New Hot-Electron Negative Resistance Effect

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A new type of negative resistance effect is described. This effect arises from the properties of hot electrons in a material with a band structure consisting of several nonspherical valleys. A semiempirical theory is developed for this effect, based on a model for hot-electron behavior which assumes (1) that the mobility and "temperature" of the electrons in each valley are determined uniquely by specifying the power delivered to the electrons, (2) that the mobility in each valley has the anisotropy of the effective-mass tensor for that valley, and (3) that the intervalley scattering time depends only on the "temperature" of the valley from which the carriers are scattered and not on that of the valley to which they scatter. This results in a criterion for the occurrence of the negative resistance which is applied to the case of n-germanium at 78°K where the parameters needed can be evaluated from other experimental data. It is shown that, under appropriate conditions, the negative resistance should occur in this material. Experimental evidence is then presented for its existence.

1. INTRODUCTION

IT is well known that the application of moderately strong electric fields to certain solids can cause currents to be produced that do not obey Ohm's law.[1–3] This is so because the charged carriers are accelerated by the electric field and, consequently, gain energy from this field. In the steady state, the rate of energy gain must equal the rate at which the carriers transfer this energy to the lattice, and, for certain materials, this can occur only if the average energy of the carriers is greater than their thermal energy at the lattice temperature. The field thus produces "hot electrons," namely, electrons whose energies are associated with temperatures that are higher than the lattice temperature. Since the mobility of the carriers usually depends on their energies, such a change in the distribution of energies will cause a change in the mobility of the carriers and a consequent non-Ohmic behavior for the solid. The variation of mobility with electric field for germanium and silicon has been investigated by many workers and is found to vary with lattice temperature,[2] impurity concentration,[4] and crystal orientation.[5,6] The average energy of the electrons in the steady state has also been investigated recently.[7]

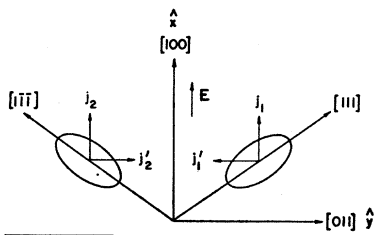

FIG. 1. Anisotropic valleys in n-germanium.

It is also known that for carriers whose surfaces of constant energy consist of several ellipsoids (rather than spheres) the application of these fields may cause unequal heating of the carriers in the various valleys.[8,9] Thus, previously equivalent valleys may no longer be equivalent after application of an electric field. This leads to situations in which the electric field and the current density are not collinear.[10] It also results in the dependence of the hot-electron effects on crystal orientation.[5,6,11] However, the possibility of obtaining negative resistance from these effects has only recently been appreciated.[12]

In Sec. 2 we give a physical picture of the negative resistance effect, and in Sec. 3 we develop the theory of the effect on the basis of a semiempirical model. The theory is applied to the case of n-type germanium in Sec. 4, and experimental confirmation for the existence of the effect is presented in Sec. 5. Finally, in Sec. 6, we discuss the information available for other materials.

2. PHYSICAL PRINCIPLES

The principle involved is illustrated in Fig. 1. Consider a material with a conduction band consisting of at least two ellipsoidal valleys as illustrated in Fig. 1. For instance, the $\hat{x}$ direction may be the [100] direction of n-germanium, and the $\hat{y}$ direction may be [011]. The valleys shown would then correspond to two of the [111] valleys in this material. If an electric field $\mathbf{E}$ is now impressed in the $\hat{x}$ direction, the carriers in each valley would be accelerated in the direction of $(\mathbf{m})^{-1}\cdot\mathbf{E}$. The resultant drift velocity would also be in this direction if the relaxation time τ were isotropic, i.e., if τ were a function of energy only. Then, for each valley, the im-

