expectation value of this magnetic field gradient is zero, and indeed the calculation that leads from (13) to (14) would give nothing if the electric and magnetic dipole operators were classical vectors, or even if they were not classical but their components commuted with each other. It is the fact that each dipole moment is proportional to $\boldsymbol{\sigma}$, the components of which do not commute, that leads to a nonvanishing interaction.[20]

If the parameter γ of Sec. I is 10^{-7} and E is 10^5 V/cm, the precession rate of He^3 nuclei caused by the magnetic moment effect is roughly half a degree per day. It seems possible that considerably smaller precession rates can be measured.[8]

[20] The same noncommutativity effect was found in connection with a *PT*-noninvariant version of electrodynamics by M. Sachs and S. L. Schwebel, Ann. Phys. (N. Y.) **8**, 475 (1959).

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Elastic Scattering of 17-MeV Protons by Heavy Nuclei*

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Absolute differential cross sections for elastic scattering of protons from Ta^{181}, W, Pb^{208}, and Bi^{209} have been measured at a center-of-mass energy of 17.00±0.05 MeV at angular intervals of five degrees ranging from 20 to 170°. The estimated relative standard deviation of each point is 3% while the absolute cross-section scale is uncertain by 5% for Pb and Bi and by 10% for Ta and W. The scattered protons were detected by a NaI(Tl) scintillation counter with an over-all energy resolution of 2.5%. Consequently, while all inelastically scattered protons are rejected for Bi and Pb, a small contribution of inelastic protons from the lowest levels in Ta and the tungsten isotopes is included in the measured cross sections. For Ta and W the diffraction pattern appears damped at backward angles relative to the heavier two targets to a greater extent than may be attributed to the effects of an inelastic scattering component.

I. INTRODUCTION

ELASTIC differential cross sections for the scattering of particles by nuclei can be measured with relative precision and have had an historic role as a formidable test of nuclear models. The present work was undertaken to aid in determining the role of a deformation parameter in the nuclear optical model. In 1955, Hahn and Hofstadter[1] found that the scattering of 183-MeV electrons by Ta, W, and U gave rise to diffraction patterns with large angle oscillations less pronounced than in Pb^{208}, Au, and Bi. In a companion paper, Downs *et al.*[2] showed that a nuclear form factor including a quadrupole charge distribution could better reproduce the observations. Margolis[3] summarized the situation in 1959 with particular reference to the success of the work of Chase, Wilets, and Edmonds[4] who reproduced some of the detailed structure of the neutron strength function using a spheroidal optical potential. Schey[5] has shown that modification of the Bjorklund-Fernbach type of optical potential improves the fit to the data of Beyster *et al.*[6] for the scattering of 7-MeV neutrons by Ta. The deformation giving the best fit was in reasonable agreement with Coulomb excitation measurements.[7] Buck[8] has extended the optical model generalization to the simultaneous prediction of proton elastic and inelastic scattering. A more recent experiment of Hudson *et al.*[9] on the scattering of 15.2-MeV neutrons by Ta, Th, and U shows once again the characteristic flattening of the diffraction structure at large angles relative to Bi and relative to predictions with a spherical optical model. Data on the four nuclei in this report supplemented by comparable earlier 17-MeV

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[1] B. Hahn and R. Hofstadter, Phys. Rev. **98**, 278(A) (1955).

[2] B. W. Downs, D. G. Ravenhall, and D. R. Yennie, Phys. Rev. **98**, 277(A) (1955).

[3] B. Margolis, Proc. Inter. Conf. Nucl. Optical Model, Florida State University Studies **32**, 34 (1959).

[4] D. M. Chase, L. Wilets, and A. R. Edmonds, Phys. Rev. **110**, 1080 (1958).

[5] H. Schey, Phys. Rev. **113**, 900 (1958).

[6] J. R. Beyster, M. Walt, and E. W. Salmi, Phys. Rev. **104**, 1319 (1956).

[7] See, for example, K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 (1956).

[8] B. Buck, Phys. Rev. **130**, 712 (1963).

[9] C. I. Hudson, Jr., W. S. Walker, and S. Berko, Phys. Rev. **128**, 1271 (1962).

proton data by Dayton and Schrank[10] for Pt and Au span the mass region $170<A<210$ which ranges from large deformations to the doubly magic spherical nucleus Pb^{208}. Comparable data for 22.2-MeV protons have been reported by Fulmer.[11]

It is instructive to point out that all the measurements mentioned above are made with energy resolution insufficient for the separation of the elastic scattering from the inelastic scattering exciting low-lying rotational states of deformed nuclei. In the treatment of Schey,[5] scattering from these states is combined with the elastic scattering to simulate the experimental conditions. The question arises as to what extent the damped structure characteristic of scattering from the deformed nuclei in these experiments may be attributed to unresolved inelastic components. Such components are absent in the Pb and Bi data which show marked structure.

