Effect of Quadrupole Vibrations on l-Forbidden $M1$ Transitions*

RAYMOND A. SORENSEN
Carnegie Institute of Technology, Pittsburgh, Pennsylvania
(Received 23 July 1963)

The effect of quadrupole vibrations on l-forbidden $M1$ transitions is computed using the wave functions resulting from the shell model with a residual pairing plus $P^{(2)}$ force. These wave functions for low states of an odd nucleus are a linear combination of a spin j quasiparticle and spin j' quasiparticles coupled to 2+ phonons to angular momentum j. The one-phonon components of the wave functions allow l-forbidden $M1$ transitions to proceed both via the single particle and collective $M1$ operators. The calculation shows that this quadrupole coupling can explain most of the observed transition rate in a number of cases, while in a few cases, particularly for the odd-N nuclei fast transitions occur which are still unexplained. In addition, it is shown that the $E2$ contribution to these transitions while expected to be enhanced to several times single particle in most cases, nevertheless usually makes only a very small contribution to the total rate.

I. INTRODUCTION

SINCE the advent of the shell model of nuclear structure, it has been the common practice to compare experimentally measured electromagnetic transition rates with the single-particle estimate.[1] Pairing theory[2] shows that this practice may be useful since the spherical shell model with a residual pairing interaction predicts rates equal to the single-particle estimate multiplied by a simple reduction factor depending on the strength of the interaction, for transitions between one quasiparticle states.[3]

Thus, l-forbidden $M1$ transitions are forbidden for quasiparticles. These transitions involve $\Delta J=\pm 1$, no parity change, and $\Delta l=\pm 2$, while the simple one-body magnetic dipole operator requires $\Delta l=0$. The characterization of the low-lying (one quasiparticle) states of odd nuclei by the orbital angular momentum quantum number l is possible in the pairing model because of the particularly simple configuration mixing caused by the pairing interaction. Nevertheless, about two dozen such transitions have been measured, with the result that in most cases the transition is predominantly $M1$, but with the rates reduced from the single-particle estimate by factors ranging from 10 to 3000. The nonobservation of $E2$ for these transitions is not surprising since even with the large reductions, the $M1$ rates far outweigh, in nearly all cases, the single-particle estimate for the $E2$ rate.

The occurrence of these l-forbidden $M1$ transitions has been explained in two ways: as being due to configuration admixtures to the wave functions[4] or changes in the form of the $M1$ operator[5] caused by the nucleon-nucleon interaction. Both of these explanations have the defect that the parameters are uncertain and thus the actual importance of these explanations is hard to ascertain. It is not the purpose of this paper to offer a new explanation, but rather to demonstrate even for these nuclei, which are not deformed, that collective quadrupole effects are important in many cases, and in some cases are large enough to be a complete explanation of the $M1$ rate. In addition, the $E2$ contribution is computed.

II. THE CALCULATION

The computations are performed unambiguously with no new parameters by the use of the wave functions derived in the pairing plus quadrupole force model.[6] In Ref. 6, the pairing force strength is determined from the energy gap and the odd-even mass difference, while the quadrupole force strength is chosen to fit the energy of the lowest 2+ state of the even nuclei. For odd nuclei, the resulting wave functions have the form

$$\psi_{jm}=C_{j00}{}^{j}\alpha_{jm}{}^{\dagger}\psi_0+\sum_{j'} C_{j'12}{}^{j}[\alpha_{j'}{}^{\dagger}B^{\dagger}]_{jm}\psi_0+\cdots, \quad (1)$$

where $\alpha_{jm}{}^{\dagger}$ and $B^{\dagger}$ are creation operators for quasiparticle and 2+ phonons respectively, ψ_0 is the quasiparticle and phonon vacuum, and the C coefficients are listed in Ref. 6. The orbital quantum number l is suppressed (j means j, l), but the sum over j' includes all quasiparticles of the same parity as j, which can couple with 2 units of angular momentum to make a state of angular momentum j. An l-forbidden $M1$ transition may take place between two such states, $j=j_i$ and $j=j_f$ where $j_i-j_f=\pm 1$, $l_i-l_f=\pm 2$, by means of the second (one-phonon) component of each wave function. (The two-phonon component is neglected.) For the terms in which $j_i'=j_f'$, the transition may proceed either by virtue of the magnetic moment of the phonon, or the j' quasiparticle. There are additional contributions from terms in which j_i' and j_f' are spin-orbit partners.

