The $\pi-\pi$ and $K-\pi$ interactions will also possess a diffraction scattering and total cross section of the form predicted above, although these interactions cannot be checked by any current experimental data. One experiment would be particularly useful at present to check our predictions. From the effect of the ω meson on the pp and $\bar{p}p$ cross sections, we conclude that $K-p$ and $\bar{K}-p$ total cross sections will be approximately the same above 10 BeV, and will have the form

$$\sigma_T=\frac{\pi}{M_K M_N} g_{KN}\left(1+\frac{\beta}{\ln w}\right) \qquad (6.2)$$

with $\beta=2.5$ as in the $N-N$ and $\pi-N$ processes.

Note added in proof. In a recent letter by Freund and Oehme [Phys. Rev. Letters **10**, 450 (1963)] it is concluded that a pole plus cut model cannot fit the data of Foley *et al.*[4] or the total cross section data.[10] Their incorrect result stems from the use of $s_0=2M_p^2$ as the normalization constant for all processes. This emphasizes the important role played by the normalization parameter $s_0=2M_1M_2$ in our theory. Freund and Oehme also remark that there is no evidence of both a pole and cut in $\pi+p$ experiments. However, new data by Brandt *et al.* [S. Brandt, V. T. Cocconi, D. R. O. Morrison, A. Wroblewski, P. Fleury, C. Kayas, F. Muller, and C. Pelletier, Phys. Rev. Letters **10**, 413 (1963)] on $\pi-p$ diffraction scattering at small t does show a "kink" in support of our theory, [Phys. Rev. **129**, 2812 (1963)].

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Photoproduction of Low-Energy Charged Pions from Deuterium

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Accurate measurements have been made of the π^-/π^+ photoproduction ratio on deuterium, in the gamma-ray energy range 165–210 MeV, for several angles: 155°, 125°, 90° (center-of-mass system) and along Baldin's kinematical line. These last data are new contributions: $\pi^-/\pi^+=1.20\pm0.03$ averaged between 165 and 180 MeV. The others are improvements of the accuracy of previous data. The comparison with Ball's theory, corrected for taking into account the $I=\frac{1}{2}$ phase shifts, gives for the coupling constant Λ for $\gamma-\pi-p$ the value: $0.25<+\Lambda/e<0.75$.

I. INTRODUCTION

LOW-ENERGY charged pion photoproduction has been studied for a decade or more. The two reactions which are of greatest interest are

$$\gamma+p\rightarrow\pi^++n\,, \qquad (1)$$

$$\gamma+n\rightarrow\pi^-+p\,, \qquad (2)$$

where the neutron is in the bound state corresponding to a deuteron. In the past, the main interest was the extrapolation of experimental data to threshold to obtain a value of the pion nucleon coupling constant, and use of the threshold values so obtained as a means to check the well-known relationship between pion photoproduction and scattering.[1] Within the present limits of accuracy of the data and of their interpretation and extrapolation, a reasonable agreement exists. The contribution of the uncertainty of the data on the π^-/π^+ ratio (about 10%) to this type of analysis is considerable. This was one of the motivations for undertaking the present experiment.

More recently, another interest in this field has been the comparison of the experimental data with the predictions of the dispersion relations of Chew, Goldberger, Low, and Nambu (C.G.L.N.).[2] Although the agreement has been reasonably good, some points are worthy of mention.

In the case of reaction (1) there is a marked tendency for the experimental data to lie below the theoretical prediction at large angles.[3] This has been interpreted by several authors in different ways. First, it was shown by Uretsky *et al.*[4] that the predictions of C.G.L.N. theory were very sensitive at backward angles to the choice of the small pion nucleon P-wave phase shifts, and that any lack of agreement of theory and experiment may be due to our lack of precise knowledge of these phase shifts. Second, Baldin[5] suggested that the discrepancy could be due to an unknown contribution

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[1] M. Beneventano, G. Bernardini, D. Carlson-Lee, G. Stoppini, and L. Tau, Nuovo Cimento **10**, 1109 (1958).
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[3] J. M. McKinley, University of Illinois, Report No. 38, 1962 (unpublished).
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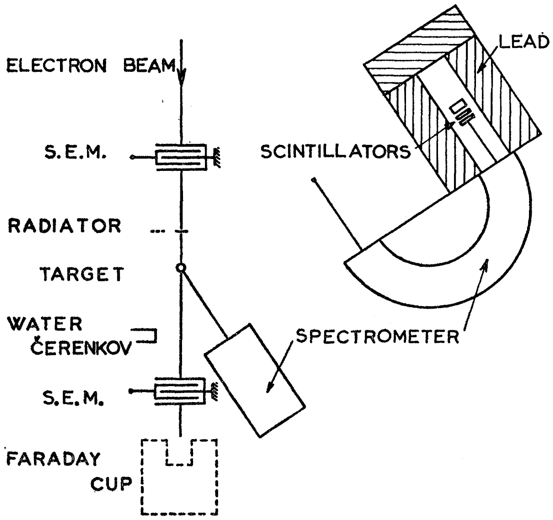

