Violation of $|\Delta I|=\frac{1}{2}$ Rule in the $K_{\pi 3}{}^0$ Decay and the $K_2{}^0 \to \gamma+\gamma$ Decay

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The experimental value of the branching ratio $\Gamma(K_2{}^0 \to \pi^+ + \pi^- + \pi^0)/\Gamma(K^+ \to \pi^+ + \pi^- + \pi^0)$ obtained by Alexander *et al.* differs from the prediction of the $\Delta I=\frac{1}{2}$ rule by 30%. We assume that the major part of the violation of the $\Delta I=\frac{1}{2}$ rule in $K_{\pi 3}$ decay comes from the chain of processes $K^0 \to \eta \to \pi^+ + \pi^- + \pi^0$, as pointed out by Bouchiat *et al.* With this assumption, we get a relation between the coupling constant $f^2{}_{K^0\eta^0}/4\pi$ and the width $\tau(\eta \to \pi^+ + \pi^- + \pi^0)$. We use the experimental value of the branching ratio $\tau(\eta \to \text{neutrals})/\tau(\eta \to \pi^+ + \pi^- + \pi^0) \approx 3.0$ to infer the unknown branching ratio $\tau(\eta^0 \to 2\gamma)/\Gamma(\eta^0 \to \pi^+ + \pi^- + \pi^0)$ and the theory of unitary symmetry to deduce the value of the width $\tau(\eta^0 \to 2\gamma)$ from the experimental decay rate of $\pi^0 \to 2\gamma$. Further, if only pion and η-meson pole terms are responsible for the mass difference $m(K_1{}^0)-m(K_2{}^0)$, we can also derive the value of the coupling constant $f^2{}_{K^0\pi^0}/4\pi$. These two coupling constants $f^2{}_{K^0\eta^0}/4\pi$ and $f^2{}_{K^0\pi^0}/4\pi$ are illustrated as a function of the deviation x from the $\Delta I=\frac{1}{2}$ rule. Under the same assumption we obtain a result that $\Gamma(K_2{}^0 \to 2\gamma)$ is comparable to $\Gamma(K_2{}^0 \to \pi^+ + \pi^- + \pi^0)$ or is negligibly small according as $f_{K^0\pi^0}$ and $f_{K^0\eta^0}$ are of the same sign or not. If the deviation $x=30\%$ is really correct, the decay rate of $\Sigma^- \to \eta + \pi^-$ can be explained with $g^2{}_{\Sigma^0 n K^0}/4\pi \lesssim 1$. On the other hand, if the extended $\Delta I=\frac{1}{2}$ rule is valid for the couplings of $K-\pi$ and $K-\eta$, we obtain the following results: $x=15\%$, $m(K_1{}^0)<m(K_2{}^0)$, $f^2{}_{K^0\pi^0}/4\pi = 2.8\times 10^{-16}$, $\Gamma(K_2{}^0 \to 2\gamma) = 8\times 10^3$ sec^{-1} and the decay rate $\Sigma^- \to n + \pi^-$ can be fitted with $g^2{}_{\Sigma^0 n K^0}/4\pi \approx 6$. Precise experiments are desired on the decay $K_2{}^0 \to 2\gamma$ as well as on the decay $K_2{}^0 \to \pi^+ + \pi^- + \pi^0$.

IF the $|\Delta I|=1/2$ rule were valid, the rate of $K_2{}^0 \to \pi^+ + \pi^- + \pi^0$ decay can be related to that of $K^+ \to \pi^+ + \pi^0 + \pi^0$ decay.

$$\Gamma_{|\Delta I|=1/2}(K_2{}^0 \to \pi^+ + \pi^- + \pi^0) = 1.032\times 2\Gamma(K^+ \to \pi^+ + \pi^0 + \pi^0) = (2.87\pm 0.23)\times 10^6 \text{ sec}^{-1}. \quad (1)$$

