Resonance Model for Photoproduction of K Mesons from Nucleons. II*

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Result of numerical calculation based on a formalism of a previous paper is given for the photoproduction of K^+ mesons and Λ hyperons on protons. Partial widths of the second πN resonance and $K\pi$ resonance are adjusted as free parameters. Differential cross section and polarization of Λ hyperon in a reasonable agreement with experiment are obtained in the case when the spin of $K\pi$ resonance is taken to be 1. The ratio of the width for the radiative decay of the $K^* \to K+\gamma$ relative to the decay mode $K^* \to K+\pi$ is estimated to be 2.5×10^{-2}.

1. INTRODUCTION

In a previous paper,[1] the photoproduction of K mesons on nucleons was considered in the approximation in which only one particle states and resonant states were taken into account in the unitarity conditions for the three related channels. In this paper, some numerical results based on the formalism obtained in I will be presented. In the numerical calculations we have limited ourselves to the photoproduction of a K^+ meson and a Λ hyperon on a proton, since this is the only process for which a sizeable amount of experimental data is available at present. In this process, the final state is in a pure $T=\frac{1}{2}$ state; therefore, there is no contribution from the first πN resonance which is in a $T=\frac{3}{2}$ state, and the resonant states which contribute to this process are the second and third πN resonances, the $K\pi$ resonance, and the πY resonance.

In the following section, the relevant formulas are briefly surveyed and the method by which the values of the parameters were determined is described. In Sec. 3, numerical results are given and discussed.

2. DETERMINATION OF THE PARAMETERS

Throughout this paper, the notation of paper I will be followed. Both $K\Lambda$ and $K\Sigma$ parity will be assumed to be negative. For the specific process in which we are interested, i.e.,

$$\gamma+p\to K^++\Lambda^0, \tag{2.1}$$

the invariant amplitudes A_i will be broken up into contributions from various particles and resonances as follows:

$$A_i=A_i^B+A_i^{\mathrm{I2}}+A_i^{\mathrm{I3}}+A_i^{\mathrm{II}}+A_i^{\mathrm{III}}, \tag{2.2}$$

where A_i^B, A_i^{I2}, A_i^{I3}, A_i^{II}, and A_i^{III} represent contributions from the Born terms, 2nd πN resonance, 3rd πN resonance, πY resonances, and $K\pi$ resonance, respectively.

The Born terms for even $\Lambda\Sigma$ parity are given by

$$A_i^B=\frac{g_\Lambda}{M^2-s}\begin{pmatrix}-e\\ 2e/(t-m_K^2)\\ \mu_p\\ \mu_p\end{pmatrix}+\frac{\mu_\Lambda g_\Lambda}{M_\Lambda^2-u}\begin{pmatrix}0\\0\\-1\\1\end{pmatrix}+\frac{\mu_T g_\Sigma}{M_\Sigma^2-u}\begin{pmatrix}M_\Lambda-M_\Sigma\\0\\-1\\1\end{pmatrix}. \tag{2.3}$$

These contain the parameters g_Λ, g_Σ, μ_p, μ_Λ, and μ_T, of which μ_p, the anomalous magnetic moment of a proton, is

$$\mu_p=1.79\mu_N, \tag{2.4}$$

where μ_N is a nuclear magneton. There are two contradictory experimental results for the anomalous magnetic moment of the Λ hyperon, i.e., one given by Kernan *et al.*,[2]

$$\mu_\Lambda=(0\pm0.5)\mu_N; \tag{2.5}$$

and another given by Cool *et al.*,[3]

$$\mu_\Lambda=(-1.5\pm0.5)\mu_N. \tag{2.6}$$

Therefore, we have made calculations for two cases, i.e., case A,

$$\mu_\Lambda=-1.5\mu_N;\quad\text{and case B,}\quad \mu_\Lambda=0.$$

The transition magnetic moment μ_T can be obtained from the rate of Σ^0 decay into a Λ^0 hyperon and photon, but as yet only the lower limit of the decay rate is

