tunneling exponent, we found the observed tunneling current to be about 10–40 times larger than that predicted by Kane's expression. This discrepancy seems too large to be accounted for by the uncertainty in the magnitude of the electron-phonon coupling constant.

It should be emphasized that the good quantitative agreement between the temperature dependence of the tunneling current calculated from Kane's theory of indirect tunneling and the experimental data does not prove the correctness of Kane's expression. It merely indicates that the coefficient of $Eg^{3/2}m^{*1/2}/F$ which appears in the exponent is at least approximately correct. Since our comparison is insensitive to the shape of the I-V characteristic, we cannot offer any evidence relating to the structure of the D function. Furthermore, no evidence for the asymmetry of the tunneling exponent with respect to forward and reverse current flow was observed.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the help of M. Cuevas, G. Rochlin, and L. Silverman with the experimental and computational work. We are also pleased to acknowledge the general support of the Institute for the Study of Metals by the U. S. Atomic Energy Commission and of the Low Temperature Laboratory of the University of Chicago by the National Science Foundation which made this work possible.

PHYSICAL REVIEW VOLUME 132, NUMBER 6 15 DECEMBER 1963

Microwave Conductivity of Semiconductors in the Presence of High Steady Electric Fields

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(Received 19 July 1963)

The distribution function of carriers in a semiconductor when subjected to a small microwave field and a high steady electric field is derived, considering both the acoustic and optical phonon scattering. Expressions for microwave conductivity and change in apparent dielectric constant are obtained from the distribution function. It is shown by numerical calculations that the conductivity evaluated from these expressions agree closely with the experimental value. The calculated value of the change in apparent dielectric constant, however, is found to be of the same order as the experimental value, but the agreement is poorer than that for the conductivity.

I. INTRODUCTION

THE microwave conductivity of semiconductors in the presence of high steady electric fields has been studied by several workers. In the first experiments reported by Arthur *et al.*[1] the attenuation of a microwave signal produced by the sample of known dimensions in the presence of a steady field was measured. The attenuation so produced was assumed to be proportional to the slope mobility and the experimental data were used to derive the high field conductivity. This assumption is apparently justified if the product of the microwave frequency and the momentum relaxation time is negligible compared to unity, as was the case in these experiments.

Later experiments carried out by Gibson *et al.*,[2] however, show that the microwave mobility in the presence of high steady fields is not the same as the slope mobility but is intermediate between the slope mobility and the dc mobility. This result that the microwave mobility is different from the slope mobility even when the product of microwave frequency and momentum relaxation time is much less than unity may be explained when the mobility expressions applicable to the problem are properly developed. Such expressions have been derived by Paranjape[3] and also by Gibson *et al.*[2]

It has been assumed that the carrier density is high enough to produce a Maxwellian energy distribution due to predominant interelectronic collisions. But, the carrier temperature which is determined by the energy and momentum balance conditions, is higher than the temperature of the lattice. It is then shown that the perturbation in the carrier temperature produced by the microwave signal is not in phase with the signal, but leads it. The lead angle is determined by the applied steady field and the product of the microwave frequency and the energy relaxation time, rather than the momentum relaxation time. Since this product is comparable to unity at the experimental frequencies, the perturbed temperature of the carriers differs appreciably in phase from the microwave signal. Hence, the microwave conductivity is much different from that derived from the slope of the conductivity versus field curves; also, a

[1] J. B. Arthur, A. F. Gibson, and J. W. Granville, J. Electron. 2 145 (1956).
[2] A. F. Gibson, J. W. Granville, and E. G. S. Paige, J. Phys. Chem. Solids 19, 198 (1961).
[3] B. V. Paranjape, Phys. Rev. 122, 1372 (1961).

positive change in apparent dielectric constant is produced.

