Evidence for the Reaction $\pi^- + p \rightarrow \bar{p} + d$ and Some Methods of Investigating Virtual Nucleon Exchange Processes*

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The cross section for the process $\pi^- + p \rightarrow \bar{p} + d$ has been studied in a spark chamber experiment and was found to be 16 ± 10 μb at 4.13 GeV/c and 8_{-8}^{+12} μb at 4.95 GeV/c. The interpretation of this process in terms of a virtual nuclear exchange is discussed and comparison is made with other possible nucleon exchange reactions such as $p + p \rightarrow \pi^+ + d$ and backward pion-nucleon elastic scattering. There is a difficulty in applying Regge theory to these reactions because unequal masses are involved.

INTRODUCTION

IN a spark chamber experiment[1,2] primarily designed and used to study elastic pion-proton scattering from 2 to 5 GeV/c, we have searched for the two-body final-state reaction $\pi^- + p \rightarrow \bar{p} + d$, which has a threshold of 3.74 GeV/c. We find the total cross section for this reaction is 16 ± 10 μb at 4.13 GeV/c and 8_{-8}^{+12} μb at 4.95 GeV/c. The first part of this paper describes the nature of this evidence.

The second part of this paper points out that this reaction may provide a way of studying virtual nucleon exchange at high energies. If the Regge concepts are correct and a single nucleon trajectory is dominant, then the nucleon Regge trajectory could be traced out. To complete the paper we have compared other methods of studying high-energy virtual nucleon exchange, namely backward elastic pion-nucleon scattering and the reaction $p + p \rightarrow d + \pi^+$.

In an Appendix the parameterization for this reaction appropriate to the Regge theory is detailed, and the limitations of some of the conventional asymptotic expressions are emphasized.

EVIDENCE FOR THE REACTION $\pi^- + p \rightarrow \bar{p} + d$

The experiment using thin plate spark chambers and a liquid-hydrogen target was carried out at the Bevatron of the Lawrence Radiation Laboratory[2,3]. In this experiment the angles of all outgoing tracks as well as the incoming tracks were measured, but the only momentum known was that of the incoming tracks.

Within very wide limits all events showing just two outgoing tracks, which we call A and B, were measured. These measurements were processed by a computer to select events of interest according to the following criteria. The first requirement was that the event be coplanar where the degree of coplanarity is defined as follows. Let $\boldsymbol{\pi}$, $\mathbf{A}$, and $\mathbf{B}$ be the unit vectors in the direction of motion of the incident pion, particle A, and particle B, respectively. Then an angle ϕ is defined by

$$\sin\phi = \boldsymbol{\pi} \cdot (\mathbf{A} \times \mathbf{B}) / |\mathbf{A} \times \mathbf{B}|,$$

so that $\phi = 0$ for exactly coplanar events. The second requirement is that the angles that A and B make with the incident pion, θ_A and θ_B, have the kinematically predicted relationship for the reaction

$$\pi^- + p \rightarrow \bar{p} + d, \qquad (1)$$

as shown in Fig. 1. The degree of conformity of θ_A and θ_B with a particular kinematics curve is defined by the distance D in degrees, i.e., the perpendicular distance of the measured point in θ_A-θ_B space to the kinematics curve. The criterion used in event selection and the precision of the measurements led to a background of two prong events from three- (or more) body final states, which could not be separated from desired two-body final states unambiguously. This background contamination averaged *5 to 10* μb. For elastic diffraction scattering which has a 5-mb cross section, this background was no problem. But for large angle elastic scattering which may amount to a few microbarns per

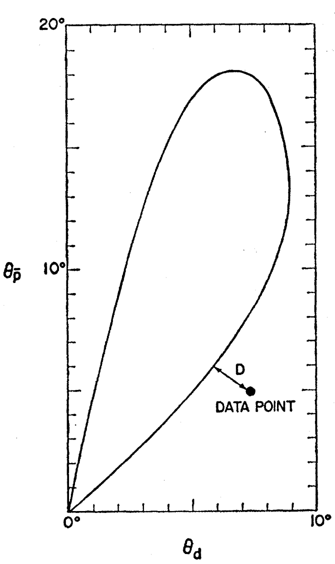

FIG. 1. The kinematics curve for the reaction $\pi^- + p \rightarrow \bar{p} + d$ at 4.0 GeV/c incident-pion laboratory momentum. The deviation, D, of a data point from the calculated kinematics curve is illustrated.

