Absorptions of Electromagnetic Waves in Electron-Phonon Systems

NARKIS TZOAR
Bell Telephone Laboratories, Whippany, New Jersey
(Received 22 May 1963)

A study of the absorption of electromagnetic waves in an electron-phonon system is described, for waves whose frequencies are high compared to the collision frequency and whose wavelengths are long compared to the Bohr (Debye) radius. The treatment rests on the introduction of the temperature-dependent Green's function and Kubo's formula for the conductivity. An exact expression for the conductivity, where collective effects are properly taken into account, is obtained, assuming the number of electrons in a Bohr (Debye) sphere is large compared with one. The application of this theory to degenerate semiconductors is suggested.

I. INTRODUCTION

THE purpose of the following work is to calculate the absorption of high-frequency electromagnetic waves by an electron-phonon system. Such a system provides one of the mechanisms of a realistic model for investigating infrared absorption by highly degenerate semiconductors as InSb, InP, GeP, etc. In these degenerate semiconductors, which are ionic to a small degree, the conducting electrons interact with the polarized vibrations of the lattice (optical phonons). The interaction between the electrons and the optical phonons is weaker here than in ionic crystals, which allows us to assume a weak coupling between the electrons and the optical phonons. An additional mechanism for the absorption of high-frequency electromagnetic waves is that due to randomly distributed frozen ions which was discussed by Ron and Tzoar[1] for the case of degenerate semiconductors. In the following, we shall be interested only in the absorption due to the interaction of electrons with optical phonons.

This phenomenon is important in the frequency range $\omega \gtrsim \omega_q$, where ω is the external field frequency and ω_q is the frequency spectrum of the optical phonons, as a function of its momentum q. Since the frequencies of interest are of the same order of magnitude as the plasma electron frequency, collective effects of the electron gas *must* be taken properly into account. Due to the interaction of the electron with the crystal, their effective mass and charge is such that the number of electrons in a Bohr sphere is larger than one. Hence, the plasma approximation for the electron gas can be used (see, for example, the article of Wolff[2]). The present problem has been treated also by Gurevich, Lang, and Firsov.[3] However, they completely neglected collective effects which, as we point out, should be taken into account.

Section II deals with the well-known relation between the Kubo[4] formula for the conductivity and the temperature-dependent Green's function. We employ the diagram technique of Luttinger and Ward[5] to obtain our result for the absorption coefficient in Sec. III. We reserve Sec. IV for discussion and conclusions.

II. EVALUATION OF THE ABSORPTION COEFFICIENT

We start from the general expression of the conductivity for a system of charged particles as given by Kubo[4] for wave number equal to zero ($\hbar$ is taken to be one):

$$\sigma(\omega)=\frac{1}{3V}\int_0^\infty d\tau\, e^{i\omega\tau}\int_0^\beta d\lambda\langle \mathbf{j}(\tau-i\lambda)\cdot\mathbf{j}(0)\rangle\,, \qquad (1)$$

where ω is the frequency of the electromagnetic wave,

$$\mathbf{j}(\tau)=e^{iH\tau}\mathbf{j}(0)e^{-iH\tau} \qquad (2)$$

is the Fourier transform of the current operator in the Heisenberg representation for wave number zero, and the average of an operator O is given by

$$\langle O\rangle=\mathrm{Tr}\{e^{\beta(\Omega+\mu N-H)}O\}\,. \qquad (3)$$

In Eqs. (2) and (3), H represents the total Hamiltonian of the system, Ω is defined by

$$e^{-\beta\Omega}=\mathrm{Tr}\{e^{\beta(\mu N-H)}\}\,, \qquad (4)$$

μ and N are the chemical potential and the number operator for the electrons, respectively, and β the inverse of the temperature in energy units. The current operator used in Eq. (2) is defined by

$$\mathbf{j}(0)=\frac{e}{m}\sum_p \mathbf{p}a_\mathbf{p}^\dagger a_\mathbf{p}\,, \qquad (5)$$

where we use the following convention for Fourier transforms:

$$\mathbf{f}(\mathbf{x},\tau)=\frac{1}{2\pi}\int d\omega\frac{1}{V}\sum_\mathbf{k}\exp(-i\omega\tau-i\mathbf{k}\cdot\mathbf{x})\,\mathbf{f}(\mathbf{k},\omega)$$

and

$$\mathbf{f}(\mathbf{k},\omega)=\int d\tau\int d\mathbf{x}\exp(i\omega\tau+i\mathbf{k}\cdot\mathbf{x})\,\mathbf{f}(\mathbf{x},\tau)\,. \qquad (6)$$

