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Muon Reactions in Liquid Hydrogen and Liquid Deuterium*†

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The reactions of muons in liquid hydrogen containing a low concentration of deuterium, $p\mu+d \rightarrow d\mu+p \rightarrow (d\mu p)^+ \rightarrow \mathrm{He}^3+\mu^-$(a) or $(\mathrm{He}^3\mu)^+ + \gamma$(b), and in liquid deuterium containing a low concentration of hydrogen, $d\mu+p \rightarrow (d\mu p)^+ \rightarrow \mathrm{He}^3+\mu^-$(c) or $(\mathrm{He}^3\mu)^+ + \gamma$(d), and $d\mu+d \rightarrow (d\mu d)^+ \rightarrow \mathrm{He}^3+n+\mu^-$(e) or $p+t+\mu^-$(f), have been investigated using the Chicago 9-in. bubble chamber. Using previously determined values for the formation and fusion rates of the $(d\mu p)^+$ ion in hydrogen in conjunction with a theoretical determination of the population of the various hyperfine levels in the $(d\mu p)^+$ ion, reaction (a) is found to occur in $(15 \pm 1.5)\%$ of all $(d\mu p)^+$ fusion reactions. The formation rate of the $(d\mu d)^+$ ion in liquid deuterium is found to be $\lambda_{dd} = (3.6\pm1.3)\times10^5$ sec^{-1}. The fusion rate of the $(d\mu d)^+$ is $\lambda_{df} = (2.5\pm0.04)\times10^5$ sec^{-1}, and the fraction of muons which, having catalyzed reaction (e), are subsequently bound in a $(\mathrm{He}^3\mu)^+$ muonic ion is (0.31 ± 0.0034). The rate of de-excitation to the lower hyperfine level of the $d\mu$ muonic atom via the reaction $d\mu(3/2)+d \rightarrow d+d\mu(1/2)$ has been investigated and found to be $(7.9\pm2.8)\times10^5$ sec^{-1} $<\lambda_{(3/2-1/2)}<(1.35 \pm0.48)\times10^6$ sec^{-1}. The interference of reactions (c), (d), (e), and (f) in the investigation of the spin-independent capture reaction $\mu^-+d \rightarrow n+n+\nu$ is discussed.

I. INTRODUCTION

It is well known experimentally that negative muons stopping in liquid hydrogen or liquid deuterium can catalyze various molecular and nuclear reactions among the hydrogen isotopes in times comparable to the mean lifetime of the muon. The sequence of muon-induced reactions in liquid hydrogen ("protium") containing a small concentration of deuterium is as follows:

$$\mu^- + \mathrm{H}_2 \rightarrow p\mu + \mathrm{H} + e^-, \quad (1)$$

$$p\mu + \mathrm{H} \rightarrow (p\mu p)^+ + e^-, \quad (2)$$

or

$$p\mu + d \rightarrow d\mu + p, \quad (3)$$

$$d\mu + \mathrm{H} \rightarrow (d\mu p)^+ + e^-, \quad (4)$$

$$(d\mu p)^+ \rightarrow \mathrm{He}^3 + \mu^-, \quad (5)$$

$$(d\mu p)^+ \rightarrow (\mathrm{He}^3\mu)^+ + \gamma. \quad (6)$$

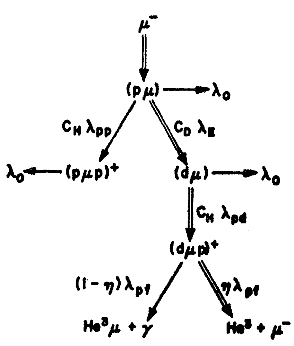

FIG. 1. The sequence of reactions for a muon stopping in liquid hydrogen. The double arrows indicate the reactions leading to the production of a rejuvenated muon. The symbol λ_0 is used to represent the normal decay of the muon $\mu^- \rightarrow e^- + \nu^- + \nu$.

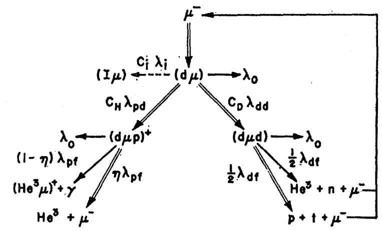

FIG. 2. The sequence of reactions for a muon stopping in liquid deuterium. The double arrows indicate those sequences which terminate in a visible event in a bubble chamber. The symbol λ_0 is used to represent the normal decay of the muon $\mu^- \rightarrow e^- + \nu^- + \nu$. The dashed arrow is used to show the transfer reaction $d\mu + I \rightarrow I\mu + d$ which is possible only in the presence of some impurity.

This chain of reactions is represented diagrammatically in Fig. 1. The sequence of reactions in liquid deuterium containing a small concentration of hydrogen is as follows:

$$\mu^- + \mathrm{D}_2 \rightarrow d\mu + \mathrm{D} + e^-, \quad (7)$$

$$d\mu + \mathrm{D} \rightarrow (d\mu d)^+ + e^-, \quad (8)$$

$$(d\mu d)^+ \rightarrow \mathrm{He}^3 + n + \mu^-, \quad (9)$$

$$(d\mu d)^+ \rightarrow p + t + \mu^-, \quad (10)$$

or, as above,

$$d\mu + \mathrm{H} \rightarrow (d\mu p)^+ + e^-, \quad (4')$$

$$(d\mu p)^+ \rightarrow \mathrm{He}^3 + \mu^-, \quad (5')$$

$$(d\mu p)^+ \rightarrow (\mathrm{He}^3\mu)^+ + \gamma. \quad (6')$$

The reaction chain in liquid deuterium is shown in Fig. 2. It should be noted that all of the reactions

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(1)–(10) are in competition with the normal decay of the muon $\mu^- \rightarrow e^- + \nu + \bar{\nu}$.

The possibility that a heavy, weakly interacting particle could catalyze a hydrogen isotope fusion reaction was first proposed by Frank.[1] Later, Zel'dovitch[2] calculated the rates for the various reactions. The muon-induced fusion between a proton and a deuteron with the ejection of the muon [reaction (5)] was first seen by Alvarez *et al.*[3] in a hydrogen bubble chamber. The reaction was identified by the appearance of a track approximately 1.7 cm long at the end point of the stopping muon, this second track being the 5.4-MeV "rejuvenated" muon from the p-d fusion reaction. They also observed that the yield of these "rejuvenated" muons per stopping muon increased as a function of deuterium concentration up to a concentration of approximately 1%, after which the yield remained constant. This effect was interpreted as the saturation of the charge-exchange reaction (3), $p\mu + d \rightarrow d\mu + p$, the rates of the other reactions in liquid hydrogen being nearly independent of the deuterium concentration (cf. above and Fig. 1).

Alvarez *et al.*[3] also observed that in some cases of reaction (5) a gap was seen between the end point of the stopping muon track and the origin of the "rejuvenated" muon track. Cresti *et al.*[4] showed that these gaps disappeared as the deuterium concentration increased. This confirmed the suggestion of Alvarez *et al.*[3] that these gaps were due to an anomalously small cross section for the scattering of $d\mu$ muonic atoms on protons. The reduction in the gap length was attributed to the increased participation of $d\mu + d$ elastic scattering in the slowing down of the $d\mu$ muonic atom.

Ashmore *et al.*[5] and more recently Bleser *et al.*[6] have investigated the time dependence of the yield of the 5.4-MeV γ ray emitted in reaction (6) in order to determine the formation rates of the various molecular species and the fusion rate for the $(d\mu p)^+$ system. Fetkovitch *et al.*[7] have investigated the muon catalysis of the deuteron-deuteron fusion reaction by stopping negative muons in a liquid-deuterium bubble chamber and detecting the 3-MeV proton arising from reaction (10).

After the experimental observations of Alvarez *et al.*[3] had proven the existence of muon molecule formation, Jackson,[8] Skyrme,[9] Cohen, Judd, and Riddell,[10] Zel'dovitch and Gershtein,[11] and Gershtein[12,13] undertook the theoretical investigation of the possible mechanisms for molecular formation and derived rates for the various mechanisms in both liquid hydrogen and liquid deuterium. Generally, the theoretical treatments yield only qualitative values for the various rates. The more recent theoretical values are in fair agreement with available experimental results. In addition to their intrinsic interest, it is essential to know the rates for the various atomic, molecular, and nuclear reactions among the hydrogen isotopes and muons in order to interpret experiments on the basic muon capture reactions (cf. Refs. 14, 15, 16, and 17)

$$\mu^- + p \rightarrow n + \nu, \tag{11}$$

and

$$\mu^- + d \rightarrow n + n + \nu. \tag{12}$$

Hildebrand[14] has shown that the rate of reaction (11) is roughly consistent with a "universal" capture theory in which the Fermi (F) and Gamow-Teller (GT) couplings have opposite phase (i.e., F−GT) but inconsistent with the variant of the theory for which these couplings have the same phase (F+GT). The existence of Gamow-Teller (spin-dependent) couplings in reaction (11) has been shown by Culligan *et al.*[18] and Winston[19] in their measurements of the time dependence of μ^- capture in ${}_9F^{19}$. A direct measurement of the ratio of GT- to F-coupling constants in μ^- capture reactions would be valuable.

An approach to this measurement has been suggested by Primakoff.[20] In reaction (12) the nucleons must normally go from an initial ${}^3S_1$ state to a final ${}^1S_0$ state. The necessity of flipping a nucleon spin should suppress the spin-independent F coupling. Thus, the rate of reaction (12) will, to first order, give the GT coupling. A comparison of this with the rate for reaction (11)

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[1] F. C. Frank, Nature **160**, 525 (1947).

[2] Ya. B. Zel'dovitch, Dokl. Akad. Nauk SSSR **95**, 493 (1954).

[3] L. W. Alvarez, H. Bradner, F. S. Crawford, Jr., J. A. Crawford, P. Falk-Variant, M. L. Good, J. D. Gow, A. H. Rosenfeld, F. Solnitz, M. L. Stevenson, H. K. Ticho, and R. D. Tripp, Phys. Rev. **105**, 1127 (1957).

[4] M. Cresti, K. Gottstein, A. H. Rosenfeld, and H. K. Ticho, University of California Radiation Laboratory, Report No. UCRL 3782, 1957 (unpublished).

[5] A. Ashmore, R. Nordhagen, K. Strauch, and B. M. Townes, Proc. Phys. Soc. (London) **71**, 161 (1958).

[6] E. Bleser, L. Lederman, J. Rosen, J. Rothberg, and E. Zavattini, Phys. Rev. Letters **8**, 128 (1962).

[7] J. G. Fetkovitch, T. H. Fields, G. B. Yodh, and M. Derrick, Phys. Rev. Letters **4**, 570 (1960).

[8] J. D. Jackson, Phys. Rev. **106**, 330 (1957).

[9] T. H. R. Skyrme, Phil. Mag. **2**, 910 (1957).

[10] S. Cohen, D. L. Judd, and R. J. Riddell, Jr., Phys. Rev. **119**, 397 (1960).

[11] Ya. B. Zel'dovitch and S. S. Gershtein, Usp. Fiz. Nauk **71**, 593 (1960) [translation: Soviet Phys.—Usp. **3**, 593 (1961)].

[12] S. S. Gershtein, Zh. Eksperim. i Teor. Fiz. **34**, 463 (1958) [translation: Soviet Phys.—JETP **34**, 318 (1958)].

[13] S. S. Gershtein, Zh. Eksperim. i Teor. Fiz. **40**, 698 (1961) [translation: Soviet Phys.—JETP **13**, 488 (1961)].

[14] R. H. Hildebrand, Phys. Rev. Letters **8**, 34 (1962).

[15] R. H. Hildebrand and J. H. Doede, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN Scientific Information Service, Geneva, 1962), p. 418.

[16] E. Bertolini, A. Citron, G. Gialanella, S. Focardi, A. Mukhin, C. Rubbia, and S. Saporetti, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN Scientific Information Service, Geneva, 1962), p. 421.

[17] E. Bleser, L. Lederman, J. Rosen, J. Rothberg, and E. Zavattini, Phys. Rev. Letters **8**, 288 (1962).

[18] G. Culligan, J. F. Lathrop, V. L. Telegdi, R. Winston, and R. A. Lundy, Phys. Rev. Letters **7**, 458 (1961).

[19] R. Winston, University of Chicago Rept. No. EFINS 62–38 (to be published).

[20] H. Primakoff, in *Proceedings of the Fifth Annual Rochester Conference on High Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1955), pp. 174–176

should then give the F coupling. There are, however, certain experimental difficulties in attempting to determine the rates of reactions (11) and (12). In reaction (11) the final state will be a two-body system, the emitted neutron having a well-defined energy of 5.2 MeV. The detection of neutrons in this energy region, while difficult, can be done with reasonable accuracy (cf. references 14–17). As the final state of reaction (12) will be a three-body system, there will be no unique energy for the emitted neutrons to distinguish them from background neutrons. Thus, in practice, an investigation of the rate of reaction (12) must rely on determining the ratio of nondecays to decays for muons stopping in deuterium. Because a muon capture reaction on any foreign nucleus present in the liquid deterium will be identical in appearance to capture on a deuteron, any competing capture reaction could significantly alter the experimental value obtained by this method. In particular, since the rate for the capture reaction

$$\mu^- + \mathrm{He}^3 \rightarrow \begin{cases} t+\nu \\ d+n+\nu \\ p+n+n+\nu, \end{cases} \tag{13}$$

is expected to be approximately five times faster than that for reaction (12), any appreciable formation of the $(\mathrm{He}^3\mu)^+$ muonic ion by trapping of the muon in a bound atomic state around a He^3 nucleus produced in a $(d\mu p)^+$ or a $(d\mu d)^+$ fusion reaction (cf. Figs. 1 and 2) could prohibit the use of this nondecay method.

