Diffraction Scattering and Singularities in the Angular Momentum Plane*

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The singularities in the angular momentum plane, which describe the vacuum intermediate state dominating high-energy scattering, are assumed to consist of a single Regge pole and a fixed branch cut. If the energy variable ($\sqrt{s}$) only enters the equations in the normalized form $w=s/2M_1M_2$, where M_1 and M_2 are masses of the particles, the value of w for lower mass interactions (e.g., $\pi-N$) will be much larger than for high mass interactions (e.g., $N-N$), for the same value of the lab energy. In the combined pole and cut model the vacuum pole dominates at lower values of w, and the cut becomes dominant as $w\to\infty$, so that the $p+p$ scattering between 7 and 20 BeV/c will be mainly described by the single vacuum pole, causing the width of the diffraction peak to shrink, whilst the π^-+p scattering between 7 and 17 BeV/c will be described by the fixed cut, which leads to no shrinkage. This permits the $p+p$ and π^-+p data to be combined into one. A fit is obtained to the combined $p+p$ and π^-+p data for the elastic differential cross section obtained recently by Foley *et al.* Another consequence of the vacuum-pole-cut model is that, in the energy range under consideration, the total cross section is not yet constant, but has the form $\sigma_T=a+b/\ln w$, which agrees well with $\pi^\pm+p$ scattering data. This behavior of σ_T should also describe the average of the $p+p$ and $\bar{p}+p$ total cross sections. Estimates of a and b using $p+p$, $\bar{p}+p$, and $\pi^\pm+p$ data agree with those obtained for $\pi^\pm+p$ alone to within 6%. This strongly supports our method of combining $N-N$ and $\pi-N$ scattering data, and is in good agreement with the pole and cut model. A test of the theory can be obtained from experiments on $K-N$ scattering.

1. INTRODUCTION

During the past year considerable attention has been devoted to high-energy-diffraction scattering and its connection with Regge poles.[1] In a recent paper, the Regge formalism was extended to include a branch cut in the angular momentum plane.[2] This extension was based on a dispersion relation for $A(t,\alpha)$, for fixed t,

$$A(t,\alpha)=\sum_i \frac{g_i(t)}{\alpha-\alpha_i}+\frac{1}{\pi}\int_c \frac{d\alpha'\,\rho(t,\alpha')}{\alpha'-\alpha}, \qquad (1.1)$$

where $g_i(t)$ are the residues of the Regge poles at $\alpha_i(t)$, and there is a branch cut along the contour c with discontinuity $\rho(t,\alpha)$. The model of high energy scattering based on (1.1) was analyzed and applied to the CERN $p+p$ data.[3] It was assumed that the dominant branch point $\alpha_2(t)$ was equal to unity everywhere in the t plane, and a fit to the CERN data was obtained.

High-energy experiments performed recently at Brookhaven[4] have yielded more accurate measurements of the elastic differential cross section from 7 to 20 BeV/c incident momentum for $p+p$ and 7 to 17 BeV/c for π^-+p over a range $0.2\leq|t|\leq 0.8$ (BeV/c)^2. In view of the improvement in accuracy, they were able to carry out a more critical evaluation of the Regge pole theory based on a simple vacuum pole trajectory.[5] According to this theory at sufficiently high energies the vacuum, or Pomeranchuk, trajectory dominates, and for any incident particle

$$d\sigma/dt=[d\sigma/dt]_{\text{opt}}F(t)[s/s_0]^{2\alpha_P(t)-2}, \qquad (1.2)$$

where s_0 is usually chosen to be $2M_p^2$ for nucleon-nucleon scattering. Eq. (1.2) gives a shrinkage of $d\sigma/dt$ with increasing s corresponding, for small t, to a growth of the radius of interaction. The new Brookhaven set of data[4] for $p+p$ scattering does show a shrinkage of the type consistent with the Regge pole prediction, but there is no shrinkage evident for the π^-+p scattering, which contradicts the assumption that strong interactions are dominated by a single vacuum pole. The three-pole model[6] was also found to be inconsistent with the Brookhaven data.

In the following, we shall investigate further the high-energy behavior of the combined pole and cut model,[2] and apply this model to the Brookhaven data.

