PHYSICAL REVIEW VOLUME 132, NUMBER 1 1 OCTOBER 1963

Decay of the π Meson and Goldberger-Treiman Relation*

MASAKUNI IDA†
Physics Department, University of Illinois, Urbana, Illinois
(Received 8 May 1963)

An extension of Goldberger and Treiman's approach to charged pion decay is attempted. Derivation of a generalized Goldberger-Treiman relation is studied without recourse to the nucleon-antinucleon pair approximation. A certain type of dispersion relation is presupposed for the annihilation amplitudes of intermediate states into a lepton pair, which contribute to the imaginary part of the decay amplitude. Also discussed is the determination, in principle, of the weak-coupling constants reponsible for the decay.

I. INTRODUCTION

FIVE years ago, Goldberger and Treiman investigated the decay processes of the charged pion[1] and of the neutral pion,[2] as well as the problem of the weak-current form factors of the baryons.[3] Their works were a great step toward the dynamical understanding of weak processes. Although they used somewhat questionable assumptions, they obtained a surprisingly good result for the charged-pion decay amplitude, which has received much attention since then, and is usually called the Goldberger-Treiman (G-T) relation.[4] These subsequent authors tried to find a physically more reasonable basis for the Goldberger-Treiman relation. The present work is closer in spirit to the original papers[1,3] than to these later works, but the connection between the various approaches will be mentioned.

We shall confine ourselves here mostly to charged-pion decay because neutral-pion decay can be discussed in almost the same way. They assumed charged-pion decay proceeds predominantly through a virtual dissociation of the pion into a nucleon-antinucleon pair, the latter annihilating through the axial-vector Fermi interaction to produce a lepton pair. It was also necessary to assume that the pion-nucleon vertex is damped for large momentum transfers. Indeed, one is led to a paradox if it is not damped.

Very recently, Barrett and Barton[5] have shown that the pion-nucleon vertex tends to a nonvanishing constant if one accepts a Regge behavior for the nucleon-antinucleon phase shifts at high energies. They propose a dispersion relation with a subtraction at infinity for the invariant decay amplitude, in order to resolve the difficulty within the one-channel approximation.

It seems worthwhile to remove the limit set by this approximation, although one then encounters the formidable barrier imposed by the intervention of strong-interaction effects. However, one can go ahead, at least formally, much farther than it appears at first sight if one supposes we know every quantity concerned with only strong interactions. Investigation of the problem from this standpoint is the aim of this paper.

The weak interaction responsible for the decay is assumed to be axial vector, which reasonably explains the experimental ratio, $\sim 1.2\times 10^{-4}$, of $\pi \to e+\nu$ decay to $\pi \to \mu+\nu$ decay. We treat the weak interaction in lowest order and neglect all electromagnetic corrections.

In Sec. II the imaginary part of the invariant decay amplitude $F(s)$ is studied by presupposing a certain type of dispersion relation for the annihilation amplitudes of intermediate states into a lepton pair. The ImF consists of two terms. One corresponds, loosely speaking, to a perturbation-theoretical result, and the other is exactly equal to what is obtained under the assumption that the axial-vector current is proportional to the derivative of the pion field. Derivation of a generalized G-T relation is discussed in Sec. III. It is important to distinguish the cases according to whether the renormalization constant Z_3 for the pion propagator is finite or zero.

In Sec. IV we discuss a possible method to determine, in principle, four axial-vector coupling constants of the baryons with the leptonic current by use of the hypothesis of universal Fermi interaction. When Z_3 vanishes, we have an additional condition on strong-interaction parameters.

Two supplementary remarks are made in the final section. First, the case of a pseudoscalar coupling is studied to see a parallelism of nonperturbational results with perturbational ones for the types of divergences in various weak couplings. Finally, neutral-pion decay, $\pi^0 \to 2\gamma$, is briefly discussed.

II. DISPERSION RELATIONS FOR π DECAY

The invariant amplitude F for the process, $\pi \to l+\nu$, is defined by

$$(2p_0)^{1/2}\langle 0|J_\lambda{}^A(0)|\pi\rangle \equiv ip_\lambda F\,, \qquad (2.1)$$

where $J_\lambda{}^A(x)$ is the strangeness-conserving axial-vector current and p_λ is the four-momentum of the pion,

* This work was supported by the National Science Foundation.

