ing remarks must be kept in mind. When the Q value is quite different from zero it is not clear whether a correlation can be made between the angle at which $d\sigma/d\Omega$ peaks and the distance of closest approach. It would be expected that particles resulting from reactions with Q values quite different from zero undergo a large deflection. This would not necessarily mean, however, that the transfers took place when the interacting nuclei were close together.

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Nuclear Elastic Scattering of Monoenergetic Neutron-Capture Gamma Rays^{*†}

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A source of monoenergetic gamma rays produced by neutron capture in various elements is described. A search was made for elastic scattering of these neutron-capture gamma rays from Ta, Hg, Pb, and Bi. A large scattered signal was observed when 7.285-MeV gamma rays produced by neutron capture in iron were incident upon a lead target. Evidence is presented which indicates that this signal results from resonance scattering of these gamma rays by a level in Pb208 with an angular momentum quantum number of unity, and with a ground-state radiative width which is less than 4 eV. An argument is presented which indicates that it is probable that the spacing near 7 MeV in Pb²⁰⁸ of levels of the type observed is of the order of magnitude of 1400 eV.

I. INTRODUCTION

UCLEAR elastic scattering of photons with energies of about 7 MeV has been observed in several investigations.¹⁻⁴ In each of these experiments the source of radiation was a continuous one, either the bremsstrahlung from an electron accelerator,^{1,3,4} or the Doppler-broadened radiation from a nuclear reaction.² In the work of Axel et al.⁴ a method was employed which limited the energy spread of the beam used to study the scattering process to about 100 keV. The interpretation of these experiments is based on the assumption that at energies below the threshold for particle emission the energy levels in a nucleus are discrete and separated by energies large compared to their widths. The observed photon scattering in this energy range is assumed to result because the incident photon spectrum overlaps one or more of these discrete levels.

In the experiments described in this paper, a search was made for nuclear elastic scattering of photons from the elements, Ta, Hg, Pb, and Bi using as a source of radiation a beam of γ rays emitted after neutron capture in various nuclei. The spectra of γ rays emitted following thermal neutron capture in various elements have been studied in many laboratories, and two compilations of the results of these studies have been published.^{5,6} From a few nuclei the capture γ -ray spectra consist of one or several intense γ -ray groups with energies between about 5 and 11 MeV, and a large number of γ rays with energies below 5 MeV. These intense, monoenergetic γ rays have been used to study some (γ, n) reactions,⁷ and an arrangement with which a beam of these γ rays could be extracted from a reactor has been described by Jarzyck et al.8

Since the spread of energy of a particular group of γ rays emitted in thermal neutron capture is only a few electron volts, it is expected that a search for nuclear elastic scattering of these γ rays should check the assumption that nuclear levels below the threshold for particle emission do not overlap. Further, it was hoped that the energies of some of the capture γ rays would

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Alamos, New Mexico. ¹ E. G. Fuller and Evans Hayward, Phys. Rev. 101, 692 (1956); Nucl. Phys. 33, 431 (1962). ² K. Riebal and A. K. Mann, Phys. Rev. 118, 701 (1960).

^aTsutomu Tohei, Masumi Sugawara, Shigeki Mori, and Motohara Kimara, J. Phys. Soc. Japan 16, 1657 (1961). ^aP. Axel, K. Min, N. Stein, and D. C. Sutton, Phys. Rev. Letters 10, 299 (1963).

⁶ G. A. Bartholomew and L. A. Higgs, Atomic Energy of Canada, Ltd. Report No. 669 (unpublished). ⁶ L. V. Groshev, U. N. Lutsenko, A. M. Demidov and V. I.

Pelekhov, Atlas of Gamma Ray Spectra from Radiative Capture of Thermal Neutrons (Pergamon Press, Inc., New York, 1959). ⁷ Robert E. Welsh and D. J. Donahue, Phys. Rev. 121, 880

^{(1961).}

⁸C. Jarczyk, H. Knoepfel, J. Land, R. Muller, and W. Wolfi, Nucl. Instr. Methods, 13, 207 (1961).