2. HIGH-ENERGY DIFFRACTION SCATTERING

The invariant amplitude is defined by

$$A=Wf, \qquad (2.1)$$

where f is the "physical" scattering amplitude and W is

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[1] S. D. Drell, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 897. This report contains numerous references to work on Regge poles.

[2] I. R. Gatland and J. W. Moffat, Phys. Rev. **129**, 2812 (1963).

[3] A. N. Diddens, E. Lillethum, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherall, Phys. Rev. Letters **9**, 108 (1962); **9**, 111 (1962); G. Cocconi, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 883.

[4] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **10**, 376 (1963); C. C. Ting, L. W. Jones, and M. L. Perl, *ibid.* **9**, 468 (1962).

[5] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **7**, 394 (1961); R. Blankenbecler and M. L. Goldberger, Phys. Rev. **126**, 766 (1962); V. N. Gribov, Zh. Eksperim. i Teor. Fiz. **41**, 1962 (1961) [translation: Soviet Phys.—JETP **14**, 1395 (1962)]; S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962).

[6] F. Hadjioannou, R. J. N. Phillips, and W. Rarita, Phys. Rev. Letters, **9**, 183 (1962); V. P. Kanavatz, I. I. Levintov, and B. V. Morosov, Phys. Letters **4**, 196 (1963); see also Ref. 1.

the total energy of the particles in the barycentric system. The differential cross section is given by

$$d\sigma/d\Omega = |(1/W)A|^2. \quad (2.2)$$

In the asymptotic region above all resonant scattering the elastic amplitude is imaginary if the dominant contribution comes from diffraction scattering of the incident-particle wave associated with absorption into many open, strongly coupled, inelastic channels. With the aid of the optical theorem

$$\mathrm{Im} f(0) = (q/4\pi)\sigma_T, \quad (2.3)$$

and assuming that A in (2.2) is the imaginary elastic amplitude, we get

$$d\sigma/d\Omega = [(q/4\pi)\sigma_T]^2, \quad (2.4)$$

and

$$\sigma_T = [16\pi(d\sigma/dt)]^{1/2}. \quad (2.5)$$

If we write $w = s/s_0$ and assume that the scale factor s_0 is given by $s_0 = 2M_1M_2$, then for sufficiently large s we have

$$d\sigma/dt = (\pi/M_1^2M_2^2)|A/w|^2. \quad (2.6)$$

From the extension of the Regge formalism introduced in our first paper,[2] we arrive at the asymptotic formula for the elastic amplitude

$$A(s,t) = g(t)\left(\frac{s}{s_0}\right)^{\alpha_p(t)} + f(t)\frac{(s/s_0)^{\alpha_2(t)}}{\ln(s/s_0)}, \quad (2.7)$$

where the second term in (2.7) arises from the cut in the α plane, and $f(t)$ is a measure of the discontinuity across the cut.[7] By substituting (2.7) into (2.6), we get

$$\frac{d\sigma}{dt} = \frac{\pi}{M_1^2M_2^2}\left|g(t)w^{\alpha_p(t)-1} + f(t)\frac{w^{\alpha_2(t)-1}}{\ln w}\right|^2, \quad (2.8)$$

and with the aid of Eq. (2.7), we obtain the total cross section

$$\sigma_T = (4\pi/M_1M_2)[g(0) + f(0)/\ln w]. \quad (2.9)$$

In (2.9), it is assumed that the maximum value allowed by unitarity, namely, $\alpha_p(0) = \alpha_2(0) = 1$ has been attained.