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known experimentally; therefore, we will use the theoretical value obtained by Dreitlein and Lee[4] based on dispersion relations,

$$\mu_T = 0.64(g_{\pi\Lambda} g_{\pi\Sigma}/g^2)\mu_V\,, \tag{2.7}$$

where

$$\mu_V = \tfrac{1}{2}(\mu_p - \mu_n) \tag{2.8}$$

and $g_{\pi\Lambda}$, $g_{\pi\Sigma}$, and g are the $\pi\Lambda\Sigma$, $\pi\Sigma\Sigma$, and πNN coupling constants, respectively. Assuming that $g_{\pi\Lambda}$ and $g_{\pi\Sigma}$ are equal to g, which would follow from globel symmetry, for the renormalized coupling constants, we obtain

$$\mu_T = 1.18\mu_N\,. \tag{2.9}$$

Gourdin and Rimpault[5] obtained the following value for the $NK\Lambda$ coupling constant from the analysis of associated production in pion-nucleon collisions:

$$g_\Lambda^2/4\pi = 0.7\,. \tag{2.10}$$

However, our preliminary calculation showed that the value

$$g_\Lambda^2/4\pi = 1.4 \tag{2.11}$$

gave better agreement with experiment and this is also closer to the value given by several other authors.[6,7] Therefore, we will give all the subsequent results for the case $g_\Lambda^2/4\pi = 1.4$. The $NK\Sigma$ coupling constant g_Σ was estimated by Ferrari and Fonda[7] to be approximately equal to g_Λ; therefore, we will assume that

$$g_\Sigma^2/4\pi = 1.4\,. \tag{2.12}$$

The contribution from the second πN resonance has the following form:

$$A_i^{I2} = \xi(s_2,t)\begin{pmatrix} 1 \\ 0 \\ 0 \\ -3 \end{pmatrix} \lambda_2/(s_2 - s)\,. \tag{2.13}$$

To obtain the term for the third πN resonance, one substitutes Eq. (I 3.18) in Eq. (I 3.16) treating Γ_3 as a constant, then noting that $\Gamma_3/W_3 \ll 1$ and $W + W_3 \approx 2W_3$ in our energy range, one obtains the following form:

$$A_i^{I3} = \xi(s_3,t)\begin{pmatrix} x_3 \\ 0 \\ 1 \\ -5x_3 \end{pmatrix}\left(\frac{W_3^2 - W^2 - \frac{1}{4}\Gamma_3^2}{W_3} + i\Gamma_3\right) \times \frac{3\lambda_3}{4W_3[(W-W_3)^2 + \frac{1}{4}\Gamma_3^2]}\,. \tag{2.14}$$

The suffices 2 and 3 indicate that the quantity is evaluated at the energy of the second and the third resonances, respectively; the λ's are defined by

$$\lambda = (\Gamma_i \Gamma_f)^{1/2}\,, \tag{2.15}$$

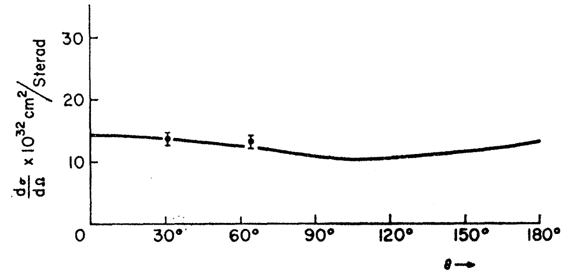

FIG. 1. Differential cross section at 980 MeV for case A.

and $\xi(s,x)$ is the matrix defined by Eq. (3.8) of I.

The ratio of λ_3 to λ_2 can be obtained from the analysis of the experiments

$$\gamma + N \to \pi + N \tag{2.16}$$

and

$$\pi + N \to K + Y\,. \tag{2.17}$$

When the Breit-Wigner formula is applied to the process (2.16), the total resonant cross section can be expressed by the following equation in the neighborhood of the resonance:

$$\sigma = \frac{2\pi}{W^2}(2J+1)\frac{\Gamma_i \Gamma}{(W-W_0)^2 + \frac{1}{4}\Gamma^2}\,, \tag{2.18}$$

where J and Γ are the spin and total width of the resonance. Therefore, we obtain the following ratio of the partial width Γ_i for

$$\gamma + N \to N^* \tag{2.19}$$

of the second and third resonance:

$$\frac{\Gamma_{i3}}{\Gamma_{i2}} = \frac{W_3^2}{W_2^2}\frac{\Gamma_3}{\Gamma_2}\frac{\sigma_3}{\sigma_2}\frac{2J_2+1}{2J_3+1} = 0.890\,. \tag{2.20}$$

