Gallium. Solid gallium shows a slight smearing of the maximum k cutoff which is probably due to anisotropy of the Fermi surface and accompanying higher momentum components of the electron wave functions. The results for liquid gallium show a very large "blurring" of the Fermi cutoff, which could imply an electron mean free path of $l \approx 1-4$ Å. It is known that the crystal structure of Ga consists of pairs of ions in the lattice.[6] If these pairs were much more tightly bound in the liquid, the molecular orbital electrons doing the binding would have a momentum distribution somewhat like the observed data. There would certainly be no Fermi cutoff. Knight, Berger, and Heine,[7] and Pashaev[8] have noted many other anomalies in the behavior of liquid and solid Ga.

Indium. The sharpness of the momentum cutoff in indium changes somewhat as the solid is heated from room temperature to about 130°C and does not change much more upon melting. If the increase in smearing above room temperature is interpreted to yield an electron mean free path, $l \approx 4-10$ Å is obtained for the solid and liquid near the melting point.

Tin. The results for tin show a slight change in the sharpness of the cutoff in heating the metal from room temperature to about 200°C. The mean free path obtained from the change in sharpness is $l \approx 25$ Å. Melting further decreases this to a value in the range $l \approx 3$–6 Å.

The mean free paths for In and Sn are approximately in accord with simple estimates from the conductivity,[1] although it is a little surprising that the thermal scattering in In appears in large part before melting. The expected mean free path for liquid gallium (≈ 17 Å) is not observed and it is possible that the measured momentum distribution cannot be usefully characterized by a Fermi cutoff and a mean free path.

We are indebted to J. J. Donaghy and J. B. Shand of this laboratory for invaluable assistance and to Dr. Philip Taylor for helpful discussions regarding positron velocities in metals. Dr. A. MacIntosh has kindly sent us a preprint of his recent measurements in liquid and solid mercury and we are indebted to Dr. N. E. Cusack for an advance copy of his review of the properties of liquid metals.

[6] H. Hendus, Z. Naturforsch. **2a**, 505 (1947).

[7] W. D. Knight, A. G. Berger, and V. Heine, Ann. Phys. (N.Y.) 8, 173 (1959).

[8] B. P. Pashaev, Fiz. Tverd. Tela **3**, 416 (1961) [translation: Soviet Phys.—Solid State **3**, 303 (1961)].

PHYSICAL REVIEW VOLUME 132, NUMBER 2 15 OCTOBER 1963

Influence of Collisions on Scattering of Electromagnetic Waves by Plasma Fluctuations*

A. Ron, J. Dawson, and C. Oberman
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey

(Received 23 May 1963)

The incoherent scattering of electromagnetic waves from electron-plasma oscillations in a thermal plasma is investigated. In particular, the effect of collisions on the shape of the isolated peak in the scattered intensity at a frequency displaced from the incident frequency by the plasma frequency is discussed making use of Nyquist's theorem and recent conductivity calculations.

In recent years considerable attention has been directed to the problem of the incoherent scattering of electromagnetic waves due to electron density fluctuations in a fully ionized plasma.[1–5] It is well established that most of the scattering arises from the low frequency fluctuations whose origin is due to the coupling of the electrons to the thermal motion of the ions. In addition to this dominant effect, there is a very sharp resonance in the vicinity of the electron plasma frequency. (See Fig. 2, Ref. 1.) For the scattering produced by long wavelength fluctuations the frequency width of this contribution is very small. It is the theoretical problem of the determination of the structure of latter isolated peak, for an equilibrium plasma, to which we turn our attention, since recently developed experimental techniques suggest this peak is resolvable.[6]

Now the expression for the differential scattering cross section is

$$\sigma d\Omega d\omega = (e^2/mc^2)^2[1-\sin^2\theta\cos^2(\phi-\phi_0)] \times S(\mathbf{k}-\mathbf{k}_0, \omega-\omega_0)d\Omega d\omega\,, \quad (1)$$

* Work accomplished under the auspices of the U. S. Atomic Energy Commission.

[1] J. P. Dougherty and D. T. Farley, Proc. Roy. Soc. (London) **A259**, 79 (1960).

[2] E. E. Salpeter, Phys. Rev. **120**, 1528 (1960).

[3] J. A. Fejer, Can. J. Phys. **38**, 1114 (1960).

[4] T. Hagfors, Stanford Electronics Laboratories, Report No. 1, 1960 (unpublished).

[5] M. N. Rosenbluth and N. Rostoker, Phys. Fluids **5**, 776 (1962).

[6] We are indebted to Dr. E. A. Frieman for pointing this fact out to us.

where the spectrum of electron density fluctuations $S(\mathbf{k},\omega)$ is given by

$$S(\mathbf{k},\omega)=V\langle|n(\mathbf{k},\omega)|^2\rangle. \tag{2}$$

In (1) and (2), $\mathbf{k}_0$ and ω_0 are the wave vector and frequency of the incident wave, σ is the average power scattered into the direction $\mathbf{k}$ at frequency ω per unit solid angle per unit incident power per unit volume per unit angular frequency, θ is the angle between $\mathbf{k}_0$ and $\mathbf{k}$, ϕ and ϕ_0 are azimuthal angles locating $\mathbf{k}$ and the incident electric vector. The scattering volume is denoted by V.