FIG. 1. General arrangement.

to the dispersion integrals of the unphysical energy region, which becomes more important at large angles. Conversely, he pointed out that the contribution of the unphysical region was zero when the momentum transfer to the nucleon was kept constant and equal to that occurring at threshold (hereafter referred to as the Baldin line), implying that good agreement between theory and experiment should exist under these kinematical conditions. The most recent comparison[6] is certainly consistent with this approach. A third and more recent approach[7] has been to assign any lack of agreement to the gamma-three pion vertex which thereby introduced new free parameters which were adjusted until agreement was obtained. From the above discussion, it is clear that major uncertainties exist in any precise comparison or interpretation of the theory of π^+ photoproduction.

When we consider the ratio of π^- to π^+ mesons in the reactions,

$$\gamma+d\rightarrow\pi^-+p+p\,,$$
$$\gamma+d\rightarrow\pi^++n+n\,,$$

the situation has similar difficulties. There are, first of all, the well-known Coulomb final-state interactions which distort the free-nucleon ratio[8] by at most a few percent in the range of interest here. Second, Walker[6,9] has shown that, although the comparison of theory and experiment is good along the Baldin line in the case of π^+ production, it is not so for the ratio π^-/π^+. Once again, the effect has been attributed to a bi-pion resonance by several workers.[10–14] In this case, however, because of the $\gamma-3\pi$ vertex contributing only to the isoscalar amplitudes, we expect the π^-/π^+ ratio to be more sensitive to this vertex. New accurate data on the π^-/π^+ ratio, particularly along the Baldin line, was desired, to study the influence of such a multipion photon vertex and to deduce a value of the coupling constant which would fit the experimental data. In an attempt to get such new information on low-energy pion physics we have made accurate measurements (4%) of the π^-/π^+ ratio between 165 and 210 MeV for several angles: Baldin line, 90°, 125°, and 155° (center-of-mass system).

II. EXPERIMENTAL METHOD

The experimental arrangement, shown in Fig. 1, is almost the same as that which we used in a previous experiment.[15] The bremsstrahlung beam was produced by the electron linear accelerator of the Ecole Normale Supérieure. The electron beam deviation and analyzing system has been described previously.[16] The γ-ray (and electron) beam passed through a liquid deuterium target. The intensity of the transmitted electron beam was measured, in order to monitor the photon flux. Pions were momentum analyzed by a 57.5-cm radius–180°-double-focusing spectrometer[16] and detected with three plastic scintillation counters operated in coincidence. Measurements on π^- and π^+ counting rates were made alternately by inverting the current in the spectrometer.

In order to have a simple experimental arrangement and to keep the full advantage of the high intensity of a linear accelerator, the electron beam was not swept after the radiator. The contribution to the pion counting rate from electroproduction was measured in the absence of the radiator.

As counting rates were taken alternately for π^+ and π^-, most systematic errors were averaged out.

Target

A detailed description of the target, which was of a very simple design, can be found elsewhere.[17] The liquid deuterium was contained in an aluminum cylinder, 22 mm in diameter, 50×10^{-3} mm thick, connected to a cooling coil, the whole being in thermal contact with a liquid-hydrogen reservoir. Two resistors, at the top and the bottom of the target, indicated whether the target was full or empty and gave the possibility of

[6] J. K. Walker, J. G. Rutherglen, D. B. Miller, and J. M. Paterson, Proc. Phys. Soc. (London) **81**, 78 (1963).
[7] C. S. Robinson, P. M. Baum, L. Criegee, and J. M. McKinley, Phys. Rev. Letters **9**, 349 (1962).
[8] A. Baldin, Nuovo Cimento **8**, 569 (1958).
[9] J. K. Walker, Nuovo Cimento **21**, 577 (1961).
[10] B. DeTollis, E. Ferrari, and H. Munczek, Nuovo Cimento **18**, 198 (1960).
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[12] M. Gourdin, D. Lurié, and A. Martin, Nuovo Cimento **18**, 933 (1960).
[13] A. E. A. Warburton and M. Gourdin, Nuovo Cimento **22**, 362 (1961).
[14] J. S. Ball, Phys. Rev. **124**, 2014 (1961).
[15] J. K. Walker and J. P. Burq, Phys. Rev. Letters **8**, 37 (1962).
[16] F. Lacoste, Laboratoire de l'Accélérateur Linéaire, Orsay, Rap. LHE 7 (unpublished); F. Lacoste and G. R. Bishop, Nucl. Phys. **26**, 511 (1961).
[17] J. K. Walker, J. P. Burq, and V. Round, Nucl. Instr. Methods (to be published).

driving the liquid out of the target and up into the cooling coil by having a steady current in the bottom resistor. In this way, background measurements were made easily (the target could be filled or emptied in a few seconds) with unchanged experimental conditions. Because of the cylindrical shape of the target, measurements could be made at different angles simply by rotation of the spectrometer. On the other hand, this shape was a source of error because of small random lateral movements of the beam, which produced variations of the irradiated deuterium volume.