However, Alexander *et al.*, by a combination of results coming from two different experiments, have obtained a value[1]

$$\Gamma(K_2{}^0 \to \pi^+ + \pi^- + \pi^0) = (1.44\pm 0.43)\times 10^6 \text{ sec}^{-1}, \quad (2)$$

which differs considerably from (1). Though, at the moment, we should perhaps not take the present experimental result too seriously, there is also some theoretical reason to expect that the violation of $|\Delta I|=1/2$ rule due to the electromagnetic correction might become enhanced particularly for the $K_2{}^0 \to 3\pi$ decay. As pointed out by Bouchiat *et al.*,[2] the contribution of η^0-meson pole term

$$K^0 \to \eta^0 \to \pi^+ + \pi^- + \pi^0, \quad (3)$$

may give rise to a large violation of $|\Delta I|=1/2$ rule, since the mass of K meson is close to that of η meson. The fact that the vertices, $K-\eta$ and $\eta-3\pi$, could proceed without centrifugal barrier suppression also seems to favor this possibility. In fact, it is now established that the decay mode $\eta \to 3\pi$, which is of the order of fine structure constant, can compete well with the mode $\eta \to 2\gamma$ which has much larger phase space but appreciable barrier suppression. In this paper we, therefore, assume as in Ref. 2 that the major part of the violation of $|\Delta I|=1/2$ rule in $K_{\pi 3}{}^0$ decay comes from Eq. (3). We write the effective Hamiltonian for the $K^0 \to \pi^+ + \pi^- + \pi^0$ process as

$$H(K^0 \to \pi^+ + \pi^- + \pi^0) = g_{K^0}(1-x)K^0(\pi^+\pi^-\pi^0)^*$$
$$x = \left(\frac{g_{\eta^0}}{g_{K^0}}\right)\left(\frac{m_p{}^2}{m^2{}_{\eta^0} - m^2{}_{K^0}}\right) f_{K^0\eta^0}, \quad (4)$$

g_{K^0} stands for the effective coupling constant of $K^0 \to \pi^+ + \pi^- + \pi^0$ decay satisfying the $|\Delta I|=1/2$ rule.[3] x corresponds to the contribution of the Eq. (3). The $K^0-\eta^0$ and $\eta^0 \to 3\pi$ vertices have been written as $f^2{}_{K^0\eta^0} m_p K^0 \eta^{0*}$ and $g_{\eta^0}\eta^0(\pi^+\pi^-\pi^0)^*$, respectively. We also assume the time-reversal invariance. Using the prediction of $|\Delta I|=1/2$ rule (1), we obtain from (4)

$$(1-x)^2 = [(K_2{}^0 \to \pi^+ + \pi^- + \pi^0) \quad \text{in} \quad 10^6 \text{ sec}^{-1}]/2.87, \quad (5)$$

g_K and g_η are related to (1) and to $\Gamma(\eta^0 \to \pi^+ + \pi^- + \pi^0)$, respectively, through[4]

$$\Gamma_{|\Delta I|=1/2}(K_2{}^0 \to \pi^+ + \pi^- + \pi^0) = [1/2\pi]^3 \tfrac{1}{4} |g_K|^2 m_{K^0} 6.36\times 10^{-3} \quad (6)$$

$$\Gamma(\eta^0 \to \pi^+ + \pi^- + \pi^0) = [1/2\pi]^3 \tfrac{1}{8} |g_\eta|^2 m_{\eta^0} 1.45\times 10^{-2}. \quad (7)$$

Eliminating g_{η^0} and g_{K^0} from (4), and using (6) and (7), we get

$$x^2 = 6.65\times 10^{-5} f^2{}_{K^0\eta^0}\Gamma(\eta^0 \to \pi^+ + \pi^- + \pi^0) \quad \text{in} \quad \text{sec}^{-1}. \quad (8)$$

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[1] G. Alexander, S. P. Almedia, and F. S. Crawford, Phys. Rev. Letters 9, 69 (1962).
[2] C. Bouchiat, J. Nuyts, and J. Prentki, Phys. Letters 3, 156 (1962).
[3] We have neglected the contributions which are not totally symmetric with respect to three final pions.
[4] We have used $m_{K^0} = 497.8$ MeV and $m_{\eta^0} = 550$ MeV.

Using the recently reported branching ratios of η meson,[5–7] $\Gamma(\eta^0 \to \text{all neutrals})/\Gamma(\eta^0 \to \pi^+ + \pi^- + \pi^0) = 3.0$, $\Gamma(\eta^0 \to \pi^0 + \pi^0 + \pi^0)/\Gamma(\eta^0 \to \pi^+ + \pi^- + \pi^0) = 1.68$, we infer, assuming $\Gamma(\eta^0 \to \text{neutrals}) = \Gamma(\eta^0 \to 2\gamma) + \Gamma(\eta^0 \to 3\pi^0)$,

$$\Gamma(\eta^0 \to 2\gamma)/\Gamma(\eta^0 \to \pi^+ + \pi^- + \pi^0) = 1.32. \quad (9)$$

