigma}\cdot\{(1-bW_0)\mathbf{L}-a\boldsymbol{\nabla}\times\mathbf{L}$$
$$+[b+(2M)^{-1}-dW_0]\boldsymbol{\nabla}L_4-d\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{L})\}$$
$$-(i/M)\mathbf{L}\cdot\mathbf{p}-(\lambda/M)L_4\boldsymbol{\sigma}\cdot\mathbf{p},\quad(9)$$

where $\mathbf{p}$ is the nucleon-momentum operator, and where we have defined parameters

$$a\equiv\lambda^{-1}[A+(2M)^{-1}],$$
$$b\equiv\lambda^{-1}B,\quad(10)$$
$$d\equiv(D/2\lambda M).$$

We employ the following spherical tensor operators, all of which have real matrix elements:

$$S_{JL}{}^M\equiv\mathbf{T}_{JL}{}^M\cdot\boldsymbol{\sigma},$$
$$P_{JL}{}^M\equiv-(i/M)\mathbf{T}_{JL}{}^M\cdot\mathbf{p},\quad(11)$$
$$W_J{}^M\equiv-(i/M)Y_J{}^M\boldsymbol{\sigma}\cdot\mathbf{p},$$

where $Y_J{}^M$ is the usual spherical harmonic and $\mathbf{T}_{JL}{}^M$ is defined by [20]

$$\mathbf{T}_{JL}{}^M=\sum_m C(L1J;M-m,m)Y_L{}^{M-m}\boldsymbol{\xi}_m,$$

and $\boldsymbol{\xi}_m$ is a unit vector in the spherical basis. The reduced nuclear matrix element of an operator such as $S_{JL}{}^M$ is represented by $\langle S_{JL}\rangle$.

In calculating spectra we observe the order-of-forbiddenness convention of classifying terms by the power of the nuclear radius associated with each term. For this purpose $1/M$ is equivalent to R^2. We uniformly include all terms of order R^2 smaller than the principal term and

[10] L. C. Biedenharn and M. E. Rose, Rev. Mod. Phys. **25**, 729 (1953).
[11] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1315 (1954).
[12] D. Tadić, Nucl. Phys. **26**, 338 (1961).
[13] M. Morita, Phys. Rev. **113**, 1584 (1959).
[14] B. Eman and D. Tadić, Period. Math.-Phys. Astron. (Zagreb) **16**, 89 (1961).
[15] E. Greuling and N. Huffaker, Trans. N. Y. Acad. Sci. **24**, 591 (1962).
[16] T. R. Fisher, Phys. Rev. **130**, 2388 (1963).
[17] R. W. Peterson and N. W. Glass, Phys. Rev. **130**, 292 (1963).
[18] L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).
[19] Morita [Ref. (13)] uses methods which we feel are incorrect in calculating GT allowed spectra with WM coupling. First, he introduces WM into the interaction by simply multiplying $-i\beta\boldsymbol{\alpha}\cdot\mathbf{L}$ by 4.7, which is like saying that strong couplings renormalize both orbital and spin contributions to the magnetic moment. Second, since he does not differentiate lepton functions, he obtains a slightly different spectral shape for WM.
[20] M. E. Rose, *Elementary Theory of Angular Momentum* (McGraw-Hill Book Company, Inc., New York, 1957), Chap. V, p. 106.

TABLE I. Gamow-Teller allowed correction factor $C_0(W)$, written in the form $C_0(W)=\langle S_{10}\rangle^{-1}\sum_k N_k\langle\Omega_k\rangle C(k)$. Upper signs refer to β^- decay, lower signs to β^+ decay.

k	N_k	Ω_k	$C(k)$
1	1	S_{10}	L_0
2	$-1/9$	r^2S_{10}	$3q^2L_0+2qN_0$
3	$-4/9$	$2^{1/2}r^2S_{12}$	qN_0
4	$\frac{2}{3}$	$(1/3)^{1/2}rW_1$	$3N_0-qL_0$
5	$\mp(2/3\lambda)$	$(2/3)^{1/2}rP_{11}$	$3N_0+qL_0$
6	$(3M)^{-1}$	S_{10}	$(W_0-V)L_0-P_0$
7	$\pm(4/3)a$	S_{10}	$(2W-W_0-V)L_0-P_0$
8	$\pm\frac{2}{3}b$	S_{10}	$C(6)-3W_0L_0$
9	$\frac{2}{3}d$	S_{10}	$(V^2-W_0V-1)L_0+(W-q)P_0$ $+\frac{1}{3}r(dV/dr)C(4)$
10	$\frac{2}{3}[\pm a\pm b-(2M)^{-1}]$	$2^{1/2}S_{12}$	$(W_0-V)L_0-P_0+C(4)$
11	$-\frac{2}{3}d$	$2^{1/2}S_{12}$	$(V^2-W_0V-1)L_0+WP_0+3R_0$ $+[\frac{1}{3}r(dV/dr)-V]C(4)$