3. EXPERIMENTAL CONSEQUENCES OF A DOMINANT BRANCH CUT IN THE α PLANE

Let us write (2.8) in the form

$$\frac{d\sigma}{dt} = \frac{\pi f^2(t)}{M_1^2M_2^2} \times \frac{w^{2[\alpha_2(t)-1]}}{\ln^2 w}\left|1 + \frac{g(t)}{f(t)}w^{[\alpha_p(t)-\alpha_2(t)]}\ln w\right|^2. \quad (3.1)$$

Then for $\alpha_2 > \alpha_p$ and for sufficiently large s the second term may be neglected so that we have

$$d\sigma/dt = (\pi/M_1^2M_2^2)[f(t)/\ln w]^2 \times \exp\{2[\alpha_2(t)-1]\ln w\}. \quad (3.2)$$

We assume that $f(t) = \text{const}$ for all t (or is slowly varying) and write

$$\alpha_2(t) = 1 + t\alpha'(0) + \cdots. \quad (3.3)$$

Let us denote by t_0 a value of t close to zero but sufficiently different from zero to allow the cut to dominate. Then the differential cross section for small $t < t_0$

$$\left(\frac{d\sigma}{dt}\right)\Big/\left(\frac{d\sigma}{dt}\right)_{t_0} = \exp[-|t-t_0|2\alpha_2'(0)\ln w], \quad (3.4)$$

exhibits a logarithmic shrinkage,[8] as in the Regge-pole model, in the region in which the branch cut dominates. Indeed, we must analyze the total cross section to distinguish between a pure Regge behavior and a pure or dominant branch-cut behavior.

If, on the other hand, we assume from the beginning that $\alpha_2(t) = 1$ for all t, in (3.2), we get

$$\frac{d\sigma/dt}{(d\sigma/dt)_{t_0}} = \left|\frac{f(t)}{f(t_0)}\right|^2. \quad (3.5)$$

Since the experimental data for $N-N$ and $\pi-N$ scattering is peaked exponentially at small t the function $f(t)$ can be described by

$$f(t) = A\exp[-\Gamma|t|]. \quad (3.6)$$

In the region in which the branch cut dominates and $\alpha_2(t) = 1$, this diffraction peak suffers no shrinkage, as is clear from (3.5).

The scale factor $s_0 = 2M_1M_2$ occurring in the variable $w = s/s_0$ is seven times smaller for $\pi-N$ scattering as compared with that for $N-N$ scattering. In view of this the high-energy limit for $\pi-N$ scattering should be reached sooner,[9] so that the shrinkage should be less evident for $\pi-N$ scattering, as is observed by Foley *et al.*[4] Field theory arguments suggest that the quantum numbers may be the same for high-energy $\pi-N$ and $N-N$ scattering. Therefore, $\alpha_p(t)$ and $\alpha_2(t)$ should be the same for $\pi-N$ and $N-N$ scattering at high energies and for small t. This leads us to suggest that we combine the $\pi-N$ and $N-N$ sets of data into one. We also assume that $[f(t)/g(t)]_{N\pi} = [f(t)/g(t)]_{NN}$ and determine the relative values of the coupling strengths to nucleons and pions (i.e.,

[7] A less phenomenological treatment of the branch cut would give a log term, in (2.7), of the form $1/[\ln(s/s_0)]^\beta$, where $\beta > 1$, but as the current experimental data cannot distinguish powers of $\ln(s/s_0)$, we ignore this refinement for the present.

[8] The width of the diffraction peak is given by $\Gamma = [\frac{1}{2}d\ln(d\sigma/dt)/dt]^{-1}$ and the shrinkage is defined by $S = (d/d\ln w)[\Gamma^{-1}]$.

[9] Because other effects, such as resonances in the s channel, which contribute to the breakdown of the theory will not depend on the normalization $s_0 = 2M_1M_2$, the minimum value of w for which the vacuum pole-cut dominates may vary with the process under consideration. Whereas the $N-N$ data may be fitted at 7 BeV the same cannot be done with the $\pi-N$ data at 1 BeV, since $\pi-N$ resonances have been observed at energies up to 2.5 BeV [T. Kycia and K. Riley, Phys. Rev. Letters **10**, 266 (1963)].