† On leave of absence from Department of Nuclear Science, Kyoto University, Kyoto, Japan.

[1] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).

[2] M. L. Goldberger and S. B. Treiman, Nuovo Cimento **9**, 451 (1958).

[3] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).

[4] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, *ibid.* **17**, 757 (1960); Y. Katayama, Progr. Theoret. Phys. (Kyoto) **26**, 878 (1961); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); Chou Kuang-Chao, Zh. Eksperim. i. Teor. Fiz. **39**, 703 (1960) [translation: Soviet Phys.—JETP **12**, 492 (1961); M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[5] B. Barrett and G. Barton (to be published).

$-p^2=\mu^2$. The pion decay rate ω is then given by

$$\omega=\frac{\mu}{4\pi}\left(\frac{m_l}{\mu}\right)^2\left(1-\frac{m_l^2}{\mu^2}\right)^2(\mu F)^2. \tag{2.2}$$

Using the standard method, one can obtain an analytic function of a variable s, such that $F=F(s=\mu^2)$. An essential assumption to be made here is that the function $F(s)$ satisfies a dispersion relation without subtractions:

$$F(s)=\frac{1}{\pi}\int_{(3\mu)^2}^{\infty}\frac{\mathrm{Im}F(s')}{s'-s-i\epsilon}ds'. \tag{2.3}$$

The imaginary part of $F(s)$ is expressed as

$$\mathrm{Im}F(s)=(\pi/s)\sum_n\langle 0|\partial_\lambda J_\lambda{}^A|s,n\rangle \times\langle s,n|J_\pi|0\rangle\delta(p_n-p), \tag{2.4}$$

where J_π is the source of the pion field, $(\mu^2-\Box)\varphi_\pi$, and n denotes all the variables other than s. The divergence of the current rather than the current itself is considered here because only the pseudoscalar states contribute to the ImF.

By summing up over spins and separating out kinematical factors, (2.4) can be written in the form

$$\mathrm{Im}F(s)=(\pi/s)\sum_m L_m{}^*(s)\rho_m(s)K_m(s),$$

or in matrix notation,

$$\mathrm{Im}F(s)=(\pi/s)\mathbf{L}^\dagger(s)\boldsymbol{\varrho}(s)\mathbf{K}(s). \tag{2.5}$$

The two invariant amplitudes, $\mathbf{K}(s)$ and $\mathbf{L}(s)$, represent virtual dissociation of the pion into intermediate states and their annihilation into a lepton pair, respectively. The kinematical factor commutes with channel projection matrices $\mathbf{P}_i(i=3\pi, N\bar{N}$, etc.):

$$[\boldsymbol{\varrho}(s),\mathbf{P}_i]=0. \tag{2.6}$$

Our approach is based on presupposing dispersion relations for $\mathbf{K}(s)$ and $\mathbf{L}(s)$ of the form

$$\frac{\mathbf{K}(s)}{s-\mu^2}=\frac{\mathbf{K}(\mu^2)}{s-\mu^2}+\frac{1}{\pi}\int_{(3\mu)^2}^{\infty}\frac{\mathbf{T}^\dagger(s')\boldsymbol{\varrho}(s')\mathbf{K}(s')}{(s'-\mu^2)(s'-s-i\epsilon)}ds', \tag{2.7}$$

$$\frac{\mathbf{L}(s)}{s}=\frac{\mathbf{L}(0)}{s}-\mathbf{K}(\mu^2)\frac{F}{s-\mu^2}+\frac{1}{\pi}\int_{(3\mu)^2}^{\infty}\frac{\mathbf{T}^\dagger(s')\boldsymbol{\varrho}(s')\mathbf{L}(s')}{s'(s'-s-i\epsilon)}ds'. \tag{2.8}$$

$\mathbf{T}(s)$ is the scattering amplitudes in the pseudoscalar sector and can be expressed as

$$\mathbf{T}(s)=\mathbf{D}^{-1}(s)\mathbf{N}(s), \tag{2.9}$$

where $\mathbf{N}(s)$ is real in the physical region and $\mathbf{D}(s)$ is given by

$$\mathbf{D}(s)=1-\frac{s}{\pi}\int_{(3\mu)^2}^{\infty}\frac{\mathbf{N}(s')\boldsymbol{\varrho}(s')}{s'(s'-s-i\epsilon)}ds'. \tag{2.10}$$

We note that $\mathbf{T}(s)$ is symmetric under time-reversal invariance.