In order to reduce this error, the tantalum radiator was covered by a thin fluorescent coat (0.2% of a radiation length), by means of which the position of the beam could continuously be controlled to about 0.5 mm with a closed circuit television system. The radiator was located in the vacuum chamber of the target, at a distance of 9 cm of the latter and had a thickness of only 8.5% of a radiation length. The effect of multiple scattering was thereby reduced. As the uncertainty in the relative position of the radiator and the target was less than 1 mm, the variation of the irradiated volume due to lateral movements of the beam did not exceed 0.5%. As the data for π^- and π^+ meson counting rates were taken by frequently changing the sign of the magnet current, we estimate this error to average out to 0.3%.

The radiator could be controlled to move out of the beam for electroproduction measurements.

The deuterium was obtained by the usual electrolytic technique using heavy water and purified by passing through a charcoal trap at liquid-nitrogen temperature. A chromatographic analysis determined the hydrogen concentration in the deuterium to be $(0.4\pm0.1)\%$. No correction was applied to the data, the error which may result in the ratio being of the same order.

Monitoring of Photon Intensity

An absolute monitor was not necessary as it was a ratio which was measured. The intensity of the γ-ray beam was proportional to the measured intensity of the electron beam. This was true in so far as the thickness of the radiator was constant during the experiment. The thickness of the tantalum plate was uniform, and irregularities in the thickness of the fluorescent coat were not sufficient to produce $>0.1\%$ variations in the photon flux.

The intensity of the transmitted electron beam was measured by means of a secondary emission monitor of standard design,[18] which was frequently calibrated with a Faraday cup.[19] The stability of the monitor was found to be good to 0.5% during the course of the experiment and we accordingly assign a 0.3% uncertainty to the final pion ratios.

Because of multiple scattering and Möller effect, beam current measurements with the radiator in the beam or not, and also with the target full or empty, were not comparable. This effect was studied with another secondary emission monitor in front of the radiator. A 4% correction was applied to the counting rates with radiator in the beam, the corresponding correction on the ratio was much smaller. The correction due to deuterium (target full or empty) was less than 0.5% to the counting rates and correspondingly much less on the final ratios, and was, therefore, neglected.

The uncertainty in the absolute energy of the electron beam was 0.6% (0.6% was also the momentum interval set by slits in the deviation system). The influence on the effective spectrum (in three-body kinematics) of photons which produce the reaction of interest is small, the error which may result is $<0.1\%$. However, for the measurements corresponding to an average γ-ray energy of 210 MeV, the energy of the beam was increased from 220 to 230 MeV.

Detection

The energy of detected pions was determined by the current in the spectrometer, and corrected for energy losses in deuterium, aluminum, and Mylar windows. The uncertainty of the absolute pion energy was about 1%, but did not produce any uncertainty on the pion ratio.

On the other hand, the counting rates were sensitive to variations of the magnetic field because of decay in flight which was important for this low-energy region. The stability of the magnetic field was about 0.1% and also the corresponding error in counting rate. The current, frequently reversed during the course of the experiment, was each time established in the same way and the magnetic field was reproduced to about 0.2%, for the same current.

The irradiated target volume could not be considered as a point source by the spectrometer. The target was centered to about 1 mm relative to the axis of rotation of the spectrometer. A calculation was made of the variations of the mean solid angle induced by small translations of the beam. The calculation, which was verified experimentally with a polyethylene target, showed that the error which could result from this effect was negligible ($<0.1\%$) in our experimental conditions. The horizontal angular resolution, which was given by the acceptance of the spectrometer, was about 2.5 deg.

Particles within the same momentum interval were detected in the focal plane of the spectrometer by three plastic scintillators connected to 56 AVP photomultipliers. The dimensions of the scintillators were 4 cm $\times$4 cm (corresponding to $\Delta p/p$ being about 1.7%). The front two scintillators were 0.3 cm thick, the third 0.5 cm, 1.5 cm, or 2.5 cm thick, depending on the energy of pions being detected. The choice was made in such a way that pions were not stopped in the third counter, thereby avoiding π^- star formation, but lost a sufficient

[18] G. W. Tautfest and R. H. Fechter, Rev. Sci. Instr. **27**, 233 (1955).
[19] K. L. Brown and G. W. Tautfest, Rev. Sci. Instr. **27**, 692 (1956).

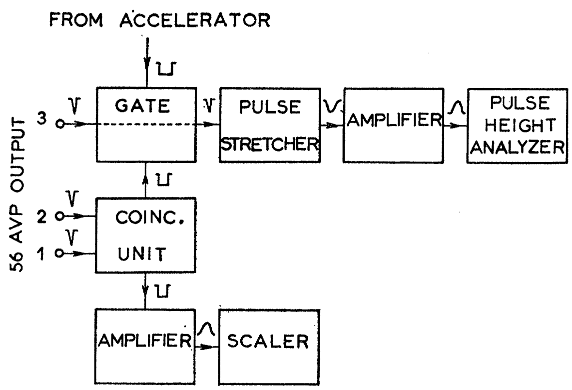

Fig. 2. Electronics.

energy to provide adequate pulse-height resolution. The data were corrected for different nuclear absorption of[20] π^- and π^+ in deuterium, windows, and scintillators. The correction for the ratio was small, from 0.1 to 0.35%.