The expression for conductivity given by Paranjape explains the experimental results qualitatively. This is because the expression was derived assuming acoustic phonon scattering only. However, there are other scattering sources, namely, optical phonons, impurity centers and intervalley phonons. It has been shown by Conwell,[4] Stratton,[5] and Yamashita and Inoue[6] that dc hot electron conductivity curves for room temperatures may be explained if, in addition to acoustic phonon scattering, the effect of optical phonon scattering only is considered. The effects of impurity scattering or intervalley phonon scattering are not of appreciable importance. It is, therefore, expected that a theory considering the effect of optical phonons, in addition to acoustic phonons, should give quantitative fit with experiments. In the theory given by Gibson *et al.*,[2] the effect of optical phonon scattering has been considered and a relatively better agreement with experiments was observed. However, like Paranjape,[3] these authors also assumed predominant interelectronic scattering though the conditions of the sample did not ensure this. An alternative approach to the development of the theory when predominant e-e scattering cannot be assumed[7,8] is to solve the Boltzmann equation assuming predominant acoustic and optical phonon scattering. The conductivity may then be obtained using this distribution function. This is the method used by Yamashita and Watanabe[7] for analysing the dc hot electron conductivity characteristics. The purpose of this paper is to analyse the microwave conductivity of semiconductors in the presence of high steady electric fields following this procedure.

In Sec. II equations giving the energy distribution function of the carriers are first derived, taking into account the effect of both acoustic and optical phonon scattering. The perturbation in the distribution function produced by the microwave field is obtained in Sec. III. In Sec. IV the expressions for the conductivity obtained from this distribution function are given. The numerical results obtained for the experimental condition of Gibson *et al.*[2] are discussed in Sec. V.

II. THE ENERGY DISTRIBUTION FUNCTION FOR THE CARRIERS

Let $f(K)$ denote the distribution function for the carriers having the wave vector K at a time t. The distribution function is assumed to depend on t because of the presence of the microwave field. The distribution function $f(K)$ satisfies the equation

$$\left.\frac{\partial f(K)}{\partial t}\right|_{\text{Field}}+\left.\frac{\partial f(K)}{\partial t}\right|_{\text{Coll}}=\frac{\partial f(K)}{\partial t}. \qquad (1)$$

Solution to (1) is obtained assuming that $f(K)$ may be expanded as

$$f(K)=f(E)+K_x g(E), \qquad (2)$$

where K_x is the component of the wave vector in the direction of the applied field F, and $f(E)$ and $g(E)$ are functions of only the energy, E, of the carriers. The total number of electrons is obtained from $f(E)$, whereas the current is obtained from $g(E)$. [The functions $f(E)$ and $g(E)$ are written henceforward as f and g, respectively, for the sake of simplicity.]

In the case of nonpolar solids like Ge and Si at room temperature, the term $\partial f(K)/\partial t|_{\text{Coll}}$ may be written as

$$\left.\frac{\partial f(K)}{\partial t}\right|_{\text{Coll}}=\left.\frac{\partial f(K)}{\partial t}\right|_{\text{ac}}+\left.\frac{\partial f(K)}{\partial t}\right|_{\text{op}}, \qquad (3)$$

where $\partial f(K)/\partial t|_{\text{ac}}$ and $\partial f(K)/\partial t|_{\text{op}}$ represent, respectively, the change in the distribution function due to the interaction of the electrons with acoustic and optical modes of lattice vibrations. The effect of impurity and e-e scattering is assumed to be negligible. The terms $\partial f(K)/\partial t|_{\text{ac}}$ and $\partial f(K)/\partial t|_{\text{op}}$ may be written as[6]

$$\left.\frac{\partial f(K)}{\partial t}\right|_{\text{ac}}=\frac{A}{(E)^{1/2}}\left[E^2\frac{\partial^2 f}{\partial E^2}+\left(\frac{E^2}{kT}+2E\right)\frac{\partial f}{\partial E}+\frac{2E}{kT}f-K_x\frac{E}{2mc^2}\cdot g\right], \qquad (4)$$

$$\left.\frac{\partial f(K)}{\partial t}\right|_{\text{op}}=\frac{B}{\hbar\omega_0(E)^{1/2}}\left[\hbar\omega_0(e^S+1)\left(\mathrm{E}\frac{\partial^2 f}{\partial E^2}+\frac{\partial f}{\partial E}\right)+2(e^S-1)\times\left(E\frac{\partial f}{\partial E}+f\right)-K_x\frac{E}{\hbar\omega_0}(e^S+1)g\right], \qquad (5)$$

where $A=8ec^2/3(\pi kT)^{1/2}\mu_a$, $S=\hbar\omega_0/kT$, $B=9/16\times(AD^2/c^2)(\hbar^2\zeta^2/2mkT)(1/e^S-1)$, c = velocity of sound in the solid, D = coupling constant between conduction electron and optical mode of vibration, C = coupling constant between conduction electron and acoustical mode of vibration, ζ = first nonvanishing reciprocal vector of the lattice, $\hbar\omega_0$ = characteristic energy of an optical phonon, E = energy of a carrier $=\hbar^2K^2/2m$, μ_a = low-field acoustical mobility, m = effective mass of the conduction electron, assumed to be isotropic.