Solutions of (2.7) and (2.8) will not be unique due to ambiguities similar to those of Castillejo, Dalitz, and Dyson.[6] We shall concern ourselves with the simplest solutions. One then finds[7]

$$\begin{aligned}\mathbf{K}(s)&=\mathbf{D}^{-1}(s)\mathbf{D}(\mu^2)\mathbf{K}(\mu^2)\\&=\mathbf{D}^{-1}(s)\mathbf{K}(0),\end{aligned} \tag{2.11}$$

$$\mathbf{L}(s)=\mathbf{M}(s)-\mathbf{K}(s)[s/(s-\mu^2)]F, \tag{2.12}$$

where

$$\mathbf{M}(s)=\mathbf{D}^{-1}(s)\mathbf{L}(0). \tag{2.13}$$

Substituting (2.12) into (2.5), we get

$$\mathrm{Im}F(s)=(\pi/s)\{\mathbf{M}^\dagger(s)\boldsymbol{\varrho}(s)\mathbf{K}(s) -[s/(s-\mu^2)]F\mathbf{K}^\dagger(s)\boldsymbol{\varrho}(s)\mathbf{K}(s)\}. \tag{2.14}$$

We note here that

$$\begin{aligned}\mathbf{K}^\dagger(s)\boldsymbol{\varrho}(s)\mathbf{K}(s)&=\sum_n|\langle s,n|J_\pi|0\rangle|^2\delta(p_n-p)\\&=(s-\mu^2)^2\sigma(s),\end{aligned} \tag{2.15}$$

where $\sigma(s)$ denotes the spectral function of Källén and Lehmann[8] for the pion propagator, and that

$$\mathbf{K}^\dagger(s)\boldsymbol{\varrho}(s)\mathbf{P}_i\mathbf{K}(s)=(s-\mu^2)^2\sigma_i(s), \tag{2.16}$$

where $\sigma_i(s)$ is the contribution from the channel i to the spectral function, so that $\sigma(s)=\sum_i\sigma_i(s)$. For later convenience another function of s for each channel will be introduced by

$$H_i(s)\equiv\mathrm{Re}\{\mathbf{M}^\dagger(s)\boldsymbol{\varrho}(s)\mathbf{P}_i\mathbf{K}(s)\}/\sigma_i(s), \tag{2.17}$$

which is defined for s larger than the channel threshold s_i. The convention of one-half the sum over "out" plus "in" states has been used in order to maintain the reality in each channel. (2.14) may then be written as

$$\begin{aligned}(1/\pi)\mathrm{Im}F(s)&=(1/s)\mathbf{M}^\dagger(s)\boldsymbol{\varrho}(s)\mathbf{K}(s)-(s-\mu^2)\sigma(s)F\\&=\frac{(s-\mu^2)^2}{s}\sum_i\sigma_i(s)\left\{H_i(s)-\frac{s}{s-\mu^2}F\right\}.\end{aligned} \tag{2.18}$$

At this point we can compare our approach with that of Bernstein *et al.*[4] These authors assumed an unsubtracted dispersion relation for $L_{N\bar{N}}(s)$ of the form,

$$L_{N\bar{N}}(s)=\frac{\mu^2F}{\mu^2-s}K_{N\bar{N}}(\mu^2)+\frac{1}{\pi}\int_{(3\mu)^2}^{\infty}\frac{\mathrm{Im}L_{N\bar{N}}(s')}{s'-s-i\epsilon}ds'. \tag{2.19}$$

By supposing the dominance of the pole term for small s,

[6] L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. **101**, 453 (1955).
[7] S. W. MacDowell, Phys. Rev. Letters **6**, 385 (1961).
[8] G. Källén, Helv. Phys. Acta **25**, 417 (1952); H. Lehmann, Nuovo Cimento **11**, 342 (1954).

they found

$$L_{N\bar{N}}(0)\equiv 2m_N G_N{}^A \simeq F K_{N\bar{N}}(\mu^2), \tag{2.20}$$

which is equivalent to the original Goldberger-Treiman relation.[1]

It is to be noted that if we assume, instead of (2.8), an unsubtracted dispersion relation for $\mathbf{L}(s)$ and if we take the simplest solution, then we would have

$$\mathbf{L}(s)=[\mu^2F/(\mu^2-s)]\mathbf{K}(s), \tag{2.21}$$

the result known to Gell-Mann and Lévy,[4] who conjectured the relation,

$$\partial_\lambda J_\lambda{}^A=\mu^2F\varphi_\pi. \tag{2.22}$$

The approximate equality, (2.20), would then be replaced by

$$L_{N\bar{N}}(0)=FK_{N\bar{N}}(0). \tag{2.23}$$

It is easily seen that $F(s)$ cannot vanish at infinity if $\mathbf{L}(s)$ is given by (2.21).