Following a description of the method and results of the present experiment, some preliminary evidence is presented that the inelastic contribution is not large enough to account for more than a small portion of the observed fill-in of the diffraction minima. Imminent improvements in detectors and in accelerator technology will lead to experiments with resolution of a few tens of keV which allow measurements of the resolved elastic and inelastic scattering from deformed nuclei and should answer the above question.

II. EXPERIMENTAL APPARATUS

Geometry

The external proton beam of the Princeton FM cyclotron was directed by focusing and steering magnets into the 60-in. scattering chamber.[12] Before reaching the target the beam passed through a collimator consisting of apertures of $\frac{1}{4}$-in. diameter, one being 1.32 m and the second 0.56 m from the target, making the beam spot at the target roughly $\frac{3}{8}$ in. in diameter. Additional baffles rejected slit scattering. Protons scattered by the target were detected by a NaI(Tl) crystal, about 0.5-in. square and cleaved to a thickness slightly exceeding the range of the proton. The crystal was mounted on the face of a Dumont 6291 photomultiplier. The detector solid angle was determined by a $\frac{3}{16}$-in.-diam aperture in front of the crystal and by the distance from this aperture to the target center which was varied between 20 and 50 cm. Angular position of the counter was easily measurable and reproducible to 0.1° while the angular resolution varied from 0.5 to 1.3° as the distance from target to counter was changed. Using the methods of Dayton and Schrank,[10] corrections were applied for the effects of finite angular resolution and multiple scattering where rapid changes in cross section made these effects significant. The experimental parameters were chosen so that the two corrections never exceeded 3% and 1.5%, respectively. In addition, an angular correction less than 0.5° was applied to the data for Ta and W where a slight displacement of the beam spot from the target center was discovered. A few points for Ta were repeated with the beam recentered to confirm the correction procedure, and satisfactory agreement was obtained for the data taken before and after the realignment.

Electronics

Pulses from the photomultiplier whose high voltage was regulated to 0.01% were passed via a preamplifier into a 20-channel pulse-height analyzer biased so that only the upper 15 to 20% of the pulse-height distribution was recorded. The system energy resolution over-all was about 2.5% while the stability was such that the pulse height varied less than 1% from day to day. The analyzer dead time of about 20 μsec exceeded the duration of beam pulses caused by the 2000-cps frequency modulation of the cyclotron so that only one proton could be analyzed per beam pulse. The counting rate was kept low enough so that dead time corrections were always less than 0.2% and hence negligible.

After the target the beam was collected by a Faraday cup maintained at a vacuum of 10^{-5} mm Hg and equipped with a large suppressor magnet. The cup was connected to polystyrene capacitors calibrated at the Bureau of Standards to 0.5% and the voltage monitored with a quadrant electrometer. The cup potential was maintained within 1 V of ground potential even though the capacitors had to be discharged several times at some angles to obtain enough counts. Previous measurements[12] had shown that secondary electron and soakage effects for this assembly could be neglected for cup potentials up to at least 6 V. The system was stable and reliable such that an over-all assignment of 1% relative uncertainty in the number of protons passing through the target is reasonable.

The beam energy was stabilized and measured by a differential ionization chamber[13] following a stack of calibrated aluminum foils. The chamber fed an error signal back to the cyclotron magnet current to control the energy to 20 keV while the foil stack gave an energy calibration good to 50 keV. Energies were selected for each target to give c.m. energies of 17.00±0.05 MeV for all. The proton beam energy resolution was about 0.24-MeV full width at half-maximum.

Targets

Commercially obtained foil targets of Ta and W were employed. The average thicknesses found by weighing were 8.30 and 12.98 mg/cm², respectively. Targets of Bi^{209} (purity 99.99%) and Pb^{208} (enriched to 90% abundance) were produced by evaporation onto polystyrene foils. The carbon content of the polystyrene

[10] I. E. Dayton and G. Schrank, Phys. Rev. **101**, 1358 (1956).
[11] C. B. Fulmer, Phys. Rev. **125**, 631 (1962).
[12] J. L. Yntema and M. G. White, Phys. Rev. **95**, 1226 (1954).
[13] G. Schrank, Rev. Sci. Instr. **26**, 677 (1955).

backing was used to determine the backing thickness by measuring its elastic scattering at 70, 90, and 155° and comparing with known absolute cross sections.[10,14] The backing thickness of 0.13 mg/cm² found in this way was then subtracted from the total weight of the foil to give the average thicknesses of the evaporated layers as 2.71 mg/cm² and 1.76 mg/cm² for the Bi and Pb, respectively.