In this experiment we have determined the rates for various muon reactions in liquid hydrogen and deuterium. From these rates we determine the influence of (13) on the investigation of (12). Specifically, we have determined the following quantities:

λ_{dd} = the rate of formation of $(d\mu d)^+$ via $d\mu+\mathrm{D}\rightarrow (d\mu d)^+ + e^- + 223$ eV

λ_{df} = the rate of the fusion reaction

$$(d\mu d)^+ \begin{cases} \nearrow \mathrm{He}^3+n+\mu^- \\ \searrow p+t+\mu^- \end{cases}$$

$\lambda_{(3/2-1/2)}$ = the rate of the reaction $d\mu(3/2)+d\rightarrow d+d\mu(1/2)$.

η = the fraction of all $(d\mu p)^+$ fusion reactions which yield a rejuvenated muon.

ξ = the fraction of all $(d\mu p)^+$ molecules formed in liquid hydrogen which have deuteron-proton spin equal to $\frac{1}{2}$.

Σ = the increase, occurring in pure deuterium, of the population of those hyperfine levels of the $(d\mu p)^+$ system which have deuteron-proton spin equal to $\frac{1}{2}$. Then by definition, the population of those levels in pure deuterium will be $\Sigma\xi$.

T = that fraction of the muons which, having catalyzed a $(d\mu d)^+$ fusion event, are trapped in a bound atomic state around a recoiling product nucleus. By "fusion event" we mean any muon stop leading to at least one $(d\mu d)^+$ fusion reaction. A muon may catalyze more than one $(d\mu d)^+$ fusion reaction in its lifetime (cf. Sec. II D), but this would still be counted as one "fusion event."

$f(\mathrm{He}^3\mu)^+$ = that fraction of muons stopping in liquid deuterium which form the $(\mathrm{He}^3\mu)^+$ muonic ion before decay. Inasmuch as this quantity is a function of deuterium and hydrogen concentration (cf. Sec. III D), it is determined for only one set of concentrations.

The remaining five sections of the paper are constituted as follows: Sec. II, General Discussion, reviews individually each of the atomic, molecular and nuclear reactions available to a muon stopping in liquid deuterium or liquid hydrogen; Sec. III, the Principles of the Experiment, presents the methods and equations used in determining our experimental values; Sec. IV, the Experimental Procedure, describes the actual procedures used in obtaining the data; and Sec. V and Sec. VI present the results and conclusions.

Following is a list of definitions for the symbols used in this paper.

$C_{\mathrm{H}}(\cdots)$ = concentration of hydrogen in liquid deuterium for a given run.

$C_{\mathrm{D}}(\cdots)$ = concentration of deuterium in liquid hydrogen for a given run.

C_i = concentration of air in liquid deuterium for run D_2I.

λ_0 = rate of the normal decay process $\mu^-\rightarrow e^-+\nu+\bar{\nu}$.

λ_e = rate of the transfer reaction $p\mu+d\rightarrow d\mu+p$.

λ_{pd} = rate of the molecular formation reaction $d\mu+\mathrm{H}\rightarrow (d\mu p)^+ + e^-$.

λ_{pp} = rate of the molecular formation reaction $p\mu+\mathrm{H}\rightarrow (p\mu p)^+ + e^-$.

λ_{pf} = rate of the fusion reaction

$$(d\mu p)^+ \begin{cases} \nearrow (\mathrm{He}^3\mu)^+ + \gamma \\ \searrow \mathrm{He}^3+\mu^-. \end{cases}$$

λ_i = rate of the transfer reaction $d\mu+I\rightarrow I\mu+d$, where the symbol I indicates some high Z impurity.

$f_{df}=\dfrac{\lambda_{df}}{\lambda_0+\lambda_{df}}$ = that fraction of $(d\mu d)^+$ formed which undergoes a fusion reaction.

$f_{pf}=\dfrac{\lambda_{pf}}{\lambda_0+\lambda_{pf}}$ = that fraction of $(d\mu p)^+$ formed with deuteron-proton spin $=\frac{1}{2}$ which undergoes a fusion reaction.

$N_{\mathrm{rej}}(\cdots)$ = the number of 5.4-MeV rejuvenated muons per stopping muon found for a given run.

TABLE I. A comparison of theoretical and experimental values for the rates of transfer and molecular ion formation in liquid protium.

	Theory			Experiment	
λ_e(sec^{-1})	1.14×10^{10}[a]	1.36×10^{10}[b]	1.3×10^{10}[d]	$(2.34\pm0.28)10^{10}$[e,f]	1.9×10^{10}[g]
λ_{pp}(sec^{-1})	$6.5\ \times10^{6}$[a]	$2.5\ \times10^{6}$[b,c]	...	$(2.04\pm0.12)10^{6}$[e]	$(1.4\pm0.5)10^{6}$[g]
λ_{pd}(sec^{-1})	$2.5\ \times10^{6}$[a]	$1.2\ \times10^{6}$[b,c]	...	$(5.9\ \pm0.6)10^{6}$[e]	$(5.5\pm1.1)10^{6}$[g]

[a] See Ref. 10.
[b] See Ref. 28.
[c] As reported by Ref. 11 in which Z is replaced by Z_{eff}, as calculated by Ref. 10.
[d] See Ref. 29.
[e] See Ref. 30.
[f] This value is obtained by combining the results given in Ref. 30 for λ_{pp} with the value of the ratio $\lambda_{pp}+\lambda_0/\lambda_e$ reported in Ref. 6.
[g] See Ref. 6.

$N_f(\cdots)$ = the number of 3.04-MeV recoil protons per stopping muon found for a given run.

The experimental quantities are:

N_{1f} = the total number of single fusions found.
N_{2f} = the total number of double fusions found.
N_{3f} = the total number of triple fusions found.
N_s = the total number of stopping muons.
ϵ_{1f} = the combined solid angle and scanning efficiency for a single fusion.
ϵ_{2f} = the combined solid angle and scanning efficiency for a double fusion.
ϵ_{3f} = the combined solid angle and scanning efficiency for a triple fusion.

II. GENERAL DISCUSSION

The sequence of atomic and molecular reactions varies as a function of deuterium concentration. For convenience we will consider two distinct systems. These will be: (A) those reactions which occur in liquid protium[21] containing small concentrations of deuterium, and (B) those reactions which occur in liquid deuterium containing small concentrations of protium. The possibility of multiple fusion reactions caused by recycling of the catalyzing muon is discussed in (D); and in (E) we discuss muon transfer reactions from hydrogen isotopes to impurities of $Z>1$.

A. Liquid Protium

In liquid protium, with a small admixture of deuterium, the successive processes are:

$$1.\ \mu^-+\mathrm{H}_2\rightarrow p\mu+\mathrm{H}+e^-. \tag{1}$$

The entering muon rapidly slows down and undergoes atomic capture and de-excitation to the ground $1S$ state of the $p\mu$ atom. It can also form a $d\mu$ atom, but since the concentration of deuterium is assumed small, this process may be neglected. These moderation and atomic absorption times of negative muons were first calculated by Wightman.[22] He calculated that a muon should go from $E=10$ MeV to a bound state in the K shell of a proton in $\sim5.5\times10^{-10}$ sec. Recent theoretical[23–25] and experimental[26,27] investigations of negative pion moderation and absorption times indicate that for weakly interacting particles Wightman's calculations are essentially correct.

2. The small neutral $p\mu$ atom will have a thermal energy of $\sim1/400$ eV and will collide with the surrounding nuclei in the liquid. It can undergo one of two types of reactions: transfer to a deuteron or molecular ion formation.

$$\text{(a)}\ p\mu+d\rightarrow d\mu^*+p+135\ \mathrm{eV}. \tag{3}$$

The asterisk is used to denote an atom with more than thermal velocity. In the transfer reaction (3) the $d\mu$ atom receives an energy of 45 eV. This process is an inelastic collision between the small neutral $p\mu$ system and a deuteron, with the difference in binding energies being converted to kinetic energy of relative motion of the two nuclei. Various authors have calculated the effective cross sections and rates for this reaction. The best treatments appear to be those of Cohen, Judd, and Riddell,[10] Belyaev *et al.*[28] and Shimizu *et al.*[29] These are compared with the recent determination of this rate by Conforto *et al.*[30] in Table I. The $p\mu$ atom may also form the $(p\mu d)^+$ molecular ion in a collision with a deuteron, but this process is much slower than transfer to a deuteron (3).

$$\text{(b)}\ p\mu+\mathrm{H}\rightarrow(p\mu p)^++e^-+93\ \mathrm{eV}. \tag{2}$$

The mechanism for this reaction has been shown by Cohen, Judd, and Riddell,[10] and Zel'dovitch and

[21] The term *protium* will refer to the mass-1 isotope of hydrogen.
[22] A. S. Wightman, Ph.D. thesis, Princeton University, 1949 (unpublished); and Phys. Rev. **77**, 521 (1950).
[23] T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Letters **3**, **61** (1959); and Phys. Rev. **118**, 864 (1960).
[24] J. E. Russell and G. L. Shaw, Phys. Rev. Letters **4**, 369 (1960).
[25] M. Leon and H. A. Bethe, Phys. Rev. **127**, 676 (1962).
[26] T. H. Fields, G. B. Yodh, M. Derrick, and J. G. Fetkovitch, Phys. Rev. Letters **5**, 69 (1960).
[27] J. H. Doede, R. H. Hildebrand, M. Israel, and M. Pyka, University of Chicago Rept. No. EFINS 62–65 (unpublished); Phys. Rev. **129**, 2808 (1963).
[28] V. B. Belyaev, S. S. Gershtein, B. N. Zakhar'ev, and S. P. Lomnev, Zn. Eksperim. i Teor. Fiz. **37**, 1652 (1959) [translation: Soviet Phys.—JETP **10**, 1171 (1960)].
[29] Shimizu, Mizuno, and Izuyama, Progr. Theoret. Phys. (Kyoto) **20**, 777 (1958); **21**, 479 (1959).
[30] G. Conforto, S. Focardi, C. Rubbia, and E. Zavattini, Phys. Rev. Letters **9**, 432 (1962); see also *ibid.* **9**, 525 (1962).

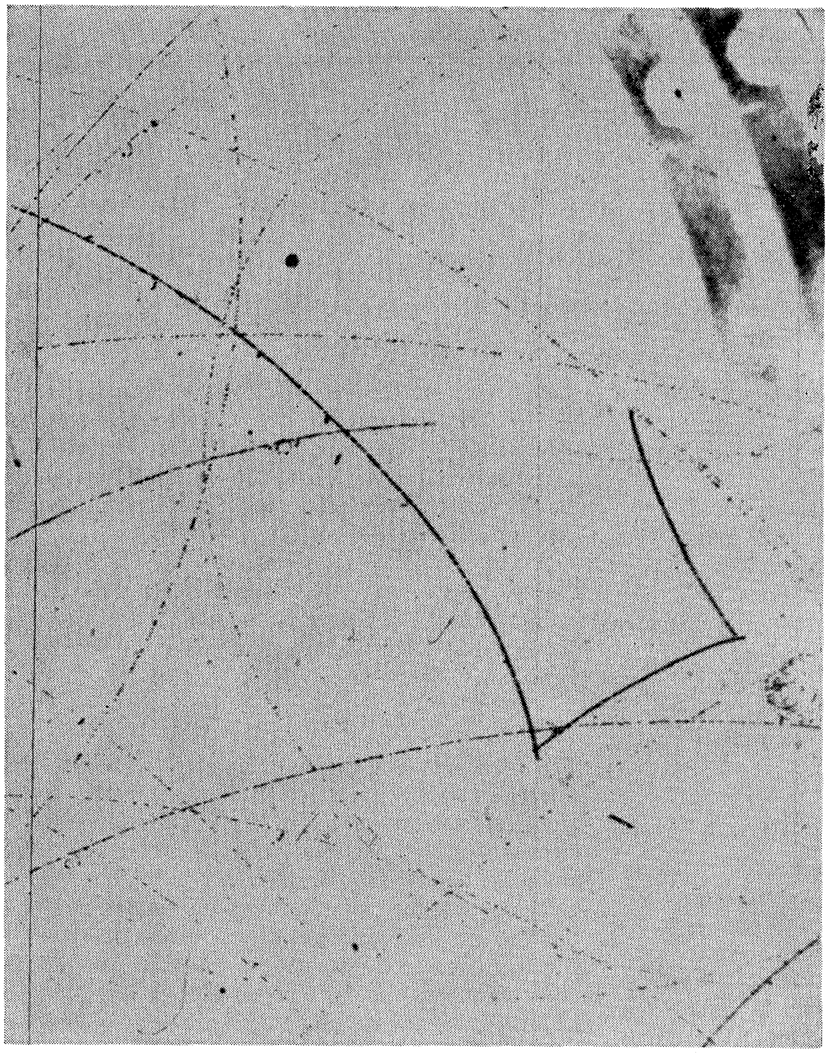

FIG. 3. An example of two successive $(d\mu p)^+$ fusion events in liquid hydrogen. Note the gaps between the stopping muon and the first rejuvenated muon and between the first and second rejuvenated muons.