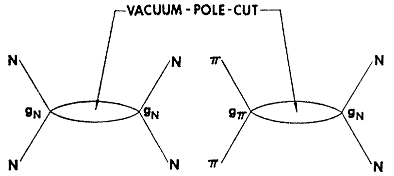

FIG. 1. Coupling of the vacuum intermediate state consisting of a combined pole and fixed cut to nucleons and pions for $N-N$ and $\pi-N$ scattering.

g_N and g_π). Below the crossover region the $N-N$ data will show a shrinkage, and above the crossover region the $\pi-N$ data will show no shrinkage in accordance with the Brookhaven data. In our combined-pole and branch-cut model the Regge pole will dominate below the crossover region and, therefore, produce the required shrinkage, while in the region in which the fixed branch cut dominates above the crossover region the diffraction scattering will show no shrinkage. This high-energy behavior is exhibited by the model investigated in our first paper.[2] The differential cross section thus takes the form

$$(d\sigma/dt)_i=(\pi g_i^2/M_1^2M_2^2)\,|w^{\alpha_p(t)-1}+\beta(t)/\ln w|^2, \quad (i=1,2) \qquad (3.7)$$

where $g_1=g_N$ and $g_2=(g_\pi g_N)^{1/2}$.

We now state the principle that all high-energy strong interactions are dominated by a pole-fixed cut with the quantum numbers of the vacuum. In other words, the vacuum intermediate state between all strongly coupled particles consists of a Regge pole and a fixed branch cut. This is shown graphically in Fig. 1.

4. TOTAL CROSS SECTIONS

In this section, we wish to discuss the total cross section in our model as given by Eq. (3.6). We see that σ_T in (2.9) will tend to a constant as $w\to\infty$. In the region of experimental interest $2<\ln w<5$, we cannot assume a constant behavior for σ_T, since the $f(0)$ term is not vanishingly small. The variation of the total cross section should not be considered fortuitous and should not be removed by normalizing the data in the form $(d\sigma/dt)/(d\sigma/dt)_{t=0}$. We shall instead proceed by using (3.6), and determine $\alpha_p(t)$, $g_i(t)$, and $\beta(t)$ from the experimental differential cross section alone; the total cross-section data can be used to determine $g(0)$ and $f(0)$ at $t=0$.

Lindenbaum *et al.*[10] have found that the π^-+p total cross section can be adequately fitted by a curve of the form $\sigma_T=a+b/\ln p$, where a and b are constants and p is the incident pion momentum in the lab system

$$p=M_2\left[\left(w-\frac{M_1^2+M_2^2}{2M_1M_2}\right)^2-1\right]^{1/2}\simeq M_2w, \qquad (4.1)$$

where M_1 is the target and M_2 is the incident particle. They found a slightly better χ^2 fit by using $\sigma_T=a+b/p$, and used this expression to normalize the differential cross sections in the Brookhaven experiments.[4] It, thus, appears that the π^-+p data exhibits a decreasing total cross section, which is consistent with the combined pole and cut model.[2] When we turn our attention to the $p+p$