Bi^{209} has a first excited state energy of 900 keV and Pb^{208} has its lowest state at 2.62 MeV so that the experimental resolution of 400 keV was sufficient to exclude all inelastic scattering. Ta^{181} has states at 6, 136, 159, 310, 482, and 500 keV from which inelastic scattering will be included in the area of the elastic peak, the highest two levels only at back angles where the target energy loss increases the peak width. Natural tungsten has three even isotopes (W^{182}, W^{184}, W^{186}) which make up 85% of the target weight and which have very similar rotational level structures with 2+ levels near 110 keV, 4+ levels near 360 keV, a third level near 700 keV and no others below 850 keV. The experimental resolution would include the inelastic scattering from the lowest two states at all angles and a portion of the third state at back angles where the target thickness spread the elastic peak by an additional 300 keV. The odd isotope W^{183} has at least 9 levels contributing but less than 15% abundance.

Errors

At least 2500 counts were accumulated in the elastic peak at each angle for each element. Allowing a relative standard deviation of 1% for charge collection and 1% for determining the area under the elastic peak, a relative standard deviation of 3% for each point may be assigned. In arriving at the absolute cross-section scale, errors in target thickness, solid angle, etc., are small in comparison to an uncertainty introduced by a correction for proton beam scattered outside of the acceptance angle of the Faraday cup. The correction was as large as 30% for the targets of Ta and W, partly because of their thickness and partly because of the beam misalignment mentioned earlier. An air lock target holder arrangement in the lid of the scattering chamber allowed the size of the correction to be found by removing the target and finding the resulting change in beam current. A cautious estimate of uncertainties in the correction so deduced sets the absolute scale error at ±5% for Pb and Bi and ±10% for Ta and W.[15]

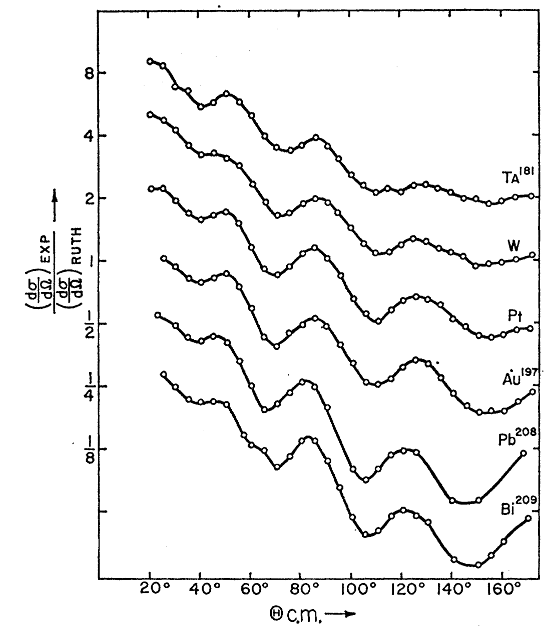

FIG. 2. Ratio of measured cross section to the Rutherford scattering cross sections for the four elements in Table I and for two elements from Ref. 10. The ordinate is logarithmic and the curves are arbitrarily displaced from one another by factors of 2 to avoid overlap. Relative errors are indicated by the size of the circles. Lines in both figures merely connect points.

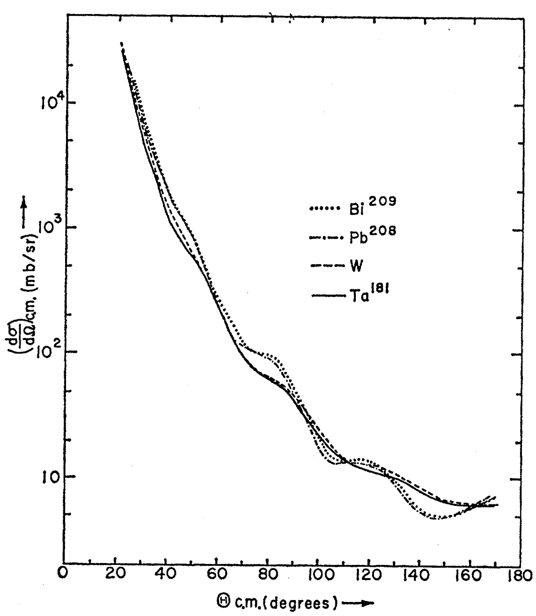

FIG. 1. Experimental differential cross sections for elastic scattering for the four elements listed in Table I. Error flags are smaller than the width of the lines.

III. RESULTS AND DISCUSSION

The cross sections obtained at each angle, transformed to the c.m. system are given in Table I. The cross sections are plotted on a semilogarithmic scale in Fig. 1, while Fig. 2 shows the ratios of the measured cross sections to the respective Rutherford cross sections. The scale in Fig. 2 is semilogarithmic and the successively heavier targets have been arbitrarily displaced in the vertical scale to avoid confusion. Data

[14] R. W. Peelle, Phys. Rev. **105**, 1311 (1957).