The ratio of the partial widths for

$$N^* \to K + Y \tag{2.21}$$

can be obtained from an analysis of the associated production by Gourdin and Rimpault[5] as follows:

$$\Gamma_{f3}/\Gamma_{f2} = 0.33\,. \tag{2.22}$$

Combining Eqs. (2.20) and (2.22), we obtain

$$(\lambda_3/\lambda_2)^2 = 0.294 \tag{2.23}$$

and

$$\lambda_3 = \pm 0.542\lambda_2\,. \tag{2.24}$$

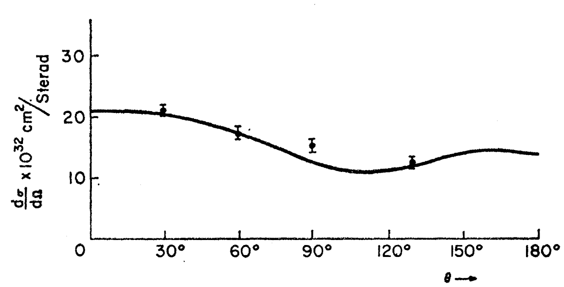

FIG. 2. Differential cross section at 1010 MeV for case A.

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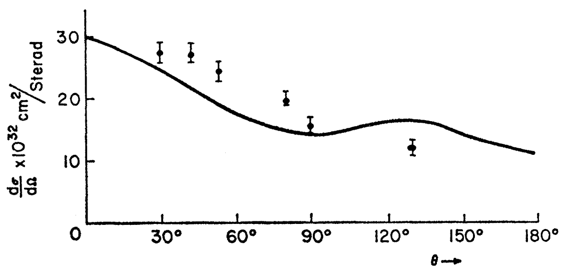

FIG. 3. Differential cross section at 1060 MeV for case A.

The contribution from the $K\pi$ resonance is given by

$$A_i^{\mathrm{III}}=\frac{1}{t_r-t}\begin{pmatrix}[(W_r^2-\Delta^2)/W_r]b\\ -(1/W_r)b\\ (\Delta/W_r)b\\ a\end{pmatrix}, \qquad (2.25)$$

where a and b are constants defined by

$$\pi\delta(W_r^2-W^2)\begin{pmatrix}a\\ b\end{pmatrix}=\frac{6\pi W_r((E_{1r}+M)(E_{2r}+M_\Lambda))^{1/2}}{p_r^2k_r(W_r+M)}\times\mathrm{Im}\begin{pmatrix}M_{11}+(2\bar{M}/W_r)\mathfrak{M}_{11}\\ \mathfrak{M}_{11}+(2\bar{M}/W_r)M_{11}\end{pmatrix}, \qquad (2.26)$$

and the suffix r indicates the value at the energy of the $K\pi$ resonance.

As for the πY resonances, their contributions are rather small due to the large distance from the physical region, unless the partial widths of these resonances are much larger than that of the πN resonance. Therefore, it is reasonable to omit these terms in the spirit of a dispersion analysis.

Now we have three parameters to determine from the kaon photoproduction experiments, i.e., λ_2, a, and b. These parameters were determined by fitting the differential cross sections at three angles;

$$\theta=0°,\quad E_\gamma=980\text{ MeV},$$

and

$$\theta=0°,\ 180°,\quad E_\gamma=1060\text{ MeV},$$

where E_γ is the photon laboratory energy. Of the eight sets of solutions thus obtained all but one can be eliminated by the following conditions:

(i) The solution must reproduce the correct differential cross section at $E_\gamma=980$ MeV and $\theta=180°$,

(ii) $|\lambda_2|\leq 1$ MeV.

The second condition was added because a large value of λ_2 gives rise to two prominent unobserved peaks in the angular distribution at $E_\gamma=1060$ MeV, due to a larger contribution from the third πN resonance.