The assumption that $\mathbf{L}(s)$ satisfies an unsubtracted dispersion relation is very appealing in that the Goldberger-Treiman relation is an almost automatic consequence. But if $F(s)$ vanishes at infinity, this assumption must be abandoned and one returns to Eq. (2.8).

III. GENERALIZED GOLDBERGER-TREIMAN RELATION

There are four baryon-antibaryon pair channels with nonvanishing contribution to the charged pion decay, which are

$$\bar{n}p,\ (1/\sqrt{2})(\bar{\Lambda}\Sigma^+ + \bar{\Sigma}^+\Lambda),\ (1/\sqrt{2})(\bar{\Sigma}^0\Sigma^+ - \bar{\Sigma}^+\Sigma^0),\text{ and } \bar{\Xi}^+\Xi^0,$$

for π^+ decay and their charge conjugated states for π^- decay. We note here that only the odd G-parity states contribute to ImF with the neglect of electromagnetic corrections. These states need special consideration because they are probably the only channels "directly" coupled to the lepton pairs, if we may use Lagrangian language. For simplicity, they will sometimes be represented by N, Λ, Σ, and Ξ. We shall give expressions for some quantities, which appeared in the previous section, for these channels. Hereafter, we assume even Λ-Σ relative parity, which seems favored by recent experiments.[9]

For the amplitudes of pion dissociation into baryon pairs, we write

$$(E_{i1}E_{i2}/M_{i1}M_{i2})^{1/2}\langle\bar{B}_{i1}B_{i2}|J_\pi|0\rangle=\bar{u}_{i2}i\gamma_5u_{i\bar{1}}K_i(s), \quad i=N,\Lambda,\Sigma,\Xi. \tag{3.1}$$

For the annihilation amplitudes of baryon pairs, we write

$$(E_{i1}E_{i2}/M_{i1}M_{i2})^{1/2}\langle\bar{B}_{i1}B_{i2}|J_\lambda{}^A|0\rangle =\bar{u}_{i2}[a_i(s)i\gamma_\lambda\gamma_5+b_i(s)p_\lambda\gamma_5+b_i'(s)\sigma_{\lambda\mu}ip_\mu\gamma_5]u_{i\bar{1}}, \tag{3.2}$$

[9] See, for instance, J. W. Cronin and O. E. Overseth, Phys. Rev. **129**, 1795 (1963).

where $s=-p^2=-(p_1+p_2)^2$, and $a_i(0)\equiv G_i{}^A$. From (3.2) we have

$$(E_{i1}E_{i2}/M_{i1}M_{i2})^{1/2}\langle\bar{B}_{i1},B_{i2}|\partial_\lambda J_\lambda{}^A|0\rangle =\bar{u}_{i2}i\gamma_5u_{i\bar{1}}L_i(s), \tag{3.3}$$

where

$$L_i(s)=(M_{i1}+M_{i2})a_i(s)+sb_i(s). \tag{3.4}$$

The functions $H_i(s)$ defined in (2.17) can be expressed for baryon-pair channels as

$$H_i(s)=\mathrm{Re}M_i(s)/K_i(s), \tag{3.5}$$

which has a meaning not only for s larger than s_i but also for s smaller than s_i. As $\mathbf{M}(0)=\mathbf{L}(0)$, we see that

$$H_i(0)=L_i(0)/K_i(0)=(M_{i1}+M_{i2})G_i{}^A/K_i(0). \tag{3.6}$$

The right-hand constants in the above equation will be called Goldberger-Treiman's constants and denoted by $F_i{}^{\mathrm{GT}}$. Elementary calculation also shows that

$$\sigma_i(s)=\frac{1}{8\pi^2}\theta[s-(M_{i1}+M_{i2})^2]\frac{s}{(s-\mu^2)^2} \times\left[1-\frac{(M_{i1}+M_{i2})^2}{s}\right]^{1/2} \times[1-(M_{i1}-M_{i2})^2/s]^{3/2}|K_i(s)|^2. \tag{3.7}$$