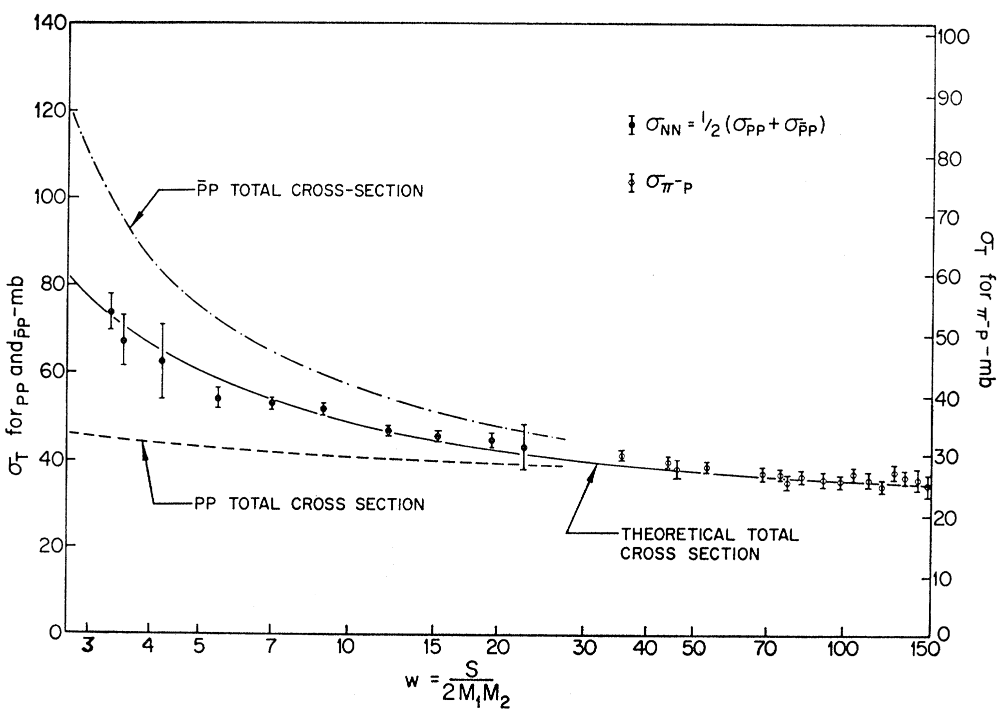

FIG. 2. Experimental data for π^-+p and the averaged $p+p$ and $\bar{p}+p$ total cross sections together with the theoretical curve. The total cross sections for $p+p$ and $\bar{p}+p$ scattering are shown for comparison.

[10] S. J. Lindenbaum, W. A. Love, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 7, 185 and 352 (1961).

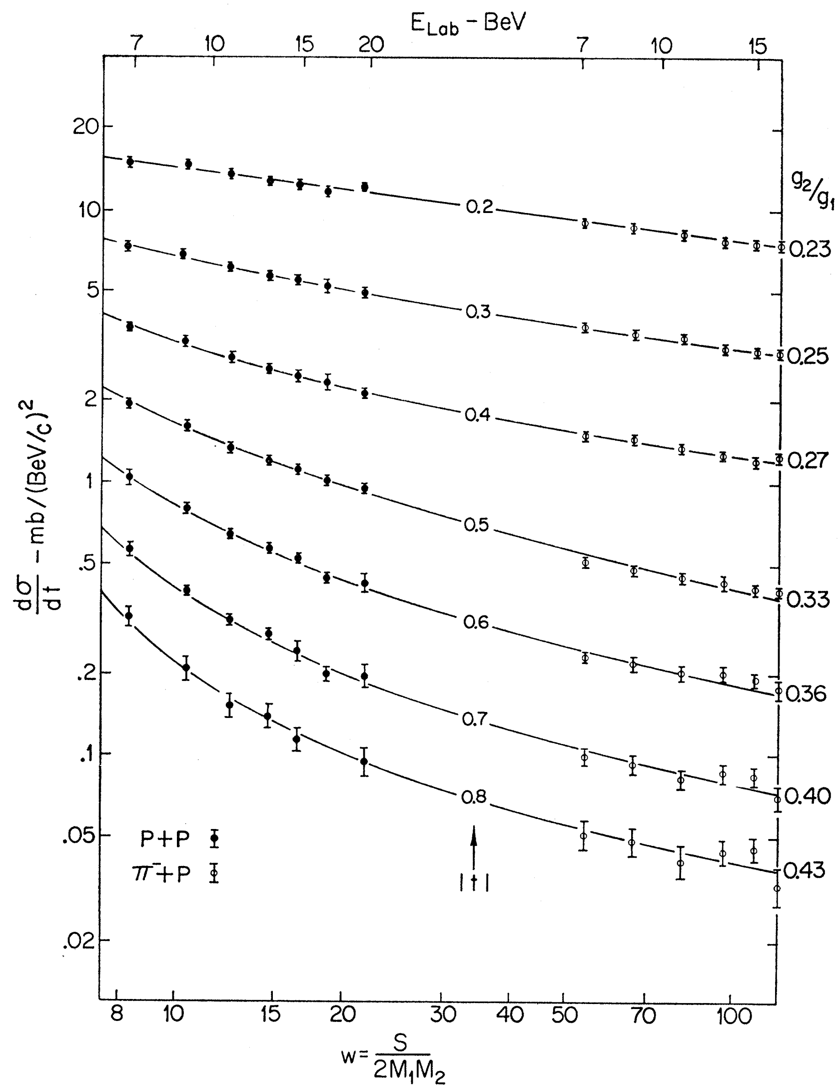

FIG. 3. Elastic differential cross sections for $p+p$ and π^-+p scattering given by Foley *et al.* and the theoretical curves for various values of t.