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[1] C. C. Ting, L. W. Jones, M. L. Perl, Phys. Rev. Letters **9**, 468 (1962).
[2] M. L. Perl, L. W. Jones, C. C. Ting, preceding paper, Phys. Rev. **132**, 1252 (1963).
[3] E. Bleuler *et al.*, in Proceedings of the High-Energy Instrumentation Conference, CERN, 1962 (to be published).

TABLE I. Number of events in intervals of $|D|$ and $|\phi|$. Events which exactly fit the $\pi^-+p\rightarrow \bar{p}+d$ kinematics curve and are perfectly coplanar have $D=\phi=0$. The intervals are in units of the standard deviations, σ; where $\sigma_\phi=0.4°$ and $\sigma_D=0.6°$ for both energies. Events falling very close to a boundary are split, giving rise to "half" events in some intervals.

$|\phi|$ \ $|D|$	$0<|D|<1\sigma_D$	$1\sigma_D<|D|<2\sigma_D$	$2\sigma_D<|D|<3\sigma_D$	$3\sigma_D<|D|<4\sigma_D$	$4\sigma_D<|D|<5\sigma_D$
4.13 GeV/*c*					
$0<|\phi|<1\sigma_\phi$	11	6	5	4	8
$1\sigma_\phi<|\phi|<2\sigma_\phi$	6.5	7.5	8	5	4.5
$2\sigma_\phi<|\phi|<3\sigma_\phi$	4	3	12	4	4
$3\sigma_\phi<|\phi|<4\sigma_\phi$	6	4	6	1	4
$4\sigma_\phi<|\phi|<5\sigma_\phi$	2	2	5.5	5	5.5
4.95 GeV/*c*					
$0<|\phi|<1\sigma_\phi$	16	6.5	11.5	10.5	12.5
$1\sigma_\phi<|\phi|<2\sigma_\phi$	8	11	15	10.5	11
$2\sigma_\phi<|\phi|<3\sigma_\phi$	11	8.5	3.5	7	7.5
$3\sigma_\phi<|\phi|<4\sigma_\phi$	15	6	11.5	9.5	8.5
$4\sigma_\phi<|\phi|<5\sigma_\phi$	8.5	7	14	5.5	6.5

steradian and for the reaction $\pi^-+p\rightarrow \bar{p}+d$ which has a small cross section, this background constituted a limit to the precision of the measurement. This entire question is more completely discussed in Ref. 2, and we shall only give the results of the $\pi^-+p\rightarrow \bar{p}+d$ analysis here.

Table I presents the numbers of events found in equal intervals of $|\phi|$ and $|D|$ about the coplanar, kinematically predicted curves. The intervals of ϕ and D used correspond to one standard deviation (about one-half degree) in each variable as determined from the analysis of the diffraction region of elastic pion-proton scattering. A peak in the first interval in D and ϕ clearly exists. However, to establish the statistical significance of this peak, the entries in Table I were fitted by a Gaussian corresponding to reaction (1) plus a background linear in D and ϕ. The triggering system biased against the detection of events where one particle emerged at less than 4° in the laboratory. In converting the number of detected events to a cross section, it was assumed that the process is isotropic in the center of mass, and the observed numbers scaled accordingly. The resulting weighted least-squares fit gives the value of cross section for reaction (1) of 16 ± 10 μb at 4.13 GeV/*c* and 8_{-8}^{+12} μb at 4.95 GeV/*c*. Thus, while a direct measurement of the events leads to a nonzero prediction for the cross section, the possibility of a large background fluctuation reduces the significance of our measurement to primarily an upper limit statement.

Eldridge[4] has made a perturbation-theory calculation for the differential and total cross section for this reaction. Of course, considering the difficulty in giving a definite total cross section, there is no possibility of this data being used for even a rough angular distribution, so that we will only compare with his total cross section prediction. At 4.13 GeV/*c*, Eldridge predicts 26 μb (to be compared with our measurement of 16 ± 10 μb), and his calculations do not extend to 4.95 GeV/*c*. Eldridge normalizes his cross section to a statistical model prediction at close to our energy, so that perhaps all that is indicated is that our measurement agrees with a statistical model prediction. Eldridge's calculation gives a definite energy dependence and angular distribution, but this we cannot check.