ignore terms of order R^4, etc. Thus, we calculate the square of the matrix element of the principal term $i\lambda\boldsymbol{\sigma}\cdot\mathbf{L}$ of $H_{\rm GT}$ and the interference between this term and each smaller term. This involves the assumptions that a, b, and d are of order of magnitude $1/M$.

Several modifications must be made in the above treatment in order to describe β^+ decay. The β^+ interaction Hamiltonian density is the Hermitian conjugate of that for the β^- interaction. If we write the H.c. term of Eq. (1) as

$$\mathcal{H}^+=2^{-1/2}G\bar{\psi}_NH^+\tau_-\psi_N\,, \tag{12}$$

then H^+ may be written as

$$H^+=[\gamma_\mu(1+\lambda\gamma_5)+i\sigma_{\mu\nu}(A-B\gamma_5)\partial/\partial x_\nu + (-C+D\gamma_5)\partial/\partial x_\mu]L_\mu{}^+, \tag{13}$$

a form similar to Eq. (4), except that the signs of the G-invariance-violating terms are reversed. Also, $L_\mu{}^+$ can be written in terms of positron creation and antineutrino annihilation operators as

$$L_\mu{}^+=\bar{\psi}_{\bar{e}}\gamma_\mu(1-\gamma_5)\psi_{\bar{\nu}}. \tag{14}$$

Therefore, correct formulas for positron decay are obtained by substitution of positron for electron functions, interchange of proton and neutron states, and changes of sign in the formulas for terms involving[2]: (1) interference between couplings which transform differently under G; and (2) interference between vector and axial vector couplings defined by the *lepton* currents.

Tables I and II list the various terms of the correction factors $C_0(W)$ and $C_1(W)$ for GT allowed and first-forbidden unique spectra, respectively. The correction factors are expressed in terms of electron energy W and potential energy V, maximum electron energy W_0, neutrino momentum q, reduced nuclear matrix elements of the operators defined in Eq. (11), and the standard tabulated electron radial function combinations.[21,22] The potential energy V of the β particle is defined to be negative for β^-, and positive for β^+ decays.

A nuclear model is required in order to evaluate the ratios of nuclear matrix elements appearing in these Tables. In Appendix A, we present formulas for evaluating appropriate matrix elements using the j-j coupling model.[23] In Appendix B, we discuss a procedure whereby lepton radial functions are averaged over the nuclear volume rather than evaluated at the nuclear surface.

III. $B^{12}-N^{12}$ *ft* RATIO

The mirror β transitions of B^{12} and N^{12} to the ground state of C^{12} have already played an important role in the study of induced effects in β decay, since the shapes of their β-energy spectra provide the best evidence of the existence of the WM coupling. The principal energy dependence of $C_0(W)$ (see Table I) for energetic tran-

TABLE II. First-forbidden unique correction factor $C_1(W)$, written in the form $C_1(W)=\langle rS_{21}\rangle^{-1}\sum_k N_k\langle\Omega_k\rangle C(k)$. Upper signs refer to β^- decay, lower signs to β^+ decay.