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pressed field would produce a drift velocity in both the $\hat{x}$ and the transverse $\hat{y}$ directions. Naturally, if the relaxation time, or mobility, of the carriers in each valley, as well as their populations, were equal, then, for the symmetric case pictured, the drift velocities in the $\hat{y}$ direction would cancel, leaving only a component in the direction of $\mathbf{E}$. The magnitude of the current density $\mathbf{j}$ in the $\hat{x}$ direction would be given by

$$j_x = e\sum_i n_i(\hat{x}\cdot\boldsymbol{\mu}_i\cdot\hat{x})E, \tag{1}$$

where n_i is the density of carriers in the ith valley, $\boldsymbol{\mu}$ is their mobility tensor, and e their charge. Now, for carriers in materials exhibiting hot-electron effects, $\boldsymbol{\mu}$ is a function of $\mathbf{E}$. One can phenomenologically treat $\boldsymbol{\mu}_i$ as composed of two parts: a tensor part giving the anisotropy of $\boldsymbol{\mu}_i$ which equals the reciprocal mass tensor $(\mathbf{m}_i)^{-1}$ for isotropic τ, and a factor $\mu(\epsilon_i)$ which depends on the power delivered to the electrons in valley i. Here $\epsilon_i=[\mathbf{E}\cdot(\mathbf{m}_i)^{-1}\cdot\mathbf{E}]^{1/2}$ is an "effective" electric field whose square is related to the power delivered by $\mathbf{E}$ to the carriers in the ith valley. For most situations, $\mu(\epsilon_i)$ decreases as ϵ increases and this is the case we discuss here. There is however, another effect which occurs. If the ϵ_i differ from valley to valley, the mean energy of the carriers in the various valleys will differ correspondingly. Usually, the electrons in the "hotter" valleys, i.e., those with larger effective fields ϵ, will tend to transfer to the "cooler" valleys, thus producing a situation in which n_i also varies between valleys. The steady state will then have n_i decreasing with increasing ϵ_i, which is the case we consider here.

For the situation pictured in Fig. 1, there is no current density in $\hat{y}$. This is so because a field in $\hat{x}$ results in the same ϵ for both valleys so that $\mu(\epsilon_i)$ and n_i are equal for the two valleys. If, however, in addition to $\mathbf{E}$ in $\hat{x}$ we add a small component E_y in $\hat{y}$, there will be three sources of $\mathbf{j}$ in $\hat{y}$. Firstly, E_y will accelerate charges as well as E_x, resulting in a component of current density in $\hat{y}$ due to E_y which we call j_{yy}. Secondly, the addition of E_y, will cause ϵ_i to change for each valley, as will $\mu(\epsilon_i)$ and n_i. Since the *total* field $\mathbf{E}$ will no longer be at the same angle to both valleys, $\boldsymbol{\mu}_i$ and n_i will no longer be equal, and therefore the components of $\mathbf{j}$ in $\hat{y}$ produced by E_x *will no longer cancel between valleys*, and a resultant j_{yx} will flow in $\hat{y}$ due to E_x. We note that for valley 1 the addition of $+E_y$ results in a smaller ϵ, while for valley 2 the result is a larger ϵ. Thus, $\mu(\epsilon_1)$ and n_1 increase, while $\mu(\epsilon_2)$ and n_2 decrease. Thus, the component of $\mathbf{j}$ in $\hat{x}$ from valley 1 becomes greater than that of valley 2, and the resultant j_{yx} will be directed *oppositely* to E_y. The total j_y will be directed oppositely to E_y if $j_{yy}<j_{yx}$ which would result in a negative resistance in this direction. A quantitative criterion for the occurrence of this effect is developed in the next section.

3. THEORY

The phenomenological theory we develop here is based on the assumptions (1) that the mobility and "temperature" of the carriers in each valley are determined uniquely by the power delivered to those carriers; (2) that the anisotropy of the mobility for each valley is independent of the electric field and, in particular, can be taken to equal the reciprocal effective-mass anisotropy; and (3) that the intervalley scattering time is determined only by the "temperature" of the initial valley. These assumptions are reasonable if the intervalley scattering is small compared with the intravalley scattering, so that an electron spends a long time in one valley before being scattered to a different valley. This is the case for n-germanium.

We can in general, write the current density as

$$\mathbf{j}=\sum_i n_i\mu(\epsilon_i)(\mathbf{m}_i)^{-1}\cdot\mathbf{E}. \tag{2}$$