FIG. 1. Experimental arrangement.

coincide with the energies of some nuclear levels, so that the properties of the elastic scattering from these levels could be studied. Recently, the observation of scattering of capture γ rays from several nuclei has been reported by Ben-David (Davis) and Huebschmann.9

II. EXPERIMENTAL ARRANGEMENT

A. Gamma-Ray Source

A schematic drawing of the γ -ray source is shown in Fig. 1. The source of neutrons is a pool reactor, which produces 200 kW of power. Adjacent to the reactor is a 4-in.-thick bismuth plug, which reduces the background in the experimental area resulting from γ rays from the reactor. Next to the bismuth plug is the material in which the neutron-capture γ rays are produced. These γ rays then pass through LiF and paraffin filters and lead and iron collimators to the room in which the scattering experiments are performed. The filters preferentially reduce the number of neutrons and low energy γ rays in the beam. With the arrangement shown in Fig. 1, in which the beam passes through four inches of water, three inches of LiF, and 16 in. of paraffin, no thermal neutrons could be detected in the beam. An upper limit of 5 neutrons/cm² sec was determined for the thermal neutron flux. The magnitude of the fast neutron flux was measured by observing the 4.4-MeV γ rays produced by fast-neutron inelastic scattering from carbon. We estimate that, with the filters enumerated above, the flux of 5-MeV neutrons is about 200/cm² sec MeV. The particular arrangement shown in Fig. 1 is perhaps not the best that could be achieved, but considerable variation of filters and collimators showed that it was a good compromise between the conflicting requirements of high γ -ray intensity and low background.

⁹ G. Ben-David (Davis) and B. Heubschmann, Phys. Letters 3, 87, (1962).

A background of 7.7-MeV γ rays is always present in the beam, and results from γ rays produced by neutron capture in the aluminum in the reactor fuel elements. However, for most sources used, this background was less than 1% of the primary γ ray peak.

A summary of the composition of the beam from several different sources is presented in Table I. The intensities of the γ rays were obtained with filters and collimators as shown in Fig. 1, and for a reactor power of 200 kW. These intensities were measured at the outlet of the beam hole. For the titanium, manganese, and copper sources, γ rays other than the primary one listed in Table I contributed to the measured signal. Corrections for the contribution of these other γ rays were made by using the percent yield of the various γ rays given in the compilations,^{5,6} and listed in the table. The numbers given in Table I could be in error by as much

TABLE I.	Properties	of	capture	gamma-ray	beam.ª
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Provide the second				
Element	Mass of source (g)	Photon energies ^b (MeV)	Percent yields ^α γ rays/100 neutron captures	Measured intensities γ/cm^2 sec
Titanium	860	6.753 6.550 6.431	45 6 33	2.0×10 ⁶
Manganese	700	7.261 7.15 7.048	12 4.8 7.8	1.0×106
Lead	2170	7.38 6.73	100 7	2.0×10 ⁵
Iron	1920	7.639 7.285	34 4.4	2.0×10 ⁶
Copper	2050	7.914 7.634 7.296	21 10 6.8	3.0×10 ⁶

^{*} Background: Thermal neutrons, $<5/\text{cm}^2$ sec; Fast neutrons, $E_n \simeq 5$ MeV, $\simeq 200/\text{cm}^2$ sec; 7.7-MeV γ -rays, $\simeq 1000/\text{cm}^2$ sec. b Energies taken from Ref. 5. e Percent yields are averages of Refs. 5 and 6.



FIG. 2. Counts versus gamma-ray energy from which the scattered signal was obtained for the case of gamma rays from a manganese source incident upon a lead target.

as a factor of 2 and are presented only to give a qualitative idea of the intensities which can be obtained from a neutron-capture γ -ray source.

Since the natural widths for emission of γ rays after thermal-neutron capture are a few tenths of an electron volt,¹⁰ and since negligible nuclear recoil results from the capture of a thermal neutron, the energy distribution of a group of neutron-capture γ rays is determined by the thermal Doppler motion of the emitting nuclei. The half-width at half-maximum of a group of 7-MeV γ rays varies from 10 eV for emitting nuclei with mass number A = 20, to 3 eV for A = 200.

For the source described above it is possible that the shape of the spectrum could be modified by Compton scattering in the source, filters, or collimators. A calculation was made to estimate the effects on the γ -ray spectrum of such Compton scattering. The results of this calculation show that the main peak of primary γ rays is accompanied by a tail of energy-degraded photons. The intensity of this tail as a function of energy is approximately constant over an interval of a few MeV below the main peak. This intensity is approximately 6% per MeV of the intensity of the primary, unscattered peak.