k	N_k	Ω_k	$C(k)$
1	1	rS_{21}	$9L_1+q^2L_0$
2	$-(1/25)$	r^3S_{21}	$30qN_1+2q^3N_0+75q^2L_1+5q^4L_0$
3	$-(4/25)$	$6^{1/2}r^3S_{23}$	$15qN_1+q^3N_0$
4	$\frac{2}{5}$	$(2/5)^{1/2}r^2W_2$	$45N_1+5q^2N_0-15qL_1-q^3L_0$
5	$\mp(2/5\lambda)$	$(3/5)^{1/2}r^2P_{22}$	$45N_1+5q^2N_0+15qL_1+q^3L_0$
6	$(2/5M)$	rS_{21}	$(W_0-V)C(1)-9P_1-q^2P_0+6qL_1-2q^2N_0$
7	$\pm(6/5)a$	rS_{21}	$(2W-W_0-V)C(1)-9P_1-q^2P_0-6qL_1-2q^2N_0$
8	$\pm\frac{4}{5}b$	rS_{21}	$C(6)-(5/2)W_0C(1)$
9	$\frac{4}{5}d$	rS_{21}	$(V^2-W_0V-1)C(1)+(W-q)(9P_1+q^2P_0)-6qVL_1$ $+2W_0N_0+2(W_0-V)R_0+\frac{1}{5}r(dV/dr)C(4)$
10	$\frac{2}{5}[\pm a\mp b-(2M)^{-1}]$	$6^{1/2}rS_{23}$	$(W_0-V)C(1)-9P_1-q^2P_0-2q^2N_0+6qL_1+C(4)$
11	$-\frac{2}{5}d$	$6^{1/2}rS_{23}$	$(V^2-W_0V-1)C(1)+W(9P_1+q^2P_0)-6qVL_1+2q^2VN_0$ $+45R_1+3q^2R_0+[\frac{1}{5}r(dV/dr)-V]C(4)$

[21] M. E. Rose, C. L. Perry, and N. M. Dismuke, Oak Ridge National Laboratory Report, ORNL-1459 (unpublished).
[22] C. P. Bhalla and M. E. Rose, Oak Ridge National Laboratory Report, ORNL-3207 (unpublished).
[23] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1326 (1954).

sitions in light nuclei is $C(7)$, the WM term, which produces a linear energy dependence with positive slope for β^- decay and with negative slope for β^+ decay. The experiments that proved the existence of WM[4–6] consisted of measuring the energy dependence of the ratio of correction factors of the B^{12} and N^{12} decays. The results are in agreement with the theoretical predictions,[3,24] obtained by estimating a from the lifetime of the analogous 15.11-MeV γ transition in C^{12}. The argument rests on the fact that the ground states of B^{12} and N^{12} and the 15.11 MeV excited state of C^{12} are members of an isotopic-spin triplet. Consequently, to the extent that nuclear forces are charge-independent, the transitions from these states to the ground state of C^{12} will have identical reduced nuclear matrix elements.

By contrast, the IT and IP coupling terms [$C(8)$, $C(9)$, and $C(11)$ in Table I] have practically no effect on the energy dependence of $C_0(W)$.[25] However, both couplings can affect the *rates* of energetic transitions, and their effect on rates is of opposite sign for β^- and β^+ decay. Therefore, as suggested by Weinberg,[2] a measured difference in B^{12} and N^{12} ft values could be due to induced couplings. The same assumption is required as was made in the analysis of WM: that the nuclear matrix elements be identical. However, this assumption is much more critical in the analysis of comparative lifetimes than it was in the analysis of spectral shapes.

The ft value for an allowed GT transition depends on the following quantities:

$$(ft)^{-1}\propto\lambda^2\left|\int\boldsymbol{\sigma}\right|^2\langle C_0\rangle_{\text{av}}, \tag{15}$$

where $|\int\boldsymbol{\sigma}|^2$, the square of the GT matrix element, is related to the reduced nuclear matrix element $\langle S_{10}\rangle$ by

$$\left|\int\boldsymbol{\sigma}\right|^2=4\pi[(2J+1)/(2J'+1)]\langle S_{10}\rangle^2, \tag{16}$$

where J' and J are initial and final nuclear spins. The symbol $\langle C_0\rangle_{\text{av}}$ represents the effect on ft values of a $C_0(W)$ differing from unity:

$$\langle C_0\rangle_{\text{av}}\equiv\int F_0(W,Z)C_0(W)pWq^2dW\bigg/\int F_0(W,Z)pWq^2dW, \tag{17}$$

where $F_0(W,Z)$ is the Fermi function.