The width of η meson is not known yet. We shall at the moment be content with the use of the prediction of unitary symmetry[8,9] which seems to work rather well in the classification of newly found resonances. We write the relations between the amplitudes of $\eta^0 \to 2\gamma$ and $\pi^0 \to 2\gamma$ as $M(\pi^0 \to 2\gamma) = \alpha M(\eta^0 \to 2\gamma)$ which yields $\Gamma(\eta^0 \to 2\gamma) = (64/\alpha^2)\Gamma(\pi^0 \to 2\gamma)$. The unitary symmetry predict[10] $\alpha = +\sqrt{3}$ and using the recently reported rate of $\pi^0 \to 2\gamma$ decay,[11] $1/\Gamma(\pi^0 \to 2\gamma) = (1.05 \pm 0.18) \times 10^{-16}$ sec, we obtain

$$\Gamma(\eta^0 \to 2\gamma) = 140 \text{ eV} \quad (\alpha = +\sqrt{3}), \quad (10)$$

which implies according to (9)

$$\Gamma(\eta^0 \to \pi^+ + \pi^- + \pi^0) = 106 \text{ eV}. \quad (11)$$

Now let us turn our attention to the problem of $K_1{}^0 - K_2{}^0$ mass difference. Since, now, evidence for the existence of bosons with spin and strangeness zero other than the pion and η meson are rather weak, it seems reasonable to assume that the main part of the mass difference comes from the contribution of pion and η-meson pole terms whose vertices may not suffer from barrier suppression effects compared with other bosons like ρ and ω meson. We obtain[12,13]

$$\Delta m_{K^0} = m(K_1{}^0) - m(K_2{}^0) = -\left(\frac{m_p{}^2}{m_{K^0}}\right)\left\{ f^2{}_{K^0\pi^0}\left(\frac{m_p{}^2}{m^2{}_{K^0} - m^2{}_{\pi^0}}\right) + f^2{}_{K^0\eta^0}\left(\frac{m_p{}^2}{m^2{}_{K^0} - m^2{}_{\eta^0}}\right)\right\}.$$

Using experimental values,[14] $|\Delta m_{K^0}| = 1.5\hbar/\tau(K_1{}^0)$, $\tau(K_1{}^0) = 1.00 \times 10^{-10}$ sec, the above relation becomes

$$(f^2{}_{K^0\pi^0} \times 10^{14}) = \pm 0.148 + 4.20 \times (f^2{}_{K^0\eta^0} \times 10^{14}). \quad (12)$$

The upper and lower signs correspond to $\Delta m_{K^0} > 0$ and $\Delta m_{K^0} < 0$, respectively. Now, if $\Gamma(K_2{}^0 \to \pi^+ + \pi^- + \pi^0)$ is measured, x can be calculated from (5) and consequently from (8) we can evaluate $f_{K^0\eta^0}$, provided that $\Gamma(\eta^0 \to \pi^+ + \pi^- + \pi^0)$ is known. Then from (12) the value of $f_{K^0\pi^0}$ can be calculated. In Fig. 1 the variations of the values $f^2{}_{K^0\pi^0}$ and $f^2{}_{K^0\eta^0}$ with x are shown by using, at the moment, the value of $\Gamma(\eta^0 \to \pi^+ + \pi^- + \pi^0)$ given by (11).

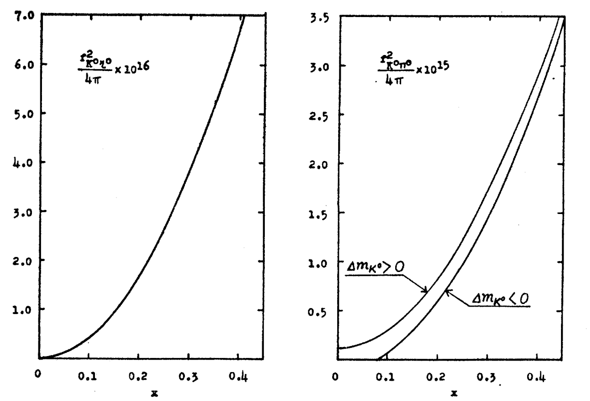

FIG. 1. The variations of the values of $f^2{}_{K^0\pi^0}/4\pi$ and $f^2{}_{K^0\eta^0}/4\pi$ with x for the choice, $\Gamma(\eta^0 \to \pi^+ + \pi^- + \pi^0) = 106$ eV, based on unitary symmetry.