[1] A. Ron and N. Tzoar, Phys. Rev. **131**, 1943 (1963).
[2] P. A. Wolff, Phys. Rev. **126**, 405 (1962).
[3] V. L. Gurevich, I. G. Lang, and Yu-A. Firsov, Fiz. Tverd. Tela **4**, 1252 (1962) [translation: Soviet Phys.—Solid State **4**, 918 (1963)].
[4] R. Kubo, J. Phys. Soc. Japan **12**, 570 (1957).
[5] J. M. Luttinger and J. C. Ward, Phys. Rev. **118**, 1417 (1960).
[6] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinsky, Zh. Eksperim. i Teor. Fiz. **36**, 900 (1959) [translation: Soviet Phys.–JETP **9**, 636 (1959)].

In order to render Eq. (1) in a more convenient form, we integrate it by parts and obtain

$$\sigma(\omega)=\sigma_0(\omega)+\sigma_1(\omega), \tag{7}$$

where

$$\sigma_0(\omega)=i\omega_p^2/4\pi\omega. \tag{8}$$

Here $\omega_p=(4\pi e^2n/m)^{1/2}$ is the plasma frequency, n, e, and m are, respectively, the electron density, charge, and mass, and

$$\sigma_1(\omega)=\frac{1}{3\omega V}\int_0^\infty d\tau\, e^{i\omega\tau}\langle[\mathbf{j}(\tau)\cdot,\mathbf{j}(0)]\rangle, \tag{9}$$

where $[\ ,\ ]$ denotes the commutator.

We next define a Green's function

$$M(u)=\frac{1}{3V}\langle T\{\mathbf{j}(u)\cdot\mathbf{j}(0)\}\rangle, \qquad -\beta<u<\beta \tag{10}$$

where T is the Dyson ordering operator and

$$\mathbf{j}(u)=e^{uH}\mathbf{j}(0)e^{-uH}. \tag{11}$$

The function $M(u)$, defined in Eq. (10) is periodic in u, i.e.,

$$M(u+\beta)=M(u),$$

and, thus, its Fourier transform with respect to u is

$$M(\omega_n)=\int_0^\beta du\, e^{\omega_n u}M(u), \tag{12}$$

where

$$\omega_n=2\pi in/\beta, \quad n=0,\pm1,\pm2,\cdots. \tag{13}$$

We now define $M(z)$ as the analytical continuation of $M(\omega_n)$ from the infinite set of points $2\pi in/\beta$ $(n>0)$ on the positive imaginary axis of z to the entire upper half-plane of z. It is then easy to show[7,8] that $\sigma_1(\omega)=(1/i\omega)M(\omega+i\epsilon)$ for $\epsilon\to0_+$. We therefore obtain

$$\sigma(\omega)=\sigma_0(\omega)+(1/i\omega)M(\omega+i\epsilon); \quad \epsilon\to0_+ \tag{14}$$

as our useful expression for the absorption coefficient.

III. EVALUATION OF THE ABSORPTION COEFFICIENT

We turn now to the calculation of $M(\omega_n)$ using a perturbation expansion technique, and then resumming *all* diagrams (terms) which contribute to the conductivity for quantum (classical) plasmas, under the condition that the number of particles in the Bohr (Debye) sphere is large, the frequency is high compared to the collision frequency, and the wavelength of the incident field is taken to be infinite. Thus in resumming the diagrams (terms) of the perturbation expansion, we consider processes proportional to the number of the electrons, N, as finite, and include them to all orders; while those processes which are not proportional to N are treated as small. This point has been discussed in detail by Balescu.[9]

Our electron-phonon system is described by the Hamiltonian:

$$H=H_0+H_I, \tag{15}$$

where

$$H_0=\sum_{\mathbf{p}}\epsilon_p a_{\mathbf{p}}^\dagger a_{\mathbf{p}}+\sum_{\mathbf{q}}\omega_q b_{\mathbf{q}}^\dagger b_{\mathbf{q}} \tag{16}$$

and

$$H_I=\frac{1}{2V}\sum_{\mathbf{pp'q}}\frac{4\pi e^2}{k^2}a_{\mathbf{p+q}}^\dagger a_{\mathbf{p'-q}}^\dagger a_{\mathbf{p'}}a_{\mathbf{p}} + \frac{1}{V^{1/2}}\sum_{\mathbf{qp}}(C_q a_{\mathbf{p+q}}^\dagger a_{\mathbf{p}}b_{\mathbf{q}}+\text{H.c.}). \tag{17}$$

Here ω_q is the phonon frequency spectrum as a function of its momentum $\mathbf{q}$, C_q is the coupling between an electron and a phonon of momentum $\mathbf{q}$, and ϵ_p is the kinetic energy of the electron having momentum $\mathbf{p}$. Here $a_{\mathbf{p}}^\dagger$, $a_{\mathbf{p}}$ and $b_{\mathbf{q}}^\dagger$, $b_{\mathbf{q}}$ are, respectively, the creation and destruction operators for electrons and phonons, which obey the usual commutation relations.