B. Scattering Geometry

The geometry used in the search for nuclear elastic scattering of the neutron-capture γ rays is illustrated in Fig. 1. The γ -ray beam which emerges from the beam hole strikes the scattering target with a divergence of 2.5 deg. Gamma rays leaving the target at an angle θ with respect to the incident beam are detected by a NaI(Tl) crystal and recorded in a 128-channel pulseheight analyzer. The crystal is shielded on its sides by 2 in. of lead, and the lead is covered with a thin layer or boron. A 2-in.-thick aluminum cylinder was placed across the face of the crystal to absorb preferentially low-energy radiation from the target, and thus to reduce pile-up in the detector. Either a 3-in.-diam by 3-in.-long crystal or a 4-in.-diam by 6-in.-long crystal was used in the experiments described below. The distance from the scattering target to the face of the detector is 2 ft.

Scattering rates were obtained by first measuring the counting rate R_1 with the detector in the position of the target, and then measuring the counting rate R_2 with the target and detector placed as shown in Fig. 1. These scattering rates are interpreted by a differential cross section given by the expression

$$\frac{d\sigma(\theta)}{d\Omega} = R_2 \frac{A}{R_1} \frac{1}{n d\Omega} \frac{XL}{1 - e^{-XL}},$$
(1)

where *n* is the number of nuclei in the target of the thickness *L*, *A* is the area of the detector used to measure R_1 , and $d\Omega$ is the solid angle subtended by the detector in the measurement of R_2 . *X* is given by

$$X = \sigma_e N(\sec\beta + \sec\alpha), \qquad (2)$$

where σ_e is the electronic absorption cross section per atom of the target, and N is the atomic density in the target. α and β , the angles between the normal to the target and, respectively, the incident and scattered γ -ray beams are shown in Fig. 1. To obtain Eq. (1) we assumed that $N\sigma(E)L$, where $\sigma(E)$ is the nuclear cross section of the target, is much less than unity.

If the nuclear scattering cross section of a sample were sufficiently smoothly varying as to be constant over the spread of γ -ray energies in the incident beam, the Eq. (1) would yield the true cross section. If many resonances were included within the energy spread of the incident beam, then Eq. (1) would give an average cross section. The negative results, which will be presented in

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¹⁰ D. J. Hughes and R. B. Schwartz, Brookhaven National Laboratories Report BNL 325 (2nd ed.) 1958 (unpublished).

the following sections, indicate that neither of these alternatives is in fact realistic. However, cross sections obtained from Eq. (1) provide a useful means for comparing our results with the average cross sections obtained by other experimenters with continuous spectra.

III. EXPERIMENTAL RESULTS

A. Cross Sections

The cross sections which were studied proved to be so small that, in general, background radiation tended to obscure any real scattered signal. The net scattered signal was determined from four separate counting-rate measurements. Rates were measured with a source of γ rays next to the reactor, with and without a scattering target in position, and with no γ -ray source in the reactor, again with and without a target. The scattered signal was taken to be the difference of the first two rates minus the difference in the second two rates. The data used to obtain the signal for the case of capture γ rays from manganese incident upon a lead target are illustrated in Fig. 2. Manganese emits intense γ rays with energies of 7.26, 7.15, and 7.05 MeV. To obtain the four counting rates mentioned above, counts in the energy interval from 6.0 to 7.5 indicated in Fig. 2, were summed for each case.

Errors in the signal result from statistical uncertainties, which are large, since the signal is the small difference of large numbers, and from uncertainties in the energy calibration used to pick the energy intervals over which the sums were taken. These latter errors are also large, since γ rays with energies of 4.4 MeV and below were used for calibrations, and extrapolations to the vicinity of 7 MeV were uncertain by as much as 1%. A 1% error in fixing the energy interval results in errors of from 20 to 40% in the cross-section values.

If the net signals deduced from spectra of the type shown in Fig. 2 are assumed to result from nuclear scattering, then the total cross sections listed in Table II are obtained. Total cross sections are calculated from the differential cross sections measured at $\theta = 120^{\circ}$ by assuming that the radiation has a dipole distribution. It can be seen from Table II that for most of the cases studied, the uncertainties in the cross sections are comparable to the cross section themselves. Even when a real signal

TABLE II. Total elastic scattering cross sections (mb).