scattering data the model appears to break down, because the $p+p$ total cross section is constant above 10 GeV/c, and certainly has less slope than the π^-+p data at somewhat higher values of w. But this difficulty is removed if we include the $\bar{p}+p$ data. The $p+p$ and $\bar{p}+p$ scattering amplitudes are assumed to differ only by a contribution given by the ω trajectory, which becomes very small compared to the vacuum trajectory as the energy increases. The effect of the vacuum pole plus cut will be given by the mean value of the $p+p$ and $\bar{p}+p$ amplitudes, which, as is shown in Fig. 2, is a continuation of the π^-+p curve. We should use the average of the π^-+p and π^++p data; however, as these only differ by a small constant factor, and only π^-+p differential cross sections are available, we shall use only the π^-+p total cross section results. Similar arguments allow us to ignore $p+n$ data, etc. A fit to the $\bar{p}+p$, $p+p$, and π^-+p data[9] gives

$$g_1(0)=4.4,\quad f_1(0)=11;\qquad g_2(0)=0.48,\quad f_2(0)=1.18. \tag{4.2}$$

The π^-+p data alone gives

$$g_2(0)=0.45,\quad f_2(0)=1.20. \tag{4.3}$$

From these results, it appears highly plausible that the π^-+p data are, indeed, a true continuation of the combined $p+p$ and $\bar{p}+p$ data. In Fig. 2, we show data on the π^-+p total cross section and the average $p+p$ and $\bar{p}+p$ total cross sections together with the theoretical curve. The experimental $p+p$ and $\bar{p}+p$ total cross sections are also shown to illustrate the effect of the ω-meson trajectory.

5. COMPARISON WITH EXPERIMENTS

We have fitted the Brookhaven $p+p$ and π^-+p elastic differential cross sections by a curve of the form (3.7),

TABLE I. Values of the parameters α_p, g_1, g_2, f_1, and f_2 for various values of t. The values at $t=0$ are obtained from total cross-section data including $\bar{p}p$ scattering. Those for $t<0$ are based on $p+p$ and π^-+p data only.

t	α_p	g_1	g_2	f_1	f_2
0	1.0	4.4	0.48	11	1.18
−0.2	0.87	4.1	0.55	0.0	0.0
−0.3	0.90	1.8	0.24	1.8	0.24
−0.4	0.99	0.35	0.053	2.6	0.39
−0.5	···	0.0	0.0	2.4	0.45
−0.6	−2.0	42	8.3	1.6	0.32
−0.7	−1.3	15	3.4	1.0	0.24
−0.8	−2.7	250	59	0.74	0.18

and obtained least-squares fits for the parameters α_p, g_1, g_2, and $f_1=g_1\beta$ and $f_2=g_2\beta$ for each value of t. The values of $f_1(t)$ and $f_2(t)$ for large $|t|$, which are determined principally by the π^-+p data, may be treated with some confidence, but the $\alpha_p(t)$, $g_1(t)$, and $g_2(t)$ are not so good, since the $\bar{p}+p$ differential cross sections have not been included. In particular, $\alpha_p(t)$ is overestimated for small $|t|$, becuase the $\bar{p}+p$ total cross section is larger than the $p+p$, and underestimated at large $|t|$, since the $\bar{p}+p$ diffraction peak is narrower than that for $p+p$. This probably accounts for the anomalous behavior of $\alpha_p(t)$ obtained from the present data.

Table I gives the values of $\alpha_p(t)$, $g_1(t)$, $g_2(t)$, $f_1(t)$, and $f_2(t)$ for $-0.8\leq t\leq -0.2$ obtained from a least-squares fit.