Next we discuss the $K_2{}^0 \to 2\gamma$ decay. It again seems reasonable to suppose that the decay is dominated by the pion as well as the η-meson pole terms. Under this assumption we obtain

$$\Gamma(K_2{}^0 \to 2\gamma) = 2\left[f_{K^0\pi^0}\left(\frac{m_p{}^2}{m^2{}_{\pi^0} - m^2{}_{K^0}}\right) + f_{K^0\eta^0}\left(\frac{1}{\alpha}\right)\left(\frac{m_p{}^2}{m^2{}_{\eta^0} - m^2{}_{K^0}}\right)\right]^2 \times \left(\frac{m_{K^0}}{m_{\pi^0}}\right)^3 \Gamma(\pi^0 \to 2\gamma). \quad (13)$$

We have plotted in Fig. 2 the variation of the rate $\Gamma(K_2{}^0 \to 2\gamma)$ with x. It is seen that $\Gamma(K_2{}^0 \to 2\gamma)$ could be large ($\gtrsim 10^5$ sec^{-1}), comparable with $\Gamma(K_2{}^0 \to \pi^+ + \pi^- + \pi^0)$, if $f_{K^0\pi^0} f_{K^0\eta^0} < 0$ and is much smaller if $f_{K^0\pi^0} f_{K^0\eta^0} > 0$. This implies that though the pion mass is not so close to the K^0 mass as the η meson, the contribution of pion pole term is, nevertheless, important, so that the interference of these two terms becomes important. The sign of $K_1{}^0 - K_2{}^0$ mass difference does not lead to a marked difference for $f_{K^0\pi^0} f_{K^0\eta^0} < 0$, although the effect is appreciable for $f_{K^0\pi^0} f_{K^0\eta^0} > 0$. For $f_{K^0\pi^0} f_{K^0\eta^0} > 0$, the boson pole approximation considered may become less reliable and

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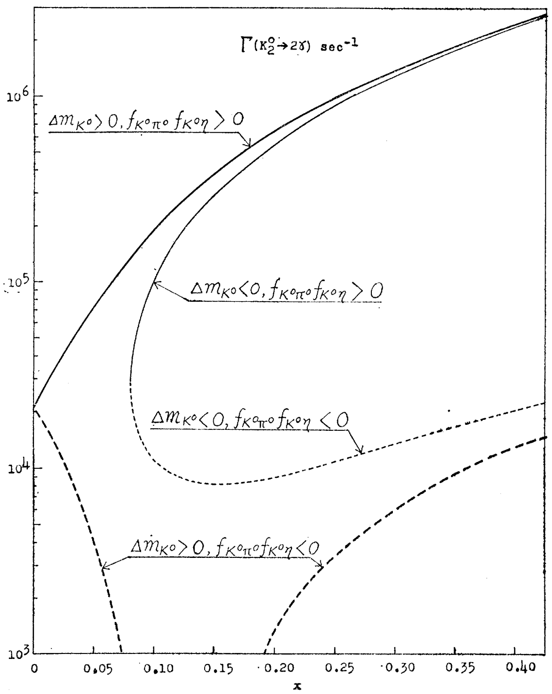

FIG. 2. The variation of the rate of $K_2^0\to\gamma+\gamma$ decay with x. The parameter, $\Gamma(\eta^0\to\gamma+\gamma)$, is taken to be 140 eV assuming the validity of unitary symmetry. The symbols $>$ and $<$ of $f_{K^0\pi^0}f_{K^0\eta}$ should be inverted in the above figure.

the contributions of higher mass states might not be neglected.

We summarize:

(i) It may not be so difficult to check whether the decay $K_2^0\to 2\gamma$ indeed takes place with a frequency comparable with that of $K_2^0\to\pi^++\pi^-+\pi^0$. This serves to determine particularly the relative sign of $f_{K^0\pi^0}$ and $f_{K^0\eta^0}$.

(ii) Simultaneous measurements of $\Gamma(K_2^0\to 2\gamma)$ and x and the comparison with Fig. 2 will give the test of the validity of unitary symmetry. If it turns out that the prediction is invalid, we can instead evaluate the width of η meson from $\Gamma(K_2^0\to 2\gamma)$ and x, by regarding $\Gamma(\eta^0\to\pi^++\pi^-+\pi^0)$ as parameter in the above analysis.

(iii) It may be stressed that the knowledge of the magnitude of K-π vertex thus obtained is also valuable, since this vertex may play an important role also, for instance, in the nonleptonic processes as will be discussed below.