Gershtein[11] to be the transition from an S state to the initial $p\mu$H system to a bound P state of the final $(p\mu p)^+$ molecular ion by an electric dipole ($E1$) transition with the ejection of the atomic electron. The electric dipole moment of a three-body system composed of two nuclei and a muon can be produced by an asymmetry in either of two separate charge distributions. These are the distribution of the nuclear charges and the distribution of the muon charge relative to the center of mass of the system. The dipole moment can be described by the equation

$$d=-\frac{e}{2}\left[\frac{M_2-M_1}{M_2+M_1}\mathbf{R}_{12}+(\mathbf{r}_1+\mathbf{r}_2)\right],$$

where M_1= mass of nucleus 1, M_2= mass of nucleus 2, $\mathbf{R}_{12}$= distance between the two nuclei, $\mathbf{r}_1$= the distance between the muon and nucleus 1, and $\mathbf{r}_2$= the distance between the muon and nucleus 2. We see that the first term will be nonzero only in the case where the two nuclei are different. The second term represents the distribution of the muon charge, and in the case of like nuclei, will completely determine the dipole moment. A further constraint is placed on the initial and final states by parity considerations. As the first term is independent of the muon wave function, it will be nonzero only for those transitions between like parity states of the muon wave function, i.e., $\Sigma_g \to \Sigma_g$ and $\Sigma_u \to \Sigma_u$. Conversely, the second term will contribute to the dipole moment only for those transitions between opposite parity states of the muon wave function, i.e., $\Sigma_g \leftrightarrows \Sigma_u$. As the final state of the bound system is a P state, the final muon wave function must have even parity (Σ_g). Thus, for like nuclei, we have only transitions for which $\Sigma_u \to \Sigma_g$. Due to the low energy of the system, only initial S-state collisions need be considered so that, in the case of like nuclei, the rate of formation must be multiplied by the statistical weight of the odd parity (Σ_u) S state. Since the Σ_u state of the muon wave function is antisymmetric with respect to the interchange of the two nuclei, the rate of formation of the $(p\mu p)^+$ ion must be multiplied by the statistical weight of the antisymmetric triplet S state ($^3S_1$) of the two protons introducing a factor of $\frac{3}{4}$. Once formed, the $(p\mu p)^+$ system can capture an electron from the surrounding medium to become a neutral system. However, at distances of the order of magnitude of a muon Bohr radius ($\sim 2.5\times 10^{-11}$ cm), the $(p\mu p)^+$ system will still exhibit a positive charge. Therefore, the close approach of a muon bound in a $(p\mu p)^+$ ion to another nucleus will be prohibited by a Coulomb barrier. This excludes the muon from further participation in transfer or molecular ion formation reactions. A comparison of the theoretically derived formation rates for the $(p\mu p)^+$ system (λ_{pp}) and a recent experimental determination by Bleser *et al.*[6] and Conforto *et al.*[30] may be found in Table I. The final P state (with rotational quantum number $k=1$) of the $(p\mu p)^+$ system is a metastable state similar to orthohydrogen, as the $k=1$ rotational level must have the total spin odd, while the $k=0$ level must have the total spin even so that a transition from the upper to the lower rotational level must change the total spin.

3. $d\mu^* + p \to d\mu + p$.

The excited $d\mu^*$ muonic atom formed in reaction (3) will collide with nuclei of the surrounding medium and become thermalized. It was suggested by Alvarez *et al.*[3] that this scattering process could be the source of the large ($\sim$1 mm) gaps observed in the bubble chamber between the end point of the stopping muon and the beginning point of the rejuvenated muon (cf. Fig. 3 and Sec. I), if the cross section for scattering

$$d\mu^* + p \to d\mu^* + p$$

were small. They observed, as did Cresti *et al.*[4] that the gap length decreased as the deuterium concentration increased. In fact, the gaps disappeared altogether in a bubble chamber containing approximately 4% deuterium.[4] This has been attributed to the increased occurrence of $d\mu + d$ scattering. Cohen, Judd, and Riddell[10] have calculated the cross sections for various muonic atom scattering processes. Their results indicate a large cross section ($\approx 3.5\times 10^{-19}$ cm^2) for the scattering of $d\mu$ muonic atoms on deuterons. For the scattering process

$$d\mu + p \to d\mu + p,$$

their calculations indicate that the scattering process exhibits a Ramsauer-Townsend effect,[31] i.e., the cross section goes to zero at a center-of-mass energy of approximately 0.2 eV. For all $d\mu$ muonic atoms which reach this energy region, the only effective scattering process would be $d\mu + d$ scattering. The Ramsauer-Townsend effect arises from distortions of the outgoing wave functions for the residual potentials operating at large distances (greater than 20 muon Bohr orbits) from the scattering center. Cohen, Judd, and Riddell[10] have also performed the calculation neglecting these long range effects and obtain a value of 5.3×10^{-21} cm^2 for the $d\mu + p$ scattering cross section. Zel'dovitch and Gershtein[11] have examined both the results of Cohen, Judd, and Riddell,[10] and those of Belyaev *et al.*[28] and conclude that good agreement with the experimentally observed gap lengths can be obtained using the value obtained by Cohen, Judd, and Riddell,[10] neglecting the long range effects.

It should be noted that, if the Ramsauer-Townsend effect exists, a reduction in deuterium concentration should increase the mean length of the gaps; while if it does not exist, decreasing the deuterium concentration below one part in 10^3 should yield an upper limit to the mean gap length, as then the only effective cross section would be that for $d\mu$ muonic atoms scattering on protons.

4. $d\mu + \mathrm{H} \to (d\mu p)^+ + e^- + 90\text{ eV}$. (4)

Once the $d\mu$ atom reaches thermal velocities, it can form $(d\mu p)^+$ in a manner similar to that of $(p\mu p)^+$ formation. For the $(d\mu p)^+$ system, there will be a contribution to the electric dipole moment from the asymmetric distribution of the nuclear charges. Thus, in the initial S-state collision, both the Σ_u and Σ_g spin configurations can contribute to the production of a bound P state. A comparison of the theoretical and experimental values for the rate λ_{pd} of formation of $(d\mu p)^+$ is presented in Table I. Having been formed in the P state, the $(d\mu p)^+$ system can undergo a rapid transition to the S ground state with an energy loss of 124 eV. This is presumed to occur in the following manner:

(a) The $(d\mu p)^+$ system quickly captures an electron in a collision with the surrounding medium. Zel'dovitch and Gershtein[11] estimate that this should occur in approximately 10^{-11} sec.

(b) The $(d\mu p)^+e^-$ system de-excites from the P state to the S state via an electric dipole $E1$ transition with the conversion on the atomic electron. Cohen, Judd, and Riddell[10] estimate the rate to be $\simeq 2.5\times 10^{10}$ sec^{-1}. Therefore, the $(d\mu p)^+$ ions will rapidly de-excite to the S ground state. As in $(p\mu p)^+$, the muon in $(d\mu p)^+$ is prevented from entering into any further transfer or molecular ion formation reactions (cf., above).

5. $(d\mu p)^+ \nearrow \mathrm{He}^3 + \mu^- + 5.4\text{ MeV}$ (5)

$ \searrow (\mathrm{He}^3\mu)^+ + \gamma + 5.4\text{ MeV}$. (6)

Due to the large mass of the muon, the distance between the nuclei in the $(d\mu p)^+$ ion is $\sim$200 times smaller than for ordinary pd molecules, and the two nuclear wave functions overlap slightly. Depending on the method of transition to the He^3 final state, either a γ ray or the

[31] See for example, N. F. Mott and H. S. W. Massey, in *The Theory of Atomic Collisions* (Oxford University Press, New York, 1952), p. 200.

muon carries off the energy of the reaction. The essential features of the $(d\mu p)^+$ fusion process are derived from the constraints placed by the He^3 final state. The exclusion principle requires that the two protons have opposite spin in the He^3 ground state. In $(d\mu p)^+$, the deuteron-proton system can have a spin equal to $\frac{3}{2}$ or $\frac{1}{2}$. In the $\frac{3}{2}$ case, the two proton spins are parallel so that one of the spins must be flipped in going to the He^3 final state. Therefore, the fusion reaction should occur with two separate rates, one for the spin $\frac{1}{2}\rightarrow$ spin $\frac{1}{2}$ reaction and the other for the spin $\frac{3}{2}\rightarrow$ spin $\frac{1}{2}$ cases. Cohen, Judd, and Riddell[10] have calculated the rates for the various electromagnetic transitions capable of transforming the deuteron-proton system to the He^3 final state. Their results indicate that for the $\frac{1}{2}\rightarrow\frac{1}{2}$ reactions, i.e., for transitions from an initial deuteron-proton spin of $\frac{1}{2}$ to a He^3 final state of spin $\frac{1}{2}$, the reaction can proceed either via a magnetic dipole ($M1$) transition emitting a γ ray with a rate $\lambda_{M1}\sim 10^7$ sec^{-1}, or by an electric monopole ($E0$) reaction ejecting the muon with a rate $\lambda_{E0}\simeq 5\times 10^5$ sec^{-1}. As the internal conversion coefficient for the $M1$ process is much too low to give the observed ratio of muon to γ-ray emission (in Ref. 11 the authors calculate the internal conversion coefficient to be $\simeq 4\times 10^{-4}$), the rejuvenated muon seen by Alvarez *et al.*[3] is assumed to arise solely as the product of the $E0$ transition, which can proceed only by ejection of the muon. Cohen, Judd, and Riddell[10] also show that, due to the necessity of flipping a proton spin, reactions from the initial $\frac{3}{2}$ spin state of the proton-deuteron system will be slow in comparison to the mean lifetime of the muon.

6. Hyperfine effects in $(d\mu p)^+$ nuclear fusion.

Zel'dovitch and Gershtein[11] have shown that the $(d\mu p)^+$ system has four nondegenerate hyperfine levels of $F(=I+J)=2$, 1 (two levels) and 0. Due to the dependence of the $(d\mu p)^+$ fusion reaction on the spin orientation of the two nuclei (see previous section), this reaction will proceed independently from each hyperfine level, the rate being proportional to the admixture of the deuteron-proton spin $\frac{1}{2}$ state to the various levels. Thus, the inclusion of hyperfine level splitting requires the calculation of the admixture of the deuteron-proton spin $\frac{1}{2}$ state to each of the hyperfine levels. We find:

(a) $F=2$ level.

It is apparent that the state $F=2$ must have a deuteron-proton spin of $\frac{3}{2}$. Therefore, those $(d\mu p)^+$ ions formed in the $F=2$ hyperfine level will not contribute to the fusion reaction.

(b) $F=0$ level.

The $F=0$ level will have a deuteron-proton spin equal to $\frac{1}{2}$. Therefore, all $(d\mu p)^+$ ions formed in the $F=0$ hyperfine level can contribute to the fusion reaction.

(c) $F=1$ level.

The $F=1$ hyperfine level will be composed of linear combinations of states with deuteron-proton spin equal to $\frac{3}{2}$ and $\frac{1}{2}$. A calculation of the admixture of the deuteron-proton spin $\frac{1}{2}$ orientation to the hyperfine levels with $F=1$ has been performed by Zel'dovitch and Gershtein.[11] They represent the spin function of the deuteron-proton system in the form

$$X_1=C_{1/2}X_1^{(1/2)}+C_{3/2}X_1^{(3/2)},$$

where the subscripts on the X refer to the spin state of the total system, and the superscripts on the X refer to the spin states of the deutron-proton system. Zel'dovitch and Gershtein[11] find coefficients $C_{1/2}=-0.41$, $C_{3/2}=+0.91$ for the lower level; and $C_{1/2}=+0.91$, $C_{3/2}=+0.41$ for the upper level.

When the statistical weights of the various hyperfine levels are included, the combined effect of the hyperfine level splitting allows only 35.5% of those $(d\mu p)^+$ ions formed to be in the fusion-favorable deuteron-proton spin-$\frac{1}{2}$ orientation.

B. Liquid Deuterium

The sequence of muon reactions in liquid deuterium is shown in Fig. 2. In comparison to the reaction sequences in liquid protium, there are two important changes in the liquid deuterium sequence. First, the formation of the $(p\mu p)^+$ ion is prohibited as the incoming muon is initially bound in a $d\mu$ atom. Second, due to the high concentration of deuterium, it is possible to form the $(d\mu d)^+$ ion. This $(d\mu d)^+$ system can then undergo a fusion reaction with the emission of a He^3 nucleus and a neutron or a triton and a proton [reaction (9) and (10)].

Fetkovitch *et al.*[7] have investigated experimentally the sequence of reactions leading to the $(d\mu d)^+$ fusion with the emission of a proton and a triton in a liquid-deuterium bubble chamber. Various authors[8–13] have undertaken the theoretical investigation of the atomic, molecular, and nuclear reactions caused by a negative muon stopping in liquid deuterium. The reactions leading to fusion in deuterium are the following:

1. $\mu^-+D_2\rightarrow d\mu+D+e^-$. (7)

This is the moderation and atomic capture of the incoming muon into the $1S$ state of a $d\mu$ muonic atom. The mechanism involved is essentially identical with that of $p\mu$ formation treated previously (cf. Sec. A1).