In the dispersion relation, (2.7), we assumed a once-subtracted form for all $L_n(s)$. According to the usual, although rather questionable, Lagrangian theory all the channels except for those of the baryon pairs have no direct coupling to the lepton pair. In dispersion theory we claim that $L_n(s)$ for channels indirectly coupled to the lepton pairs satisfy unsubtracted dispersion relations. $L_n(0)$ for those channels will then be determined by $G_i{}^A$, F, and quantities concerned with strong interactions. Therefore we have the five weak-coupling constants, $G_N{}^A$, $G_\Lambda{}^A$, $G_\Sigma{}^A$, $G_\Xi{}^A$, and F.

Determination of F in terms of the axial-vector coupling constants can be done by the unsubtracted dispersion relation for $F(s)$. In carrying out the integration of (2.3), with ImF given by (2.18), we have to distinguish the two alternative cases according to whether

$$Z_3{}^{-1}=1+\int_{(3\mu)^2}^{\infty}\sigma(s)ds \tag{3.8}$$

is finite or infinite.

A. $Z_3\neq 0$

On substituting (2.18) into (2.3) and putting $s=\mu^2$, it immediately follows that

$$F=\int_{(3\mu)^2}^{\infty}\frac{s-\mu^2}{s}\sum_i\sigma_i(s)H_i(s)ds\bigg/1+\int_{(3\mu)^2}^{\infty}\sigma(s)ds, \tag{3.9}$$

which may be called a generalized Goldberger-Treiman formula.

In the nucleon-antinucleon pair approximation one finds from (3.5) that $H_N(s)$ is constant since $D^{-1}(s)$ is not a matrix in this approximation, and from (3.6) that it is equal to F_N^{GT}. Thus, (3.9) becomes identical to the formula obtained by Goldberger and Treiman,[1,3]

$$F=\left[\int_{(2M)^2}^{\infty}\frac{s-\mu^2}{s}\sigma_N(s)ds\Big/1+\int_{(2M)^2}^{\infty}\sigma_N(s)ds\right]F_N^{\mathrm{GT}}. \tag{3.10}$$

They derived the G-T relation,

$$F\approx F_N^{\mathrm{GT}}, \tag{3.11}$$

from (3.10) under the assumption that the denominator in (3.10) is much larger than unity. This last assumption means that Z_3 in their approximation is much smaller than unity, although it does not vanish. If Z_3 vanishes in this approximation, one encounters a difficulty that will soon be discussed.

B. $Z_3=0$

Substitution of (2.18) into (2.3) causes a divergence of the Z_3^{-1} type if no cancellation occurs in the high-mass limit, and the generalized G-T formula is no longer valid. In order that the dispersion relation for $F(s)$ need no subtractions, or in other words, in order that the expression

$$F=\int_{(3\mu)^2}^{\infty}\left[\frac{s-\mu^2}{s}\sum\sigma_i(s)H_i(s)-F\sigma(s)\right]ds \tag{3.12}$$

be meaningful, we must have

$$F=\lim_{s\to\infty}\left[\sum_i\sigma_i(s)H_i(s)/\sigma(s)\right], \tag{3.13}$$

which can be regarded as a generalized G-T relation.

In the nucleon-antinucleon pair approximation, (3.13) means that

$$F=F_N^{\mathrm{GT}},$$

which leads to a difficulty because we then obtain the absurd result,

$$1=-\int_{(2M)^2}^{\infty}\frac{\mu^2}{s}\sigma_N(s)ds<0,$$

as was noted by Barrett and Barton[5] and by Nishijima.[10] In order to avoid this difficulty, the former abandoned the unsubtracted form of the dispersion relation for $F(s)$ and the latter thought electromagnetic corrections should be important, all of them working in the one-channel approximation.