The term $\partial f(K)/\partial t|_{\text{op}}$ written above is obtained assuming that the average energy of an electron is much greater than the characteristic optical phonon energy in the solid. This assumption is valid in the field range higher than 1 kV/cm and at lattice temperatures, at which the experiments have been conducted.

[4] E. M. Conwell, J. Phys. Chem. Solids 8, 234 (1959).
[5] R. Stratton, J. Electron. Control 5, 157 (1958).
[6] J. Yamashita and K. Inoue, J. Phys. Chems. Solids 12, 1 (1960).
[7] J. Yamashita and M. Watanabe, Progr. Theoret. Phys. (Kyoto) 12, 443 (1954).
[8] R. Stratton, Proc. Roy. Soc. (London) A242, 157 (1958).

The term $\partial f(K)/\partial t|_{\text{Field}}$ may be written as

$$\left.\frac{\partial f(K)}{\partial t}\right|_{\text{Field}}=\frac{eF}{\hbar}\left[K_x\frac{\partial f}{\partial E}+g+\tfrac{2}{3}E\frac{\partial g}{\partial E}\right]. \qquad (6)$$

One obtains from (1), (3), (4), (5), and (6)

$$E\frac{\partial^2 f}{\partial E^2}+\left(\frac{E}{kT}+2\right)\frac{\partial f}{\partial E}+\frac{2}{kT}f+\frac{B}{2A}\left\{\left[\hbar\omega_0(e^S+1)\times\left(E\frac{\partial^2 f}{\partial E^2}+\frac{\partial f}{\partial E}\right)+2(e^S-1)\left(E\frac{\partial f}{\partial E}+f\right)\right]\right\}$$
$$=\frac{1}{A}\cdot\frac{1}{(E)^{1/2}}\left[\frac{\partial f}{\partial t}-\frac{eF}{\hbar}\left(g+\tfrac{2}{3}E\frac{\partial g}{\partial E}\right)\right], \qquad (7)$$

$$g=-\frac{2mc^2}{A\Omega}\cdot\frac{1}{(E)^{1/2}}\left[\frac{eF}{\hbar}\frac{\partial f}{\partial E}+\frac{\partial g}{\partial t}\right], \qquad (8)$$

where

$$\Omega=1+(B/A)(2mc^2/\hbar\omega_0)(e^S+1).$$

Since the microwave field is in the same direction as the dc field the total field F may be written as

$$F=\text{Re}F_0(1+\lambda e^{j\omega t}), \qquad (9)$$

where F_0= the steady field, λF_0= the amplitude of the microwave field, and ω= the microwave frequency. The effect of the microwave field would be to perturb both f and g. Since λ is a small quantity, this perturbation may be considered to be small and f and g may be written as

$$f=f_0+\lambda f_1 e^{j\omega t}, \qquad (10)$$

$$g=g_0+\lambda g_1 e^{j\omega t}. \qquad (11)$$

On substituting (10) and (11) in (7) and (8), and collecting the first order terms only, one obtains

$$\frac{d}{dE}\left[E^2\frac{df_0}{dE}+\frac{E^2}{kT}f_0\right]+\frac{d}{dE}\left[\frac{BkT}{2A}(e^S+1)S\left(E\frac{df_0}{dE}\right)+\frac{B}{A}(e^S-1)(Ef_0)\right]$$
$$=-\frac{2}{3}\frac{eF_0}{A\hbar}\frac{d}{dE}(E^{3/2}g_0), \qquad (12)$$

$$g_0=-e\hbar/m\cdot 2mc^2/A\Omega\cdot\frac{1}{(E)^{1/2}}\cdot F_0\frac{df_0}{dE}, \qquad (13)$$