2. $d\mu+H\rightarrow(d\mu p)^++e^-+90$ eV. (4′)

As there is generally some protium present as an isotopic impurity, it is possible to form the $(d\mu p)^+$ ion. The mechanism of formation is identical with that in liquid hydrogen (cf. Sec. A2).

3. $d\mu+D\rightarrow(d\mu d)^++e^-+223$ eV. (8)

Cohen, Judd, and Riddell[10] have concluded that $(d\mu d)^+$ can be formed by an electric dipole ($E1$) transition from an incident S state of the $(d\mu\mathrm{D})$ system to a bound P state of the final $(d\mu d)^+$ ion. This mechanism is similar to that for $(p\mu p)^+$ formation. However, there are two important differences: (i) the deuterons obey Bose-Einstein statistics, and (ii) the $(d\mu d)^+$ system has a vibrationally excited S state of low-binding energy relative to the ground state of the $d\mu$ muonic atom ground state. As the electric dipole moment of the ions composed of identical nuclei was shown to be nonzero only for a transition of the type $\Sigma_u \rightarrow \Sigma_g$ (cf. Sec. IIA 4), the rate of formation for the $(d\mu d)^+$ system corresponds to a statistical weight of the Σ_u state for a deuteron-deuteron system of $\frac{1}{3}$ and not $\frac{3}{4}$ as in the proton-proton system. Also, the existence of the vibrationally excited S state causes some cancellation of matrix elements in the expression of the dipole moment. Both of these factors tend to reduce the electric dipole rate of formation of the $(d\mu d)^+$ system relative to the formation rate for the $(p\mu p)^+$ system. Specifically, Cohen, Judd, and Riddell[10] find $\lambda_{dd}(E1)=5.9\times10^4$ sec^{-1}. Zel'dovitch has investigated the possibility of molecular ion formation via an electric monopole transition. While the rates for an electric monopole transition are generally small compared to those for electric dipole transitions, Zel'dovitch[2] points out that if the bound system has an S state with an excited vibrational mode of low-binding energy, the collision will occur under conditions very close to resonance. Then the probability of molecular ion formation via an $E0$ transition will be large. Cohen, Judd, and Riddell[10] and Gershtein[32] have calculated that the $(d\mu d)^+$ system should have such an excited S state with a binding energy of 40 eV relative to the ground state of the $d\mu$ muonic atom. Thus, Zel'dovitch and Gershtein[11] arrive at a rate $\lambda_{dd}(E0)=3\times10^4$ sec^{-1} for the formation of the $(d\mu d)^+$ system via an $E0$ transition in the reaction

$$d\mu+\mathrm{D}\rightarrow(d\mu d)^+ + e^- + 40\ \mathrm{eV}.$$

Those $(d\mu d)^+$ ions formed by the electric dipole transition will be in a metastable P state similar to that of the $(p\mu p)^+$ system, while those formed by the electric monopole transition will be in a vibrationally excited S state.

$$4.\ (d\mu p)^+ \begin{cases} \nearrow \mathrm{He}^3+\mu^-\ (5.4\ \mathrm{MeV}) & (5') \\ \searrow (\mathrm{He}^3\mu)^+ + \gamma. & (6') \end{cases}$$

In liquid deuterium this reaction will proceed in exactly the same manner as in liquid hydrogen (cf. Sec. IIA 5).

$$5.\ (d\mu d)^+ \begin{cases} \nearrow \mathrm{He}^3+n+\mu^-+3.3\ \mathrm{MeV} & (9) \\ \searrow p+t+\mu^-+4.0\ \mathrm{MeV}. & (10) \end{cases}$$

[32] S. S. Gershtein, dissertation, Institute for Physics Problems, 1958 (unpublished); as reported in Ref. 11.

The fusion of the $(d\mu d)^+$ system will proceed via nuclear penetration of the Coulomb barrier and subsequent de-excitation by particle emission. The P state of the ion, formed in the electric dipole process, will have a greater internuclear distance than the S state of the system formed by the electric monopole process. As the barrier penetration coefficient is a function of the internuclear separation, the fusion reaction will proceed with different rates for the two rotational states of the $(d\mu d)^+$ system. Zel'dovitch and Gershtein,[11] using the results of Jackson's investigation,[8] have investigated the reaction rates for the $(d\mu d)^+$ ions formed in the S state, the nuclear fusion reaction should be very fast with a rate $\lambda_{df}(S)\sim10^{10}$–$10^{11}$ sec^{-1}. They also give a qualitative statement that rates for fusion reactions from the P state should be no slower than $\lambda_{df}(P)\sim10^7$–10^8 sec^{-1}. Therefore, they predict that nearly all $(d\mu d)^+$ systems formed will undergo a fusion reaction since the rate of the fusion reaction is at least 20 times faster than the decay rate of the muon.

C. Hyperfine Effects in Liquid Deuterium

Gershtein[13] has pointed out that in liquid deuterium the possibility of the reaction

$$d\mu(3/2)+d\rightarrow d+d\mu(1/2)+0.046\ \mathrm{eV}, \qquad (14)$$

depopulating the upper (3/2) hyperfine level of the $d\mu$ muonic atom could considerably enhance the frequency of the spin-dependent $(d\mu p)^+$ fusion reaction. Increasing the population of the lower hyperfine state would increase the percentage of $(d\mu p)^+$ systems formed in the fusion-favorable $dp\frac{1}{2}$ state. Clearly, if this reaction were very fast, essentially all of the $d\mu$ muonic atoms would be in the lower $\frac{1}{2}$ hyperfine level and only the $F=1, 0$ hyperfine levels of the $(d\mu p)^+$ system could be formed. Gershtein[13] calculates an increase by a factor of approximately 1.8 in the frequency of $(d\mu p)^+$ fusion reactions if all $d\mu$ atoms reach the spin $\frac{1}{2}$ state.

D. Recycling and Trapping of Muons

Since the fusion reaction between hydrogen isotopes does not affect the catalyzing muon, the muon should be capable of re-entering the reaction sequence and occasionally catalyzing several fusion reactions before its ultimate decay. However, the muon may become bound in an atomic orbit around one of the charged nuclei produced in the fusion reaction, forming an excited muonic atom. If the muon is "trapped" in an atomic state around the recoiling He^3 nucleus, it is prevented from entering into any further reactions because the $(\mathrm{He}^3\mu)^+$ ion has a net positive charge which prevents its close approach to any other nucleus in the medium.

Since the dominant branch of the $(d\mu p)^+$ fusion reaction is the emission of a γ ray, the muon will generally be bound to the He^3 nucleus; and, therefore, the effect of the recycling of the muon in the $(d\mu p)^+$ fusion reactions may be neglected.

For $(d\mu d)^+$ fusions, recycling can occur. Assuming a 1:1 branching ratio between reaction (9) and (10) in the $(d\mu d)^+$ fusion process, a He^3 nucleus with a recoil energy of approximately 0.75 MeV will be produced in one-half of the $(d\mu d)^+$ fusion reactions. The probability of the muon being bound in a 1S state of the recoiling He^3 atom has been calculated by Jackson[8] and by Judd.[33] They find that the muon should be trapped in the 1S level in approximately 15% of the cases of reaction (9). They also calculate the probability for trapping the muon around the recoiling triton produced in reaction (10) and find that it should be trapped in approximately 2% of the cases of triton production. Therefore, in calculating the effect of recycling of the muon in $(d\mu d)^+$ fusion reactions some allowance must be made for the trapping probability. It should be noted that the theoretical percentages are only for trapping into the ground state. Since both the $(t\mu)$ and $(He^3\mu)^+$ systems have bound excited states, the fraction of muons trapped may be considerably higher than calculated for the ground state alone. As is shown in Sec. VI, the comparison of our results with the theoretical predictions shows that the contribution of these excited states is appreciable.

E. Transfer Reaction with Atoms of $Z>1$

Schiff[34] has examined the rates of transfer of muons from protons and deuterons to neon atoms dissolved in liquid hydrogen.

$$p\mu+\mathrm{Ne}\rightarrow \mathrm{Ne}\mu+p$$
$$d\mu+\mathrm{Ne}\rightarrow \mathrm{Ne}\mu+d.$$

If these reactions occur with a rate comparable to either the molecular ion formation rate or the $p\mu+d$ transfer rate, then the number of fusion reactions should decrease. Due to the Coulomb barrier presented by the net positive charge on a muonic ion of $Z>1$, the transfer of the muon prevents it from entering into any further transfer or molecular ion formation reactions.

Schiff[34] has shown that the transfer rates for neon are of the same order of magnitude as the proton-deuteron transfer rate. Further investigation of high-Z transfer reactions have been carried out by Dzhelepov *et al.*[35] in a diffusion chamber. They examined the rates for transfer to carbon and oxygen by detecting the Auger electron arising from conversion of the $2p\rightarrow 1s$ γ ray in the de-excitation of the $C\mu$ and $O\mu$ muonic atoms formed in the transfer reaction. They find that the transfer reactions proceed with a rate $\lambda_Z=(1.2_{-0.5}^{+0.8})10^{10}$ sec^{-1}.

Using Schiff's[34] results for transfer reactions with neon, Conforto *et al.*[30] measured the rates of formation of the $(p\mu p)^+$ ion (λ_{pp}) and the $(d\mu p)^+$ ion (λ_{pd}) (cf. Fig. 1) by detecting the 2P-1S 214-keV x ray arising in the de-excitation of the neon muonic atom. Because the molecular ion formation reactions are the only one competing with the transfer reaction, (λ_{pp}) and (λ_{pd}) may be determined by varying the concentration of neon and deuterium in liquid protium. This method has the advantage that it is independent of the $(d\mu p)^+$ fusion reaction and any hyperfine level splitting effects. Thus, it can be used to check the assignment of rates in experiments which detect the fusion γ ray.

III. PRINCIPLE OF THE EXPERIMENT

A. η-The Fraction of $(d\mu p)^+$ Fusion Reactions which Produce Rejuvenated Muons

η is defined as

$$\eta=\frac{\text{yield of rej. }\mu^-}{\text{yield of }\gamma\text{ rays}+\text{yield of rej. }\mu^-}$$

for reactions (5) and (6). Note that η is not an internal conversion coefficient for the γ ray but is the ratio between the $E0$ transition and the sum of the $E0$ and $M1$ transitions. Using the results of the experiments by Bleser *et al.*[6] and Conforto *et al.*[30] we can determine the value of the product $\xi\eta$ where ξ= the fraction of $(d\mu p)^+$ formed with dp spin equal to $\frac{1}{2}$ by using the bubble chamber to detect the rejuvenated muon. Using the value $\xi=0.355$ derived by Zel'dovitch and Gershtein,[11] we can obtain a result for η. The experimental method used to determine ξ (cf. Sec. IIIB) yields only an upper limit which is completely consistent with this value (cf. Sec. VI). Setting N_{rej}= the number of rejuvenated muons per stopping muon, we have:

$$N_{\mathrm{rej}}=\frac{C_{\mathrm{D}}\lambda_e}{\lambda_0+C_{\mathrm{H}}\lambda_{pp}+C_{\mathrm{D}}\lambda_e}\,\frac{C_{\mathrm{H}}\lambda_{pd}}{\lambda_0+C_{\mathrm{H}}\lambda_{pd}}\,\xi\frac{\lambda_{pf}}{\lambda_0+\lambda_{pf}}\eta. \quad (15)$$

In terms of experimentally observable quantities, N_{rej} is given by

$$N_{\mathrm{rej}}=N\mu/\epsilon\mu N_s,$$

where N_s= the total number of stopping muons, ϵ_μ= the detection efficiency for seeing a 5.4 MeV rejuvenated muon in the bubble chamber, and $N\mu$= the total number of rejuvenated muons found.

An alternative method for determining η is to compare the yields of γ rays and rejuvenated muons from muon-catalyzed fusions at a specific concentration of deuterium. Since all fusions lead to either γ ray or μ emission, we have, at a given deuterium concentration, C_{D}, the relationship

$$N_\gamma/N_{\mathrm{rej}}=(1-\eta)/\eta, \quad (16)$$

where N_γ= yield of γ rays per stopping muon. Inasmuch as ξ applies to both N_{rej} and N_γ, it disappears from the expression of the ratio $N_\gamma/N_{\mathrm{rej}}$. We can determine η

[33] D. L. Judd (private communication).
[34] M. Schiff, Nuovo Cimento 22, 66 (1961).
[35] V. P. Dzhelepov, P. F. Yermolov, E. A. Kushnirenko, V. I. Moskalev, and S. S. Gershtein, Dubna Report No. D-812 (to be published).

TABLE II. The result of this experiment and the comparison to other theoretical and experimental values (the symbols are defined in the text, Sec. I).