If $\mathbf{E}$ is changed, we can express the change in $\mathbf{j}$ as

$$\boldsymbol{\delta}\mathbf{j}=\boldsymbol{\delta}\mathbf{j}_1+\boldsymbol{\delta}\mathbf{j}_2+\boldsymbol{\delta}\mathbf{j}_3, \tag{3}$$

where

$$\boldsymbol{\delta}\mathbf{j}_1=\sum_i n_i\mu(\epsilon_i)(\mathbf{m}_i)^{-1}\cdot\boldsymbol{\delta}\mathbf{E}, \tag{4}$$

$$\boldsymbol{\delta}\mathbf{j}_2=\sum_i n_i(d\mu(\epsilon_i)/d\epsilon_i)\nabla_E\epsilon_i\cdot\boldsymbol{\delta}\mathbf{E}E_x(\hat{y}\cdot\mathbf{m}_i^{-1}\cdot\hat{y}), \tag{5}$$

$$\boldsymbol{\delta}\mathbf{j}_3=\sum_i\mu(\epsilon_i)\mathbf{m}_i^{-1}\cdot\mathbf{E}\sum_j(\partial n_i/\partial\epsilon_j)\nabla_E\epsilon_j\cdot\boldsymbol{\delta}\mathbf{E}. \tag{6}$$

In these expressions, all quantities except $\boldsymbol{\delta}\mathbf{E}$ are evaluated at the initial field $\mathbf{E}$. Note that the component of $\boldsymbol{\delta}\mathbf{j}_1$ in $\hat{y}$ equals j_{yy} while the sum of the components of $\boldsymbol{\delta}\mathbf{j}_2+\boldsymbol{\delta}\mathbf{j}_3$ in $\hat{y}$ equals j_{yx}. Thus, if $\mathbf{E}=E_x\hat{x}$ and $\boldsymbol{\delta}\mathbf{E}=E_y\hat{y}$ we have

$$j_{yy}=E_y\sum_i n_i\mu(\epsilon_i)(\hat{y}\cdot\mathbf{m}_i^{-1}\cdot\hat{y}), \tag{7}$$

$$\begin{aligned}j_{yx}=E_y\sum_i n_i(d\mu(\epsilon_i)/d\epsilon_i)(d\epsilon_i/dE_y)E_x(\hat{y}\cdot\mathbf{m}_i^{-1}\cdot\hat{x})\\ +E_y\sum_i(\hat{y}\cdot\mathbf{m}_i^{-1}\cdot\hat{x})E_x\mu(\epsilon_i)\\ \times\sum_j(\partial n_i/\partial\epsilon_j)(d\epsilon_j/dE_y).\end{aligned} \tag{8}$$

Now,

$$d\epsilon_i/dE_y=(\epsilon_i/E_x)(\hat{y}\cdot\mathbf{m}_i^{-1}\cdot\hat{x}),$$

when evaluated at $\mathbf{E}=E_x\hat{x}$. Furthermore, we have decided to assume that the intervalley scattering time is determined by the "temperature" or, equivalently, by ϵ of the initial valley. Therefore, the steady-state number of carriers per valley is determined by the I equations,

$$\begin{aligned}\sum_{j\neq i} n_j/\tau(\epsilon_j)&=(I-1)n_i/\tau(\epsilon_i),\quad i=1\text{ to }I-1\\ \sum_j n_j&=N,\end{aligned} \tag{9}$$

where I is the total number of valleys, N the total number of electrons, and $\tau(\epsilon_i)$ the intervalley scattering time for carriers in valley i with effective electric field ϵ_i. These equations merely state that the number of carriers entering valley i from all other valleys $j\neq i$ equals the number leaving valley i. The solution of

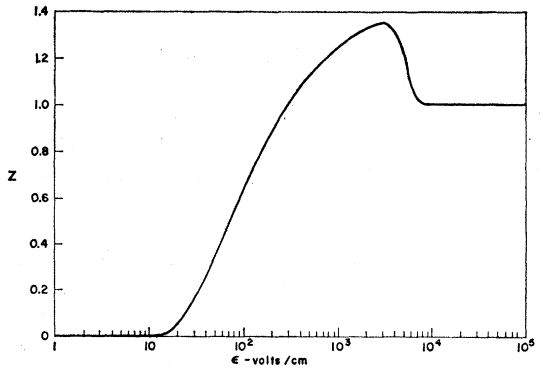

FIG. 2. Variation of Z with effective electric field for n-germanium at $T=78°$K. This curve is derived from the data of Nathan (Ref. 11) with a donor concentration of 10^{15}/cc.

these equations is

$$n_i/N=\tau(\epsilon_i)/\textstyle\sum_k \tau(\epsilon_k) \tag{10}$$

and

$$\partial n_i/\partial\epsilon_j=n_i(d\ \ln\tau(\epsilon_i)/d\epsilon_i)\delta_{ij} - (n_in_j/N)[d\ \ln\tau(\epsilon_j)/d\epsilon_j]. \tag{11}$$