The schematic arrangement of electronic circuits is shown in Fig. 2. The outputs of counters 1 and 2 were connected, by 50-Ω cables, to a fast coincidence circuit, which had a resolving time of 20×10^{-9} sec. This circuit was followed by a discriminator and a fast amplifier which gave a standard pulse of 6 V in amplitude and 50×10^{-9} sec width. The dead time of the assembly, measured with a pulse generator, was 60×10^{-9} sec. The standard pulse output from the coincidence unit was made to open one side of a gate, while the other side was opened by a standard pulse of 10 V and 2×10^{-6} sec wide which straddled the time during which the beam from the accelerator was produced. The output of counter 3, after passing through this gate, was lengthened, amplified, and displayed on a 100-channel pulse-height analyzer. At low energy (~12 MeV), the range of the pion is so short that two counters only were placed in the focal plane of the spectrometer. In this case, the output of counter 2 was pulse-height analyzed. The linearity of the pulse-height analysis system was checked with a pulse generator in the range 0.3 V (bias of the gate) and 6 V (amplitude of the smallest of the two-gate pulses). (See Fig. 3.)

With this simple system, the background of the room was almost eliminated (≪1% of the pion rate), and the ability to detect pions in a background of electrons and muons was rather good. This latter feature was particularly necessary at small angles, where scattered electrons became numerous.

As well as the corrections due to monitoring and nuclear absorption, the following corrections were applied to the data: (1) Correction due to the dead time of the coincidence unit. For this purpose the number of coincidences was measured by a one megacycle scalar and the length of the beam pulse (about 0.4×10^{-6} sec) monitored by means of a water Čerenkov located in the neighborhood of the beam. This correction did not exceed 2.5% and produced less than 1% correction in the pion ratio. (2) Counting-rate corrections. Because of the pulse lengthener, no more than one event could be counted per beam pulse (this event corresponded to the largest pulse). This correction, of about 1%, was always less than 2% for any counting rate. (3) Relative contribution of electrons and positrons to meson identification. For the most part, the π^+ and π^- spectra showed a peak-to-valley ratio of about 40 and were identical within the precision of the measurements. No correction was made, except for those measurements along the Baldin line, where the electron contamination was estimated to be (4±1)%. This correction was arrived at by extrapolating the electron background underneath the pion peak. The estimated error involved has been included in the statistical one.

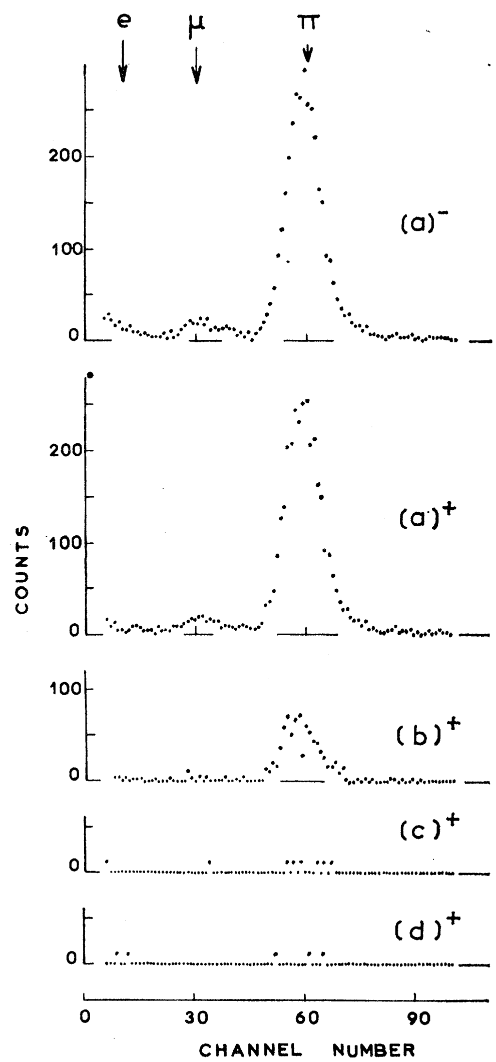

Fig. 3. Samples of spectra (T_π~25 MeV, $\theta=90°$). (a) and (c) are for radiator in the beam, target full and empty. (b) and (d) are for radiator out, target full and empty. ( )$^+$ is for π^+, ( )$^-$ is for π^-.

[20] D. H. Stork, Phys. Rev. **93**, 858 (1954).

TABLE I. Experimental errors on π^-/π^+ ratio.