	This experiment	Others		
$f_{df}\lambda_{df}^{(\mathrm{sec}^{-1})}$	$(1.26\ \pm 0.45)10^5$		$8.5\times 10^{4\,\mathrm{a,c}}$	
$\lambda_{dd}^{(\mathrm{sec}^{-1})}$	$(3.6\ \pm 1.3)10^5$	$5.9\times 10^{4\,\mathrm{b,d}}$	$\sim 3\times 10^{4\,\mathrm{b,e}}$	$\sim 9\times 10^{4\,\mathrm{b,f}}$
f_{df}	(0.357 ± 0.008)		$\sim 1^{\mathrm{b,g}}$	
$\lambda_{df}^{(\mathrm{sec}^{-1})}$	$(2.5\ \pm 0.04)10^5$			
$\lambda_{(3/2-1/2)}^{(\mathrm{sec}^{-1})}$	$<(1.35\ \pm 0.5)10^6$		$\sim 6\times 10^{6\,\mathrm{b,h}}$	
	$>(7.9\ \pm 2.8)10^5$			
Σ	$2.5\ \pm 0.8$		$1.86^{\mathrm{b,h}}$	
ξ	$\leq(0.4\ \pm 0.13)$		$\simeq 0.355^{\mathrm{b,g}}$	
T	$0.31\ \pm 0.0034$	$0.09^{\mathrm{b,i}}$	$0.09^{\mathrm{b,j}}$	
η	$0.15\ \pm 0.015$	$\sim 0.12^{\mathrm{b,d}}$	$0.078\pm 0.013^{\mathrm{a,k}}$	$0.068\pm 0.012^{\mathrm{a,l}}$
$f(\mathrm{He}^3\mu)^+$	$(0.04\ \pm 0.005)$			

[a] Experimental result.
[b] Theoretical result.
[c] See Ref. 7.
[d] See Ref. 10.
[e] See Ref. 2.
[f] Obtained by combining the results of Ref. 10 for the $E1$ formation process and Ref. 2 for the $E0$ formation process.
[g] See Ref. 11.
[h] Derived from values presented in Refs. 13 and 11.
[i] See Ref. 8.
[j] See Ref. 33.
[k] This value is obtained by dividing the $(d\mu p)^+$ fusion γ ray yield of Ref. 6 by the yield of rejuvenated muons from Ref. 34 (cf. text Sec. IIIA).
[l] This is obtained as per (k) with the values reported by Ref. 5 and Ref. 4.

from this expression even if N_γ and N_{rej} are determined in experiments at somewhat different deuterium concentration, provided that C_{D} is high enough ($\sim 1\%$) in each case so that the transfer reaction (3) is saturated but low enough so that $C_{\mathrm{H}}\approx 1$.

Our experimental value for η presented in Sec. V and Table II is derived using the value $\xi=0.355$, as reported by Zel'dovitch and Gershtein.[11] Section VI and Table II also present values for η derived by combining previous experimental determinations of N_γ and N_{rej} as per Eq. (16).

It should be noted that, in principle, a combination of these two methods would allow experimental determination of the value of ξ. Unfortunately, the experimental determination of N_γ in both Refs. 5 and 6 has been done in such a manner as to include the value of ξ implicitly (cf. Table II and Sec. VIA).

B. $(d\mu p)^+$-Hyperfine Effect in Deuterium

We wish to determine the increase in the yield of the $(d\mu p)^+$ fusion reaction in liquid deuterium due to the interaction

$$d\mu(3/2)+d\rightarrow d+d\mu(1/2)\,. \qquad (14)$$

As we have seen (Sec. IIA 5), the $(d\mu p)^+$ fusion reaction is highly spin-dependent, proceeding only from those states in which the deuteron-proton spin is equal to $\frac{1}{2}$. In liquid protium, the $(d\mu p)^+$ system will be formed in the hyperfine levels $F=2$, 1 (two levels), and 0 with statistical population of each level. Only the $F=1$, 0 levels can contribute to the fusion reaction.

In liquid protium, the fraction of $(d\mu p)^+$ ions which undergo a fusion reaction is given by ξf_{pf}, while the possibility of reaction (14) occurring in liquid deuterium will increase the effective value of ξ. If the rate of reaction (14), $\lambda_{(3/2-1/2)}$, is fast, then all the $d\mu$ muonic atoms will be in the lower (1/2) hyperfine level, and only the $F=1$, 0 hyperfine levels of $(d\mu p)^+$ can be formed. This situation would give the maximum population for these fusion-favorable hyperfine levels.

In liquid deuterium, the fraction of $(d\mu p)^+$ ions which undergo a fusion reaction can be expressed as $\Sigma\xi f_{pd}$. Now we equate the quantity Σ to the increase, in pure deuterium, of the population of those hyperfine levels of the $(d\mu p)^+$ ion which have deuteron-proton spin$=\frac{1}{2}$. Then, by definition, the population of those levels will be $\Sigma\xi$.

The fraction Γ of all $d\mu(3/2)$ muonic atoms which undergo reaction (14) in a pure deuterium medium will be given by

$$\Gamma_{\mathrm{pure}}=\frac{C_{\mathrm{D}}\lambda_{(3/2-1/2)}}{\lambda_0+C_{\mathrm{H}}\lambda_{pd}+[1-(1-T)f_{df}]C_{\mathrm{D}}\lambda_{dd}+C_{\mathrm{D}}\lambda_{(3/2-1/2)}}, \qquad (17)$$

while in the presence of some impurity,

$$\Gamma_{\mathrm{impure}}=\frac{C_{\mathrm{D}}\lambda_{(3/2-1/2)}}{\lambda_0+C_{\mathrm{H}}\lambda_{pd}+C_{\mathrm{D}}\lambda_{dd}+C_{\mathrm{D}}\lambda_{(3/2-1/2)}+C_i\lambda_i}. \qquad (18)$$

The term $[1-(1-T)f_{df}]$ is included in the denominator of the expression for Γ_{pure} to account for the recycling of the muon. This was included by Fetkovitch *et al.*[7] as $(1-f_{df})$. In order to take into account the loss of muons via trapping on the product nuclei, it is necessary to insert the term $(1-T)$. The term $[1-(1-T)f_{df}]$ is removed from the denominator of the expression for Γ_{impure} because the presence of the impurity reduces the contribution of recycling of the muon to the value of N_f (cf. Sec. IIIC). The increase in the population of the lower hyperfine levels of the $(d\mu p)^+$ ion can be suppressed if $C_i\lambda_i>C_{\mathrm{D}}\lambda_{(3/2-1/2)}$, i.e., $\Gamma_{\mathrm{impure}}\simeq 0$. Then the hyperfine levels of the $d\mu$ muonic atom will maintain

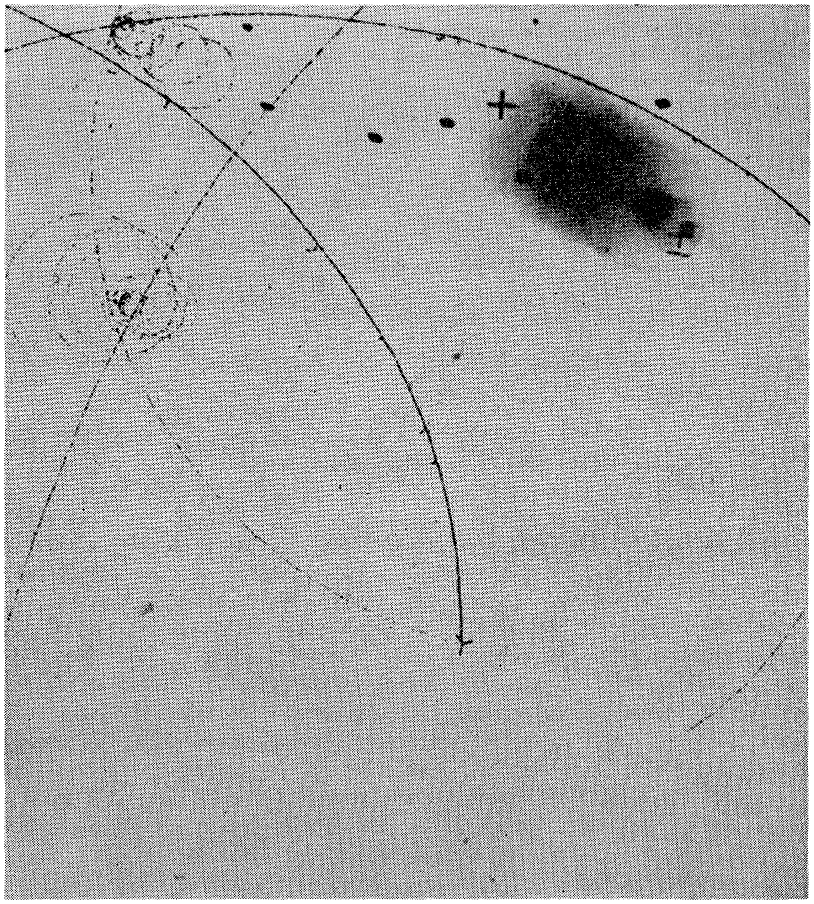

FIG. 4. An example of three successive fusions of the type $(d\mu d)^+ \to p+t+\mu^-$. The decay electron is also visible.

their statistical population, and the frequency of the $(d\mu p)^+$ fusion reaction will be identical to that in liquid protium.

Assuming that in impure deuterium we have suppressed reaction (14) as indicated above, we can determine Σ experimentally from the equation

$$\Sigma = \frac{N_{\rm rej}(P)}{N_{\rm rej}(I)} \cdot \frac{N_f(I)}{N_f(P)} \cdot \frac{C_{\rm H}(I)}{C_{\rm H}(P)}, \tag{19}$$

where (P) refers to experiments in pure deuterium, and (I) refers to experiments in impure deuterium. Gershtein[13] has calculated that Σ should be 1.86 if $\Gamma_{\rm pure}$ is equal to one. We may also determine an upper and a lower limit to the value of $\lambda_{(3/2-1/2)}$ by assuming that $\Gamma_{\rm pure}=1$ and $\Gamma_{\rm impure}=0$. Under these assumptions

$$\lambda_0 + C_{\rm H}(P)\lambda_{pd} + [1-(1-T)f_{df}]\lambda_{dd} < \lambda_{(3/2-1/2)} < \lambda_0 + C_{\rm H}(I)\lambda_{pd} + \lambda_{dd} + C_i\lambda_i,$$

where $C_{\rm D}(I) = C_{\rm D}(P) \simeq 1$. An upper limit to the value of ξ may be derived from the constraint that the product $\Sigma\xi$ must be less than one, i.e., that no more than 100% of all $(d\mu p)^+$ ions can be formed in the fusion-favored hyperfine levels in pure deuterium. The experimental upper limit to ξ is presented in Sec. VI and Table II.

C. $(d\mu d)^+$-Formation and Fusion Rates

When negative muons stopping in a liquid-deuterium bubble chamber catalyze the fusion reaction (10) giving a proton and a triton, the reaction can be detected by observing the 3.04-MeV proton (cf. Fig. 4). If the fusion proceeds by the other branch, yielding a neutron and a He^3 nucleus [reaction (9)], then no visible tracks are formed. If we assume, by charge independence, that the two branches are equally probable, then the total number of deuteron-deuteron fusions will be twice the number of fusions producing 3.04-MeV protons. Thus, N_f = [fraction of $d\mu$ muonic atoms which form $(d\mu d)^+$] $\times$ [fraction of $(d\mu d)^+$ which undergo a fusion reaction]. An inspection of Fig. 2 shows that N_f will be given by

$$N_f = \frac{C_{\rm D}\lambda_{dd}f_{df}}{\lambda_0 + C_{\rm H}\lambda_{pd} + C_{\rm D}[1-(1-T)f_{df}]\lambda_{dd}}. \tag{20}$$

Experimentally, the value for N_f may be obtained by the equation

$$N_f = \frac{2N_{1f}/\epsilon_{1f} + 4N_{2f}/\epsilon_{2f} + 6N_{3f}/\epsilon_{3f}}{N_s}.$$

The factors of 2, 4, and 6 contained in the numerator arise from considerations of the effects of multiple fusions and the branching ratio between reactions (9) and (10). The equation is derived in the Appendix, Sec. B. T, the trapped fraction, is determined experimentally by the following method. If we set N_F equal to the probability of a stopping muon catalyzing at least one visible fusion reaction, then if there were no muons removed from the reaction sequence by trapping, the probability of seeing a double fusion N_d should be simply N_F^2. However, if the trapping effect exists, the expression is

$$N_d=(1-T)N^2{}_F,$$

where

$$N_d=N_{2f}/\epsilon_{2f}N_s$$

$$N_F=\frac{N_{1f}/\epsilon_{1f}+N_{2f}/\epsilon_{2f}+N_{3f}/\epsilon_{3f}}{N_s}.$$

Note that N_F is not the total probability of a fusion reaction but is the probability that a stopping muon will undergo at least one visible fusion event. We may now include the branch which emits a neutron and a He^3 nucleus quite simply as it enters only as a constant factor of 2. The rate equation

$$N_f=\frac{C_D\lambda_{dd}f_{df}}{\lambda_0+C_H\lambda_{pd}+C_D[1-(1-T)f_{df}]\lambda_{dd}}, \tag{20}$$

will describe the deuteron-deuteron fusion reaction for all experiments performed at relatively high deuterium concentrations. Therefore, in order to determine the values of λ_{dd} and f_{df} separately, it is necessary to insert some mechanism which will alter the denominator in the expression. The method employed is based on the work of Schiff[34] and Dzhelepov *et al.*[35] on the transfer reactions between hydrogen isotopes and high Z materials. In the presence of nitrogen and oxygen, the $d\mu$ muonic atom can undergo a transfer reaction with the formation of $N\mu$ or $O\mu$ muonic species in which the muon can only decay or undergo nuclear capture. These reactions will reduce the probability of $(d\mu d)^+$ molecular ion formation, thereby reducing the fusion probability. If the probability of a fusion reaction is small, it is possible to neglect the recycling of the muon, therefore, we may remove the term $[1-(1-T)f_{df}]$ from the denominator of the equation. As the rates of the transfer reaction are very large, it is possible to add an amount of air to a bubble chamber which will sufficiently reduce the probability of $(d\mu d)^+$ molecular ion formation. Then the expression for λ_{df} and f_{df} becomes

$$N_f=\frac{C_D\lambda_{df}f_{df}}{\lambda_0+C_H\lambda_{pd}+C_D\lambda_{dd}+C_i\lambda_i}. \tag{21}$$

The denominator in the expression may be evaluated from the yield of rejuvenated muons in the same sample (cf. Appendix, Sec. A). Thus, we obtain two equations in λ_{dd} and λ_{df} which may be solved simultaneously for both variables. The results of our investigation using the above method are presented in Sec. VI and Table II.