Source element	Energy interval (MeV)	Source energy (MeV)	Ta(1.3)	Target (thickness in cm) Hg(3)	Pb(0.6)	Bi(1.3)
Ti	5.0-7.0	6.41			0.6 ± 0.4	
Mn	6.0–7.5	7.26 7.15 7.05	<0.3	0.5 ± 0.3	$0.9\pm~0.5$	$0.8\pm\!0.4$
Fe	6.0-7.6	$7.64 \\ 7.28$	0.7 ± 0.4	2.4 ± 1.3	125 ±20ª	2.0 ± 1.1
Cu	7.6-8.2	7.91	<0.2	<0.4	<0.2	<0.2

* Calculated using the intensity of 7.64-MeV γ rays produced by neutron capture in iron,



FIG. 3. Pulse-height spectrum obtained when gamma rays from an iron source are scattered from natural lead, radiolead, and bismuth.

is seen, it is possible that it could result from processes other than nuclear elastic scattering, for example, from pile-up of low-energy γ rays which have been scattered by electrons in the target, from neutron induced reactions, or from Thomson scattering. The only case for which we can say with confidence that nuclear scattering was observed is that in which γ rays from an iron source are scattered by a lead target. This case will be discussed in detail below.

B. Scattering of Iron Gamma Rays from Lead

The signals obtained with an iron γ -ray source from targets of lead, radiolead and bismuth, each with the same number of electrons, are shown in Fig. 3. The difference between the signals from the natural lead and from the bismuth targets is shown in Fig. 4. This figure shows that γ rays with energies of 7.3 ± 0.1 MeV are incident upon the crystal. The ratio of the signal with a natural lead target to that with a radiolead target is 14 ± 4 . Since the ratio of abundances of the isotope Pb²⁰⁸ in natural lead and in radiolead is about 17, the data indicate that the 7.3-MeV γ rays come from this isotope.

Various checks were made to insure that the signal shown in Fig. 4 did not result from the interaction of neutrons with the lead target. The magnitude of the signal was measured, (a) with and without the iron capture γ -ray source in the reactor, (b) with and without a 15-cm-thick paraffin absorber in the beam, (c) with the lead target replaced by a copper target, and (d) as a function of the thickness of the lead target. The re-



FIG. 4. The difference between the scattered signals from targets of natural lead and of bismuth, when gamma rays from an iron source are used versus the pulse height (in energy units) from a 3-in.-diam by 3-in.-long NaI (Tl) crystal.

sults of all of these experiments indicate that the signal results from the scattering by the lead target of γ rays from the iron source. Since neutron capture in iron produces γ rays with energies of 7.285 ± 0.009 MeV,⁵ we conclude that the observed scattering results from the fact that these γ rays nearly coincide in energy with a nuclear level in Pb²⁰⁸. The fact that the signal is not seen when capture γ rays from copper, nickel, and several other sources are incident on a lead target shows that the signal does not result from the scattering of γ rays in the energy-degraded tail of the incident spectrum.

The data shown in Fig. 4 also indicate that scattering from 7.285-MeV level in Pb²⁰⁸ is predominantly elastic. The shape of the spectrum is very nearly that which would be expected from a single group of γ rays. The points at low energies are, of course, small differences of large numbers, so that their precision is not good. However, any inelastic scattering events in Pb²⁰⁸ would have to produce some γ rays with energies greater than 2.6 MeV. Such γ rays, if present with intensities comparable to that of the 7.285-MeV γ rays, would certainly have been detected.

The cross section for scattering by a lead target of neutron-capture γ rays produced in iron listed in Table II was calculated from Eq. (1) using the intensity of 7.64-MeV γ rays from iron. Since the weak 7.28-MeV γ rays could not be resolved from the 7.64-MeV γ rays, their intensity could not be measured in these experiments. Instead, the average of the ratio of the intensities of the 7.28- and 7.64-MeV γ rays measured in highresolution experiments, ^{5,6} 0.14±0.03, together with our measured intensity for the 7.64-MeV γ rays was used to calculate the cross section for scattering of 7.28MeV γ rays by Pb²⁰⁸. The resulting cross section is $(1.7\pm0.5)\times10^{-24}$ cm².

Scattering from this level was observed by Ben-David (Davis) and Huebschmann.⁹ Assuming a dipole distribution, and using the number of target nuclei in a natural lead target, they obtain a cross section of 0.8×10^{-24} cm². This is in good agreement with our cross section for scattering by Pb²⁰⁸ nuclei.

The strong signal obtained from iron neutron-capture γ rays scattered from lead made it possible to measure the angular distribution of the scattered γ rays. Figure 5 shows the relative yield of 7.28-MeV γ rays at a given angle θ , plotted versus $\cos^2\theta$. The points plotted as dots represent data collected between the angles of 90° and 180°, and the ×'s are for points at angles less than 90°. The background increases rapidly as the scattering angle is decreased below 90°, so that most of the data were taken at angles greater than 90°. All points plotted the figure have been corrected for differences in γ -ray in absorption in the target.