(iv) The reported rate of $K_2^0\to\pi^++\pi^-+\pi^0$, (2), corresponds to $x=0.3$. This predicts $\Gamma(K_2^0\to 2\gamma)=1.4\times10^6$ sec^{-1} for $f_{K^0\pi^0}f_{K^0\eta^0}<0$ and 1.4×10^4 sec^{-1} or 6×10^3 sec^{-1} for $f_{K^0\pi^0}f_{K^0\eta^0}>0$. Corresponding values of $f_{K^0\pi^0}$ and $f_{K^0\eta^0}$ are large, $f^2_{K^0\eta^0}/4\pi=3.8\times10^{-16}$ and

$$\begin{aligned} f^2_{K^0\pi^0}/4\pi&=1.7\times10^{-15} \quad \text{for} \quad \Delta m_{K^0}>0,\\ f^2_{K^0\pi^0}/4\pi&=1.5\times10^{-15} \quad \text{for} \quad \Delta m_{K^0}<0. \end{aligned} \tag{14}$$

Supposing that the $\Sigma^-\to n+\pi$ decay is explained by the pole term $\Sigma^-\to n+K^-\to n+\pi^-$ we obtain

$$\begin{aligned}\Gamma(\Sigma^-\to n+\pi^-)\\ &=(g^2_{\Sigma^0n^0K^0}/4\pi)\times5.80\times10^9\ \text{sec}^{-1}\ (\Delta m_{K^0}>0)\\ &=(g^2_{\Sigma^0n^0K^0}/4\pi)\times5.1\times10^9\ \text{sec}^{-1}\ (\Delta m_{K^0}<0).\end{aligned} \tag{15}$$

Experiments indicate $\Gamma(\Sigma^-\to n+\pi^-)\approx6\times10^9$ sec^{-1}. Thus if $x=0.3$ is correct, we have $(g^2_{\Sigma^0n^0K^0}/4\pi)\lesssim1.0$.

(v) So far, we have not specified the theory which leads to the usual $|\Delta I|=1/2$ rule. Complete knowledge of both the values of $f_{K^0\pi^0}$ and $f_{K^0\eta^0}$ is very desirable at this point, so that we present a result of such a theory here.[15] Assume that the strangeness nonconserving weak nonleptonic interaction is invariant in the unitary space by introducing a spurion which behaves like the K^0 meson which belongs to the eight-dimensional representation.[8,9] Then the relation between $f_{K^0\pi^0}$ and $f_{K^0\eta^0}$ is fixed as $K^+(\pi^+)^*-[1/(6)^{\frac{1}{2}}K^0\eta^0+1/\sqrt{2}K^0\pi^0]$. Thus, from (12) we obtain

$$m(K_1^0)<m(K_2^0),\quad (fK^2{}_{0\pi^0}/4\pi)=2.8\times10^{-16}, \tag{16}$$

and predict $x=0.15$ and $\Gamma(K_2^0\to2\gamma)=8\times10^3$ sec^{-1}, assuming (10) and (11). Corresponding to (15) we have $\Gamma(\Sigma^-\to n+\pi^-)=(g^2_{\Sigma^0\eta K^0}/4\pi)9.6\times10^8$ sec^{-1} so that if $(g^2_{\Sigma^0nK^0}/4\pi)\approx6$ the $\Sigma^-\to n+\pi^-$ may be explained by this model. For the $\Lambda^0\to n+\pi^0$ decay, this pole term may not be so important unless $(g^2_{nK}/4\pi)$ is very large.

(vi) Finally we might add the following remarks. If we assume the dominance of pion pole term $K\to\pi\to3\pi$ for the $K\to3\pi$ decays, the above obtained values of $f^2_{K^0\pi^0}$, (15) and (16), will lead to a faster rate than the experimental one by more than an order of magnitude, if we take $\lambda=-0.15$ for the S-wave pion-pion interaction.[16] However, we may stress that this is not a difficulty if the unitary symmetry is valid. That is, if the symmetry works well, the contribution of other possible chains through unitary symmetric strong interaction, $4\pi\lambda(\boldsymbol{\pi}\cdot\boldsymbol{\pi}+\eta^2+\bar{K}K+K\bar{K})^2$, and weak K-π vertex,

$$K\underset{\lambda}{\to}K+\pi+\pi\underset{f_{K\pi}}{\to}\pi+\pi+\pi,$$

will just cancel that of the pion pole term considered above.[17] It is interesting to notice that the similar situations also take place for the $\eta\to3\pi$ mode.[17]

These points seem to urge the precise experiments on the rates of $K_2^0\to2\gamma$ decay as well as of $K_2^0\to3\pi$ decay.

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