In this way, we obtain the following parameters:

Case A

$$\begin{aligned}\mu_\Lambda&=-1.5\mu_N,\\ \lambda_2&=-0.640\text{ MeV},\\ a&=1.18\times10^{-3}\ (\text{MeV})^{-1},\\ b&=0.380\times10^{-3}\ (\text{MeV})^{-1},\end{aligned}\qquad (2.27)$$

and Case B

$$\begin{aligned}\mu_\Lambda&=0,\\ \lambda_2&=-0.630\text{ MeV},\\ a&=4.01\times10^{-3}\ (\text{MeV})^{-1},\\ b&=0.485\times10^{-3}\ (\text{MeV})^{-1}.\end{aligned}\qquad (2.28)$$

In both cases, the following numbers were used for the other parameters:

$$g_\Lambda^2/4\pi=g_\Sigma^2/4\pi=1.4, \qquad (2.11,\ 12)$$

$$\mu_T=1.18\mu_N, \qquad (2.9)$$

$$\lambda_3=\pm0.542\lambda_2. \qquad (2.24)$$

Taking either sign in Eq. (2.24) hardly affects the differential cross sections due to the small interference between the third πN resonance and other terms. However, since only the third πN resonance has an imaginary part, the polarization of the Λ hyperon is proportional to λ_3 and its sign changes by a different choice of sign in Eq. (2.24).

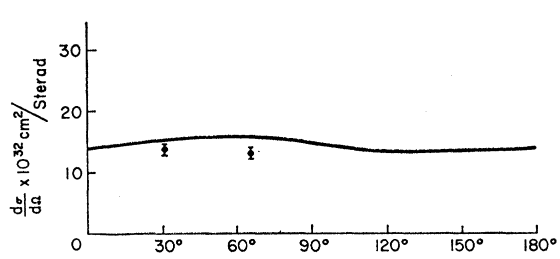

FIG. 4. Differential cross section at 980 MeV for case B.

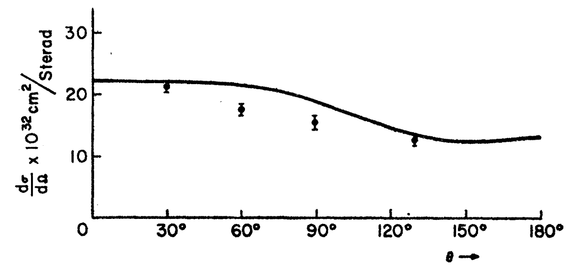

FIG. 5. Differential cross section at 1010 MeV for case B.

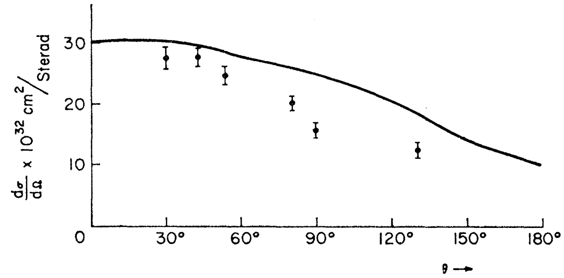

FIG. 6. Differential cross section at 1060 MeV for case B.

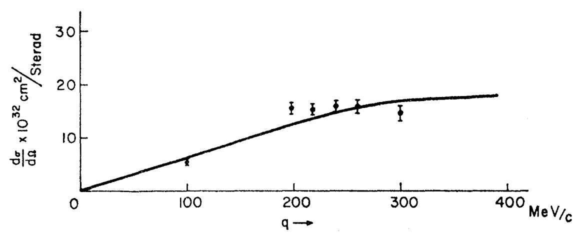

FIG. 7. Excitation curve at 90° for case A.

3. NUMERICAL RESULTS AND DISCUSSIONS

With the parameters obtained in the previous section, the differential cross section was calculated at three energies $E_\gamma=980$, 1010, and 1060 MeV. The results were plotted in Figs. 1–3 for case A and in Figs. 4–6 for case B, together with the experimental data by Anderson *et al.*[8] The theoretical curves are in good agreement with the experimental data in both case A and case B. However, we prefer case A to case B for two reasons: first because, in case B, the predicted width for the radiative decay of K^* is unreasonably large [see Eqs. (2.28) and (3.5)], and secondly because unitary symmetry predicts a nonzero lambda anomalous moment. An excitation curve at 90° is shown in Fig. 7 for case A.