The straight line in Fig. 5 is a least-square fit to the data of an equation of the form $Y=1+A\cos^2\theta$. The value for A obtained from the analysis is $A=0.94\pm0.10$.

Self-absorption measurements were made by observing the scattered 7.28-MeV signal as a function of the thickness of lead absorber placed in the incident beam. The quantity Q is defined by the equation

$$Q = \frac{R(\theta, T, 0)^{-} e^{+\sigma_{e}NS} R(\theta, T, S)}{R(\theta, T, 0)}, \qquad (3)$$

where $R(\theta,T,0)$ and $R(\theta,T,S)$ are the scattered signals from a lead target observed at an angle θ with respect

TABLE III. Results of self-absorption experiments.

Absorber (cm) thickness	$e^{+\sigma}e^{SN}$	$Q/S \ (cm^{-1})$
0.58	1.36	0.14
1.16	1.85	0.23
1.27	1.96	0.16
1.27	1,96	0.10
2.54	3.84	0.26

to the incident beam, without and with a lead absorber of thickness S in the incident beam. The exponential term is included to correct for attenuation of the incident beam by electronic processes in the absorber. Table III lists the results of several measurements of Q.

The spread of the measurements results mostly from our inability to determine accurately the backgrounds to be subtracted from the observed signals. We adopt a value of $Q/S=0.18\pm0.04$ cm⁻¹. The use of this number to obtain the natural width of the 7.285-MeV level in Pb²⁰⁸ will be described in the next section.

C. Search for Other Discrete Levels

A search was made for other levels in heavy nuclei, besides the 7.285-MeV level in Pb²⁰⁸, which could be excited by γ rays produced in neutron capture. To enumerate the possibilities investigated, we assume that a signal one-tenth as large as that of the 7.285-MeV γ rays scattered from lead could be observed. That is, it is assumed that if, for a particular combination of incident γ rays and target isotope, the product of the intensity of the incident γ rays and the number of target nuclei is one-tenth as large as the same product for the 7.285-MeV γ rays from iron incident on Pb²⁰⁸, and if the incident γ rays excite a nuclear level with properties similar to those of the 7.285-MeV level in Pb²⁰⁸, an observable signal should result. Since the signal from radiolead shown in Fig. 3, which is only 1/14 of the signal from natural lead, is readily observed, this assumption seems reasonable.

The results of this search are summarized in Table IV. The first column lists the γ -ray sources used. The other columns show the targets investigated. The numbers in these columns are the combinations of γ rays

TABLE IV.	Survey	of scattering	experiments
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Source	Target					
element	Ta	$_{\rm Hg}$	\mathbf{Pb}	Bi		
Chromium	5	6	9	4		
Manganese	5	12	10	5		
Iron	1	• • •	2	- 1		
Copper	4	10	11	4		
Zinc	3	5	6	3		
Silver	• • •	• • •	10	• • •		
Lead			3	1		
Titanium	• • •	• • •	15	6		
Total-each target	18	33	66	24		
Total		14	1			



FIG. 5. Relative yield of 7.285-MeV γ rays versus $\cos^2\theta$, where θ is the angle between the incident beam and the scattered γ rays. The points are for angles greater than 90°, the \times 's are for angles less than 90°.

from the source and isotopes in the target which satisfy the above criterion. Only γ rays with energies between 6.5 MeV and the threshold for neutron production in the target isotopes were counted. To obtain the relative intensities of the γ rays, the yields of γ rays per neutron capture tabulated in Refs. 5 and 6 were used. The total number of possibilities investigated was 141, and from these possibilities, only scattering from the 7.285-MeV level in Pb²⁰⁸ was observed.

IV. DISCUSSION OF RESULTS

A. Cross Sections

A comparison of our results obtained with the 7.26-, 7.15- and 7.05-MeV γ rays from a manganese source with cross sections at γ -ray energies of about 7 MeV measured with continuous sources is shown in Table V.

The cross sections which we observe are roughly from 2 to 10% of those measured with continuous sources. Since we calculate that the intensity of the photons in the incident beam whose energy has been degraded by Compton scattering is about 6% per MeV of the intensity of the unscattered photons in the beam, reasonable agreement with other measurements results if it is assumed that all of the signal which we observe is from scattering of degraded-energy photons. Thus, it is con-

	This work	Ref. 2	Ref. 1ª	Ref. 3	Ref. 4 ^t
Та	< 0.3		2		
Hg	0.5 ± 0.3	3.5			
Рb	0.9 ± 0.5	15	17	60	55
Bi	0.8 ± 0.4	17.5	19	35	17

TABLE V. Cross sections at about 7 MeV (mb).