D. Formation of $(He^3\mu)^+$

The $(He^3\,\mu)^+$ ion will be produced in each $(d\mu p)^+$ fusion reaction which proceeds via the emission of a γ ray [reaction (6)]. It can also be formed in a $(d\mu d)^+$ fusion reaction where the catalyzing muon is bound in an atomic state around the recoiling He^3 product nucleus produced in reaction (9).

We may obtain an upper limit to the percentage of $(He^3\,\mu)^+$ muonic ions formed at a specific deuterium and hydrogen concentration in liquid deuterium if we neglect trapping by the triton produced in reaction (10) and assume that muons are trapped only on He^3 product nuclei. Then the fraction of stopping muons which ultimately form the $(He^3\,\mu)^+$ muonic ion would be given by

$$f(He^3\mu)^+=\left[T(N_f)+N_{\rm rej}\left(\frac{1}{\eta}\right)-1\right]. \tag{22}$$

The first term accounts for the trapping in $(d\mu d)^+$ fusion reactions, while the second term arises from $(d\mu p)^+$ fusion reactions which emit a γ ray. The experimental value of $f(He^3\,\mu)^+$ for one concentration of deuterium is presented in Sec. VI.

IV. EXPERIMENTAL METHOD

The two reactions which produce a visible product in a hydrogen or deuterium bubble chamber are:

$$(d\mu p)^+\to He^3+\mu^- \tag{a}$$

$$(d\mu d)^+\to p+t, \tag{b}$$

where reaction (a) can be detected by the rejuvenated muon and reaction (b) by the recoil proton. The experiment is divided into three runs corresponding to the three different media in the bubble chamber. They are:

run (H_2) Liquid protium containing 22 parts per million (ppm) of deuterium. The sequence of reactions in this medium is shown in Fig. 1.

run (D_2) Liquid deuterium containing 0.96% protium. The sequence of reactions under these conditions is shown in Fig. 2.

run (D_2I) Liquid deuterium containing 1.2% hydrogen as an isotopic impurity and a small amount of air as an impurity. The sequence of reactions is shown in Fig. 2.

Negative muons from the University of Chicago synchrocyclotron were stopped in the Chicago 9-in. bubble chamber operating in a magnetic field of 20.5 kG. The muon beam contained less than 1% pions. These pions were excluded in the data analysis by range versus curvature measurements.

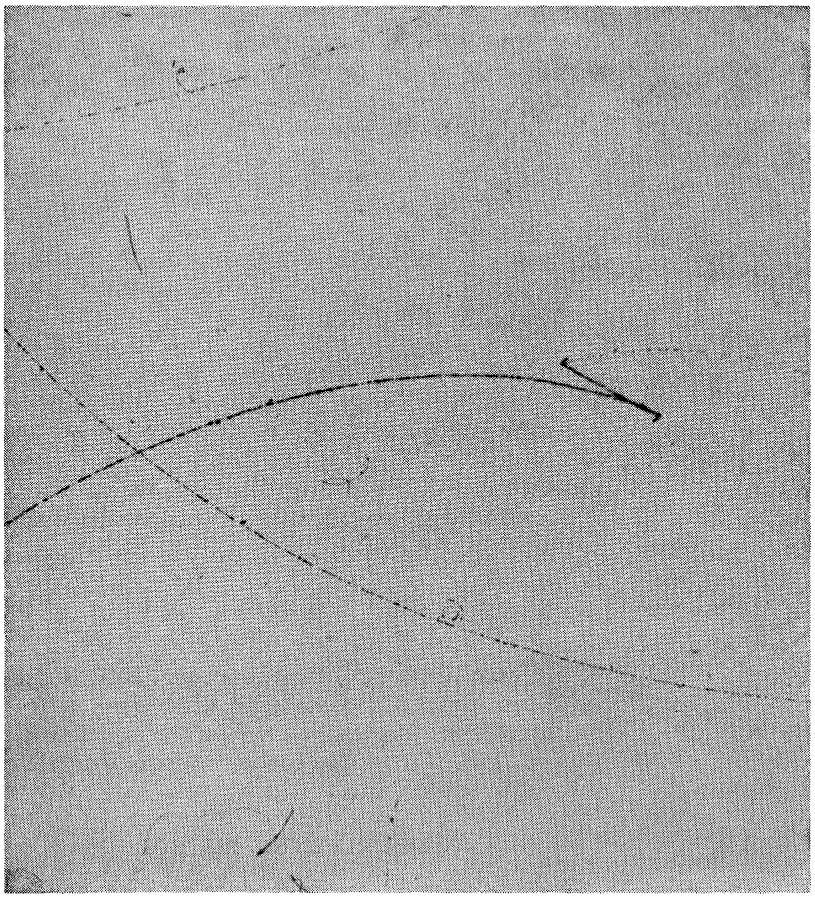

FIG. 5. An example of three successive fusions of the type $(d\mu d)^+ \to p+t+\mu^-$, $(d\mu p)^+ \to He^3+\mu-$, $(d\mu d)^+ \to p+t+\mu^-$.

Approximately 275 000 pictures were taken in three stereo views. They were scanned in commercial microfilm readers with a projection twice life size. All frames were scanned by two scanners working independently. This procedure gave efficiencies of 99.97% for decaying muons, 99.04% for protons from reaction (b), 99.16% for the rejuvenated muons from reaction (a), and 99.72% for all types of double and triple fusions.

The selection criteria for recoil protons were the following: (i) Projected length >0.5 mm and <3.0 mm. (ii) Angle between recoil and stopping meson track $\geqslant 20°$ and $\leqslant 160°$.

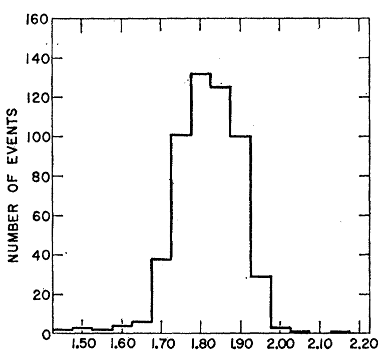

FIG. 6. Range distribution of rejuvenated muons from the $(p\mu d)^+$ fusion reaction in run H_2 (pure hydrogen containing 22 ppm D_2).

In addition, events were separated as to whether or not a decay electron originated from the vertex of the muon and recoil tracks. The selection of rejuvenated muons was based on the following: (i) Track length >5.0 mm and <36 mm. (ii) Angle between meson track and recoil track $\geqslant 20°$ but $\leqslant 160°$. (iii) Absence of decay electron at vertex of meson and recoil tracks. The events were separated as to whether or not a decay electron was seen at the end of the event track. This separation was made to check on the frequency of nuclear capture interactions.

Any muon track which had two recoiling particles which met either of the above sets of criteria was classified as a double fusion. In addition, in runs D_2 and D_2I, six meson tracks were found to have three recoil tracks which satisfied the selection criteria (cf. Figs. 5 and 6). All events were measured on a conventional digital projection measuring machine and then analyzed by a digital computer.

The isotopic ratios of the media were determined by mass spectrometric measurements of representative samples of the gas used in the chamber. In run D_2I, sampling was performed while the chamber was in

TABLE III. Experimental data and results listed by run (the symbols are defined in the Appendix, Sec. A1).

		Run H_2	Run D_2	Run D_2I
Observed numbers of events	N_s	270,967	11,465	13,542
	N_{1f}	...[b]	547 ± 23	375 ± 19
	N_{2f}	...[b]	42 ± 6.5	32 ± 5.6
	N_{3f}	...[b]	2 ± 1.4	$1_{-0.5}^{+1.0}$
	N_μ	652 ± 25	37 ± 6	13 ± 3.6
Numbers of events corrected	$2N_{1f}/\epsilon_{1f}$	...	1634 ± 69	1120 ± 57
	$4N_{2f}/\epsilon_{2f}$	...	179 ± 28	136 ± 24
	$6N_{3f}/\epsilon_{3f}$	...	13.2 ± 9.2	$6.6_{-3.3}^{+6.6}$
	N_μ/ϵ_μ	838 ± 32	47.5 ± 7.7	16.7 ± 4.6
	N_f[a]	...	$(15.92\pm0.62)10^{-2}$	$(9.32\pm0.46)10^{-2}$
	N_{rej}[a]	$(3.09\pm0.12)10^{-3}$	$(4.14\pm0.67)10^{-3}$	$(1.23\pm0.27)10^{-3}$

[a] $Nf/2$ is the number of $(d\mu d)^+$ fusion reactions per stopping muon. N_{rej} is the number of $(d\mu p)^+$ fusion reactions which emitted a rejuvenated muon per stopping muon (cf. Sec. III of text).
[b] Due to the low concentration of deuterium, no scanning was done for $(d\mu d)^+$ fusion events.

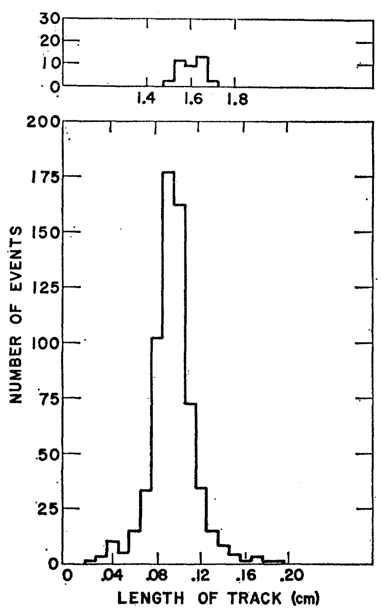

FIG. 7. Range distribution of events in run D_2 (pure D_2 containing 0.96% H_2). The upper section shows the rejuvenated muons from $(p\mu d)^+$ fusion, and the lower section the recoil protons from the $(d\mu d)^+$ fusion reaction.

operation and on the exhausting gas while the chamber was being emptied. The results of the analysis for run D_2I were treated so as to minimize the effect of fractionation of the isotopes. The deuterium concentrations C_D for the three runs were found to be

$$C_D(H_2)=(2.2\pm0.2)10^{-5},$$
$$C_D(D_2)=0.9904\pm0.001,$$
$$C_D(D_2I)=0.988\pm0.001.$$

V. RESULTS

The range distribution of recoil protons and rejuvenated muons for runs H_2, D_2, and D_2I are presented in Figs. 6, 7, and 8, respectively. A summary of events found in each run and the values of N_{rej} and N_f derived from them are presented in Table III.

Various sources of background events for both types of visible fusion reactions have been investigated. In the $(d\mu p)^+$ fusion reaction with the emission of a rejuvenated muon, there are two possible sources of background events.

A negative pion, entering the chamber, can be mistaken for a muon and can decay in flight in the forward direction. If the resulting muon stops in the chamber and decays, and if the range of the decay product muon is sufficiently large, this event could satisfy the scanning selection criteria for a rejuvenated muon. Also, an incoming muon undergoing a large angle (>20°) scattering reaction before it decays can appear to be a rejuvenated muon if the range of the muon after the scattering is in the acceptable length interval. An inspection of the spectra for the three runs shows that the combined background from both sources is negligible.

There are two processes which could produce an event that is indistinguishable from the recoil proton produced in a $(d\mu d)^+$ fusion reaction. These are: (1) the chance production in a small region (a circle with a radius of 0.5 cm) around the end point of a stopping

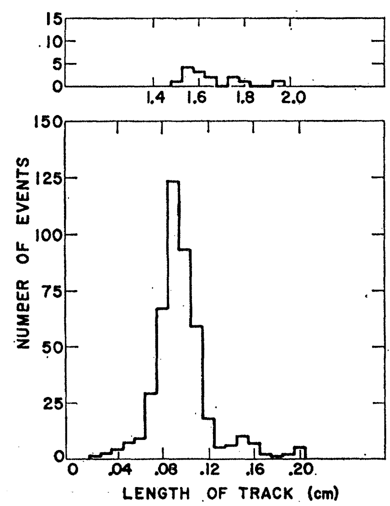

FIG. 8. Range distribution of events in run D_2I (deuterium containing 1.2% H_2 and $<5\times10^{-5}$ parts of air). The upper section shows the rejuvenated muons from the $(p\mu d)^+$ fusion reaction, and the lower section the recoil protons from the $(d\mu d)^+$ fusion reaction.

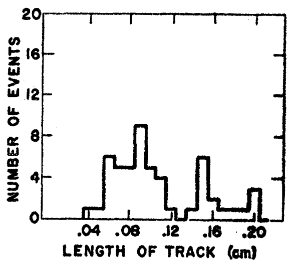

FIG. 9. Range distribution of recoil protons from the $(d\mu d)^+$ fusion reaction in run D_2I (deuterium containing 1.2% H_2 and $<5\times10^{-5}$ parts of air) which did not have an associated decay electron.

muon track of a recoil proton in the allowable range interval by an energetic neutron, and (2) the possibility that a muon having undergone a transfer reaction to some high-Z element could undergo nuclear capture with the production of one or more charged recoil particles of the correct range.