[4] O. C. Eldridge, Jr., University of California Radiation Laboratory Report No. UCRL 9128, 1960 (unpublished).

METHODS OF STUDYING VIRTUAL NUCLEON EXCHANGE AT HIGH ENERGIES

One method of investigating the dynamics of high-energy interactions consists of looking for channels in which the most important process is the exchange of a single virtual particle. This method, sometimes called the peripheral model, has been widely discussed recently.[5–7] If the simplest Regge concepts are correct, then this peripheral model can be generalized so that the single particle is replaced by the exchange of a Regge trajectory which represents all intermediate states with the same quantum numbers (except for angular momentum).[8–10]

To apply these ideas to virtual nucleon exchange, one may look for reactions in which there is some hope that the nucleon-exchange diagram will be most important and in which the peripheral model can be used.

[5] S. D. Drell, Rev. Mod. Phys. **33**, 458 (1961).
[6] F. Salzman and G. Salzman, Phys. Rev. **121**, 1541 (1961).
[7] E. Ferrari and E. Selleri, Phys. Rev. Letters **7**, 387 (1961).
[8] S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962).
[9] D. Amati and S. Fubini, Ann. Rev. Nucl. Sci. **12**, 359 (1962).
[10] M. Gell-Mann, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 533.

Three such reactions are:

$$\pi^- + p \rightarrow \bar{p} + d, \tag{1}$$

$$p + p \rightarrow d + \pi^+, \tag{2}$$

$$\pi^+ + p \rightarrow p + \pi^+ \text{ (backward elastic scattering)}, \tag{3}$$

corresponding to the Feynman diagrams of Fig. 2. Reactions (1) and (2) have a particular virtue. The isotopic spin change from the proton to the deuteron is $\frac{1}{2}$ to 0. Therefore, the exchanged particle must have $T=\frac{1}{2}$ and the $T=\frac{3}{2}$, $J=\frac{3}{2}$ nucleon isobar cannot be exchanged, which simplifies the calculation. This is in contrast to reaction (3) where the interpretation in terms of the nucleon-exchange diagram is complicated by the possible exchange of the $\frac{3}{2}$, $\frac{3}{2}$ isobar. Data on all three reactions

$$\pi^- + p \rightarrow p + \pi^- \text{ (180° scattering)},$$
$$\pi^+ + p \rightarrow p + \pi^+ \text{ (180° scattering)},$$
$$\pi^- + p \rightarrow n + \pi^0 \text{ (180° scattering)},$$

could possibly resolve this complication.

At a fixed energy the differential cross section is a function of a single variable. For the peripheral model this variable is t, the square of the four-momentum transfer. One condition for the validity of the peripheral model is that $|t|$ be small. In terms of the three-momenta ($\mathbf{p}$) and the total energies (E) designated in Fig. 2, for reaction (1)

$$t = (E_d - E_p)^2 - (\mathbf{p}_d - \mathbf{p}_p)^2. \tag{4}$$

In the laboratory systems in which the proton is at rest,

$$t = M_d^2 + M^2 - 2ME_{dL} \cong -2M(T_{dL} - \tfrac{1}{2}M), \tag{5}$$

where M_d and M are the deuteron and proton masses, respectively, and E_{dL} and T_{dL} are the total and kinetic energies of the deuteron in the laboratory system. Equation (4) also holds for reaction (2), but here the p subscript may stand for either the incident or the target proton. Therefore, there are two possible values of t. If the target proton connects with the deuteron vertex,

$$t = M_d^2 + M^2 - 2ME_{dL} \cong -2M(T_{dL} - \tfrac{1}{2}M), \tag{6}$$

exactly as in (5). If the incident proton connects with the deuteron vertex,

$$t' = m^2 + M^2 + 2ME_{dL} - s, \tag{7}$$

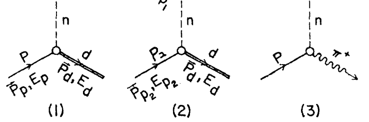

FIG. 2. Feynman diagrams for the exchange of a single nucleon in the three reactions (1) $\pi^- + p \rightarrow \bar{p} + d$, (2) $p + p \rightarrow \pi^+ + d$, and (3) backward $\pi^+ + \text{p}$ elastic scattering.