The functions $f_1(t)$ and $f_2(t)$ for $t<-0.5$ are well approximated by an equation of the form

$$f_i(t)=A_i\exp[-\Gamma_i|t|], \tag{5.1}$$

where

$$A_1=16, \qquad A_2=1.9,$$
$$\Gamma_1=3.9\ (\text{BeV}/c)^{-2}, \quad \Gamma_2=3.0\ (\text{BeV}/c)^{-2}. \tag{5.2}$$

Equation (5.1) can be used to obtain more realistic values of $f_1(t)$, $f_2(t)$ for small $|t|$. The values of the five parameters for $t=0$, obtained from π^-+p, $p+p$, and $\bar{p}+p$ total cross-section data, are given for comparison. The ratios of the π^-+p and $p+p$ form factors $g_2/g_1 = f_2/f_1$ are the functions best determined by the data, and may be fitted by

$$g_2/g_1=f_2/f_1=0.12\exp(-0.9t). \tag{5.3}$$

The π^-+p and $p+p$ coupling to the vacuum-pole-cut intermediate state also differs by the factor $\pi/M_1^2M_2^2$ and the ratio of the π^-+p and $p+p$ differential cross sections can be given by

$$\left(\frac{d\sigma}{dt}\right)_{\pi p}\left(\frac{d\sigma}{dt}\right)_{pp}^{-1}=\left(\frac{M_1}{M_2}\cdot\frac{g_2}{g_1}\right)^2=0.65\exp(-1.8t). \tag{5.4}$$

The $p+p$ and π^-+p data and the theoretical curves are exhibited in Fig. 3.

It is interesting to note that this analysis favors $g_i=0$ for $t=-0.5$ and $\alpha_p(t)$ passes through zero in the region $-0.6<t<-0.4$. This result may correspond to the physical exclusion of the "ghost pole,"occurring when $\alpha_p(t)=0$ for negative values of t. If we assume that, for small t,

$$g(t)\approx g(0)\,,\quad f(t)\approx A\exp(\Gamma t)\,,\quad \alpha_p(t)\approx 1+\alpha'_p(0)t\,, \tag{5.5}$$

then the elastic cross section will be given by

$$\sigma_{\text{el}}=\int_0^{-4p^2}dt\frac{d\sigma}{dt}\approx\frac{\pi}{M_1^2M_2^2}\left[\frac{g^2(0)}{2\alpha_p'(0)\ln w}+\frac{2g(0)A}{[\Gamma+\alpha_p'(0)\ln w]\ln w}+\frac{A^2}{2\Gamma\ln^2 w}\right]. \tag{5.6}$$

Thus, as $w\to\infty$ we have

$$\sigma_{\text{el}}\propto\frac{g^2(0)}{\alpha_p'(0)}\left(\frac{1}{\ln w}\right) \tag{5.7}$$

and $\sigma_T\to$ const as in the pure Regge-pole theory.

6. CONCLUSIONS

We observe that the normalized variable $w=s/2M_1M_2$ implies that the $\pi-N$ data will reach its asymptotic limit sooner than the $N-N$ data. We assume that the two interactions differ only by a multiplicative constant and combine the two sets of data into one. The width of the forward peak associated with a Regge pole has a shrinkage proportional to $\alpha_p'(0)\ln w$, causing a logarithmic decrease in the elastic cross section. This decrease is a general feature of the Regge asymptotic behavior. The cut in the α plane gives a diffraction peak with a shrinkage proportional to $\alpha_2'(0)\ln w$, and the cut dominates over the Regge pole at large w and $|t|$ provided $\alpha_2>\alpha_p$. We now note the absence of any shrinkage of the π^-+p diffraction peak in the Brookhaven data, and observe that this implies $\alpha_2'(0)=0$ and, therefore, $\alpha_2(t)=1$ [as we have assumed $\alpha_2(0)=1$] and this gives us the fixed cut. The diffraction-scattering pattern then requires $f(t)=A\exp(-\Gamma|t|)$.

The vacuum pole plus fixed-cut model causes the total cross section to take the form $\sigma_T=a+b/\ln w$. We obtain support for the combined $N-N$ and π^-p data when we apply this prediction to the π^-p and $NN=\frac{1}{2}(pp+\bar{p}p)$ total cross sections. We now adopt the principle that all strong interactions at high energies are dominated by a vacuum-pole-cut intermediate state; and therefore, all such interactions should possess a total cross section of the form $\sigma_T=a+b/\ln w$. Thus, within a multiplicative factor, the pole+fixed cut describes all averaged strong interactions of the type

$$\begin{aligned}&\tfrac{1}{4}(\pi^+p+\pi^-p+\pi^+n+\pi^-n)\,,\\&\tfrac{1}{4}(pp+\bar{p}p+pn+\bar{p}n),\\&\tfrac{1}{4}(Kp+\bar{K}p+Kn+\bar{K}n)\,,\ \text{etc.}\end{aligned} \tag{6.1}$$