* This work supported in part by the U. S. Office of Naval Research.

[1] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), Chap. XII, No. 6.

[2] A. Bohr, B. R. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958); B. R. Mottelson, Notes from Cours de L'Ecole d'Été de Physique Théorique des Houghes, 283 (1958).

[3] L. S. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **32**, No. 9, 1 (1960).

[4] A. Arima, H. Horie, and M. Sano, Progr. Theoret. Phys. (Kyoto) **17**, 567 (1957).

[5] R. G. Sachs and N. Austern, Phys. Rev. **81**, 705, 710 (1951); J. H. D. Jensen and M. G. Mayer, *ibid.* **85**, 1040 (1952).

[6] L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. **35**, 854 (1963).

The reduced $M1$ transition rate may be written

$$B_{\rm th}(M1)=\sum_{\mu m_f}|\langle\psi_{j_f m_f}|\mu_\mu|\psi_{j_i m_i}\rangle|^2\frac{3}{4\pi}$$
$$=\frac{3}{4\pi(2j_i+1)}|\langle\psi_f||\mu||\psi_i\rangle|^2, \quad (2)$$

where

$$\mu=\mu_{\rm particle}+\mu_{\rm collective}$$
$$=\mu_P+g_R R. \quad (3)$$

The gyromagnetic ratio g_R for the phonon was computed as in Ref. 6 except for the heavy nuclei for which $g_R=0.4$ was used. Thus, we need

$$\langle\psi_f||\mu||\psi_i\rangle$$
$$=\sum_{jj'} C_{j12}{}^{j_i}C_{j'12}{}^{j_f}(2j_i+1)^{1/2}(2j_f+1)^{1/2}(-1)^{j+j_i}$$
$$\times\{W(j'j_f j j_i;21)\langle\psi_0\alpha_{j'}||\mu_P||\alpha_j{}^\dagger\psi_0\rangle$$
$$+W(2j_f 2j_i;j1)\delta_{jj'}g_R\frac{e\hbar}{2mc}\langle\psi_0 B||R||B^\dagger\psi_0\rangle\}. \quad (4)$$

The reduced matrix elements are

$$\langle\psi_0\alpha_{j'}||\mu_P||\alpha_j{}^\dagger\psi_0\rangle$$
$$=\frac{e\hbar}{2mc}\left[\frac{(2j+1)(j+1)}{j}\right]^{1/2}[\tfrac{1}{2}g_s+(j-\tfrac{1}{2})g_l], \quad \text{for } j=j'=l+\tfrac{1}{2}$$
$$=\frac{e\hbar}{2mc}\left[\frac{j(2j+1)}{j+1}\right]^{1/2}[-\tfrac{1}{2}g_s+(j+\tfrac{3}{2})g_l], \quad \text{for } j=j'=l-\tfrac{1}{2}$$
$$=\frac{e\hbar}{2mc}(-1)^{l+\frac{1}{2}+j}\left[\frac{2l(l+1)}{2l+1}\right]^{1/2}(g_s-g_l)(U_jU_{j'}+V_jV_{j'}),$$
$$\text{for } l=l',\quad j=j'\pm1, \quad (5)$$

and

$$\langle\psi_0 B||R||B^\dagger\psi_0\rangle=(30)^{1/2}. \quad (6)$$

The U, V factors are obtainable from Ref. 6. For comparison purposes we use the single-particle estimate

$$B_{\rm sp}(M1)=\frac{3}{4\pi}\left(\frac{e\hbar}{2mc}\right)^2\frac{2l(l+1)}{(2l+1)(2j_i+1)}(g_s-g_l)^2, \quad (7)$$

computed as if j_i and j_f were spin-orbit partners equal to $l\pm\frac{1}{2}$.