The basic rules for the perturbation expansion of $M(\omega_n)$ and their diagrammatical representation are given essentially by Luttinger and Ward,[5] with the addition that here we also have an electron-electron interaction via a phonon. The essential ingredients of the perturbation expansion are the free electron propagator given by

$$G_p(\zeta_l)=(\zeta_l-\epsilon_p)^{-1}, \quad \zeta_l=(2l+1)\pi i/\beta+\mu; \quad l=0,\pm1,\pm2,\cdots \quad \epsilon_p=p^2/2m, \tag{18}$$

and indicated diagrammatically by a solid line; and the Coulomb and phonon interaction lines, respectively, given by dotted and dash-dotted lines. We represent a Coulomb interaction by means of its matrix element $4\pi e^2/k^2$ and a phonon interaction by means of the product $|C_k|^2D_k(\alpha_m)$, where $\mathbf{k}$ and α_m are the momentum and "complex energy" transferred by the interaction, and $D_k(\alpha_m)$ is the free phonon propagator given by

$$D_k(\alpha_m)=2\omega_k/(\alpha_m^2-\omega_k^2). \tag{19}$$

The coupling term, $|C_k|^2=2\pi e^2k^{-2}\epsilon_0^{-1}\omega_l\omega_t^{-2}(\omega_l^2-\omega_t^2)$, between two electrons via a phonon is considered to be, for degenerate semiconductors, of the order of e^2, and of long range because of the k^{-2} term. Here ω_l, ω_t, respectively, represent the frequencies of the longitudi-

[7] A. I. Larkin, Zh. Eksperim. i Teor. Fiz. **37**, 264 (1959) [translation: Soviet Phys.—JETP **10**, 186 (1960)].

[8] A. Ron and N. Tzoar, Phys. Rev. **131**, 12 (1963).

[9] R. Balescu, Phys. Fluids **4**, 95 (1960). Many references concerning quantum and classical plasmas can be found here.

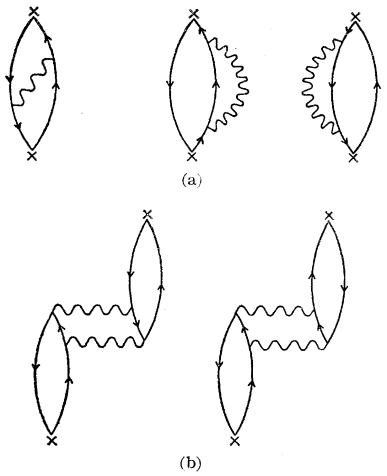

FIG. 1. The class of diagrams which contribute to the high-frequency conductivity.

nal and transverse optical vibrations and ϵ_0 represents the lattice static dielectric constant. (See Ref. 3.)

In order to determine the diagrams (terms) which contribute to the absorption coefficient at frequencies high compared to the collision frequency and wavelengths long compared to the Bohr (Debye) radius, we must recognize the three parameters of the system; the number of particles in the Bohr (Debye) sphere, the strength of the electron-phonon coupling and the number of excited phonons which is merely a measure of the crystal temperature.

For the practical case of degenerate semiconductor at low temperature one may assume a weak electron-phonon coupling and small number of excited phonons.

However, for the sake of generalization, we first solve the symmetric problem in which the dielectric function of the medium is affected by the presence of the phonons. Furthermore, we assume that the number of particles in the Bohr (Debye) sphere is *much* larger than one and thus plasma effects are dominant. This results in strong shielding of the Coulomb potential as well as the phonons via the effective field of the medium. We therefore find that the Green's function $M_s(\omega_n)$ for this problem is approximated by the diagram given in Fig. 1. However, at low temperature and weak electron-phonon coupling (i.e., the case of degenerate semiconductor) our Green's function $M(\omega_n)$ should have only one-phonon interaction and, thus, is given by the first-order term in the expansion of $M_s(\omega_n)$ in the phonon propagator.