Then,

$$\textstyle\sum_j (\partial n_i/\partial\epsilon_j)(d\epsilon_j/dE_y)=D_1+D_2, \tag{12}$$

where

$$D_1=(n_i/E_x)[d\ \ln\tau(\epsilon_i)/d\ \ln\epsilon_i]M_i,$$
$$D_2=(n_i/NE_x)\textstyle\sum_j n_j[d\ \ln\tau(\epsilon_j)/d\ \ln\epsilon_j]M_j =n_iD_0/NE_x, \tag{13}$$
$$M_i=\hat{y}\cdot\mathbf{m}_i^{-1}\cdot\hat{x}/\hat{x}\cdot\mathbf{m}_i^{-1}\cdot\hat{x}.$$

If we substitute D_2 into the last term of Eq. (7), we get

$$(E_yD_0/NE_x)\textstyle\sum_i n_i\mu(\epsilon_i)\hat{y}\cdot\mathbf{m}_i^{-1}\cdot\hat{x}E_x. \tag{14}$$

The term in the summation, however, is equal to the current initially flowing in the $\hat{y}$ direction when the field was limited to the $\hat{x}$ direction. This term vanishes for the case in which $\hat{x}$ is a symmetry direction of the crystal, so that initially there is no current in $\hat{y}$. Then, using (8), (12), and (13) in (7), we get

$$j_{yx}=E_y\textstyle\sum_i n_i\mu(\epsilon_i)M_i(\hat{y}\cdot\mathbf{m}_i^{-1}\cdot\hat{x}) \times d\ \ln[\tau(\epsilon_i)\mu(\epsilon_i)]/d\ \ln\epsilon_i, \tag{15}$$

and

$$j_{yx}/j_{yy}=\textstyle\sum_i A_iZ_i, \tag{16}$$

where

$$A_i=n_i\mu(\epsilon_i)M_i\hat{y}\cdot\mathbf{m}_i^{-1}\cdot\hat{x}/\textstyle\sum_j n_j\mu(\epsilon_j)\hat{y}\cdot\mathbf{m}_j^{-1}\cdot\hat{y}, \tag{17}$$

$$Z_i=d\ \ln[\tau(\epsilon_i)\mu(\epsilon_i)]/d\ \ln\epsilon_i. \tag{18}$$

This is the desired result. We note that if $j_{yx}/j_{yy}>1$, we have a negative resistance (see end of Sec. 2). The problem has thus been reduced to one of determining the A_i and Z_i for particular materials under particular conditions. We note that what is basically required is a knowledge of $\mu(\epsilon_i)$ and of $\tau(\epsilon_i)$. These quantities may be obtained experimentally from measurements such as $\boldsymbol{\mu}$ versus $\mathbf{E}$ for various crystallographic directions of $\mathbf{E}$. Such measurements have been made for n-germanium by various experimenters.[5,6,11] In the next section we apply the data of Nathan[11] to n-germanium at liquid nitrogen temperatures.

In many cases, it turns out that all valleys for which A_i is nonvanishing are equivalent, i.e., have equal values of A_i and Z_i. Then, one can simplify Eq. (16) to read

$$j_{yx}/j_{yy}=AZ. \tag{19}$$

In Table I we list values of A for both germanium and silicon and various directions of $\hat{x}$ and $\hat{y}$. Also included are values of A for cases in which certain valleys are depopulated by strain, a situation discussed later.

4. APPLICATION TO GERMANIUM

Nathan[11] has measured $\mathbf{j}$ versus $\mathbf{E}$ for various directions of E in a crystal of n-germanium with $\sim10^{14}$ donors/cc at 78°K. Since the valleys for electrons in germanium are along the $\langle111\rangle$ directions, a field in $[100]$ will treat all valleys equivalently. A measurement of μ versus E for $\hat{x}=[100]$, thus directly provides a curve of μ versus ϵ. With this information, one can analyze the data of μ versus E for $\hat{x}=[111]$ to determine the steady-state electron concentration in the $[111]$ valley and in the three other equivalent valleys. These data of n_i versus ϵ_i can then be used to determine $\tau(\epsilon_i)$. Such an analysis has been carried through by Nathan and checked for consistency with data for $\hat{x}=[110]$. We use his data on $\mu(\epsilon)$ and $\tau(\epsilon)$ to determine Z for this particular material. In Fig. 2, we plot Z versus ϵ and note that it rises to a maximum of ~1.35 before approaching unity in the region of the saturated drift velocity. We thus require A to be greater than 0.74 for negative resistance to occur, a situation which does not naturally prevail in this material. However, if two of the valleys are depopulated of electrons, then one can get $A=0.80$ which would give rise to negative resistance. The effect of depopulation can be visualized with the aid of Fig. 1. If the valleys pictured are the two $\langle111\rangle$ valleys in the $(0\bar{1}1)$ plane, the other pair of valleys lies in the (011) plane, perpendicular to the plane of the paper. These two valleys have their low

TABLE I. Values of A for various field directions in germanium and silicon.