The experimental reduced transition rates may be obtained from the lifetimes with the relations

$$T_\gamma(M1)=1.44(1+\alpha_{\rm tot})(1+E2/M1)T_{1/2}, \quad (8)$$

and

$$1/T_\gamma(M1)=(16\pi/9)(E^3/\hbar^4c^3)B_{\rm exp}(M1). \quad (9)$$

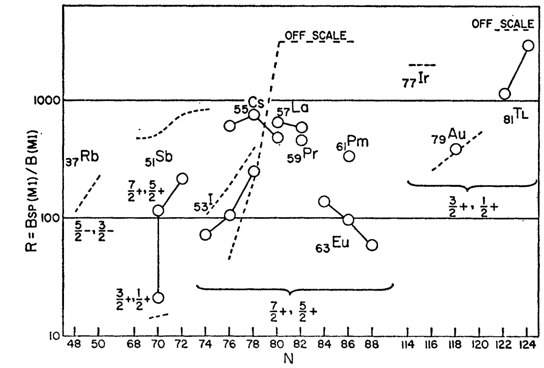

FIG. 1. The reduction factor for l-forbidden transitions in odd-Z nuclei. The ratios of the single-particle reduced rate given by Eq. (7) to the experimental rate [the main sources used for the experimental points were: R. L. Graham and R. E. Bell, Can. J. Phys. **31**, 377 (1953); and Phys. Rev. **84**, 380 (1951); T. Alvager, B. Johansson, and W. Zuk, Ark. Fysik **14**, 373 (1958); H. DeWaard and T. R. Gerholm, Nucl. Phys. **1**, 281 (1956); L. V. Groshev and A. M. Demidov, Atomnaya Energ. **7**, 321 (1959); R. W. Bauer, L. Grodzins, and H. H. Wilson, Phys. Rev. **128**, 694 (1962); R. Leonard (private communication of the $I^{127}-I^{129}$ results); Refs. 7, 8, 9] determined from Eqs. (8) and (9) are plotted as open circles, the isotopes being connected by lines. The theoretical ratios obtained from Eqs. (2)–(7) for the corresponding transitions are indicated by dashed lines.

III. RESULTS

The experimental reduction factor, $R=B_{\rm sp}(M1)/B_{\rm exp}(M1)$ is shown in Figs. 1 and 2, together with $B_{\rm sp}(M1)/B_{\rm th}(M1)$, the theoretical reduction factor due to the quadrupole coupling. From Fig. 1, it is seen that there is reasonable agreement between the experimental systematics and the calculated rates for $Z=51$, 53, 55, 79, and 81. On the other hand, for $Z=57$, 59, and 61, while the experimental reduction factor is large in all cases, the calculated rate is much lower yet owing to the weak quadrupole coupling occurring near $N=82$. It is seen from Fig. 2, that for the odd-neutron cases, the calculated rate is much too slow in most cases.

From the wave functions of Ref. 6, the $E2$ contribution to these same transitions may also be obtained. The one quasiparticle-to-one quasiparticle transition is

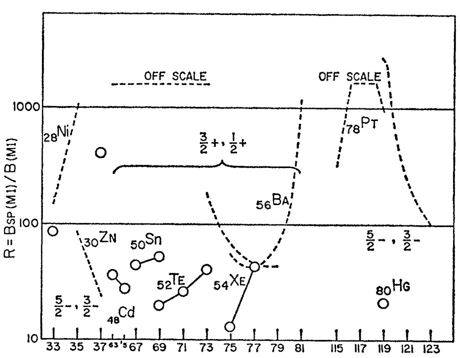

FIG. 2. The reduction factor for l-forbidden transitions in odd-N nuclei. See caption of Fig. 1.