$$\frac{d}{dE}\left[E^2\frac{df_1}{dE}+\frac{E^2}{kT}f_1\right]+\frac{d}{dE}\left[\frac{BkT}{2A}(e^S+1)S\left(E\frac{df_1}{dE}\right)+\frac{B}{A}(e^S-1)(Ef_1)\right]$$
$$=-\frac{2}{3}\frac{eF_0}{A\hbar}\cdot\frac{d}{dE}[E^{3/2}(g_1+g_0)]+\frac{j\omega(E)^{1/2}}{A}f_1, \qquad (14)$$

$$g_1=-e\hbar/m\cdot 2mc^2/A\Omega\cdot\frac{F_0}{(E)^{1/2}}\times\left(\frac{df_0}{dE}+\frac{df_1}{dE}\right)\Bigg/1+j\frac{2mc^2}{A\Omega}\cdot\frac{\omega}{(E)^{1/2}}. \qquad (15)$$

Putting

$$\tau_e=(kT)^{1/2}/A, \qquad (B/2A)(e^S+1)S=q,$$
$$\tau_m=2mc^2/A\Omega(kT)^{1/2}, \qquad (B/A)(e^S-1)=r,$$
$$p=(3\pi/16c^2)\mu_a{}^2F_0{}^2, \qquad (p/\Omega)+(B/2A)(e^S+1)S=p',$$
$$E/kT=z^2,$$

and eliminating g_0 and g_1 from (12) through (15) one obtains

$$\frac{d}{dz}\left[(z^3+p'z)\frac{df_0}{dz}+(2z^4+2rz^2)f_0\right]=0, \qquad (16)$$

$$\frac{d}{dz}\left[\left(z^3+\frac{z\cdot p/\Omega}{1+j(\omega\tau_m/z)}+qz\right)\frac{df_1}{dz}+(2z^4+2rz^2)f_1\right]$$
$$=j4\omega\tau_e z^2 f_1-\frac{2p}{\Omega}\frac{d}{dz}\left(z\frac{df_0}{dz}\right). \qquad (17)$$

Equation (16) is the same as that obtained for the steady field case.[6]

The functional form of the solution of (16) is

$$f_0=(z^2+p')^{p'-r}\exp(-z^2). \qquad (18)$$

The perturbation in the energy distribution function f_1 is, however, given by Eq. (17).

III. THE PERTURBATION IN THE DISTRIBUTION FUNCTION BY THE MICROWAVE FIELD

The quantities symbolized by τ_e and τ_m may be identified at this stage, respectively, with the energy and momentum relaxation time. Their values for germanium are found to be about 2.24×10^{-10} and 2.50×10^{-13} sec. The value of $\omega\tau_e$ and $\omega\tau_m$ at a frequency of 2.18×10^{11} rad/sec (the experimental frequency of Gibson *et al.*) are, respectively, 48.6 and 0.056. Evidently then, the effect of $\omega\tau_m$ on f_1 is negligible for all energies except for very small values. Neglect of this term in Eq. (17) will, therefore, introduce errors only for low values of E. However, since f_1 is very near zero in this range of energy the ultimate result will be very little in error if $\omega\tau_m/z$ is neglected in Eq. (17).

It may be noted, further, that for values of the steady field in the range of 1 kV/cm to 4 kV/cm, the value of p is of the order of 30–400. Hence, one may reasonably introduce the approximation $p/\Omega\gg z^2$, since Ω is of the order of 2. With this approximation Eq. (18) may be reduced to

$$f_0=\exp\left[-\left(\frac{z^4+2rz^2}{2p'}\right)\right]. \qquad (19)$$

Also, Eq. (17) reduces to

$$\frac{d}{dz}\left[zp'\frac{df_1}{dz}+(2z^4+2rz^2)f_1\right]=j4\omega\tau_e z^2 f_1-\frac{2p}{\Omega}\frac{d}{dz}\left(z\frac{df_0}{dz}\right). \quad (20)$$

The perturbation in the distribution function due to microwaves may, hence, be obtained from (20) in the presence of steady fields in the range of 1–4 kV/cm.