Now the density fluctuations may be related to the dissipation[1] by Nyquist's theorem to obtain

$$S(\mathbf{k},\omega)=\frac{\Theta k^2}{\pi e^2\omega^2}\operatorname{Re}\{Z_{\text{ext}}^{-1}(\mathbf{k},\omega)\}, \tag{3}$$

where Θ is the temperature in energy units and Z_{ext} is the impedance of the system to an external electric field, $Z_{\text{ext}}(\mathbf{k},\omega)\mathbf{j}(\mathbf{k},\omega)=\mathbf{E}_{\text{ext}}(\mathbf{k},\omega)$.

All previous calculations of S have been equivalent to computing Z_{ext} from the Vlasov description. That is, the only dissipative mechanism considered is Landau damping. While these calculations are valid for the low-frequency regime, they are inadequate to describe the structure of the isolated spike occurring in the vicinity of the electron plasma frequency, $|\omega-\omega_0|\simeq\omega_p$, when $|\mathbf{k}-\mathbf{k}_0|\ll\kappa_D$, where κ_D is the Debye wave number $\kappa_D=(4\pi ne^2/\Theta)^{1/2}$. As pointed by Salpeter[2] this structure must be determined by the collisional contribution to Z_{ext} since the Landau damping is negligibly small by comparison in this regime.

In order to describe the structure of this isolated spike near the plasma frequency we rewrite (3) in the form

$$S=\frac{\Theta k^2}{\pi e^2\omega^2}\frac{R}{X^2+R^2}, \tag{4}$$

where $Z_{\text{ext}}=iX+R$. Near ω_r, defined by $X(\omega_r)=0$, $R(\omega)$ is a small slowly varying function of ω. Hence, S has the Lorentzian shape

$$S=\frac{\Theta k^2}{\pi e^2\omega^2}\left|\frac{\partial X(\omega_r)}{\partial\omega}\right|^{-1}\frac{\epsilon(\omega_r)}{(\omega-\omega_r)^2+\epsilon^2(\omega_r)}, \tag{5}$$

where $\epsilon(\omega_r)=R/|\partial X(\omega_r)/\partial\omega|$. Note that the integrated power over the spike is independent of the absorption mechanism. For the case of small collisional damping we may write

$$Z_{\text{ext}}\cong Z_{\text{Vlasov}}+Z_{\text{collisions}},\quad |Z_C|\ll|Z_V| \tag{6}$$

independent of the nature of the collisions. Recently, the collisional impedance[7–9] for long-wavelength plasma oscillations has been systematically derived for a fully ionized plasma. If we make use of these results we have

$$X(k,\omega)=\frac{4\pi}{\omega}\left\{1+\frac{3k^2}{\kappa_D^2}-\frac{\omega^2}{\omega_p^2}+O\left(\frac{k^4}{\kappa_D^4}\right)+O\left(\frac{\kappa_D^3}{n}\right)\right\}, \tag{7}$$

$$R=R_V+R_C, \tag{8}$$

$$R_C=\frac{8}{3}\frac{e^2}{m\omega_r^3}\operatorname{Im}[I(\omega)]+O\left(\frac{\kappa_D^3}{n}\frac{k^2}{\kappa_D^2}\right), \tag{9}$$

$$R_V=\frac{4\pi}{\omega_p}\left(\frac{\pi}{2}\right)^{1/2}\frac{\kappa_D^3}{k^3}\left[\exp\left(-\frac{\omega_r^2}{2\omega_p^2}\frac{\kappa_D^2}{k^2}\right)\right]\times\left[1+O\left(\frac{k^2}{\kappa_D^2}\right)\right], \tag{10}$$

$$\partial X(\omega_r)/\partial\omega=8\pi/\omega_p^2, \tag{11}$$

and

$$\omega_r^2=\omega_p^2[1+3k^2/\kappa_D^2+O(k^4/\kappa_D^4)]. \tag{12}$$

$I(\omega)$ is given in Refs. 7 and 8 and is well approximated for a hydrogenic plasma, when $\omega\sim\omega_p$, by

$$I(\omega)=(\tfrac{1}{2}\pi)^{1/2}\kappa_D^3\ln(k_{\max}/2\kappa_D). \tag{13}$$

Note that when $k\lesssim0.1\kappa_D$, $R_L\ll R_C$ for most typical plasmas and may be neglected.

Thus, the width is given by

$$\epsilon=\frac{\omega_p}{24\pi^2}(2\pi)^{1/2}\frac{\kappa_D^3}{n}\ln\left(\frac{k_{\max}}{2\kappa_D}\right), \tag{14}$$

and the height is given by

$$S_{\max}=6(2\pi)^{1/2}n\frac{n}{\kappa_D^3}\left(\frac{k^2}{\kappa_D^2}\right)\omega_p/\omega^2\ln\left(\frac{k_{\max}}{2\kappa_D}\right). \tag{15}$$

The peak although much broader and lower than that given by Landau damping is still very sharp. The above results apply for infinitely sharp spectral resolving power and uniform average plasma density. Average density nonuniformities and the finite resolving power of instruments in both k and ω will add to the width of the observed line and must be seriously taken into account in any measurement of this effect.

---

[7] J. Dawson and C. Oberman, Phys. Fluids **5**, 517 (1962).
[8] C. Oberman, A. Ron, and J. Dawson, Phys. Fluids **5**, 517 (1962).
[9] J. Dawson and C. Oberman, Phys. Fluids **6**, 394 (1963). Here and in Refs. **7** and 8 the impedance calculated relates the current to the net average field in the plasma. These impedances are simply related by $Z_{\text{ext}}=Z_{\text{int}}+4\pi i/\omega$.