If we assume that all nuclear matrix elements are the same for B^{12} and N^{12} decays, the theoretical ft ratio is

$$ft(N^{12})/ft(B^{12})=\langle C_0(B^{12})\rangle_{\text{av}}/\langle C_0(N^{12})\rangle_{\text{av}}. \tag{18}$$

For energetic β transitions in light nuclei, where $W_0^2\gg1$ and $(\alpha Z)^2\ll1$, $\langle C_0\rangle_{\text{av}}$ can be obtained in closed form by ignoring the slight variation of $F_0(W,Z)$ from unity:

$$\langle C_0\rangle_{\text{av}}\approx1+C_1+aC_2+bC_3+dC_4, \tag{19}$$

where, according to the methods described in Appendix B,

$$\begin{aligned}C_1=&-R^2\{(4/35)W_0\pm\tfrac{5}{8}UW_0\\&+(1+2\xi)[(1/105)W_0^2\pm(1/25)UW_0]\}\\&-\eta[\tfrac{2}{3}W_0\pm(6/5)U]+\zeta(6/5)U\\&+(3M)^{-1}[W_0\pm(1+\tfrac{1}{2}\xi)(6/5)U],\end{aligned} \tag{20}$$
$$C_2=(1+\tfrac{1}{4}\xi)(8/5)U,$$
$$C_3=\mp\tfrac{4}{3}W_0+(1+\tfrac{1}{2}\xi)\tfrac{4}{5}U,$$
$$C_4=[(51/70)+(67/105)\xi]U^2\pm(1+\tfrac{1}{5}\xi)\tfrac{2}{3}UW_0.$$

Upper signs refer to β^- emission, lower signs to β^+ emission. The quantities U, ξ, η, and ζ are defined by

$$\begin{aligned}&U\equiv\alpha Z/R, &&\xi\equiv2^{1/2}\langle S_{12}\rangle/\langle S_{10}\rangle,\\&\eta\equiv\tfrac{1}{3}^{1/2}\langle rW_1\rangle/\langle S_{10}\rangle, &&\zeta\equiv\lambda^{-1}\tfrac{2}{3}^{1/2}\langle rP_{11}\rangle/\langle S_{10}\rangle.\end{aligned} \tag{21}$$

We adopt[22] $R=0.426\alpha A^{1/3}$, and evaluate the matrix-element ratios ξ,η, and ζ as described in Appendix A, obtaining the following values for $p_{1/2}\to p_{3/2}$ transitions:

$$\xi=-\tfrac{1}{2},\quad\eta\approx(9/64)R^2U\Lambda,\quad\zeta=(2\lambda M)^{-1}, \tag{22}$$

where $\Lambda\approx2\pm1$ [see Eq. (A8) in Appendix A]. The WM parameter a, evaluated[2] from the rate of the analogous γ transition in C^{12}, is $a\approx1.16\times10^{-3}$. Expressing $\langle C_0\rangle_{\text{av}}$ in terms of b and d, we obtain

$$\begin{aligned}\langle C_0(B^{12})\rangle_{\text{av}}&=1.013-32.4b+113d,\\\langle C_0(N^{12})\rangle_{\text{av}}&=1.021+47.9b-110d.\end{aligned} \tag{23}$$

Inserting Fisher's measured value,[16]

$$ft(N^{12})/ft(B^{12})=1.14\pm0.025, \tag{24}$$

into Eq. (18), we obtain the following relation between b and d:

$$0.152\pm0.026=-(87.0\pm1.2)b+(238\pm3)d. \tag{25}$$

If b vanishes, as is required by G invariance, Eq. (25) has the solution $d=(6.4\pm1.0)\times10^{-4}$, which corresponds to $D/\lambda=2.4\pm0.4$ [see Eq. (10)]. On the other hand, if the Goldberger-Treiman estimate[7] of D/λ [Eq. (6)] is used, then Eq. (25) yields $b=-(1.8\pm0.3)\times10^{-3}$.

The above analysis is quite insensitive to the matrix-element ratios ξ, η, and ζ, and to the value of a. There is a somewhat greater dependence upon the choice of nuclear radius. For $b=0$ (as required by G invariance) the value of d is roughly proportional to the nuclear radius assumed. For the Goldberger-Treiman estimate of D/λ, a 10% increase in R produces only a 1% change in b.

[24] M. Gell-Mann and S. M. Berman, Phys. Rev. Letters **3**, 99 (1958).

[25] By treating the square of the IP coupling, J. M. Pearson, Can. J. Phys. **40**, 656 (1962), obtains a term proportional to W. He fails to observe that a value of d large enough to make this term significant would drastically affect the ratio of B^{12} and N^{12} ft values.