It has not been possible to obtain an analytical solution of Eq. (20). In order to study the nature of the function f_1, numerical solution of (20) was obtained by a digital computer retaining only the terms due to acoustic phonon scattering and for a value of $\omega\tau_e/(2p)^{1/4}=12.5$, corresponding to $F_0=2$ kV/cm. The plots of f_1 so obtained are shown in Fig. 1. Because of the presence of $j\omega$ in Eq. (20) evidently f_1 consists of both a real and an imaginary component which are shown separately in the figure. It may be mentioned that the real component gives the in phase component of microwave current, while the imaginary part gives the out of phase component of current.

It is of interest to note here that the perturbation in the distribution function due to the microwave field is of oscillatory nature, which means an increased concentration of the carriers at certain controllable energy levels. It is conjectured that this concentration may be made more intense by enhancing the strength of the microwave field, and an application of the microwave hot electron property of semiconductors to amplification by arranging for the interchange of energy with the desired signal may be realized. However, in the present problem, these distribution function curves could not be further utilized. This was due to the fact that the calculation of the final current involves an integration of these curves, and due to their oscillatory nature, the final accuracy obtained in numerical integration was rather poor. Hence, the microwave current was calculated using a different procedure outlined in the next section.

IV. MICROWAVE CONDUCTIVITY AND CHANGE IN DIELECTRIC CONSTANT

The distribution function giving the number of carriers may be written as

$$f=N[f_0+\lambda(f_{1r}+jf_{1i})], \quad (21)$$

where f_{1r} and f_{1i} represent respectively the real and imaginary components of f_1 and N is the normalization constant.

The normalization constant N is given by

$$N=N_0/[1+\lambda(n_r+jn_i)], \quad (22)$$

where N_0 is the normalization constant in the absence of the microwave field and n_r and n_i represent, respectively, the integrals

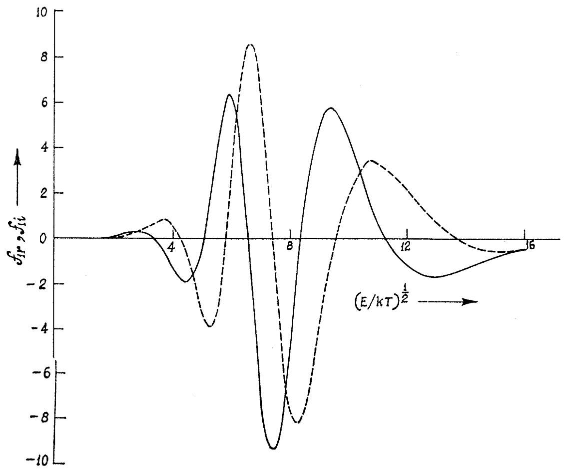

FIG. 1. Perturbation in the distribution function of carriers due to the microwave field. Solid line: real part of the function; dashed line: imaginary part of the function.

$$n_r=\frac{1}{n_0}\int f_{1r}z^2dz \quad (23)$$

and

$$n_i=\frac{1}{n_0}\int f_{1i}z^2dz, \quad (24)$$

where

$$n_0=\int f_0z^2dz. \quad (25)$$

The integrals n_r and n_i may be evaluated directly from Eq. (20). On integrating both sides of (20) between 0 and ∞, one readily obtains

$$\int_0^\infty f_1z^2dz=0, \quad (26)$$

since f_0, f_1 and their derivatives are zero at infinity.

Hence n_r and n_i are each equal to zero and N is same as N_0.

The part of the distribution function, contributing to the current, may also be written as

$$g_1=-\frac{e\hbar}{2mkT}\tau_m\frac{F_0}{z^2}\left(1-j\frac{\omega\tau_m}{z}\right)\times\left(\frac{\partial f_0}{\partial z}+\frac{\partial f_{1r}}{\partial z}+j\frac{\partial f_{1i}}{\partial z}\right). \quad (27)$$