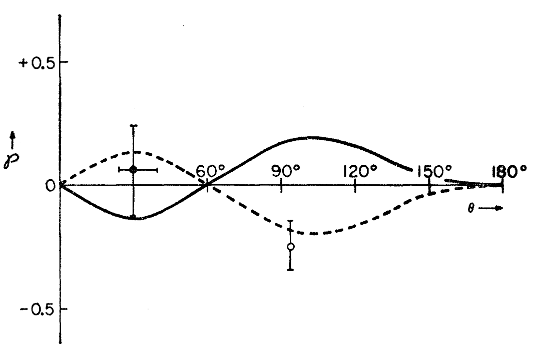

FIG. 8. Polarization of Λ hyperon at 980 MeV for case A. The data are at 1000 MeV from Ref. 9 (black circle) and Ref. 10a (open circle).

The polarization of the Λ hyperon was calculated for the case A and is given in Figs. 8–10. The real lines are for the case $\lambda_3=+0.542\lambda_2$, while the broken lines are for the case $\lambda_3=-0.542\lambda_2$. A measurement of the polarization has been made by McDaniel *et al.*[9] at $E_\gamma=1000$ MeV and $\theta=32°$, and the value obtained was

$$\alpha P=-0.04\pm0.11. \tag{3.1}$$

where α is the asymmetry parameter of Λ decay and the direction of the vector $\mathbf{k}\times\mathbf{q}$ is taken to be positive, $\mathbf{k}$ and $\mathbf{q}$ being the momenta, respectively, of the photon and K meson in the center-of-mass system.

If our result is interpolated to the energy $E_\gamma=1000$ MeV, we obtain

$$P=\mp0.19 \tag{3.2}$$

at $\theta=32°$, and this, combined with the value of α obtained by Beall *et al.*[10]

$$\alpha=-0.67_{-0.24}^{+0.18} \tag{3.3}$$

gives

$$\alpha P=\pm(0.13_{-0.04}^{+0.05}), \tag{3.4}$$

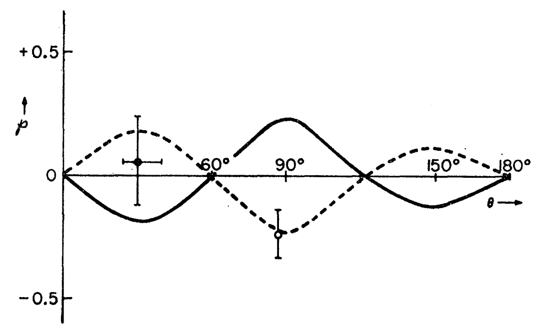

FIG. 9. Polarization of Λ hyperon at 1010 MeV for case A. The data are at 1000 MeV from Ref. 9 (black circle) and Ref. 10a (open circle).

where the upper and lower signs are for $\lambda_3=\pm0.542\lambda_2$, respectively.

Although this value of αP with the lower sign, which corresponds to the case $\lambda_3=-0.542\lambda_2$, agrees with the data given by McDaniel *et al.*[9] within experimental error, a more precise experiment is needed[10a] before any final conclusion can be drawn about the polarization, since this experiment has large errors in the

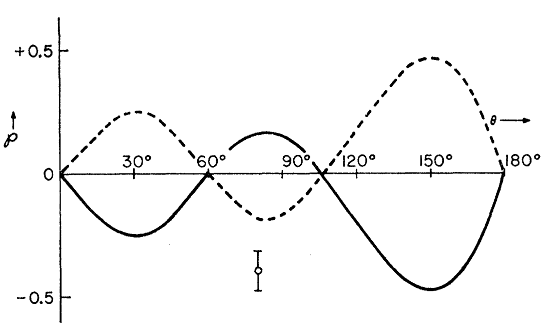

FIG. 10. Polarization of Λ hyperon at 1060 MeV for case A. The datum shown is at 1054 MeV from Ref. 10a.

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[10a] The following data on the polarization of Λ hyperons were reported by B. D. McDaniel to the Conference on Photon Interactions in the BeV-energy Range, January 26–29, 1963, Cambridge, Massachusetts (unpublished):

E_γ (MeV)	$\theta_{\text{c.m.}}$	$\mathcal{P}$
1000±25	93	0.24±0.10
1025±30	87	0.22±0.09
1055±35	80	0.39±0.08,

where the polarization $\mathcal{P}$ is positive in the direction of $\mathbf{p}_\gamma\times\mathbf{p}_\Lambda$.

determination of energy (± 30 MeV) and angle ($+10°$, $-6°$).