An important weakness in the above analysis lies in the assumption that the matrix elements $\langle S_{10}\rangle^2$ for the two transitions are exactly equal. Actually, one would expect the matrix elements to differ somewhat because of Coulomb effects. It is by no means easy to estimate the size of this effect. Weinberg[2] estimated that the difference in matrix elements $\langle S_{10}\rangle^2$ might be as much as 1%, basing his estimate upon Wilkinson's analysis[26] of isotopic spin impurities in forbidden $E1$ γ transitions.

In an attempt to resolve the indeterminancy of Eq. (25) and to obtain a rough check on the accuracy of our assumption concerning the equality of the B^{12} and N^{12} matrix elements, we discuss in the next two sections μ capture in C^{12} and the β spectrum of N^{16}.

IV. μ CAPTURE IN C^{12}

We assume that the μ-capture interaction is given by Eq. (4), in which the muon replaces the electron in L_μ. Due to the large rest mass of the muon, the IP coupling will have a much larger effect in μ capture than in β processes.[7]

The interaction in which a negative muon is captured in C^{12} and the resulting B^{12} nucleus is formed in its ground state has been thoroughly analyzed[9] and measured.[8] Since the analyses have consistently omitted effects of IT coupling, we have added terms in b to the formulas of Morita and Fujii.[9]

The resulting transition rate as a function of b and d is

$$\mathcal{W}=[7.82-(121b+1.33)^2+(5.27\times10^4d-295b+1.54)^2]\times10^3\ \mathrm{sec}^{-1}. \quad (26)$$

Figure 1 shows values of b and d consistent with the measurements of Reynolds *et al.*[8]:

$$\mathcal{W}=(6.6\pm0.9)\times10^3\ \mathrm{sec}^{-1}. \quad (27)$$

On the same Figure, we have indicated the values of b and d which satisfy Eq. (25). The intersection of the two shaded areas in Fig. 1 is approximately given by

$$b=-(1.9\pm0.35)\times10^{-3},\quad d=-(4\pm2)\times10^{-5}. \quad (28)$$

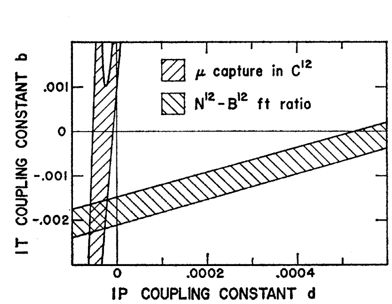

FIG. 1. Values of induced coupling parameters b and d producing the observed rate of μ capture in C^{12} [simultaneous solutions of Eqs. (26) and (27)], and values of these parameters producing the observed N^{12}–B^{12} ft ratio [solutions of Eq. (25)].

[26] D. W. Wilkinson, in *Proceedings of the Rehovoth Conference on Nuclear Structure* (North-Holland Publishing Company, Amsterdam, 1958), p. 175.

Clearly, the requirement of G invariance, that $b=0$, is inconsistent with our analyses of the μ capture rate in C^{12} and of the B^{12}–N^{12} ft ratio. The value $D/\lambda=-0.15\pm0.08$ obtained from Eqs. (10) and (28) is of the order of magnitude of the Goldberger-Treiman estimate [Eq. (6)].

V. β SPECTRUM OF N^{16}

The energy dependence of the first-forbidden unique correction factor $C_1(W)$, unlike that of $C_0(W)$, is sensitive to the amounts of IT and IP couplings. This dependence increases with W_0, and it may be possible to detect effects of induced couplings in the 10.40-MeV β spectrum of N^{16}.

In order to calculate these effects, we obtained nuclear matrix element ratios using methods described in Appendix A. We used point-nucleus electron radial functions averaged over the nuclear volume, as described in Appendix B. Other assumptions were that $a=1.16\times10^{-3}$, as for B^{12}, and that the nuclear radius $R=0.426\alpha A^{1/3}$.