[15] The Faraday cup assembly has since been rebuilt by Dr. W. W. Daehnick to increase the acceptance angle and reduce the importance of this effect.

TABLE I. Experimental differential cross sections transformed to the center-of-mass system. The angles are in degrees and the cross sections in mb/sr.

Ta^{181} $\theta_{c.m.}$	Ta^{181} $(d\sigma/d\Omega)_{c.m.}$	W $\theta_{c.m.}$	W $(d\sigma/d\Omega)_{c.m.}$	Pb^{208} $\theta_{c.m.}$	Pb^{208} $(d\sigma/d\Omega)_{c.m.}$	Bi^{209} $\theta_{c.m.}$	Bi^{209} $(d\sigma/d\Omega)_{c.m.}$
20.5	27 300	20.5	31 000				
25.1	11 900	25.4	12 500	23.1	20 600	25.1	15 600
25.4	11 200						
30.5	4280	30.5	5440	30.2	6440	30.2	6660
35.5	2270	35.5	2560	35.2	3100	35.2	3240
40.5	1140	40.5	1380	40.2	1760	40.2	1850
45.2	762			45.2	1200	45.2	1200
45.7	758	45.6	885				
50.7	559	50.7	564	50.2	752	50.2	772
55.7	362	55.7	364	55.2	430	57.2	340
60.8	226	60.8	216	60.3	237	60.3	253
65.3	131					65.3	178
65.8	136	65.8	133	65.3	137		
70.8	93.0	70.8	89.5	70.3	111	70.3	115
75.8	70.7	75.8	72.5	75.3	99.8	75.3	102
80.7	61.0	80.7	64.9	80.3	90.5	80.3	97.2
85.7	54.4	85.7	57.0	85.3	70.2	85.3	79.2
90.3	41.9	90.3	45.9	90.3	47.1	90.3	53.3
90.3	41.3						
94.9	31.4	94.6	35.7			95.3	33.6
99.9	22.5	99.6	25.5	100.3	17.3	100.3	21.0
104.9	17.1	104.6	18.8	105.3	13.4	105.3	15.0
109.7	14.1	109.5	14.9	110.3	13.3	110.3	13.7
114.5	13.2	114.5	13.4				
115.3	12.6			115.3	13.7	115.3	14.4
119.5	11.5	119.5	13.0	120.3	12.9	120.3	13.9
124.4	11.0	124.4	12.6	125.2	11.6	125.2	11.8
129.4	10.2	129.4	11.2			130.2	9.79
134.4	9.11	134.4	9.72				
135.2	9.10						
139.4	8.10	139.4	8.69	140.2	5.44	140.2	5.79
144.4	7.14	144.4	7.75				
149.4	6.73	149.4	6.65				
150.2	6.63			150.2	4.92	150.2	4.90
154.3	6.18	154.4	6.54			155.1	5.21
159.3	6.11	159.4	6.39			160.1	5.87
165.4	6.18	165.4	6.35				
170.1	6.24			168.1	7.29	170.1	7.26
171.3	6.10	171.3	6.48				

from Dayton and Schrank[10] for Pt and Au have been included in this plot for comparison. The most forward angles reached are still too large to permit use of the transition to Rutherford scattering at sufficiently large impact parameters to check the absolute cross section scale. In Fig. 2 one can follow the growth in amplitude of the large angle oscillations as the mass increases.

A scattering experiment on Ho^{165} with 55-keV resolution in progress at this laboratory shows the cross section for excitation of the first excited state at 95 keV to be an order of magnitude less than the elastic cross section at back angles.[16] Higher states are more weakly excited. A rough estimate based on this result would indicate that the cross section for exciting a 2^+ state near 100 keV in one of the even isotopes of W, for example, would also be considerably less than the elastic cross section, perhaps 20% at large angles. Referring to Fig. 2, we see that the minima and maxima for Pb^{208} and Bi^{209} differ in height almost by factors of two. Thus, if we were to assume that the curves for Ta and W are a superposition of a curve similar to that for the spherical nuclei plus an incoherent inelastic contribution of 20% amplitude, the result could not reproduce the very flat distributions at large angles observed for Ta and W. Apparently the damped diffraction structure in elastic scattering from highly deformed nuclei arises primarily from the cross terms in the coupling of direct elastic and inelastic processes rather than from experimental inclusion of inelastic states. It would be interesting to see whether a generalized optical model such as that of Buck[8] could predict the elastic shapes here and in Refs. 9 and 11, and whether the inelastic predictions arising therefrom would then agree with experiments which resolve these states.

[16] We are indebted to Dr. A. Lieber for showing us these data prior to publication and to Dr. E. Rost for a discussion of the applications to other nuclei.