^a See also E. G. Fuller and Evans Hayward, Phys. Rev. Letters 1, 465 (1958). ^b Differential cross sections at 135^o were multiplied by 11.2.

sistent with our results to conclude that the photon scattering observed with the continuous spectra is resonance scattering from discrete levels which are separated by energies large compared to their widths, and which are not excited by the monoenergetic γ rays produced by neutron capture in manganese.

B. 7.285-MeV Level in Pb²⁰⁸

The 7.285-MeV γ rays produced by neutron capture in iron were observed to be strongly scattered by nuclei of the isotope Pb²⁰⁸. The angular distribution of this scattered radiation was shown to fit the expression $W(\theta) = 1 + (0.94 \pm 0.1) \cos^2\theta$. We conclude that the scattering is resonance scattering from a level in Pb²⁰⁸, which has an energy of 7.285 \pm 0.009 MeV,⁵ and an angular momentum quantum number of unity. The total nuclear scattering cross section for this process obtained from Eq. (1) and the measured angular distribution is $\sigma_m = (1.7 \pm 0.5) \times 10^{-24}$ cm².

Self-absorption measurements were made by interposing lead absorbers in the incident beam. A quantity Q/S, where Q is defined by Eq. (3) and S is the thickness of the lead absorber, was determined to be 0.18 ± 0.04 cm⁻¹. The two measured quantities, σ_m and Q/S, can be used to obtain two parameters of the nuclear level causing the scattering, namely, its natural width for transitions to the ground state Γ_0 and the separation between the median energy of the beam and the peak energy of the resonance, $|E_B - E_0|$. This is done as follows: The intensity, as a function of energy, of the incident beam is

$$I(E) = B \exp\left[-(E - E_B)^2 / \Gamma_B^2\right], \qquad (4)$$

where $\Gamma_B = E\{2kT/Mc^2\}^{1/2}$, and M and T are the nuclear mass and effective temperature of the γ -ray source. The constant B is related to the integrated flux by the equation $I_0 = \int I(E)dE = B\pi^{1/2}\Gamma_B$. When this flux is incident upon a target of thickness L, containing n scattering nuclei, the number of γ rays scattered into a solid angle $d\Omega$ at θ is

$$G(\theta,L) = Bn\omega(\theta)d\Omega \int \exp\left[-(E - E_B)^2/\Gamma_B^2\right]\sigma(E) \frac{1 - \exp\left[-(2\sigma_e N + \sigma(E)N')L \sec\alpha\right]}{(2\sigma_e N + \sigma(E)N')L \sec\alpha}dE,$$
(5)

where $[1/\omega(\theta)](d\sigma/d\Omega) = \sigma$, σ_e and $\sigma(E)$ are the electronic and nuclear cross sections of the target, and N and N' are the densities of lead and Pb²⁰⁸ atoms in the target. Equation (5) is for an arrangement in which the angles α and β in Fig. 1 are equal. To obtain this expression, we have assumed that the total radiation width of the 7.285-MeV level is equal to Γ_0 . This assumption is a reasonable one, since no inelastically scattered γ -rays can be observed in Fig. 4. Equation (5) can be written in terms of the experimentally measured quan-

tity, σ_m , by noting that

$$\frac{G(\theta,L)d\Omega}{Bn\omega(\theta)d\Omega} = \sigma_m \frac{1 - e^{-XL}}{XL} \pi^{1/2} \Gamma_B.$$
(6)

A second relationship between measured quantities and the properties of the scattering level is obtained from the self-absorption experiments. Equation (3) can be written

$$Q = 1 - \frac{\int \exp[-(E - E_B)^2 / \Gamma_B^2] \exp[-N'\sigma(E)S]\sigma(E) \left\{ \frac{1 - \exp[-(2\sigma_e N + \sigma(E)N')L \sec\alpha]}{(2\sigma_e N + \sigma(E)N')L \sec\alpha} \right\} dE}{\int \exp[-(E - E_B)^2 / \Gamma_B^2]\sigma(E) \left\{ \frac{1 - \exp[-(2\sigma_e N + \sigma(E)N')L \sec\alpha]}{(2\sigma_e N + \sigma(E)N')L \sec\alpha} \right\} dE}$$
(7)

With the assumption that the total radiation width of the scattering level is Γ_0 , the cross sections in Eqs. (5) and (7) are the same.