where m is the pion mass and s is the square of the total energy in the center of mass, given by

$$s = 2M^2 + 2ME_{pL}, \tag{8}$$

where E_{pL} is the laboratory energy of the incident proton. Thus, for a particular value of E_{dL} one would have to write the scattering amplitude as $f(t) + f(t')$, where t and t' are given by (6) and (7). This is the same kind of ambiguity as exists in proton-proton elastic scattering. In proton-proton elastic scattering analyses the $f(t)$ belonging to the smaller $|t|$ is always used on the argument that the scattering is diffractive so that the larger $|t'|$ value contributes a very small $f(t')$. In the case of reaction (2), the reaction might be mainly peripheral with T_{dL} small. Then $|t|$ would be much less than $|t'|$ and one could try to neglect $f(t')$ compared to $f(t)$. There is some evidence by Turkot *et al.*[11] at 1.55 to 2.50 GeV that reaction (2) may be peripheral, so that this assumption may be justified.

Thus, in addition to the previously discussed isotopic spin considerations, the comparative values of using these reactions to examine nucleon exchange through the peripheral model are as follows: The reaction $p + p \rightarrow d + \pi^+$ gives an indication of being peripheral and may be easier to study because intense external proton beams exist. The reaction $\pi^- + p \rightarrow \bar{p} + d$ and backward elastic scattering have no t ambiguity. However, there is no evidence regarding the possible peripheral nature of the reaction $\pi^- + p \rightarrow \bar{p} + d$. Finally, the evidence on the peripheral nature of the backward elastic scattering, which should be manifest as a backward peak in the elastic differential cross section, is ambiguous.[2]

There may be no preference for any of these methods as far as size of the cross section goes. With incident particles above several GeV/c, the cross sections for $\pi^- + p \rightarrow \bar{p} + d$, $p + p \rightarrow d + \pi^+$, and for backward pion-proton elastic scattering are all less than 100 μb and probably less than 20 or 30 μb.

The final question is the use of Regge concepts. The uncertainty in the application of Regge theory to the simpler system of elastic scattering and the lack of data on the above processes prevents application at this time. We should like to point out that ideally one would hope to trace out the nucleon trajectory on the $\alpha(t)$ versus t Chew-Frautschi[12] diagram. That trajectory is roughly fixed in the region $\alpha(t) > 0$ by the nucleon and $T=\frac{1}{2}$, $J=\frac{5}{2}$ nucleon isobar. If the nucleon trajectory in the $\alpha(t) < 0$ region could be traced out by the above processes, then one could examine how the two parts of the trajectory connect. Unfortunately there is a particular difficulty (Appendix) associated with the application of the Regge concepts to the nucleon trajectory. This difficulty is caused by the change of mass of the

[11] F. Turkot, G. B. Collins, T. Fujii, M. A. R. Kemp, Z. Menes, and J. Oostens, Bull. Am. Phys. Soc. **7**, 620 (1962).
[12] G. F. Chew, Rev. Mod. Phys. **34**, 394 (1962).

real particles at each vertex, and prevents the simple association of large values of the cosine of the scattering angle in the unphysical t channel with large values of s.

Therefore, at present it appears that the experiments must lead the way in investigating high-energy virtual nucleon exchange, first, to see if a behavior indicative of nucleon exchange can be found, and second, to see if the Regge concepts are valid.