Material	Initial direction of E	Direction of E_y	A
Ge	[100]	any	0.215
Ge	[110]	at angle θ from $[1\bar{1}0]$	$\sin^2\theta/[1.25+0.58\cos^2\theta +(n_1\mu_1/n_2\mu_2)(1.25-0.58\cos^2\theta)]$
Si	[111]	any	0.135
Ge	[100]	[011]	0.80 if the valleys in the (011) plane are depopulated by uniaxial stress in [011].
Ge	[011]	[100]	0.80 if the valleys in the (011) plane are depopulated by uniaxial stress in [011].
Si	[110]	$[1\bar{1}0]$	0.46 if the valleys in [001] are depopulated by stress.

mass principal axis parallel to $\hat{y}$ and, therefore, do not contribute at all to j_{yx} while contributing greatly to j_{yy}. If the electrons can be transferred from the valleys in the (011) plane to the $(0\bar{1}1)$ plane, one would have a large increase in A, as indicated by Table I. Such a redistribution of charge can be effected in n-germanium by applying a large uniaxial stress in the [011] direction.[13] For this case, then, one would expect to find a negative resistance effect.

It should be noted that the application of stress is not the only means by which one can depopulate the (011) valleys. In fact, if one chooses $\hat{x}=[011]$ and $\hat{y}=[100]$, then the initial field will deliver much more power to the electrons in the (011) plane than to those in the $(0\bar{1}1)$ plane, and they will consequently become "hotter." They will then naturally tend to transfer to the $(0\bar{1}1)$ plane which is the situation we seek. In fact, one sees from Table I, that for this situation $A \rightarrow 0.80$ as $n_1\mu_1/n_2\mu_2 \rightarrow 0$. Unfortunately, for the material used, $n_1\mu_1/n_2\mu_2$ does not get small enough for A to exceed 0.74, but, at different temperatures, or with different impurity concentrations one may be able to get $Ay>1$ without applying stress.

5. EXPERIMENTAL RESULTS

Evidence for this negative resistance effect comes from noise-temperature experiments reported previously. It was observed in these experiments that noise temperatures of hot electrons, measured in [011] with the heating field in [100], increased with uniaxial stress in [011].[14] For these observations, the lattice temperature was 78°K. This previously mysterious dependence can now be explained as due to a variation of transverse mobility as a result of the depopulation of the (011) valleys with stress. We note that as stress increases, A varies from 0.215 to 0.80 with a consequent large decrease of transverse mobility. At a certain critical heating field in [100], oscillations were observed in [011] which we believe to be due to the negative resistance predicted above.

We note the following corroborating evidence. The oscillations first occur at a field of 380 V/cm. If one assumes that the stress is sufficient to completely depopulate the valleys in the (011) plane, the predicted field for negative resistance is 340 V/cm, in excellent agreement with observation. Furthermore, the oscillations disappear when the stress is removed and they do not occur at all for several other orientations of stress and initial electric field. The same is true of the variation of noise temperature with stress. This is as expected from the results of the theory presented above.

6. OTHER MATERIALS

Unfortunately, data similar to those of Nathan for germanium, do not exist for other materials which might exhibit this effect. We, therefore, content ourselves with stating the following general requirements for a material to exhibit this effect.

(1) It must have a band structure for the majority carrier consisting of several valleys.

(2) The valleys, must have a large ellipticity. Any admixture of valleys with spherical energy surfaces will reduce the likelihood of the effect.

(3) The material must exhibit hot-electron effects such that μ *decreases* as ϵ increases. Ionized impurity scattering should therefore not be dominant.[2,4]

(4) If there are too many carriers, as, for instance, in semimetals, too much power would probably be required before hot-electron effects arise. However, theoretically, the effect is not limited to semiconductors.

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