Equations (5) and (7) have been solved with a computer in an attempt to find the values of Γ_0 and $|E_0-E_B|$ which satisfy the experimental results. In the Appendix these equations are put into the form in which they were numerically integrated. The procedure

used consisted of picking a value for Γ_0 , and having the computer find values of $|E_0-E_B|$ which satisfy Eqs. (5) and (7). If Γ_0 is plotted versus the resultant values of $|E_0-E_B|$, two curves should result, one for each equation, and the intersection of these two curves should give a unique value for Γ_0 and $|E_0-E_B|$.

These two curves are shown in Fig. 6. Curve A is for the scattering experiment, and curve B for the self-



Pb²⁰⁸ versus the separation between the peak energy of the level and median beam energy, $|E_0-E_B|$. Values of Γ_0 and $|E_0-E_B|$ indicated by curve A satisfy the scattering experiments, and those indicated by curve B satisfy the self-absorption experiments. Uncertainties resulting from experimental errors are indicated by the flags.

FIG. 6. The ground state radiative width Γ_0 of the 7.285-MeV level in

absorption measurements. The uncertainties shown in the figure result from the uncertainty in the value of Q/S. Similar curves have been plotted for other values of σ_m , values which differ from the measured cross section by our quoted error for the measurement. Although the slopes of both curve A and curve B are influenced by the value used for σ_m , neither the value of Γ_0 at which they seem to diverge nor the rate of divergence of the curves is sensitive to this quantity, within the limits investigated.

As can be seen from Fig. 6, an intersection between the two curves is not clearly defined. However, they do seem to diverge as Γ_0 and $|E_0-E_B|$ increase, and at a width $\Gamma_0=4$ eV, the two curves are separated by more than twice the standard deviation of curve B resulting from the uncertainty in Q/S. As mentioned above, the value of Γ_0 at which the two curves separate is not noticeably influenced by the uncertainty in σ_m . Calculations, not shown in Fig. 6, have been made for values of Γ_0 as large as 30 eV, and the curves continue to diverge in the manner indicated in the figure.

A lower limit to the width of the level can be obtained from the Doppler form of Eq. (5). With $\Gamma_0 \ll \Delta$, and with the assumption $\sigma(E)N'L \sec \alpha \ll 1$, Eq. (5) can be integrated directly, giving σ_m as a function of Γ_0 and $|E_0 - E_B|$. The effect on the solution of Eq. (5) of the assumption that $\sigma(E)N'L \sec \alpha \ll 1$ has been investigated and found to be small compared to experimental uncertainties. If, in the Doppler form, $|E_0 - E_B|$ is set equal to zero, a lower limit $\Gamma_0 \ge 0.1$ eV is obtained. Thus, it seems possible to conclude that

$$\begin{array}{l} 0.1 \leq \Gamma_0 \leq 4 \text{ eV}, \\ |E_0 - E_B| \leq 26 \text{ eV}. \end{array}$$

The limit on $|E_0 - E_B|$ has been made large enough to

include the uncertainty in that quantity resulting from the uncertainty in the value of σ_m .

Axel, Min, Stein, and Sutton⁴ have recently reported the observation of a resonance near 7.3 MeV in the cross section for scattering of γ rays from Pb²⁰⁸. They state that their results are consistent with the interpretation that the scattering is from a single level in Pb²⁰⁸ with an energy of 7.29 MeV and a width Γ_0 of about 40 eV. Although this energy is very nearly the same as the energy of the level observed in our work, the widths seem to be sufficiently different to suggest that we are looking at different levels in Pb²⁰⁸.

C. Search for Other Levels

The limit $|E_0 - E_B| \leq 26$ eV obtained above, together with the negative result of the search for other levels which could be detected by scattering of neutroncapture γ rays, allows an estimate to be made of the separation in Pb²⁰⁸ of levels with properties similar to those of the 7.285-MeV level. Of the 66 possibilities for scattering from lead listed in Table IV, 28 are for the isotope Pb²⁰⁸. Since the separation of the 7.285-MeV iron capture γ rays from the state in Pb²⁰⁸ is less than 26 eV, a scattered signal should have been observed if any of these groups of γ rays came within plus or minus about 26 eV of a state like the 7.285-MeV state. If none of the γ rays used in the search overlap, then the total energy range covered in Pb²⁰⁸ was 28×52 eV~1450 eV. Since only one level was observed, it is probable that at energies just below the threshold for neutron production, the spacing in Pb²⁰⁸ of levels with properties similar to those of the 7.285-MeV level is of the order of magnitude of 1400 eV. To make such an estimate, it is necessary to assume that levels excited in Pb²⁰⁸ de-excite a reasonable fraction of the time by transitions to the ground state, as does the 7.285-MeV level. If a level were to deexcite predominantly by cascades through intermediate states, elastic scattering from that level would have been difficult to detect in these experiments.