The above expression is obtained from (15), expanding the denominator in the binomial form and retaining only the first term. It should be mentioned here that the term $\omega\tau_m/z$, which was neglected while writing the equation for f_1, is retained here, since its contribution to the out of phase component of current may be appreciable in comparison to that contributed by f_{1i}. The microwave current is given by

$$J=F_1e^{j\omega t}N_0\frac{e\hbar}{m}\int K_x^2g_1dK_xdK_ydK_z. \quad (28)$$

On substituting g_1 from (27), one obtains

$$J=F_1e^{j\omega t}\sigma_{\mathrm{dc}}[1+m_r/m_0+(j)m_i/m_0], \quad (29)$$

where σ_{dc} is the dc conductivity,

$$m_0=\int f_0zdz, \quad (30)$$

$$m_r=\int\{f_{1r}z+\tfrac{1}{2}(\omega\tau_m)f_{1i}\}dz, \quad (31)$$

and

$$m_i=\int\{f_{1i}z-\tfrac{1}{2}\omega\tau_m(f_{1r}+f_0)\}dz. \quad (32)$$

The microwave conductivity and change in dielectric constant are given by

$$\sigma_m=\sigma_{\mathrm{dc}}(1+m_r/m_0), \quad (33)$$

and

$$\Delta\epsilon=(\sigma_{\mathrm{dc}}/\omega\epsilon_0)(m_i/m_0). \quad (34)$$

In the above expressions m_0 may be directly evaluated, since f_0 is known. For evaluating m_r and m_i, the method of momenta, used by Dykman and Tomchuk[9] for solving a similar problem has been used. This is detailed below.

Substitute

$$y(z)=\frac{df_1}{dz}+\frac{2z^3+2rz}{2p'}f_1. \quad (35)$$

The undetermined function f_1 may be written in terms of $y(z)$ as

$$f_1=\left[a_0+\int_0^z(f_0)^{-1}y(z)dz\right]f_0. \quad (36)$$

f_0 is as given by Eq. (19), and a_0 is a complex constant. On substituting $y(z)$ from (35), Eq. (20) is converted into the integral equation

$$zy(z)=j\frac{4\omega\tau_e}{p'}\int_0^z z^2f_1dz-\frac{2p}{p'\Omega}\left(z\frac{df_0}{dz}\right). \quad (37)$$

Using Eq. (26) one writes Eq. (37) as

$$zy(z)=-j\frac{4\omega\tau_e}{p'}\int_z^\infty z^2f_1dz-\frac{2p}{p'\Omega}\left(z\frac{df_0}{dz}\right). \quad (38)$$

Since f_1 is finite for all values of z, the above equation may be solved putting

$$y(z)=(a_1+2a_2z+3a_3z^2+\cdots), \quad (39)$$

[9] I. M. Dykman and P. M. Tomchuk, Fiz. Tverd. Tela **2**, 2228 (1960) [translation: Soviet Phys.—Solid State **2**, 1988 (1960)].

where a_1, a_2, etc., are constants like a_0.

On putting (39) into (36) one also obtains for f_1

$$f_1=(a_0+a_1z+a_2z^2+\cdots)f_0. \quad (40)$$

To evaluate a_0, a_1, $a_2\cdots$ etc., one may convert Eq. (31) into a set of linear simultaneous algebraic equations. This is done by multiplying both sides of Eq. (38) by z^n, integrating between the limits 0 to ∞, and varying n from 1 to m, if a_m should be the highest coefficient chosen in (39).

The nth row of this set of simultaneous equations is given by

$$\sum_{r=1}^m ra_rI_{n+r}+j\frac{4\omega\tau_e}{p'}\sum_{r=0}^m\frac{a_r}{n+1}I_{r+n+3}=\frac{2p}{p'\Omega}(n+1)I_n, \quad (41)$$

where

$$I_K=\int_0^\infty z^Kf_0dz. \qquad K=n+r,\quad n+r+3,\text{ etc.} \quad (42)$$

It should be noted that by varying n from 1 to m, one obtains from (41) m equations, whereas there are $m+1$ unknowns. The other required equation is obtained from Eq. (20).

$$\sum_{r=0}^m a_rI_{r+2}=0. \quad (43)$$

After evaluating the constants a_0, a_1, etc., from the above-mentioned set of linear algebraic equations, the constants m_r and m_i of (31) and (32) are obtained from the following equations:

$$m_r=\sum_{K=1}^{m+1}a_{(K-1)r}I_K+\tfrac{1}{2}\omega\tau_m\sum_{K=0}^m a_{Ki}I_K, \quad (44)$$

and

$$m_i=\sum_{K=1}^{m+1}a_{(K-1)i}I_K-\tfrac{1}{2}\omega\tau_m(I_{K=0}+\sum_{K=0}^m a_{Kr}I_K). \quad (45)$$

The above equations are derived from (31) and (32) replacing f_{1r} and f_{1i} by

$$f_{1r}=(a_{0r}+a_{1r}z+a_{2r}z^2+\cdots)f_0 \quad (46)$$

and

$$f_{1i}=(a_{0i}+a_{1i}z+a_{2i}z^2+\cdots)f_0. \quad (47)$$

The subscripts r and i identify, respectively, the real and imaginary parts of the constants.