Figure 2 shows the function $C_1(W)/(p^2+q^2)$ calculated from these assumptions and Table II. Calculations were performed for three sets of induced parameters: (1) $b=d=0$,[27] which corresponds to ascribing the $N^{12}-B^{12}$ ft ratio entirely to a difference of matrix elements $\langle S_{10}\rangle$; (2) $b=0$, $d=6.4\times10^{-4}$, corresponding to the assumption of G invariance, for which the ft ratio may be explained as due entirely to the IP coupling; and (3) $b=-1.9\times10^{-3}$, $d=-4\times10^{-5}$ as given by Eq. (28).

Due to the considerable branching to the 6.14-MeV excited state of O^{16}, only the upper half of the 10.4-MeV β spectrum of N^{16} can be analyzed. In this region, the plot for case (3) has a slope of nearly 0.6% per MeV. It should be possible with present techniques[5] to measure $C_1(W)/(p^2+q^2)$ with sufficient accuracy to distinguish

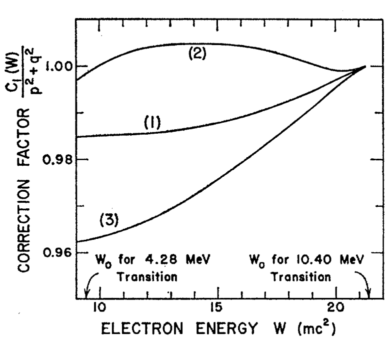

FIG. 2. Correction factors $C_1(W)/(p^2+q^2)$ for the 10.40-MeV β transition of N^{16}, calculated with $a=1.16\times10^{-3}$ and the following amounts of IT and IP couplings: (1) $b=d=0$; (2) $b=0$, $d=6.4\times10^{-4}$; (3) $b=-1.9\times10^{-3}$, $d=-4\times10^{-5}$. All curves are normalized to unity at W_0.

[27] Curve (1) of Fig. 2 shows a smaller variation with energy than calculated by J. F. Drietlein, Phys. Rev. **116**, 1604 (1959), since he ignored Coulomb effects in the WM term [cf., Ref. (15)].

between cases (1) and (3). Such a measurement, coupled with a careful analysis of possible errors arising from the various assumptions we have made, could provide positive evidence of IT coupling, and consequently of the inapplicability of G invariance to the β interaction.

ACKNOWLEDGMENTS

The authors are indebted to Dr. F. C. Michel for a private communication informing us of Fisher's recent measurements of the ft values of B^{12} and N^{12}.

APPENDIX A: NUCLEAR MATRIX ELEMENTS

In order to identify induced effects by comparing experimental results with calculated spectra, it is necessary to evaluate ratios between the various nuclear matrix elements. Rose and Osborn[23] have calculated reduced nuclear matrix elements for the j-j coupling model, expressing the results in terms of vector coupling coefficients and radial integrals F_m and $G_{mn}^{\pm}$, defined by

$$F_m \equiv \int r^{m+2}\rho(r)\rho'(r)dr\,, \qquad \text{(A1)}$$

$$G_{mn}^{\pm} \equiv \int r^{m+2}\rho(r)D_{\pm}(n)\rho'(r)dr\,, \qquad \text{(A2)}$$

where $\rho(r)$ and $\rho'(r)$ are final and initial radial functions of the transforming nucleon and $D_{\pm}(n)$ is given by

$$D_{\pm}(n) \equiv (d/dr) + r^{-1}[\tfrac{1}{2} \pm (n+\tfrac{1}{2})]. \qquad \text{(A3)}$$

The operators we employ [Eq. (11)] are similar to operators treated in reference (23), and the results of that paper can easily be applied to our operators: For a transition with shell model assignments $l'(j') \to l(j)$,

$$\langle r^n S_{JL}\rangle = (-1)^L\left[\frac{(2J+1)(2l+1)}{2\pi(2j+1)}\right]^{1/2} \times C(lLl';00)A\begin{bmatrix} l & \frac{1}{2} & j \\ l' & \frac{1}{2} & j' \\ L & 1 & J\end{bmatrix}F_n\,, \qquad \text{(A4)}$$

$$\begin{aligned}\langle r^n P_{JJ}\rangle = {} & M^{-1}(4\pi)^{-1/2}(-1)^{l'+j'-\frac{1}{2}} \\ & \times[(2l+1)(2j'+1)(l'+1)]^{1/2} \\ & \times(2J+1)(2l'+1)W(ljl'j';\tfrac{1}{2}J) \\ & \times C(lJl'+1;00)W(JJl'+1,l';1l)F_{n-1}\,, \qquad \text{(A5)}\end{aligned}$$