The contribution of random recoil protons to the recoil proton spectrum from reaction (10) can be estimated by geometric considerations. The number of background events produced by random recoil protons should increase as r^3, where r is the distance from the end point of the stopping muon track to the first point of the recoil track. As there is no reason to anticipate a gap being produced in a $(d\mu d)^+$ fusion reaction, the recoil proton is required to originate within a radius of 5×10^{-2} cm of the stopping muon. In practice, this gap length cutoff removed only three events from a total of 1079 acceptable recoils, indicating that the contribution of random recoil protons is negligible.

The possible contribution of high Z nuclear capture can also be shown to be small. In run D_2, there were only four fusion recoils which did not have an associated decay electron. Assuming that all four were due to high Z nuclear capture with charged particle emission, the ratio of capture recoils to fusion recoils is 4/637; far less than the statistical fluctuation on the total number of recoils. In run D_2I, there were 42 out of 445 fusion recoils which did not have an associated decay electron. The energy spectrum of recoil protons from muon capture in a high Z nucleus is expected to be an evaporation spectrum. A histogram of the 42 suspected events is shown in Fig. 9 As may be seen by comparing Figs. 8 and 9, the possible contribution of nuclear capture is small.

Using the experimental values of N_f and N_{rej} presented in Table III, we have obtained the results shown in Table II. The equations used in the data analysis are derived in the Appendix, Sec. A. The errors quoted for the various results arise from two sources: (1) the statistical errors of the experimental results, and (2) the errors associated with the rates λ_{pp}, λ_e, λ_{pd} as reported by Conforto *et al.*[30] and λ_{pf} as reported by Bleser *et al.*[6]

The chief source of the quoted error for the reported value of η is in the errors of values for the four rates given above. For Σ, $\lambda_{dd}f_{df}$, λ_{dd}, and λ_{df}, the major contribution to the error comes from the statistical error of N_{rej} for run D_2I, which is based on thirteen events. Table II also gives a comparison of our results with those predicted by various theoretical treatments and previous experiments.

VI. CONCLUSIONS

Our conclusions concerning the various measured quantities and their effect on the measurement of μ capture in deuterium may be summarized as follows:

A. η (cf. Sec. IIIA), the Production of Rejuvenated Muons in the $(d\mu p)^+$ Fusion Reaction

As may be seen in Table II, the value $\eta=(15.0\pm1.5)10^{-2}$ obtained from the solution of Eq. (15) for $N_{\mathrm{rej}}(H_2)$ is considerably higher than the results obtained by a simple comparison of the yields of γ rays and rejuvenated muons. However, the two methods give good agreement if our value is multiplied by the value of $\xi(=0.355)$. Inasmuch as the method based on comparison of the yields is independent of any hyperfine effect (cf. Sec. IIIA), these results indicate that the reported yield of γ rays from $(d\mu p)^+$ fusion reactions are high by a factor of ξ. That such is, in fact, the case is indicated in Ref. 36.

In the two counter experiments which have investigated the yield of γ rays from $(d\mu p)^+$ fusion reactions (Refs. 5 and 6), it has been the practice to fit the experimental yield versus time curve to a "parent-daughter" radio-active decay equation, the solution giving the two exponential rates. These rates have then been used to determine the number of stopping muons in the experimental sample. In both cases, neglecting the hyperfine level effect increases the yield rates by a factor of $1/\xi$. It should be noted that an accurate experimental determination of the yield of γ rays from $(d\mu p)^+$ fusions would allow a determination of the value of ξ.

B. Σ (cf. Sec. IIIB), the Hyperfine Effect in Liquid Deuterium

Our results indicate that all $d\mu$ muonic atoms formed in liquid deuterium quickly de-excite to the lower (1/2) hyperfine level, presumably by reaction (14) as predicted by Gershtein.[13] Our value for Σ, the increase in the yield of the $(d\mu p)^+$ fusion reaction in pure deuterium, is in qualitative agreement with the increase predicted by Gershtein,[13] who assumes that all $d\mu$ muonic atoms are in the (1/2) hyperfine level before formation of $(d\mu p)^+$. The limits on the rate for reaction (14) itself are somewhat lower than the limits predicted by Gershtein,[13] but they are in general agreement.

[36] E. Bleser (private communication).

C. λ_{dd}, the Rate of Formation of $(d\mu d)^+$

The value of $f_{df}\lambda_{dd}$ determined in this experiment is in good agreement with the previous experiments of Fetkovitch *et al.*[7] However, the value of λ_{dd} is somewhat higher than the theoretical predictions of Cohen, Judd, and Riddell[10] for a pure $M1$ transition. It is probable that the actual formation process includes a contribution from $E0$ transitions,[11] as the combined rate for the two processes is in better agreement with the experimental value.

D. λ_{df}, the Rate of Nuclear Fusion Reactions of $(d\mu d)^+$

The experimental values for λ_{df} and f_{df} are significantly lower than the qualitative estimates of Zel'dovitch and Gershtein.[11]

If the $(d\mu d)^+$ ion is formed in both the P and S states, as is indicated above, than the most likely explanation for the discrepancy between the experimental and theoretical values for f_{df} and λ_{df} is that the fusion reaction proceeds very slowly, if at all, from the initial P state, while the S state reaction is quite fast.

Assuming that the mixture of S and P state is approximately 1:2 as indicated by comparison of the theoretical estimates for the $M1$ and $E0$ formation rates given above, then $\frac{1}{3}$ of the $(d\mu d)^+$ ions will be formed in the S state. This then gives for the S-state fusion rate

$$\lambda_{df}(S) \gtrsim 10^7,$$

and for the P state

$$\lambda_{df}(P) \lesssim 10^4.$$

E. The Effect of Fusion Reactions on Muon Capture Reactions in Liquid Deuterium

The investigation of the capture rate of muons in liquid deuterium requires that very few of the stopping muons form $(\mathrm{He}^3\,\mu)^+$ (cf. Sec. I). Capture by He^3 nuclei is expected to have a rate[37] $\bar{\Lambda}_{\mathrm{He}^3} = 2500\ \mathrm{sec}^{-1}$, a factor of five higher than the known capture rate in protium.[14] Thus, to obtain a meaningful upper limit to the muon capture rate in deuterium, the maximum permissible value of $f(\mathrm{He}^3\,\mu)^+$ is approximately 4%. Using the data of run D_2, we find

$$f(\mathrm{He}^3\,\mu)^+ = (4.7\pm0.5)10^{-2}$$

which is derived from Eq. (22) (Sec. IIID).

$$f(\mathrm{He}^3\mu)^+ = \left[TN_f + N_{\mathrm{rej}}\left(\frac{1}{\eta}-1\right)\right] \tag{22}$$
$$= [(2.3\pm0.75)+(2.4\pm0.4)]10^{-2}.$$

We see that there is a substantial contribution to $f(\mathrm{He}^3\,\mu)^+$ in run D_2 from the second term. As N_{rej} is linearly dependent on the protium concentration, a reduction in the concentration by a factor of five (from 1% to 0.2%) should drop the value of $f(\mathrm{He}^3\,\mu)^+$ to approximately 3%. Specifically, an investigation of muon capture in deuterium of 99.8% isotopic purity would have an inherent background due to He^3 capture $\sim$67 sec^{-1}.

[37] H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959).

APPENDIX

A. Derivation of Equations

1. Equation for η.

From

$$N_{\mathrm{rej}}(\mathrm{H}_2) = \frac{C_{\mathrm{D}}(\mathrm{H}_2)\lambda_e}{\lambda_0+\lambda_{pp}+C_{\mathrm{D}}(\mathrm{H}_2)\lambda_e}\,\frac{\lambda_{pd}}{\lambda_0+\lambda_{pd}}\,\xi f_{pf}\eta,$$

we obtain directly

$$\eta = N_{\mathrm{rej}}(\mathrm{H}_2)\frac{[\lambda_0+\lambda_{pp}+C_{\mathrm{D}}(\mathrm{H}_2)\lambda_e](\lambda_0+\lambda_{pd})}{C_{\mathrm{D}}(\mathrm{H}_2)\lambda_e\lambda_{pd}\xi f_{pf}}.$$

2. Equations for λ_{dd} and λ_{df}.

From run D_2I

$$N_{\mathrm{rej}}(\mathrm{D}_2I) = \frac{C_{\mathrm{H}}(\mathrm{D}_2I)\lambda_{pd}\xi f_{pf}\eta}{\lambda_0+\lambda_{dd}+C_{\mathrm{H}}(\mathrm{D}_2I)\lambda_{pd}+C_i\lambda_i}$$

$$\lambda_0+\lambda_{dd}+C_{\mathrm{H}}(\mathrm{D}_2I)\lambda_{pd}+C_i\lambda_i = \frac{C_{\mathrm{H}}(\mathrm{D}_2I)\lambda_{pd}\xi f_{pf}\eta}{N_{\mathrm{rej}}(\mathrm{D}_2I)}$$

$$N_f(\mathrm{D}_2I) = \frac{\lambda_{dd}f_{df}}{\lambda_0+\lambda_{dd}+C_{\mathrm{H}}(\mathrm{D}_2I)\lambda_{pd}+C_i\lambda_i}$$

$$\lambda_{dd}f_{df} = N_f(\mathrm{D}_2I)\left[\frac{C_{\mathrm{H}}(\mathrm{D}_2I)\lambda_{pd}f_{pf}\eta\xi}{N_{\mathrm{rej}}(\mathrm{D}_2I)}\right].$$

Substituting for η

$$\lambda_{dd}f_{df} = N_f(\mathrm{D}_2I)\left[\frac{C_{\mathrm{H}}(\mathrm{D}_2I)}{C_{\mathrm{D}}(\mathrm{H}_2)\lambda_e}\,\frac{N_{\mathrm{rej}}(\mathrm{H}_2)}{N_{\mathrm{rej}}(\mathrm{D}_2I)} \times\{\lambda_0+\lambda_{pp}+C_{\mathrm{D}}(\mathrm{H}_2)\lambda_e\}(\lambda_0+\lambda_{pd})\right].$$

From run D_2 we have

$$N_f(\mathrm{D}_2) = \frac{\lambda_{dd}f_{df}}{\lambda_0+C_{\mathrm{H}}(\mathrm{D}_2)\lambda_{pd}+[1-(1-T)f_{df}]\lambda_{dd}}$$

$$\lambda_{dd} = \left[\frac{1}{N_f(\mathrm{D}_2)}+(1-T)\right]\lambda_{dd}f_{df} - [\lambda_0+C_{\mathrm{H}}(\mathrm{D}_2)\lambda_{pd}].$$

Now, substituting for $\lambda_{dd}f_{df}$ from run D_2I

$$\lambda_{dd}=\left[\frac{1}{N_f(D_2)}+(1-T)\right]\left\{N_f(D_2I)\frac{N_{\text{rej}}(H_2)}{N_{\text{rej}}(D_2I)}\frac{C_H(D_2I)}{C_D(H_2)\lambda_e}\times[\lambda_0+\lambda_{pp}+C_D(H_2)\lambda_e](\lambda_0+\lambda_{pd})\right\}-[\lambda_0+C_H(D_2)\lambda_{pd}].$$

Knowing λ_{dd}, we can substitute the value in the expression for $\lambda_{dd}f_{df}$ to obtain a value for f_{df}.

By definition, $f_{df}=\lambda_{df}/\lambda_0+\lambda_{df}$, so we can determine λ_{df}.

3. Equation for Σ *and* $\lambda_{(3/2-1/2)}$.

From run D_2I we have

$$N_f(D_2I)=\frac{\lambda_{dd}f_{df}}{\lambda_0+C_H(D_2I)\lambda_{pd}+\lambda_{dd}+C_i\lambda_i}$$

$$N_{\text{rej}}(D_2I)=\frac{C_H(D_2I)\lambda_{pd}f_{pf}\eta\xi}{\lambda_0+C_H(D_2I)\lambda_{pd}+\lambda_{dd}+C_i\lambda_i}$$

$$\frac{N_f(D_2I)}{N_{\text{rej}}(D_2I)}=\frac{\lambda_{dd}f_{df}}{C_H(D_2I)\lambda_{pd}f_{pf}\eta\xi}.$$

From D_2 we have

$$N_f(D_2)=\frac{\lambda_{dd}f_{df}}{\lambda_0+C_H(D_2)\lambda_{pd}+[1-(1-T)f_{df}]\lambda_{dd}}$$

$$N_{\text{rej}}(D_2)=\frac{C_H(D_2)\lambda_{pd}f_{pf}\Sigma\xi\eta}{\lambda_0+C_H(D_2)\lambda_{pd}+[1-(1-T)f_{df}]\lambda_{dd}}$$

$$\therefore\ \Sigma=\frac{N_{\text{rej}}(D_2)}{N_f(D_2)}\frac{\lambda_{dd}f_{df}}{C_H(D_2)\lambda_{pd}f_{pf}\eta\xi}$$

$$\Sigma=\frac{N_{\text{rej}}(D_2)}{N_f(D_2)}\cdot\frac{N_f(D_2I)}{N_{\text{rej}}(D_2I)}\cdot\frac{C_H(D_2I)}{C_H(D_2)}.$$

We can obtain upper and lower limits to the value of $\lambda_{(3/2-1/2)}$ in the following manner.