One, thus, obtains the conductivity and change in dielectric constant using Eqs. (33), (34), (30), (44), and (45).

V. NUMERICAL RESULTS AND DISCUSSION

The determination of conductivity according to the theory given in the previous section requires evaluation of the integrals I_K and solution of $m+1$ simultaneous equations, if m is the highest order of the constants

TABLE I. Values of the constants a_0, a_1, etc., for predominant acoustic phonon scattering.

a_0		$a_1\times(3.9094)^{-1}$		$a_2\times(3.9094)^{-2}$		$a_3\times(3.9094)^{-3}$		$a_4\times(3.9094)^{-4}$	
a_{0r}	a_{0i}	a_{1r}	a_{1i}	a_{2r}	a_{2i}	a_{3r}	a_{3i}	a_{4r}	a_{4i}
0.040950	0.043574	−0.047365	0.086709	−0.074253	0.464759	0.107237	−0.216700	−0.395210	−0.295473

$a_5\times(3.9094)^{-5}$		$a_6\times(3.9094)^{-6}$		$a_7\times(3.9094)^{-7}$		$a_8\times(3.9094)^{-8}$		$a_9\times(3.9094)^{-9}$	
a_{5r}	a_{5i}	a_{6r}	a_{6i}	a_{7r}	a_{7i}	a_{8r}	a_{8i}	a_{9r}	a_{9i}
−0.398596	−0.045964	0.019437	−0.072918	−0.213871	−0.028022	0.176261	0.062792	−0.043573	−0.016469

chosen. If one considers the effect of acoustic and optical phonons together, f_0 has such a form that I_K cannot be directly integrated. One may evaluate each of the I_K in series form or numerically. To keep numerical work to a minimum, numerical results have been obtained considering two special cases as discussed below.

A. Acoustic phonon scattering predominant

It is assumed that optical phonon scattering is completely absent. This is also the assumption made by Paranjape. Hence, the results obtained from the approximation may serve as a basis of comparison between the method used by this author and that of the present paper.

Under the above assumption, f_0 may be written as

$$f_0=\exp(-z^4/2p). \qquad (48)$$

On substituting (48) in (42) one obtains

$$I_K=\tfrac{1}{4}(2p)^{(K+1)/4}\Gamma\left(\frac{K+1}{4}\right). \qquad (49)$$

The value of m was chosen to be 9. The value of p was taken to be 100, which corresponds to an applied dc field of 2 kV/cm and experimental temperature of 300°K. The values of the different parameters of the Ge sample were assumed to be as given below:

$$e=1.6\times10^{-19}\ \text{C},\qquad m=0.12\times9\times10^{-31}\ \text{kg},$$
$$c=5.4\times10^{3}\ \text{m/sec},\qquad \mu_a=0.38\ \text{m}^2/\text{v sec},$$
$$S=1.333,\qquad B/A=63\ \text{(from Refs. 4, 10)}.$$

The values of the constants obtained numerically are shown in Table I. The values of $\Delta\sigma/\sigma$ and $\Delta\epsilon$ calculated using Eqs. (44) and (45) are also given at the bottom of the table.

Neglecting $\omega\tau_m$

$$\frac{\Delta\sigma}{\sigma_{dc}}=\frac{\sigma_{dc}-\sigma}{\sigma_{dc}}=0.0029,$$

$$\Delta\epsilon=0.336.$$

Considering $\omega\tau_m$

$$\frac{\Delta\sigma}{\sigma_{dc}}=\frac{\sigma_{dc}-\sigma}{\sigma_{dc}}=0.0013,$$

$$\Delta\epsilon=0.219.$$

The value of $\Delta\sigma/\sigma_{dc}$ and $\Delta\epsilon$ obtained from Paranjape's formula are, respectively, 0.0013 and 0.149. The experimental values are, however, 0.229 and 1.33. It is thus observed that the results of this present analysis are very close to those of Paranjape, though both are much different from the experimental values.