Irradiated deuterium volume	±0.3%
Hydrogen contamination	−0.6%
Monitoring	±0.4%
Spectrometer stability	±0.1%

Experimental errors are summarized in Table I, and corrections in Table II.

III. PROCEDURE, DATA

For each pion energy, the high voltages on each of the counters was adjusted until the pion peak was comfortably above the coincidence circuit pulse height threshold.

Four distinct types of measurements were then made for π^- and four for π^+, in the following conditions:

(1) N_{R+e} = target full, radiator in the beam;

(2) N_e = target full, radiator out;

(3) N_{R+e}^0 = target empty, radiator in;

(4) N_e^0 = target empty, radiator out.

N measurements were repeated three times or more, N^0 measurements twice or more. Although the notation is obvious, we must note that there is a small contribution of photoproduction in N_e due to the foils which remain in the beam when the radiator is out and to deuterium itself, the whole being evaluated to 0.7% of a radiation length. There is also a contribution from the deuterium gas in N^0. But these contributions disappear in evaluating

$$R=[(N_{R+e}-N_{R+e}^0)-(N_e-N_e^0)]^-/[(N_{R+e}-N_{R+e}^0)-(N_e-N_e^0)]^+,$$

which represents the ratio of π^- and π^+ photoproduction cross sections on deuterium. All the contributions of pion electroproduction on deuterium, photoproduction and electroproduction on aluminum are subtracted.

Data are tabulated in Table III, where all the corrections mentioned above have been made. Our results are in agreement with previous data.[1,21–23] However-

TABLE II. Corrections.[a]

Monitoring	<0.5%
Counting rate	<1%
Dead time of coincidence circuit	<1%
Electron contamination (only on Baldin's line)	<4%
Nuclear absorption	<0.4%

[a] The magnitudes of the corrections indicated are the magnitudes on the π^-/π^+ ratio.

[21] W. R. Hogg and E. H. Bellamy, Proc. Phys. Soc. (London) **72**, 895 (1958).

[22] M. Sands, J. G. Teasdale, and R. L. Walker, Phys. Rev. **101**, 1159 (1956).

[23] J. G. Rutherglen and J. K. Walker, Proc. Phys. Soc. (London) **76**, 431 (1960).

TABLE III. Summary of data. Values of N are for an arbitrary integrated beam current which is the same for one line, but not always the same in one column. R is the measured π^-/π^+ ratio on deuterium corrected for effects shown in Table II.

		π^+				π^-				
$\theta_{c.m.}$	E_γ (MeV)	N_{R+e}	N_e	N_{R+e}^0	N_e^0	N_{R+e}	N_e	N_{R+e}^0	N_e^0	$R=\frac{[(N_{R+e}-N_{R+e}^0)-(N_e-N_e^0)]^-}{[(N_{R+e}-N_{R+e}^0)-(N_e-N_e^0)]^+}$
68°	165	196.91± 3.48	53.98± 2.38	8.39±1.75	1.25±0.88	247.12± 4.62	83.12± 3.58	7.28±2.74	5.83±2.20	1.195±0.065
65°	170	268.90± 3.82	64.07± 2.92	5.94±1.33	1.33±0.94	339.46± 5.35	89.03± 3.45	12.07±2.19	1.67±0.96	1.199±0.045
60°	180	509.41± 8.15	126.33± 5.85	10.40±6.00	0	656.26± 8.25	174.35± 6.60	36.47±4.35	0	1.195±0.049
90°	170	198.57± 3.56	55.63± 3.05	6.72±0.93	2.86±1.27	278.97± 4.74	70.85± 3.28	6.52±1.63	2.35±1.18	1.466±0.068
90°	180	350.03± 5.80	84.53± 3.94	5.94±1.87	4.00±2.00	487.86± 6.86	124.66± 5.28	10.74±2.68	5.00±2.68	1.356±0.052
90°	190	505.56± 7.75	102.58± 5.76	5.20±2.12	0	695.12±11.02	149.30± 7.75	19.80±4.52	5.01±3.54	1.335±0.049
90°	200	536.47± 8.50	132.13± 4.73	8.43±2.18	2.22±1.57	680.13± 9.67	142.38± 7.02	6.52±2.06	3.34±1.92	1.347±0.046
90°	210	811.01±13.31	206.28±14.58	18.20±6.88	0	1080.71±15.34	269.56±16.60	15.66±6.36	0	1.356±0.062
90°	190	270.92± 4.37	61.87± 3.02	7.73±2.14	1.00±1.00	420.86± 6.45	100.96± 4.50	11.90±2.65	2.00±1.41	1.578±0.063
125°	200	363.19± 6.53	93.98± 4.58	10.40±4.24	0	560.65±10.00	136.65± 7.28	8.68±3.88	0	1.605±0.075
125°	210	519.10± 7.91	106.74± 4.90	12.48±2.94	3.33±2.36	765.23±10.93	169.71± 7.70	16.69±3.40	10.03±4.08	1.460±0.051
155°	200	216.61± 3.48	47.00± 2.43	6.24±1.36	1.82±0.81	361.31± 5.68	94.69± 4.37	13.74±2.39	2.78±1.24	1.548±0.062
155°	210	317.54± 5.19	66.64± 3.34	3.10±2.43	4.00±2.83	509.71± 7.78	120.00± 5.17	15.05±2.87	2.50±2.50	1.534±0.061