If we assume that only a very small fraction of the $d\mu$ muonic atoms formed in the (3/2) hyperfine level undergo reaction (14) (cf. Sec. IIIC),

$$d\mu(3/2)+d\to d+d\mu(1/2). \qquad (14)$$

In run D_2I we have

$$\Gamma=\frac{\lambda_{(3/2-1/2)}}{\lambda_0+C_H(D_2I)\lambda_{pd}+\lambda_{dd}+C_i\lambda_i+\lambda_{(3/2-1/2)}}$$

and since Γ is small,

$$\lambda_{(3/2-1/2)}<\lambda_0+C_H(D_2I)\lambda_{pd}+\lambda_{dd}+C_i\lambda_i$$

$$\lambda_{(3/2-1/2)}<\frac{\lambda_{dd}f_{df}}{N_f(D_2I)}.$$

Likewise, if we assume that for run D_2 that Γ is large, we have where

$$\Gamma=\frac{\lambda_{(3/2-1/2)}}{\lambda_0+C_H(D_2)\lambda_{pd}+[1-(1-T)f_{df}]\lambda_{dd}+\lambda_{(3/2-1/2)}}$$

$$\lambda_{(3/2-1/2)}>\lambda_0+C_H(D_2)\lambda_{pd}+[1-(1-T)f_{df}]\lambda_{dd}$$

$$\lambda_{(3/2-1/2)}>\frac{\lambda_{dd}f_{df}}{N_f(D_2)}.$$

B. Derivation of the Expression·

$$N_f=\frac{2N_{1f}/\epsilon_{1f}+4N_{2f}/\epsilon_{2f}+6N_{3f}/\epsilon_3}{N_s}.$$

Experimentally in each run we detected a certain number of $(d\mu d)^+$ fusion events. These may be represented by N_{1f}=number of single fusions detected; N_{2f}=number of double fusions detected; N_{3f}=number of triple fusions detected, and N_s=total number of stopping muons. For each type of event there is an associated solid angle and length cutoff detection efficiency, i.e., ϵ_{1f}, ϵ_{2f}, and ϵ_{3f}, respectively. Assuming a 1:1 branching ratio for the $(d\mu d)^+$ fusion reaction

$$(d\mu d)^+ \nearrow \mathrm{He}^3+n+\mu^- \qquad (9)$$

$$(d\mu d)^+ \searrow p+t+\mu^-, \qquad (10)$$

we will see only $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$ or $\frac{1}{8}$ of all triple fusions. Thus, we have $(8/\epsilon_{3f})N_{3f}$ as the true number of triple fusions in our sample. However, $\frac{3}{8}$ of these $(8/\epsilon_{3f})$ triple fusions will have appeared as double fusions. If we define $C=(8/\epsilon_{3f})N_{3f}$, then we subtract $\frac{3}{8}\epsilon_{2f}C$ from N_{2f}, the number of double fusions in our sample, and $\frac{3}{8}\epsilon_{1f}C$ from N_{1f}, the number of single fusions in our sample. Now for the double fusions, we have $(N_{2f}-\frac{3}{8}\epsilon_{2f}C)$ in our sample. Due to the branching ratio between (9) and (10), we will detect only $\frac{1}{4}$ of the double fusions. If B is defined as the true number of double fusions in our sample,

$$B=\frac{4}{\epsilon_{2f}}(N_{2f}-3/8\epsilon_{2f}C).$$

As before, we must subtract from N_{1f}, the number of single fusions, the quantity $\frac{1}{2}\epsilon_{1f}B$.

Therefore, we have $(N_{1f}-\frac{1}{2}\epsilon_{1f}B-\frac{3}{8}\epsilon_{1f}C)$ single fusions in our sample. Correcting for the branching ratio and detection efficiency, we find that the true number of single fusions is $2/\epsilon_{1f}(N_{1f}-\frac{1}{2}\epsilon_{1f}B-\frac{3}{8}\epsilon_{1f}C)$. Thus, we have for a total of each event type in our sample:

$$\text{true number of triple fusions}=\frac{8}{\epsilon_{1f}}N_{3f},$$

$$\text{true number of double fusions}=\frac{4N_{2f}}{\epsilon_{2f}}-12\frac{N_{3f}}{\epsilon_{3f}},$$

$$\text{true number of single fusions}=\frac{2N_{1f}}{\epsilon_{1f}}-\frac{4N_{2f}}{\epsilon_{2f}}+\frac{6N_{3f}}{\epsilon_{3f}}.$$

Now N_f is defined as the number of fusion reactions per stopping muon. Therefore,

$$N_f=\frac{1\left(\frac{2N_{1f}}{\epsilon_{1f}}-\frac{4N_{2f}}{\epsilon_{2f}}+\frac{6N_{3f}}{\epsilon_{3f}}\right)+2\left(\frac{4N_{2f}}{\epsilon_{2f}}-12\frac{N_{3f}}{\epsilon_{3f}}\right)+3\left(\frac{8}{\epsilon_{3f}}N_{ef}\right)}{N_s}$$

$$=\frac{\frac{2N_{1f}}{\epsilon_{1f}}+\frac{4N_{2f}}{\epsilon_{2f}}+\frac{6N_{3f}}{\epsilon_{3f}}}{N_s}.$$

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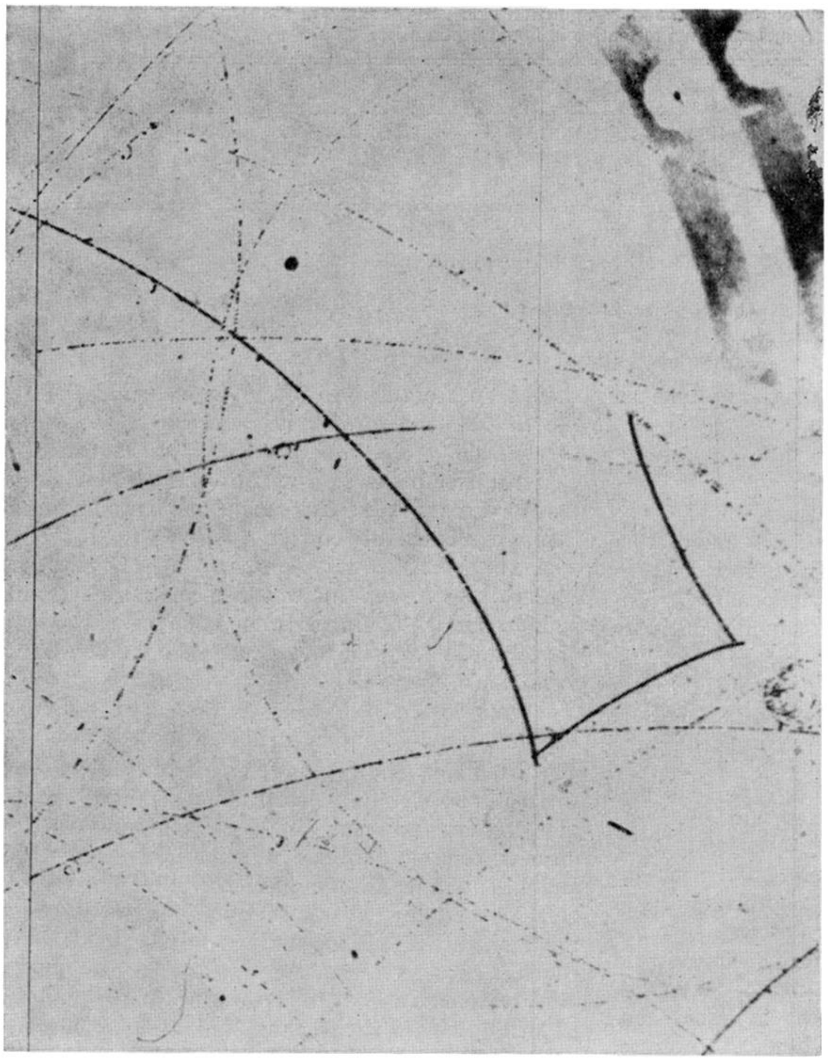

FIG. 3. An example of two successive $(d\mu p)^+$ fusion events in liquid hydrogen. Note the gaps between the stopping muon and the first rejuvenated muon and between the first and second rejuvenated muons.

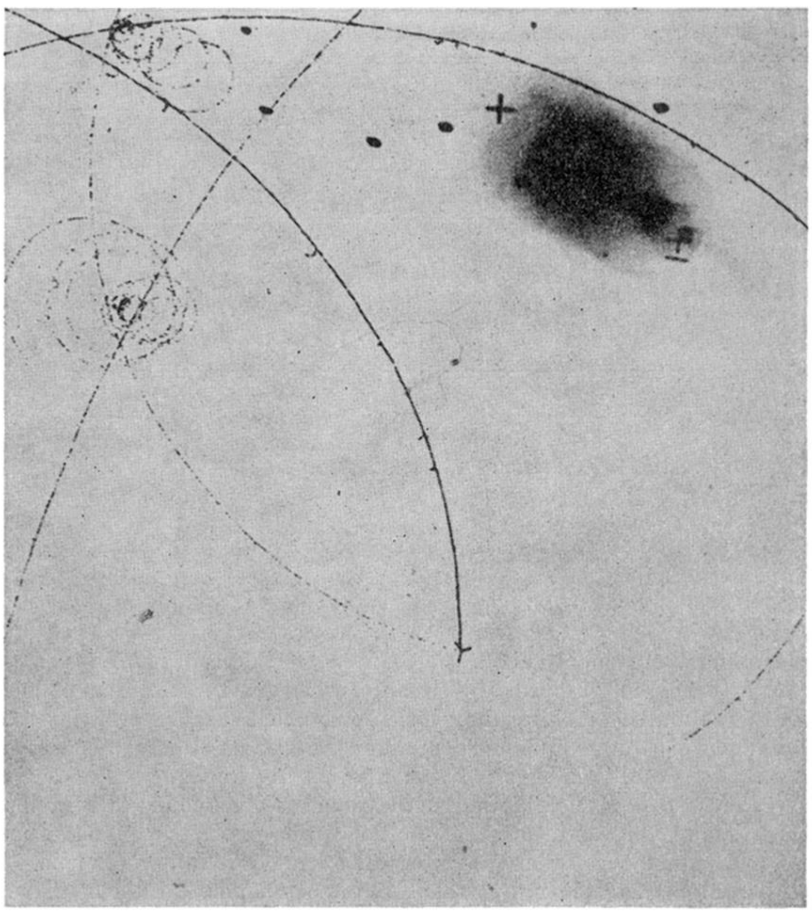

FIG. 4. An example of three successive fusions of the type $(d\mu d)^+ \rightarrow p+t+\mu^-$. The decay electron is also visible.

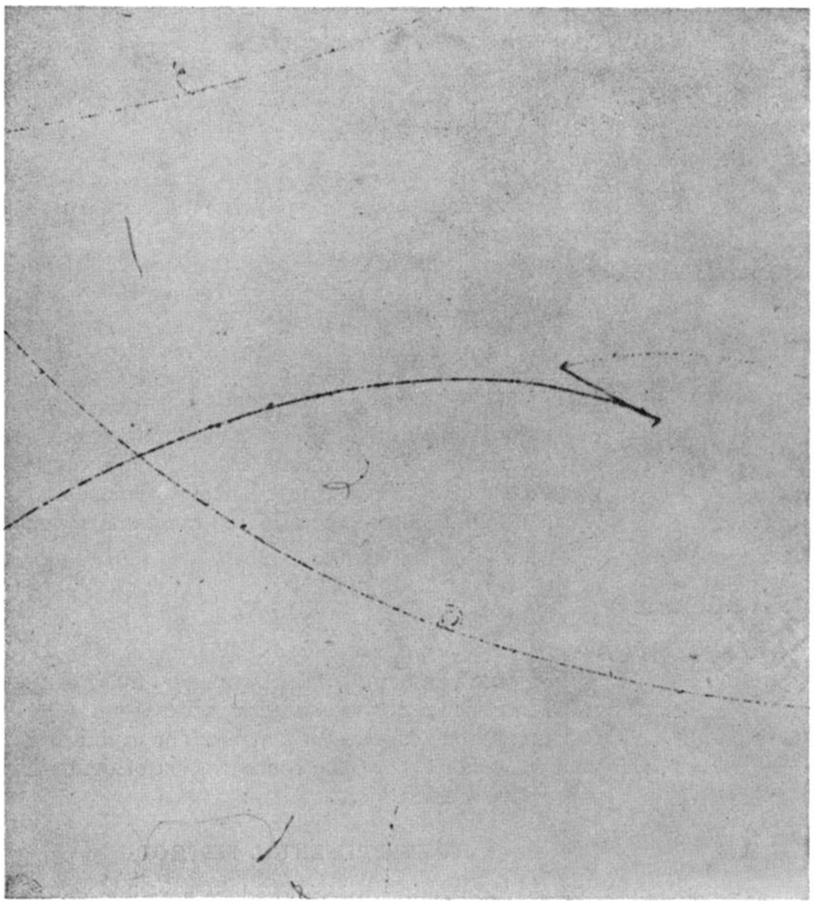

FIG. 5. An example of three successive fusions of the type $(d\mu d)^+ \rightarrow p+t+\mu^-$, $(d\mu p)^+ \rightarrow He^3+\mu-$, $(d\mu d)^+ \rightarrow p+t+\mu^-$.