It should be noted that, although we included $D_{3/2}$ and $F_{5/2}$ πN resonances, we could still reproduce the experimental angular distribution which can be represented approximately by $A+B\cos\theta+C\cos^2\theta$. In order to account for this rather isotropic distribution, Kuo[11] assumed that the second and the third πN resonances do not contribute to photokaon production, and instead assumed the existence of a P-wave $K\Lambda$ resonance introduced by Kanazawa[12] in the analysis of the associated production of $K\Lambda$ pairs by πN collisions. Both Kuo's model with P-wave $K\Lambda$ resonance and our model with D and F-wave πN resonances can reproduce the differential cross sections equally well, but the polarizations predicted are quite different. Although the polarization obtained by Kuo has the same sign at all angles for $E_\gamma=1054$ MeV, our theory predicts a polarization which oscillates with angle and changes rapidly with energy in a way which is characteristic of the polarization on passing through a resonance region.

In this connection it is interesting to note that the phase-shift analysis by Bertanza *et al.*[13] of the production of $K\Lambda$ pairs in πN collision shows that, even though both S-P wave and S-P-D wave give equally good fit to the differential cross section, inclusion of D wave is necessary to obtain a good fit to the polarization. Therefore, in photoproduction, more accurate data on polarization is needed to determine the angular momentum of the states which contribute to the polarization.

The above calculations are based on the assumption that the spin of the $K\pi$ resonance is 1 and if its spin is 0, it would not contribute to the process in question. The fact that we only obtained fits to the data with nonzero a and b indicates that a resonance model favors spin 1 for the $K\pi$ resonance. This agrees with the conclusion reached by Chinowsky *et al.*[14] from an analysis of the production and decay of K^*.

Since we have found the effective coupling constants in the K^* pole graph, we can make a rough estimate of the decay rate

$$K^*\rightarrow K+\gamma.$$

The radiative decay rate $\Gamma_{\gamma K}$ of K^* can be expressed in terms of a and G_V or in terms of b and G_T, where G_V (or G_T) is the coupling constant of $N\Lambda K^*$ by means of γ_μ (or $\sigma_{\mu\nu}$). The result is

$$\Gamma_{\gamma K}=\frac{1}{\pi}k_0^3(a/2G_V)^2 \tag{3.5a}$$

$$=\frac{1}{\pi}k_0^3(b/2G_T)^2, \tag{3.5b}$$

where k_0 is the photon momentum in the K^* rest system and is given by

$$k_0=(W_r^2-m_K^2)/2W_r.$$

We will make use of the value of G_V obtained by Chan[15]; the calculation using (3.5b) can give a check of our results when there is a reliable estimate of the tensor coupling of $N\Lambda K^*$. From the experimental width[16] $\Gamma_{\pi K}=16$ MeV of K^* and (3.5a), we obtain for the branching ratio the following tentative estimate

$$\Gamma_{\gamma K}/\Gamma_{\pi K}=2.5\times10^{-2}. \tag{3.6}$$

We would like to close with some general remarks. First, we want to emphasize the need for additional measurements, particularly of the polarization, to provide stringent tests for models of the same type. Measurements of the cross-section and angular distribution near the observed plateau in the cross section would serve to give additional information on the important angular momentum states in this region.

It is not apparent at a glance whether Eq. (2.13) and Eq. (2.14) for the second and third πN resonance terms have a correct threshold behavior. One should note, however, that these $A_i^{\mathrm{I}2}$ and $A_i^{\mathrm{I}3}$ do not represent pure multipole amplitudes, but rather these contain all the waves up to D and F waves, respectively. When Eq. (2.13) and (2.14) are decomposed into multipole amplitudes, one sees that each amplitude has the correct threshold behavior. This result holds more generally, i.e. when a resonance with spin J is introduced in Channel I by the Cini-Fubini method,[1] the resulting amplitude contains, in addition to a resonant amplitude with spin J, all the nonresonant partial waves with total angular momentum smaller than J, each having a correct threshold behavior. This is due to the fact that the constant momentum transfer dispersion relation is used. When the zero width approximation is made, the resonant part agrees with the perturbation result.

We close with a reminder to the reader that resonance models should be considered convenient, approximate descriptions of the data, particularly since the approximations made do not preserve unitarity.

ACKNOWLEDGMENTS

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