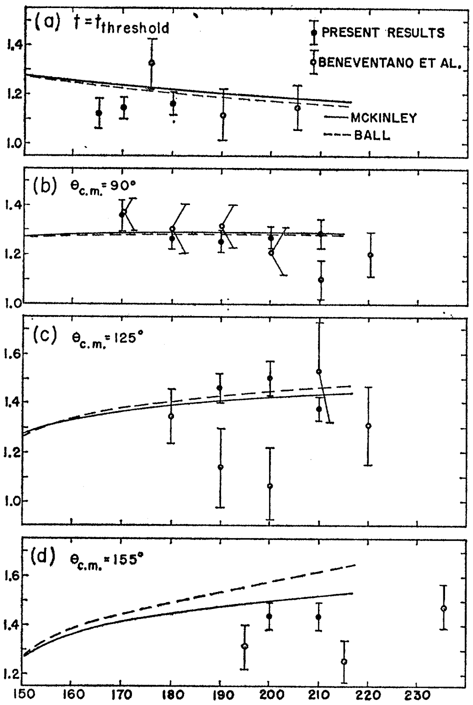

FIG. 4. π^-/π^+ ratio. Experimental data are corrected for Coulomb interactions in the final state. The data of Beneventano that are shown were mostly for slightly different angles (less than 10 deg difference from the present experiment). A two-body kinematic fit is used to determine the photon energy and c.m. angle of the produced pion. Curves are calculated from McKinley (see Ref. 25), with set X of phase shifts, and from Ball, for $\Lambda=0$.

ever, the values we have found on the Baldin line are clearly smaller than the values extracted from previous data by Walker,[9] but they are in agreement with the results of Swanson.[24]

IV. DISCUSSION

We must deduce from the measured values of the ratio R on deuterium, the values R_0 for photoproduction from free nucleons. A detailed analysis of this problem has been given.[1]

The three-body nature of the reaction implies that it is a spectrum of photons which produce the reaction and the neutronproton mass difference produces a shift of this spectrum. However, the influence on the ratio due to these effects is very small, the ratio varying slowly with γ-ray energy in the energy range considered here. Two-body kinematics are, therefore, used to deduce the energy of the incoming photon.

The only important corrections to be made to R are Coulomb corrections due to interactions in the final state. These corrections, as calculated by Baldin,[8] have been applied to R and the values R_0 obtained are given in Figs. 4 and 5. Unfortunately, the uncertainty of this correction is extremely difficult to estimate accurately. In our case, the total Coulomb corrections lie between 3 and 7% throughout the energy and angular range we have investigated: 0 to 4% of which is due to the positive correction produced by the proton-proton interaction and 3 to 11% for the negative one produced by the meson-proton interaction. The most comprehensive experiment which has previously been done in this energy range is that of Beneventano *et al.*,[1] whose results (after making the above Coulomb corrections) are also shown in Fig. 4. There is general agreement within the precision of the experiments.

We may now compare these values of R_0 with theoretical predictions. First, cross sections have been evaluated from C.G.L.N., with some corrections. The most recent calculations have been made from

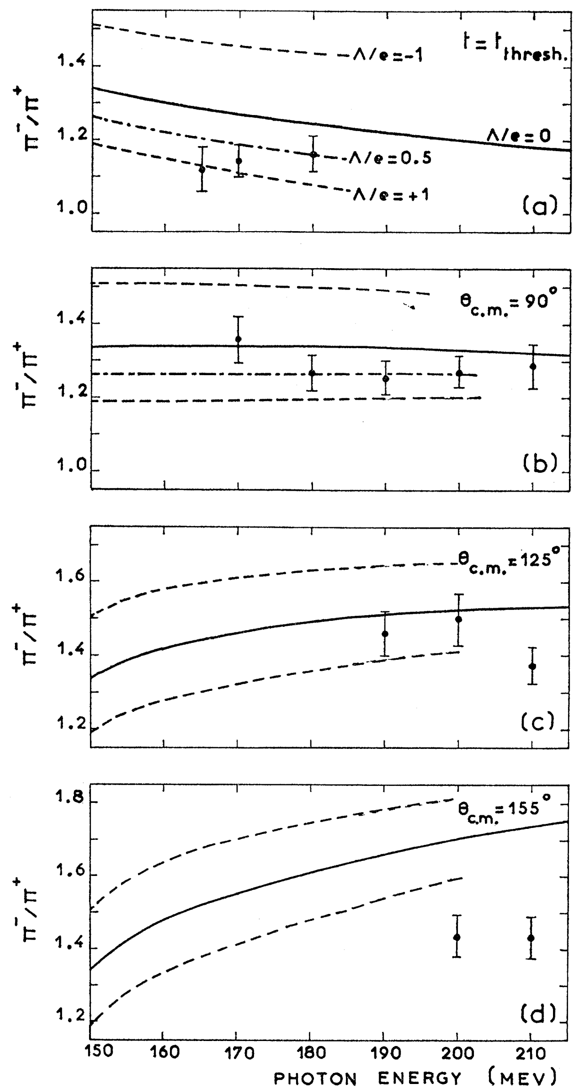

FIG. 5. π^-/π^+ ratio. Curves are calculated from Ball, the isoscalar amplitudes being corrected for the $I=\frac{1}{2}$ phase shifts and for different values of Λ.