allowed as an $E2$, but more important in most cases is the enhancement coming from the collective part of the operator which connects wave function components differing by the addition of one phonon in the initial or final state. As the no phonon to one phonon matrix element of the $E2$ operator is just the matrix element involved in the collective $E2$ rate $0+ \rightarrow 2+$ in the neighboring even nuclei we write the result in terms of this reduced rate:

$$\begin{aligned} B_{i\rightarrow f}(E2) &= (2j_f+1)\Bigg\{ \frac{\langle f|r^2|i\rangle}{2\sqrt{\pi}} C_{\frac{1}{2}-\frac{1}{2}0}{}^{j_i j_f 2} C_{j_f 00}{}^{j_f} C_{j_i 00}{}^{j_i} \\ &\quad \times (U_{j_f}U_{j_i} - V_{j_f}V_{j_i})(-1)^{j_f-1/2} + \left[\frac{B_{0+\rightarrow 2+}(E2)}{5}\right]^{1/2} \\ &\quad \Big[(2j_f+1)^{-1/2} C_{j_f 00}{}^{j_f} C_{j_f 12}{}^{j_i} (-1)^{j_i - j_f} \\ &\qquad + (2j_i+1)^{-1/2} C_{j_i 12}{}^{j_f} C_{j_i 00}{}^{j_i} \Big]\Bigg\}. \end{aligned} \quad (10)$$

The reciprocal lifetime is given by

$$1/T_\gamma(E2) = (8\pi/150)(E^5/\hbar^6 c^5) B(E2). \quad (11)$$

The result is that for these transitions, in most cases the contribution is but a few percent of the $M1$, and in many cases is less than one percent, in agreement with the $M1$ character of the transitions. Nevertheless, the calculated $E2$ rate is, in general, five to fifteen times the single-particle estimate for an $E2$ transition, except for a few cases in which both the initial and final levels are so near the Fermi energy that both the U, V, factor and the one-phonon amplitudes of Eq. (10) are particularly small. This is in qualitative agreement with the experimental results on $B(E2)$'s for these levels.[7] A detailed discussion of $B(E2)$ values in odd nuclei will be made in a future publication.

[7] M. Schmorak, A. C. Li, and A. Schwarzschild, Phys. Rev. **130**, 727 (1963); R. C. Ritter, P. H. Stelson, F. K. McGowan, and R. L. Robinson, *ibid.* **128**, 2320 (1962); F. K. McGowan and P. H. Stelson, *ibid.* **109**, 901 (1958).

IV. DISCUSSION

There are no doubt various causes for the occurrence of l-forbidden $M1$ transitions as mentioned in the Introduction, but we have shown that the effect considered here must be the most important effect in a number of cases. Also, in all cases but two the experimental transition is faster than the calculated one, consistent with the idea of quadrupole coupling being one of a number of causes. A particularly interesting case is the recent measurement in ${}_{51}\mathrm{Sb}_{70}{}^{121}$ of two different l-forbidden transitions[8] one going from the $1/2+$ to $3/2+$ levels and the other from $7/2+$ to $5/2+$. The latter has a reduction factor of 120 consistent with systematics. The former transition which is between excited states is unusually fast having a reduction factor of about 20. Both of these are explained in large part by the quadrupole coupling which has a particularly large effect on these excited states, which have very large one-phonon amplitudes in their wave functions. The relatively fast transitions in the Eu isotopes[9] were included in Fig. 1, even though no calculations were performed owing to the fact that these isotopes were considered to be deformed. Since the quadrupole effects are large for these isotopes, this may well explain these fast transitions as suggested in Ref. 9. On the other hand, except for the Ni and Xe isotopes for which the quadrupole effect is comparable with the experimental rate, the experimental l-forbidden $M1$ rates for the odd-N nuclei are almost all too fast for the quadrupole effect to be important. Thus, these surprisingly fast transitions must be due to some other cause.

ACKNOWLEDGMENT

The author wishes to acknowledge the hospitality of the Physics Division at the Aspen Institute for Humanistic Studies where part of this work was performed.

[8] E. R. Metzger and H. Langhoff (to be published).

[9] E. Berlovich, Y. K. Gusev, V. V. Ilyin, V. V. Nikitin, and M. K. Nikitin, Nucl. Phys. **37**, 469 (1962).