IV. WEAK-COUPLING CONSTANTS

In the previous section the four axial-vector coupling constants, G_i^A with $i=N$, Λ, Σ, and Ξ, were regarded as given parameters. It will be very appealing to see how they can, at least in principle, be determined, except for a scaling factor, if we assume we know every quantity concerned with strong interactions. It seems difficult to present a unique method since our knowledge of weak interactions is still very restricted. We shall suggest a method based on the hypothesis of the universality of weak Fermi interactions.[11]

According to this hypothesis, the "bare-" coupling constants of the vector and the axial-vector currents are equal to the coupling constant of the μ-e decay interaction, G. It should be noted that bare coupling constants of weak interactions seem to have a physical meaning, although those of strong interactions do not. Indeed, it is to explain the experimental fact of no renormalization for the vector coupling constant, $G_N^V=G$, that the hypothesis of vector current conservation was introduced.[11,12]

The vector and the axial-vector form factors of the nucleon, $c_N(s)$ and $a_N(s)$, are, under the universality principle, supposed to tend to Z_NG as $s\to\infty$, where Z_N is the renormalization constant Z_2 for the nucleon.[13] It will be convenient to introduce the function defined by

$$Z_N^{-1}(s)=1-(s-M_N^2)\int_{(M_N+\mu)^2}^{\infty}\frac{\sigma_{N1}(s')}{s'-s-i\epsilon}ds', \tag{4.1}$$

where $\sigma_{N1}(s)$ is one of the spectral functions of Källén and Lehmann for the nucleon propagator:

$$S_{FN}'(x)=S_{FN}(x)+\int_{(M_N+\mu)^2}^{\infty}[S_F(x,s)\sigma_{1N}(s)+\Delta_F(x,s)\sigma_{2N}(s)]ds.$$

As we know that

$$\lim_{s\to\infty}Z_N^{-1}(s)=Z_N^{-1}, \tag{4.2}$$

we now express the condition that $a_N(s)$ tends to Z_NG as $s\to\infty$ in the form

$$\lim_{s\to\infty}Z_N^{-1}(s)a_N(s)=G. \tag{4.3}$$

Similar equations may be written for the form factors of other baryons. We thus claim that

$$\lim_{s\to\infty}Z_i^{-1}(s)a_i(s)=G, \tag{4.4a}$$

for i, $i=N$, Σ, Ξ, and that

$$\lim_{s\to\infty}Z_\Lambda^{-1/2}(s)Z_\Sigma^{-1/2}(s)a_\Lambda(s)=G. \tag{4.4b}$$

[10] K. Nishijima (private conversation)

[11] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[12] S. S. Gershtein and I. B. Zel'dovich, Zh. Eksperim. i Teor. Fiz. **29**, 698 (1955) [translation: Soviet Phys.—JETP **2**, 576 (1956)].

[13] G. Källén, *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. 2, p. 187. K. Symanzik, Nuovo Cimento **11**, 269 (1959); M. Gel-Mann and F. Zachariasen, Phys. Rev. **123**, 1065 (1961).

Let us suppose we know $a_i(s)$ in terms of the axial-vector coupling constants, G_i^A, and strong-interaction parameters. Then, from the above four equations, we may, in principle, determine the G_i^A's by G (and strong-interaction parameters).

The functions $H_i(s)$ are now expressed by G and F. What we have to do is to express F by G. It can be done by use of the generalized G-T formula, (3.9), if $Z_3\neq 0$, or by use of the generalized G-T relation, (3.13), if $Z_3=0$. In the latter case, however, one is put in a somewhat paradoxical situation because there is one more equation, (3.12). If we divide the both sides of (3.12) by F, we obtain

$$1=\int_{(3\mu)^2}^{\infty}\left[\frac{s-\mu^2}{s}\sum_i\sigma_i(s)\frac{H_i(s)}{F}-\sigma(s)\right]ds, \quad (4.5)$$

which should be regarded as a condition imposed on strong-interaction parameters.[14]

V. SUPPLEMENTARY REMARKS

It might be of theoretical interest to see the case of a primary pseudoscalar Fermi interaction, although the axial-vector character has been established experimentally by the measurement of $\pi\to\mu+\nu$ and $\pi\to e+\nu$ decay rates. As the argument is almost the same as before, we shall omit all its detail. Corresponding to (3.12) we here have

$$F^P=\int_{(3\mu)^2}^{\infty}\left[\frac{s-\mu^2}{\mu^2}\sum_i\sigma_i(s)H_i^P(s)-\frac{s}{\mu^2}\sigma(s)F^P\right]ds, \quad (5.1)$$

where F^P is the invariant decay amplitude of the pion in the case of pseudoscalar coupling and $H_i^P(s)$ is defined in the same way as $H_i(s)$. Therefore, (5.1) has self-mass type of divergence if there occurs no cancellation in the high-energy limit. Even when we have a condition for cancellation similar to (3.13), (5.1) may still have Z_3^{-1} type of divergence if Z_3 vanishes. Such a situation certainly corresponds to the well-known fact that in a perturbation-theoretic treatment of pion decay one encounters a quadratic divergence for pseudoscalar coupling, while one has only a logarithmic divergence for axial-vector coupling.