[24] W. P. Swanson, Lawrence Radiation Laboratory, University of California, Berkeley, Report UCRL-9194, 1960 (unpublished).

McKinley's report,[25] with no $1/M$ expansion and use of experimental phase shifts rather than effective range formulas for both the large and small P-wave phase shifts. McKinley, besides, has introduced a nonconstant comparison function into the relationship between the scattering and photoproduction partial-wave amplitudes, the effect of which is small. It can be seen on Fig. 4 that the agreement for the whole is reasonable.

To introduce the bi-pion interaction, we have followed Ball.[14] In this work, the isovector amplitudes F^- are calculated with no $1/M$ expansion, but with more approximations than above (small P-wave shifts put equal to zero and use of effective-range formulas). It can be seen on Fig. 4 that the two approaches give approximately the same answer in the forward hemisphere. So, we shall confine the comparison with the data to the Baldin line and 90° where the influence of phase shifts in minimized (and also the contribution of the nonphysical region to the dispersion integrals). In the calculation of the isoscalar amplitudes F^0, Ball neglected the $I=\frac{1}{2}$ phase shifts. This effect has been evaluated by Warburton and Gourdin[13] and should be taken into account, as well as the bi-pion contribution. Therefore, we have calculated the π^-/π^+ ratio from Ball, but we have corrected the F^0 as in Ref. 13. In Fig. 5 the solid curve shows Ball's prediction for the π^-/π^+ ratio after being corrected by the addition of the $I=\frac{1}{2}$ phase shifts. The dashed curves indicate the deviations from the solid curve with various assumptions for the value of Λ. Figures 5(a) and 5(b) show immediately that the sign of Λ/e must be positive to provide agreement with the experimental data. The magnitude of Λ/e can be roughly estimated from Figs. 5(a) and 5(b) to be 0.5 ± 0.25. A precise definition of Λ is given by Ball.[14] In the backward hemisphere, the difference between the predictions of Ball's theory and the data becomes quite large. It is seen, however, that this is not so for the theory where more realistic values of phase shifts have been assumed. In fact, the difference between the latter theory and the data remains approximately constant over the entire angular range considered. This gives a strong confidence that, if we were able to then calculate the bi-pion correction to this more realistic theory, we would get the same order of magnitude for Λ. So far this calculation has not been done.

The value of $\Lambda/e=+0.50\pm0.25$, which we have obtained is in good accord with some previous estimates[11,13] but is in disagreement with a recent result of an experiment on π^+ photoproduction from hydrogen[7] which gave $\Lambda/e=-1.2\pm0.4$. The interpretation of this experiment rested on the assumption that the deviation of the experimental data from the C.G.L.N. predictions for backward meson angles was due essentially to bi-pion effects. We would like to point out that, to the extend that Baldin's remark, on the influence of the unphysical region being important at backward angles is correct, then, to the same extent, the above assumption is questionable. Second, it should be noted that the evaluation of the bi-pion contribution by McKinley[3] and Ball do not agree, particularly at large angles. We do not understand the reason for this apparent discrepancy.

[25] D. J. Drickey (private communication).

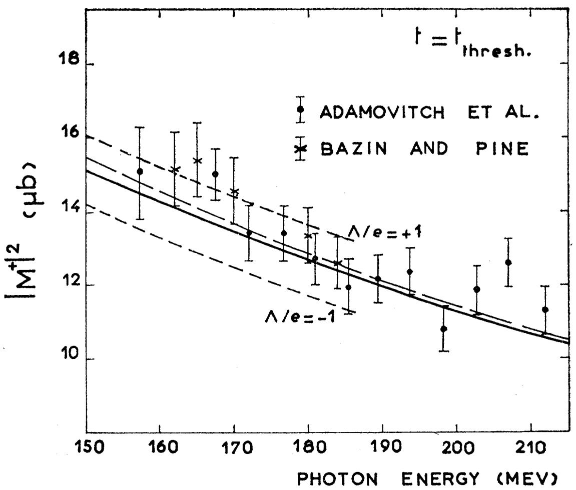

FIG. 6. Matrix element for π^+ photoproduction along the Baldin line. The matrix element is given by $|M^+|^2=(k/q)(d\sigma/d\Omega)$, where k and q are the photon and meson momentum (c.m. system). Curves are calculated from Ball, the notation is the same as in Figs. 4(a) and 5(a).