The $\pi-\pi$ and $K-\pi$ interactions will also possess a diffraction scattering and total cross section of the form predicted above, although these interactions cannot be checked by any current experimental data. One experiment would be particularly useful at present to check our predictions. From the effect of the ω meson on the pp and $\bar{p}p$ cross sections, we conclude that $K-p$ and $\bar{K}-p$ total cross sections will be approximately the same above 10 BeV, and will have the form

$$\sigma_T=\frac{\pi}{M_K M_N}g_{KN}\left(1+\frac{\beta}{\ln w}\right) \tag{6.2}$$

with $\beta=2.5$ as in the $N-N$ and $\pi-N$ processes.

Note added in proof. In a recent letter by Freund and Oehme [Phys. Rev. Letters **10**, 450 (1963)] it is concluded that a pole plus cut model cannot fit the data of Foley *et al.*[4] or the total cross section data.[10] Their incorrect result stems from the use of $s_0=2M_p^2$ as the normalization constant for all processes. This emphasizes the important role played by the normalization parameter $s_0=2M_1M_2$ in our theory. Freund and Oehme also remark that there is no evidence of both a pole and cut in $\pi+p$ experiments. However, new data by Brandt *et al.* [S. Brandt, V. T. Cocconi, D. R. O. Morrison, A. Wroblewski, P. Fleury, C. Kayas, F. Muller, and C. Pelletier, Phys. Rev. Letters **10**, 413 (1963)] on $\pi-p$ diffraction scattering at small t does show a "kink" in support of our theory, [Phys. Rev. **129**, 2812 (1963)].

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Photoproduction of Low-Energy Charged Pions from Deuterium

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Accurate measurements have been made of the π^-/π^+ photoproduction ratio on deuterium, in the gamma-ray energy range 165–210 MeV, for several angles: 155°, 125°, 90° (center-of-mass system) and along Baldin's kinematical line. These last data are new contributions: $\pi^-/\pi^+=1.20\pm0.03$ averaged between 165 and 180 MeV. The others are improvements of the accuracy of previous data. The comparison with Ball's theory, corrected for taking into account the $I=\frac{1}{2}$ phase shifts, gives for the coupling constant Λ for $\gamma-\pi-p$ the value: $0.25<+\Lambda/e<0.75$.

I. INTRODUCTION

LOW-ENERGY charged pion photoproduction has been studied for a decade or more. The two reactions which are of greatest interest are

$$\gamma+p\rightarrow\pi^++n\,, \tag{1}$$

$$\gamma+n\rightarrow\pi^-+p\,, \tag{2}$$

where the neutron is in the bound state corresponding to a deuteron. In the past, the main interest was the extrapolation of experimental data to threshold to obtain a value of the pion nucleon coupling constant, and use of the threshold values so obtained as a means to check the well-known relationship between pion photoproduction and scattering.[1] Within the present limits of accuracy of the data and of their interpretation and extrapolation, a reasonable agreement exists. The contribution of the uncertainty of the data on the π^-/π^+ ratio (about 10%) to this type of analysis is considerable. This was one of the motivations for undertaking the present experiment.

More recently, another interest in this field has been the comparison of the experimental data with the predictions of the dispersion relations of Chew, Goldberger, Low, and Nambu (C.G.L.N.).[2] Although the agreement has been reasonably good, some points are worthy of mention.

In the case of reaction (1) there is a marked tendency for the experimental data to lie below the theoretical prediction at large angles.[3] This has been interpreted by several authors in different ways. First, it was shown by Uretsky *et al.*[4] that the predictions of C.G.L.N. theory were very sensitive at backward angles to the choice of the small pion nucleon P-wave phase shifts, and that any lack of agreement of theory and experiment may be due to our lack of precise knowledge of these phase shifts. Second, Baldin[5] suggested that the discrepancy could be due to an unknown contribution

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