Finally, one may wonder why no scattered signals were observed from other target nuclei, where the level spacings are smaller than in Pb²⁰⁸. However, in these other nuclei the number of levels at lower energies is greater than in Pb²⁰⁸, and the probability that a 7-MeV state would de-excite by cascades through intermediate states could be large. For example, for levels near 7

MeV in the mercury isotopes, $Axel^{11}$ quotes a value of 0.036 for the ratio of the average of the ground-state radiative widths to the total widths. Thus, even though other states may have been struck by neutron-capture γ rays, elastic scattering from these levels was probably weak, and therefore, not detected.

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APPENDIX

In this Appendix we write Eqs. (5) and (7) of the text in the form in which they were numerically integrated. Using the notation of Metzger,¹² the nuclear cross section can be written

$$\sigma(E) = \sigma_{\max}^{0} \frac{\Gamma_{0}}{2\pi^{1/2}\Delta} \int_{E'} \frac{\exp[-(E-E')^{2}/\Delta^{2}]}{1 + (4/\Gamma^{2})(E'-E_{0})^{2}} \frac{2}{\Gamma_{0}} dE' = \sigma_{\max}^{0} \psi(x,t) , \qquad (A1)$$

where $\sigma_{\max}^{0} = 4\pi \lambda^{2} [(2J_{1}+1)/2(2J_{0}+1)]$ and $\psi(x,t)$ is a tabulated function¹³ of $x = (2/\Gamma_{0})(E-E_{0})$ and $t = (\Delta/\Gamma_{0})^{2}$. Inserting this cross section into the combination of Eqs. (5) and (6) and making the substitution $z = (2/\Gamma_{0}) |E_{0}-E_{B}|$ we obtain

$$2\frac{\sigma_m}{\sigma_{\max}^0}\Gamma_B \pi^{1/2} \left(\frac{1-e^{-XL}}{XL}\right) = \Gamma_0 \int \exp[-(x-z)^2 \Gamma_0^2 / 4\Gamma_B^2] \psi(x,t) f(\psi) dx, \qquad (A2)$$
$$f(\psi) = \frac{1-\exp[-(2N\sigma_e + N'\sigma_{\max}^0\psi)L \sec\alpha]}{(2N\sigma_e + N'\sigma_{\max}^0\psi)L \sec\alpha}.$$

where

From Eq. (A2) a series of values of Γ_0 and $|E_0 - E_B|$ can be obtained which give the measured cross section σ_m . A second expression is obtained from Eq. (7) which can be written

$$\Gamma_{0} \int \exp\left[-(x-z)^{2} \Gamma_{0}^{2} / 4 \Gamma_{B}^{2}\right] \exp\left[-N' \sigma_{\max}^{0} \psi(x,t) S\right] \psi(x,t) f(\psi) dx$$

$$\Gamma_{0} \int \exp\left[-(x-z)^{2} \Gamma_{0}^{2} / 4 \Gamma_{B}^{2}\right] \psi(x,t) f(\psi) dx$$
(A3)

The denominator of this equation is just the right-hand side of Eq. (A2) so that

$$(1-Q)\left\{2\frac{\sigma_m}{\sigma_{\max}^0}\Gamma_B\pi^{1/2}\left(\frac{1-e^{-XL}}{XL}\right)\right\} = \Gamma_0\int \exp\{-(x-z)^2(\Gamma^2_0/4\Gamma_B^2) - N'\sigma_{\max}^0\psi(x,t)S\}\psi(x,t)f(\psi)dx.$$
(A4)

¹¹ Peter Axel, Phys. Rev. **126**, 671 (1962).

¹² Franz R. Metzger in Progress in Nuclear Physics, (Pergamon Press, Inc., New York, 1959), Vol. I, pp. 54-88.

¹³ M. E. Rose, W. Miranker, P. Leak, L. Rosenthal, and J. K. Hendrickson, Westinghouse Atomic Power Division Report No. SR-506, 1954 (unpublished).