It is interesting to look at the data on π^+ photoproduction on the Baldin line, in the same way as for the π^-/π^+ ratio. Figure 6 shows recent data,[26,27] with the values of the matrix element $|M^+|^2$ calculated from Ball, as in Figs. 4(a) and 5(a). It can be seen on this figure that the effect of the $I=\frac{1}{2}$ phase shifts corrections to the isoscalar amplitudes is very small, and that the agreement is still good for positive Λ/e and not at all for $\Lambda/e=-1$.

The threshold values of the π^-/π^+ ratio and of $|M^+|^2$, as extrapolated in Figs. 5 and 6, for $0.25<\Lambda/e<0.75$:

$$\pi^-/\pi^+=1.26\pm0.04\,,\quad |M^+|^2=(15.85\pm0.25)\mu\text{b}\,,$$

are consistent with the other experimental parameters of low-energy pion physics. In particular, if we take[9] as a value for the Panofsky ratio $P=1.53\pm1\%$ and for the pion nucleon S-wave scattering lengths $a_1-a_3=0.245\pm0.007$, then together with the above value of $|M^+|^2$ we would predict a ratio of $\pi^-/\pi^+=1.145\pm0.04$ which is reasonably consistent with the above threshold value. Clearly, however, the accuracy of the data in this field has now reached the stage where within the framework of a charge independent theory small discrepancies may appear. A considerable extension of

[26] M. I. Adamovitch, E. G. Gorghevskaya, V. G. Larionova, N. M. Panova, S. P. Kharlamov, and F. R. Yagudina, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 207.

[27] M. Bazin and J. Pine (private communication) (to be published).

the present theory would be required to overcome this shortcoming.

V. CONCLUSION

Low-energy photoproduction of π^+ and π^- mesons seems internally consistent and in agreement with theoretical predictions of dispersion relations, particularly if the analysis is made for constant nucleon momentum transfer, equal to that occurring at threshold. The introduction of a bi-pion interaction and a correction to the isoscalar amplitudes for the $I=\frac{1}{2}$ phase shifts gives $\Lambda/e\approx+0.50\pm0.25$. For large angles, a better knowledge of the isovector amplitudes seems neccessary before drawing detailed conclusions. Nevertheless, on the basis of the present interpretation, the constant, Λ/e, is certainly not negative and not greater than $+e$.

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Cluster Formulation of the Exact Equation for the Evolution of a Classical Many-Body System

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An exact non-Markoffian equation is derived for the evolution of an infinite homogeneous system. This equation—which may be viewed as a time-dependent analog of the equilibrium virial expansion—may be readily applied when the forces between particles include infinite repulsions. The derivation of this equation from Liouville's equation is analogous to Mayer's derivation of the virial expansion from the partition function. In this way the formal development of nonequilibrium statistical mechanics is placed on a similar footing to that of equilibrium statistical mechanics, and a many-body problem is reduced to understanding the dynamics of isolated groups of particles. Fourier expansions and expansions in powers of the interaction potential are avoided by dealing with s-body Green functions (propagators) which are always convergent functions of the interaction potential. These functions correspond to multiplet collisions in ordinary configuration space between s isolated particles and are time-dependent analogs of the irreducible clusters well known in equilibrium statistical mechanics. The kernel (memory) of the equation of evolution consists of a linear sum of the time-dependent irreducible clusters. The non-Markoffian behavior of the equation of evolution is, thus, directly given by the time dependence of these clusters, and is explicitly related to incompleted collisions. The equation of evolution is solved in the asymptotic limit of long times. In this limit it is found (because the kernel rapidly vanishes) that the equation reduces to a Markoffian master equation involving a scattering operator for both completed and incompleted collisions in configuration space.

I. INTRODUCTION

Ever since Van Hove[1] derived an exact non-Markoffian equation for the irreversible evolution of a many-body system there has been an increased activity in the field of nonequilibrium statistical mechanics. This activity was inspired by Van Hove's demonstration that, with sufficient determination, it is possible to obtain exact (if formal) solutions of difficult statistical mechanical problems.

More recently, Prigogine and co-workers[2] have obtained exact equations for all the Fourier components of the distribution function and for a wide class of initial states.

The methods used by Van Hove and Prigogine are necessarily characterized by their excessive complexity and their use of topological notions whose physical content is somewhat obscure. Their results are correspondingly complicated and difficult to apply—except in certain limiting cases (weak interactions, low density).

Another characteristic of these equations is that they are not *directly* applicable to interaction potentials with infinite repulsions (hard cores). This is because the kernels of these equations are expressed as expansions in the interaction potential. Hence, in order to rigorously apply these equations to hard-core interactions one must first sum higher Born approximations (sum

[1] L. Van Hove, Physica **23**, 441 (1957).
[2] I. Prigogine and P. Balescu, Physica **27**, 629 (1961). This paper refers to earlier related work.