The final remark is concerned with neutral pion decay, $\pi^0\to 2\gamma$. This decay process[2] is evidently more complicated than that of the charged pion, for two photons come out from two different points, while in the latter decay a lepton pair come out from a single point if we neglect electromagnetic corrections. We shall apply, without proof, to the neutral-pion decay the method used in the previous sections.

[14] E. R. McCliment and K. Nishijima, Phys. Rev. **128**, 1970 (1962).

The invariant amplitude F is defined by

$$(4k_0k_0')^{1/2}\langle k,\epsilon,k',\epsilon'|J_\pi|0\rangle = i\delta_{\mu\nu\lambda\sigma}\epsilon_\mu\epsilon_\nu'k_\lambda k_\sigma'F[-(k+k')^2], \quad (5.2)$$

where ϵ and ϵ' denote polarization vectors of the two photons. We demand that $F(s)$ satisfies an unsubtracted dispersion relation:

$$F(s)=\frac{1}{\pi}\int_{(3\mu)^2}^{\infty}\frac{\mathrm{Im}F(s')}{s'-s-i\epsilon}ds'. \quad (5.3)$$

$\mathrm{Im}F(s)$ is expressed in terms of matrix notation as

$$\mathrm{Im}F(s)=(\pi/s)\mathbf{L}^\dagger(s)\boldsymbol{\varrho}(s)\mathbf{K}(s), \quad (5.4)$$

where $s^{-1}\mathbf{L}(s)$ denotes the annihilation amplitudes of intermediate states into the two photons, the factor s^{-1} being separated just to keep a parallelism with π-μ decay. It has been assumed here that in the annihilation matrix elements the four-momenta and the polarization vectors of the two photons form a pseudoscalar invariant by themselves, not with the vectors of intermediate states, because the two photons are in a pseudoscalar state.

$\mathbf{L}(s)$ are then supposed to satisfy the dispersion relations of the form

$$\frac{\mathbf{L}(s)}{s}=\frac{\mathbf{a}}{s}-\frac{F}{s-\mu^2}\mathbf{K}(\mu^2)+\frac{1}{\pi}\int_{(3\mu)^2}^{\infty}\frac{\mathbf{T}^\dagger\boldsymbol{\varrho}\mathbf{L}}{s'(s'-s-i\epsilon)} +\frac{1}{\pi}\int_\Gamma\frac{\mathbf{b}(s')}{s'(s'-s-i\epsilon)}ds', \quad (5.5)$$

where $\mathbf{a}=\mathbf{L}(0)$ and Γ represents unphysical cuts. The last term in the right-hand side of the above equation is due to the two-point character of the decay interaction. Discarding again CDD ambiguities, one finds

$$\mathbf{L}(s)=\mathbf{M}(s)-\mathbf{K}(s)[s/(s-\mu^2)]F, \quad (5.6)$$

with

$$\mathbf{M}(s)=\mathbf{D}^{-1}(s)\mathbf{a}+\frac{s}{\pi}\mathbf{D}^{-1}(s)\int_\Gamma\frac{\mathbf{D}(s')\mathbf{b}(s')}{s'(s'-s-i\epsilon)}ds'. \quad (5.7)$$

$\mathrm{Im}F(s)$ is then given by (2.14) with $\mathbf{M}(s)$ defined above. The difficulty in practice here is that we must express $\mathbf{a}$ and $\mathbf{b}(s)$ in terms of the fine-structure constants and strong-interaction parameters, even when $\mathbf{D}(s)$ is supposed to be known.

The rest of the argument is almost the same as before. We obtain a generalized G-T formula [see Eq. (3.9)] for the decay amplitude when Z_3 does not vanish, and a generalized G-T relation [see Eq. (3.13)] if it vanishes. We may note that although no divergence occurs in the lowest order perturbational calculation of π^0 decay one encounters a logarithmic divergence if one takes into account the anomalous magnetic moments of the nucleons.