In Fig. 1 the wavy line represents the effective potential shown in Fig. 2 and given by

$$\begin{aligned} U_q(\alpha_m) &= B_q(\alpha_m)[1-B_q(\alpha_m)Q_q(\alpha_m)]^{-1} \\ &= B_q(\alpha_m)[P_q(\alpha_m)]^{-1}, \end{aligned} \tag{20}$$

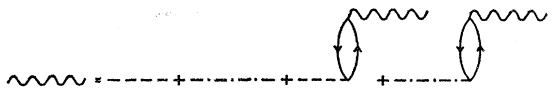

FIG. 2. The integral equation for the effective interaction.

where

$$Q_q(\alpha_m)=\frac{1}{(2\pi)^3}\int d\mathbf{p}\frac{f_{p+q/2}-f_{p-q/2}}{\epsilon_{p+q/2}-\epsilon_{p-q/2}-\alpha_m}, \tag{21}$$

$$\alpha_m=2\pi i m/\beta, \quad m=0, \pm1, \pm2, \cdots,$$

$$f_p=[e^{\beta\epsilon_p-\beta\mu}+1]^{-1}. \tag{22}$$

and

$$B_q(\alpha_m)=4\pi e^2/q^2+|C_q|^2D_q(\alpha_m). \tag{23}$$

We now calculate $M_s(\omega_n)$ for our symmetrical model by summing all the diagrams given in Fig. 1 and obtain

$$M_s(\omega_n)=\frac{1}{3V}\frac{e^2}{m^2}\sum_{\mathbf{pp'}}(\mathbf{p}\cdot\mathbf{p'})\sum_{i=1}^{5}K_{pp'^{(i)}}(\omega_n), \tag{24}$$

where $K_{pp'^{i}}(\omega_n)$ corresponds to the ith diagram of Fig. 2.

$$\begin{aligned}
K_{pp'^{(1)}}(\omega_n)&=\frac{1}{V}\frac{1}{\beta}\sum_m U_{p-p'}(\alpha_m)\frac{1}{\beta}\sum_l G_p(\zeta_l)G_p(\zeta_l+\omega_n) \\
&\quad\times G_{p'}(\zeta_l+\omega_n+\alpha_m)G_{p'}(\zeta_l+\alpha_m), \\
K_{pp'^{(2)}}(\omega_n)&=\delta_{pp'}\frac{1}{V}\sum_m U_q(\alpha_m)\frac{1}{\beta}\sum_l [G_p(\zeta_l)]^2 \\
&\quad\times G_{p-q}(\zeta_l-\alpha_m)G_p(\zeta_l-\omega_n), \\
K_{pp'^{(3)}}(\omega_n)&=\delta_{pp'}\frac{1}{V}\sum_q\frac{1}{\beta}\sum_m U_q(\alpha_m)\frac{1}{\beta}\sum_l [G_p(\zeta_l)]^2 \\
&\quad\times G_{p+q}(\zeta_l+\alpha_m)G_p(\zeta_l+\omega_n), \\
K_{pp'^{(4)}}(\omega_n)&=\frac{1}{V^2}\sum_q\frac{1}{\beta}\sum_m U_q(\alpha_m)U_q(\alpha_m-\omega_n) \\
&\quad\times\frac{1}{\beta}\sum_l G_p(\zeta_l)G_p(\zeta_l-\omega_n)G_{p-q}(\zeta_l-\alpha_m) \\
&\quad\times\frac{1}{\beta}\sum_{l'} G_{p'}(\zeta_{l'})G_{p'}(\zeta_{l'}-\omega_n)G_{p'-q}(\zeta_l-\alpha_m), \\
K_{pp'^{(5)}}(\omega_n)&=\frac{1}{V^2}\sum_q\frac{1}{\beta}\sum_m U_q(\alpha_m)U_q(\alpha_m+\omega_n) \\
&\quad\times\frac{1}{\beta}\sum_l G_p(\zeta_l)G_p(\zeta_l-\omega_n)G_{p-q}(\zeta_l-\alpha_m) \\
&\quad\times\frac{1}{\beta}\sum_{l'} G_{p'}(\zeta_{l'})G_{p'}(\zeta_{l'}+\omega_n)G_{p'+q}(\zeta_{l'}+\alpha_m).
\end{aligned} \tag{25}$$