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APPENDIX

If four different masses are involved in a reaction in the s channel, Fig. 3, the equation for t is

$$t=t_0-2q_iq_f(1-\cos\theta). \tag{A1}$$

Here q_i and q_f are the three-momenta of the particles in the initial and final states in the barycentric system

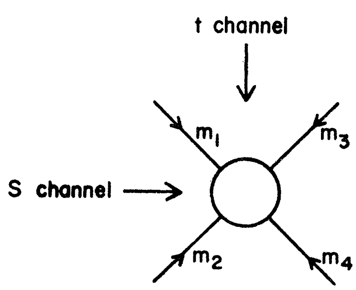

FIG. 3. Nomenclature convention in labeling s, t, and the masses when all masses are different for the calculation of t and $\cos\theta_t$.

in the s channel and θ is the angle between particles 1 and 3 in that system. q_i, q_f, and t_0 are functions of the masses and s as follows:

$$q_i=\tfrac{1}{2}[(s-m_1^2-m_2^2)^2-4m_1^2m_2^2]^{1/2}/s^{1/2}, \tag{A2}$$

$$q_f=\tfrac{1}{2}[(s-m_3^2-m_4^2)^2-4m_3^2m_4^2]^{1/2}/s^{1/2}. \tag{A3}$$

$$t_0=\{[m_4^2-m_3^2-m_2^2+m_1^2]-[q_i-q_f]^2\}/4s. \tag{A4}$$

In the simple case of forward elastic scattering, $m_3=m_1$, $m_4=m_2$, $q_i=q_f$, and therefore, $t_0=0$, which simplifies (A1). Then as $\theta=0$, $t=0$, which is the case one usually thinks of. However, in general, $t_0\neq 0$. For example, at large s,

$$t_0\cong-(m_3^2-m_1^2)(m_4^2-m_2^2)/s. \tag{A5}$$

For backward elastic scattering and for the reaction $p+p\to d+\pi^+$ the mass relationships are:

$$m_3>m_1,\ m_4<m_2 \quad\text{or}\quad m_3<m_1,\ m_4>m_2. \tag{A6}$$

Thus, $t_0>0$ for large s (and can be shown for all s) for these two reactions. On the other hand, for the reaction $\pi^-+p\to\bar{p}+d$, $t_0<0$. In fact $t_0\neq 0$ must occur for all cases of virtual nucleon exchange, because baryon conservation requires $m_3\neq m_1$ and $m_2\neq m_4$. The occurrence of $t_0>0$ for the first two reactions may be useful because it in principle enables one to trace out more of the unphysical region of the trajectory. However, the mass inequalities which lead to $t_0\neq 0$ also lead to a complication in the Regge concepts.

In the application of the Regge concepts to relativistic processes, it is usual to require that the cosine of the scattering angle in the unphysical t channel (Fig. 3), denoted by $\cos\theta_t$, be expressed in terms of the s and t of the physical s channel; and that $\cos\theta_t$ be large when s is large. The general expression for $\cos\theta_t$ is

$$\cos\theta_t=1+\frac{4st-\{(m_4^2-m_3^2-m_2^2+m_1^2)^2-([(t-m_1^2-m_3^2)^2-4m_1^2m_3^2]^{1/2}-[(t-m_2^2-m_4^2)^2-4m_2^2m_4^2]^{1/2})^2\}}{2[(t-m_1^2-m_3^2)^2-4m_1^2m_3^2]^{1/2}[(t-m_2^2-m_4^2)^2-4m_2^2m_4^2]^{1/2}}. \tag{A7}$$

For forward elastic scattering (A7) becomes

$$\cos\theta_t=1+\frac{s-([(t/4)-m_1^2]^{1/2}-[(t/4)-m_2^2]^{1/2})^2}{2[(t/4)-m_1^2]^{1/2}[(t/4)-m_2^2]^{1/2}}. \tag{A8}$$

For large s and small t (A8) becomes

$$\cos\theta_t\cong-s/2m_1m_2, \tag{A9}$$

leading to the association of large $\cos\theta_t$ with large s.

However, in the case of the reaction $\pi^-+p\to\bar{p}+d$, $\cos\theta_t=1+$

$$\frac{4st-\{(2M^2+m^2)^2-([(t-m^2-M^2)^2-4m^2M^2]^{1/2}-[(t-5M^2)^2-16M^4]^{1/2})^2\}}{2[(t-m^2-M^2)^2-4m^2M^2]^{1/2}[(t-5M^2)^2-16M^4]^{1/2}}. \tag{A10}$$

For large s, small t, and neglecting m compared to M, this becomes

$$\cos\theta_t \cong 1 + (2st/3M^4). \qquad \text{(A11)}$$

(A11) has a different behavior from (A9), because in (A11) even if s is large, a small t, namely t close to t_0, can still keep $\cos\theta_t$ small.