and

$$\begin{aligned}\langle r^n W_J\rangle & \\ = {} & M^{-1}(2\pi)^{-1/2}(-1)^{l-l'}[3(2J+1)(2l+1)(2j'+1)]^{1/2} \\ & \times[(l'+1)^{1/2}C(lJl'+1;00)W(1\tfrac{1}{2}l'j';\tfrac{1}{2}l'+1) \\ & \times W(l'+1,j'lj;\tfrac{1}{2}J)G_{nl'}^{-} - (l')^{1/2}C(lJl'-1;00) \\ & \times W(1\tfrac{1}{2}l'j';\tfrac{1}{2}l'-1)W(l'-1,j'lj;\tfrac{1}{2}J)G_{nl'}^{+}]. \qquad \text{(A6)}\end{aligned}$$

Published tables can be used to evaluate the C and W coefficients[28] and the A coefficient.[29]

We approximate $F_m/F_{m'}$ by using the value of this ratio for constant radial functions:

$$F_m/F_{m'} \approx [(m'+3)/(m+3)]R^{m-m'}\,, \qquad \text{(A7)}$$

where R is the nuclear radius. We further assume that the ratio $G_{mn}^{\pm}/F_m$ is a constant. With the aid of (A7), we express this ratio in terms of the usual parameter Λ, expected[23] to lie in the range $1 \leq \Lambda \leq 3$:

$$G_{mn}^{\pm}/(2MF_m) \approx \tfrac{3}{16}\alpha Z\Lambda. \qquad \text{(A8)}$$

APPENDIX B: AVERAGING LEPTON FUNCTIONS OVER THE NUCLEAR VOLUME

In handling the IP spectral terms, one is required to evaluate the function $r(dV/dr)$. Inside a uniformly charged nucleus of radius R the potential energy $V(r)$ of an electron is given by

$$V(r) = \tfrac{1}{2}V_R(3 - r^2/R^2), \qquad \text{(B1)}$$

where $V_R = -\alpha Z/R$ ($+\alpha Z/R$ for a positron). If we define the average of an operator $\Omega(r)$ to mean

$$\langle\Omega(r)\rangle_{\text{av}} \equiv \int r^2\rho(r)\Omega(r)\rho'(r)dr\,, \qquad \text{(B2)}$$

then from Eqs. (A7) and (B1) we obtain

$$\langle r(dV/dr)\rangle_{\text{av}} \approx -\tfrac{3}{5}V_RF_0\,, \quad \langle V\rangle_{\text{av}} \approx (6/5)V_RF_0\,, \qquad \text{(B3)}$$

as the values to be used in $C_0(W)$, and

$$\langle r^2(dV/dr)\rangle_{\text{av}} \approx -\tfrac{2}{3}V_RF_1\,, \quad \langle rV\rangle_{\text{av}} \approx (7/6)V_RF_1\,, \qquad \text{(B4)}$$

to be used in $C_1(W)$.

It would be more consistent with this treatment of V if the electron radial functions were also averaged over the nucleus, instead of merely being evaluated at the nuclear radius. For instance, evaluated in the usual way, L_0 has the following approximate form for a point-charge nucleus[13]:

$$L_0 \approx 1 - \tfrac{1}{3}p^2R^2 - \tfrac{1}{3}\alpha Z[5W + (1/W)]R\,, \qquad \text{(B5)}$$

where p is the electron momentum. If, on the other hand, electron radial functions are averaged over the nucleus according to Eq. (A7), L_0 for allowed transitions takes the form

$$L_0 \approx 1 - \tfrac{1}{5}p^2R^2 - \tfrac{1}{4}\alpha Z[5W + (1/W)]R. \qquad \text{(B6)}$$

Since the small terms in Eqs. (B5) and (B6) are of the same order of magnitude as induced effects, effects of averaging should be taken into account with regard to the spectra of B^{12}, N^{12}, and especially N^{16}. A further refinement of the approach outlined above would be to combine such an averaging process with electron functions calculated for a uniform charge distribution.[22]

[28] L. C. Biedenharn, J. M. Blatt, and M. E. Rose, Rev. Mod. Phys. **24**, 249 (1952).
[29] G. E. Lee-Whiting, Can. J. Phys. **36**, 1199 (1958).