We now carry out the summation over l and l' by converting the sums into integrals (see Ref. 5). After considerable manipulation and using the symmetry properties under the transformation $\alpha_m\rightarrow-\alpha_m-\omega_n$,

we obtain for $M_s(\omega_n)$

$$M_s(\omega_n)=\frac{1}{3\omega_n{}^2}\int\frac{d\mathbf{q}}{(2\pi)^3}q^2\frac{e^2}{m^2}\frac{1}{\beta}\sum_m|C_q|^2 \times D_q(\alpha_m)[Q_q(\alpha_m+\omega_n)-Q_q(\alpha_m)] \times[P_q(\alpha_m)P_q(\alpha_m+\omega_n)]^{-1}. \quad (26)$$

Equation (26) is not suitable for the analytical continuation to the upper z half plane. In order to perform the continuation we first have to evaluate the summation over m. We use essentially a method developed by Perel' and Eliashberg[10] [see also Appendix B of Ref. 8] to carry out this summation. We obtain the absorption coefficient which results from the contribution of the diagrams in Fig. 1:

$$\sigma_s(\omega)=\sigma_0(\omega)+i\frac{e^2}{3\omega^3m^2(2\pi)^3}\int d\mathbf{q}q^2|C_q|^2\frac{P}{4\pi i}\int_{-\infty}^{+\infty}dx \times\coth(\tfrac{1}{2}\beta x)\frac{1}{P_q{}^+(x+\omega)}\Big\{\frac{1}{P_q{}^+(x)} \times[Q_q{}^+(x-\omega)-Q_q{}^+(x)][D_q{}^+(x+\omega)-D_q{}^+(x)] -\frac{1}{P_q{}^-(\omega)}[Q_q{}^+(x+\omega)-Q_q{}^-(x)] \times[D_q{}^+(x+\omega)-D_q{}^-(x)]\Big\}, \quad (27)$$

where P stands for the principal values of the integral. In Eq. (25)

$$Q_q{}^\pm(x)=\frac{1}{(2\pi)^3}\int d\mathbf{p}\frac{f_{p+q/2}-f_{p-q/2}}{\epsilon_{p+q/2}-\epsilon_{p-q/2}-x\mp i\epsilon}, \qquad D_q{}^\pm(x)=\frac{2\omega_k}{(x\pm i\epsilon)^2-\omega_k{}^2}, \quad (28)$$

and

$$P_q{}^\pm(x)=1-\Big[\frac{4\pi e^2}{q^2}+|C_q|^2D_q{}^\pm(x)\Big]Q_q{}^\pm(x). \quad (29)$$

The first order term of $M_s(\omega_n)$ expanded in a power series in the phonon interaction is a suitable approximation for the Green's function in the high-frequency and low-temperature case. It is given by

$$M(\omega_n)=\frac{1}{3\omega_n{}^2}\int\frac{d\mathbf{q}}{(2\pi)^3}q^2\frac{e^2}{m^2}\frac{1}{\beta}\sum_m|C_q|^2 \times D_q(\alpha_m)[Q_q(\alpha_m+\omega)-Q_q(\alpha_m)] \times[\mathcal{E}_q(\alpha_m)\mathcal{E}_q(\alpha_m+\omega_n)]^{-1}, \quad (30)$$

where

$$\mathcal{E}_q(\alpha_m)=1-\frac{4\pi e^2}{q^2}Q_q(\alpha_m) \quad (31)$$

is the electron dielectric function.

We now evaluate the sum over m in the expression for $M(\omega_m)$ by employing the same technique as in Eq. (26). We obtain the absorption coefficient for the low-temperature case as

$$\sigma(\omega)=\sigma_0(\omega)+i\frac{e^2}{3\omega^3m^2(2\pi)^3}\int d\mathbf{q}\,q^2|C_q|^2\frac{P}{4\pi i}\int_{-\infty}^{+\infty}dx \times\coth(\tfrac{1}{2}\beta x)\frac{1}{\mathcal{E}_q(x+\omega)}\Big\{\frac{1}{\mathcal{E}_q(x)} \times[Q_q{}^+(x+\omega)-Q_q{}^+(x)][D_q{}^+(x+\omega)-D_q{}^+(x)] -\frac{1}{\mathcal{E}_q{}^*(x)}[Q_q{}^+(x+\omega)-Q_q{}^-(x)] \times[D_q{}^+(x+\omega)-D_q{}^-(x)]\Big\}, \quad (32)$$

where

$$\mathcal{E}_q(x)=1-\frac{4\pi e^2}{q^2}Q_q{}^+(x). \quad (33)$$

Equation (32) can be simplified if we express the Q's in terms of the $\mathcal{E}$'s, and we obtain