ACKNOWLEDGMENTS

The author would like to thank Professor K. Nishijima for stimulating discussions, without which this work could not have been done. He is grateful to Professor J. D. Jackson for valuable comments and to Dr. R. L. Schult for a careful reading of the manuscript. He would also like to express his sincere thanks to the Fulbright Commission for a travel grant.

PHYSICAL REVIEW VOLUME 132, NUMBER 1 1 OCTOBER 1963

Scattering of 300-MeV Positrons from Cobalt and Bismuth*

J. GOLDEMBERG, J. PINE,† AND D. YOUNT‡
High-Energy Physics Laboratory, Stanford University, Stanford, California

(Received 16 May 1963)

Positrons and electrons from the Stanford Mark III linear accelerator have been scattered from cobalt and bismuth at 300 MeV. The ratio R, equal to $(\sigma_- - \sigma_+)/(\sigma_- + \sigma_+)$, has been measured at a number of angles from 10° to 45° for cobalt and from 5° to 45° for bismuth. Two experiments are reported: a high-percision experiment with poor energy resolution, suitable for measuring the small values of R found at small angles, where inelastic scattering is not important; and an experiment with somewhat lower precision but better energy resolution, suitable for measuring the larger values of R found at angles where inelastic scattering must be taken into account. The elastic scattering data are in good agreement with phase-shift calculations of Herman, Clark, and Ravenhall, who used nuclear charge distributions which fit earlier electron scattering data. The inelastic data, for which no reliable predictions exist, indicate that $R_{\text{inelastic}}$ is generally smaller than R_{elastic}. This suggests that the inelastic scattering is better described by the first Born approximation, in which $R=0$, than is the elastic scattering.

INTRODUCTION

THE elastic scattering of positrons by nuclei differs from that of electrons. For point nuclei with no magnetic moment, Feshbach[1] has computed σ_+/σ_-, the ratio of positron to electron cross sections, at a given angle and energy. For backward scattering by high-Z nuclei, this ratio is $\approx 1/5$ and it approaches 1 as the scattering angle and the atomic number are made small. The effect can be understood in terms of different spin-orbit interactions arising from the different classical trajectories of positrons and electrons scattered through the same angle. Alternatively, the effect can be ascribed to different distortions of the incident and of the scattered waves by the Coulomb field of the nucleus. In the first Born approximation, which takes into account only 1-photon exchanges, such distortions are neglected, and positron and electron scattering are identical. The difference in scattering is, thus, a measure of the importance of the exchange of two or more photons.

For finite nuclei, the difference between positron and electron scattering is sensitive to the distribution of nuclear charge. Figure 1 shows qualitatively the expected behavior of the difference in positron and electron scattering as a function of the scattering angle θ for fixed and equal incident energies. The difference is characterized by the quantity R, defined by

$$R=(\sigma_- - \sigma_+)/(\sigma_- + \sigma_+)\,, \tag{1}$$

where σ_- and σ_+ are the differential scattering cross sections for electrons and positrons. The initial increase of R corresponds to the point nucleus behavior. At angles where the classical trajectories begin to penetrate the nuclear charge distribution, R becomes negative. In terms of the classical trajectories, the deeper penetration of electrons into the charge distribution causes the electron cross section to become smaller than the positron cross section. Finally, as the angle is further increased, R oscillates. The de Broglie wavelengths of the positrons and electrons differ at the nucleus, and

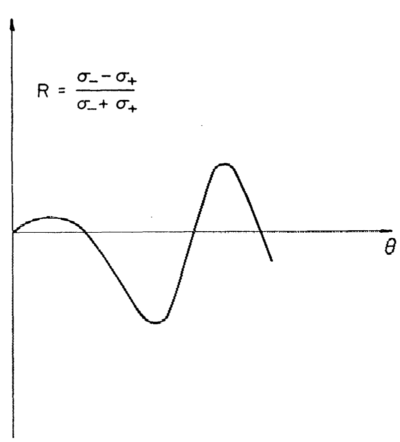

FIG. 1. Qualitative behavior of the ratio R as a function of scattering angle for nuclei of finite size.

* This work was supported in part by the U. S. Office of Naval Research, the U. S. Atomic Energy Commission, and the U. S. Air Force Office of Scientific Research.

† Present address: California Institute of Technology, Pasadena, California.

‡ Present address: Department of Physics, Princeton University, Princeton, New Jersey.

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