B. Optical phonon scattering predominant

It is assumed that in the expression for f_0 the term z^4 is negligible compared to $2rz^2$ so that f_0 may be written as

$$f_0=\exp(-rz^2/p'). \qquad (50)$$

The integral I_K may then be written as

$$I_K=\frac{1}{2}\left(\frac{p'}{r}\right)^{(K+1)/2}\Gamma\left(\frac{K+1}{2}\right). \qquad (51)$$

The values of the constant, $\Delta\sigma/\sigma_{dc}$ and $\Delta\epsilon$ calculated for the same condition as in Case A are shown in Table II.

$$\frac{\Delta\sigma}{\sigma_{dc}}=0.2368,\quad \Delta\epsilon=0.4311.$$

The values of $\Delta\sigma/\sigma_{dc}$ and $\Delta\epsilon$ as calculated in this case fit quite closely with the experimental values. $\Delta\sigma/\sigma_{dc}$ agrees to within 5% of the experimental value. The agreement in the value of $\Delta\epsilon$, though better than that obtained considering acoustic phonons only is, however, poorer than that for $\Delta\sigma/\sigma_{dc}$. It is difficult to decide at this stage whether this is a shortcoming of the theory or due to any experimental error.

It should be noted that in this case though B/A has been considered to be much larger, the effect of acoustic phonon scattering has been partially taken into consideration through the choice of p'. In dc conductivity calculations, also, agreement with theory and experiment was found with similar assumptions by Yamashita and Inoue.

[10] J. Yamashita, Progr. Theoret. Phys. (Kyoto) 24, 357 (1960).

TABLE II. Values of constants a_0, a_1, etc., for predominant optical phonon scattering. $F_0=2$ kV/cm, $T=300$°K.

a_0		$a_1\times(0.7817)^{-1/2}$		$a_2\times(0.7817)^{-1}$		$a_3\times(0.7817)^{-3/2}$		$a_4\times(0.7817)^{-2}$	
a_{0r}	a_{0i}	a_{1r}	a_{1i}	a_{2r}	a_{2i}	a_{3r}	a_{3i}	a_{4r}	a_{4i}
−0.812519	0.260418	0.241280	−0.021382	−0.009446	0.003038	0.638935	−0.037532	−3.345988	−1.276863

$a_5\times(0.7817)^{-5/2}$		$a_6\times(0.7817)^{-3}$		$a_7\times(0.7817)^{-7/2}$		$a_8\times(0.7817)^{-4}$		$a_9\times(0.7817)^{-9/2}$	
a_{5r}	a_{5i}	a_{6r}	a_{6i}	a_{7r}	a_{7i}	a_{8r}	a_{8i}	a_{9r}	a_{9i}
0.252538	0.719359	3.487547	−0.733107	−1.278321	−0.633259	1.524858	2.258067	−0.039384	−0.221570

It should be mentioned that the present analysis has been made with the assumptions: (i) Effective mass of carriers is isotropic. (ii) Impurity, intervalley and e-e scattering are negligible. (iii) For fields of or above 2 kV/cm, optical phonon scattering is predominant. (iv) Average energy of a carrier is much larger than the characteristic energy of optical phonons. It is, however, found that even with these assumptions the theory shows good agreement with experimental results which probably indicates that the deviation from the above assumptions is either small or of negligible importance in the calculation of conductivity.

It may be further noted that the numerical results given here are for a particular value of steady field. Calculations may be extended to cover other values of steady field and it is expected because of the general nature of the expression that similar agreement would be obtainable.

ACKNOWLEDGMENTS

The authors are indebted to Professor J. N. Bhar for his kind supervision of the present study. Numerical results quoted in this paper were obtained by the digital computer at the computation laboratory of Indian Statistical Institute. The authors wish to express their gratitude to Professor S. K. Mitra for making the computer facilities available, to Dr. A. Ghosh for his general interest, and to M. Roy for programming and running the problem on the computer.