For the reaction $p+p \to d+\pi^+$ and for backward elastic scattering a similar difficulty appears. In these cases since $t_0 > 0$ there is a point $t=0$ corresponding to $\theta > 0$; and putting $t=0$ into (A7) for the particular unequal mass conditions (A6), yields $\cos\theta_t = -1$ for all s! Thus, the simplicity of $\cos\theta_t$ being necessarily large when s is large is lost, and with that loss of simplicity goes the usual direct argument that that Regge trajectory will dominate the process.

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Pion-Nucleon Scattering and the $J=2$, $T=0$ Pion-Pion Interaction*

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The effect of the $J=2$, $T=0$ $\pi-\pi$ interaction on the $\pi-N$ invariant amplitude, $B^{(+)}$ is analyzed. It is found that the $\pi-N$ scattering data is inconsistent with a $J=2$, $T=0$ $\pi-\pi$ phase $\delta_2{}^0$ which rises to above 13° around 650 MeV. The data are consistent with a δ-function contribution at 1200 MeV but it is impossible to say whether this corresponds to a resonant phase or only a sharp peak in the corresponding absorptive part of the amplitude.

1. INTRODUCTION

In the $\pi+\pi \to N+\bar{N}$ channel of the pion-nucleon system only states with isospin $T=0$ and angular momentum $J \geqslant 2$ contribute to the pion-nucleon total invariant amplitude $B^{(+)}$. Since this amplitude affords a means of investigating the $J=2$, $T=0$ $\pi-\pi$ interaction without interference from the $J=0$, $T=0$ state it is of interest to consider possible methods of studying $B^{(+)}$.

The $\pi-N$ total invariant amplitudes, $A^{(\pm)}$ and $B^{(\pm)}$ have been studied at fixed angles in both the forward and backward directions[1,2] since no difficulties due to divergences of Legendre series are encountered in these cases. It is necessary, however, to approximate unitarity by retaining only a small number of terms in the partial-wave expansions of the amplitudes. The resulting errors may be considerable if the convergence of these series is slow, as is to be expected if there are appreciable low-energy $\pi-\pi$ effects. Accordingly, it is of interest to consider the amplitudes formed by integrating the total amplitudes over all physical angles. These amplitudes have distant singularities which cannot be calculated in terms of convergent Legendre series but have the advantage that the contributions of alternate terms of the partial-wave expansion are much reduced in the low-energy physical region. Thus, it is possible to calculate nearby singularities more accurately than in the fixed-angle case at the expense of introducing distant singularities which must be represented by some approximation scheme.

Hence, a dispersion relation is written for the amplitude formed by integrating $B^{(+)}$ over all angles and the results are analyzed by methods similar to those which have been successfully applied to the analysis of $\pi-N$ partial waves[3] so as to give values for $\delta_2{}^0$, the $J=2$, $T=0$ $\pi-\pi$ phase. The dispersion relation is described in Sec. 2; the contribution from the $\pi+\pi \to N+\bar{N}$ channel and its relation to the $J=2$, $T=0$ $\pi-\pi$ interaction are considered in Sec. 3, and the analysis of the results in terms of the phase $\delta_2{}^0$ is discussed in Sec. 4.

2. THE DISCREPANCY

(i) Kinematics

The notation follows the standard usage. The total amplitude with isospin T is given by

$$B^{(T)}(s,x) = 8\pi W\left[\frac{f_1{}^{(T)}(s,x)}{(W+M)^2-\mu^2} + \frac{f_2{}^{(T)}(s,x)}{(W-M)^2-\mu^2}\right], \qquad (1)$$

where $f_1{}^{(T)}$ and $f_2{}^{(T)}$ are expressible in terms of partial-wave expansions

$$f_1{}^{(T)}(s,x) = \sum_{l=0}^{\infty} f_{l+}{}^{(T)}(s) P_{l+1}'(x) - \sum_{l=2}^{\infty} f_{l-}{}^{(T)}(s) P_{l-1}'(x), \qquad (2)$$

$$f_2{}^{(T)}(s,x) = \sum_{l=1}^{\infty} \left(f_{l-}{}^{(T)}(s) - f_{l+}{}^{(T)}(s)\right) P_l'(x). \qquad (3)$$

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