$$\sigma=\sigma_0(\omega)-i\frac{4\pi}{3\omega^3m^2(2\pi)^3}\int_0^\infty dq\,q^6|C_q|^2\frac{P}{4\pi i} \times\int_{-\infty}^{+\infty}dx\coth(\tfrac{1}{2}\beta x)\Big\{\Big[\frac{1}{\mathcal{E}(x)}-\frac{1}{\mathcal{E}(x+\omega)}\Big] \times[D_q{}^+(x+\omega)-D_q{}^+(x)]+\Big[\frac{1}{\mathcal{E}^*(x)}-\frac{1}{\mathcal{E}(x+\omega)}\Big] \times[D_q{}^+(x+\omega)-D_q{}^-(x)]\Big\},$$

$$=\sigma_0(\omega)-i\frac{4\pi}{3\omega^3m^2(2\pi)^3}\int_0^\infty dq\,q^6|C_q|^2\frac{P}{4\pi i} \times\int_{-\infty}^{+\infty}dx\coth(\tfrac{1}{2}\beta x)\Big\{D_q{}^+(x+\omega)2i\,\mathrm{Im}\frac{1}{\mathcal{E}_q(x)} +\frac{1}{\mathcal{E}_q(x+\omega)}2i\,\mathrm{Im}D_q{}^+(x)-2i\,\mathrm{Im}\Big[\frac{D_q{}^+(x)}{\mathcal{E}_q(x)}\Big]\Big\}. \quad (34)$$

In Eq. (34), $\mathcal{E}_q(x)$ is defined by Eq. (33) for a degenerate electron gas. However, for a nondegenerate electron gas (classical limit), $\sigma(\omega)$ is still given by Eq. (34) with the understanding that $\mathcal{E}_q(x)$ is the classical limit of Eq.

[10] V. I. Perel' and G. M. Eliashberg, Zh. Eksperim. i Teor. Fiz. **41**, 886 (1961) [translation: Soviet Phys.—JETP **14**, 633 (1962)].

(33) and is given by

$$\mathcal{E}_q(x)=1-\frac{\omega_p^2}{q^2}\int du'\frac{1}{u'-x-i\epsilon}\frac{\partial}{\partial u'}f(u'), \quad (35)$$

where $f(u)$ is the one-dimensional Maxwell-Boltzman distribution.

Since the last term in Eq. (34) is purely imaginary and does not contribute to the absorption, our final result for the resistivity $R(\omega)=$real part of $1/\sigma(\omega)$ is

$$R(\omega)=\frac{16}{3\omega_p^4 m^2\omega}\int_0^\infty dq\, q^6|C_q|^2\frac{P}{4\pi}\int_{-\infty}^{+\infty}dx\coth(\tfrac{1}{2}\beta x)\times\left[\mathrm{Im}D_q^+(x+\omega)\,\mathrm{Im}\frac{1}{\mathcal{E}_q(x)}+\mathrm{Im}\frac{1}{\mathcal{E}_q(x+\omega)}\mathrm{Im}D_q^+(x)\right]. \quad (36)$$

Our result for $R(\omega)$ is complicated and cannot be evaluated analytically. Moreover, in order to evaluate the integral in Eq. (36) we must know the energy spectrum of the phonons as a function of their momentum. We hope to submit a computation of Eq. (36) for real semiconductors in a future communication.

IV. DISCUSSION

In this paper we have derived a general expression for the absorption coefficient of electromagnetic waves by a plasma-phonon system. We have restricted ourselves to applied fields of high frequency and long wavelength and we have properly accounted for collective effects. Our approximation rests on the fact that the phonons are weakly coupled to the electrons and our results are given in Eq. (32) for the case where the population of the phonons is smaller than that of the electrons, and by Eq. (27) for the more general case, i.e., the number of the phonons is comparable to that of the electrons.

We now compare our result for the high frequency conductivity with that given in Ref. 3. The proper inclusion of the collective effects gives rise in the right-hand side of our Eq. (32), to the factors $[\mathcal{E}_q(x+\omega)\mathcal{E}_q(x)]^{-1}$ or $[\mathcal{E}_q(x+\omega)\mathcal{E}_q^*(x)]^{-1}$ which do not appear in the result given in Ref. 3 [their Eq. (A16)]. These factors modify the expression for the absorption coefficient to represent the screening effect due to the self-consistent field.

ACKNOWLEDGMENT

We would like to thank Dr. P. M. Platzman for an interesting discussion and some helpful remarks.