violate unitary symmetry; and secondly, to compare their predictions for η decay modes other than $\eta\rightarrow\pi^0\rightarrow\pi^+\pi^-\pi^0$.

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APPENDIX

(i) The following identity can be proved by means of the commutation relations in Eq. (1):

$$4A_\lambda{}^1A_1{}^\lambda\equiv 2A:A+2(A_1{}^1)^2+2(3A_1{}^1-A_\mu{}^\mu)-2A_3{}^2A_2{}^3-2A_2{}^3A_3{}^2-(A_3{}^3-A_2{}^2)^2-(A_1{}^1-A_\mu{}^\mu)^2. \quad (49)$$

From (5), (7), and (8), it then follows that

$$A_\lambda{}^1A_1{}^\lambda\equiv\tfrac{1}{2}A:A-\tfrac{3}{2}Q+\tfrac{1}{4}Q^2-\mathbf{K}^2. \quad (50)$$

Similarly,

$$A_\lambda{}^2A_2{}^\lambda\equiv\tfrac{1}{2}A:A+\tfrac{3}{2}Y_L+\tfrac{1}{4}Y_L{}^2-\mathbf{L}^2. \quad (51)$$

The identities in (50) and (51) are the analogs of Okubo's identity[15] for $A_\lambda{}^3A_3{}^\lambda$.

(ii) Equation (39) is an *ad hoc* result which applies to the states forming a basis for the representation $U(1,0,-1)$ of $U(3)$. It can be verified with the aid of Tables I and II, but the author has not found a proof for it.

(iii) For reasons of charge conjugation invariance, we require $H(\omega\rho)$ to be of the form[17]

$$\bar{\rho}^0\omega+\bar{\omega}\rho^0.$$

Equation (41) is then a simple consequence of Eq. (39).

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Theory of Spin-$\frac{1}{2}$ Particles with Parity-Nonconserving Interactions*

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It is shown that when interactions are not invariant under parity conjugation, both the self-mass term $(-\delta m\bar{\psi}\psi)$ and the term $(-a\bar{\psi}\gamma_\mu\gamma_5\partial\psi/\partial x_\mu)$ are induced by the self-interaction of any spin-$\frac{1}{2}$ particle with nonvanishing mass. (For simplicity T invariance is assumed.) When $a^2>1$, the spin-$\frac{1}{2}$ particle propagates in vacuum faster than the velocity of light. When $a^2=1$, the observed mass should be zero. Therefore, it follows that $a^2<1$ for any spin-$\frac{1}{2}$ particle with nonvanishing mass. Since $1>a^2>\frac{1}{3}$ implies the existence of ghost states, one must require $a^2\leqslant\frac{1}{3}$. Although a has no physical meaning for free particles, as an example, it is also shown that it has a physical meaning when a charged particle is interacting with an external electromagnetic field. The value of a is estimated for the electron and the muon.

1. INTRODUCTION AND SUMMARY

THE purpose of this work is to study the properties spin-$\frac{1}{2}$ particles possess as a result of parity-nonconserving interactions. To outline our discussions given here, we shall tentatively start from the Lagrangian density

$$L=L_1+L_2,\quad L_1=-\bar{\psi}(x)\left[\gamma_\mu\frac{\partial}{\partial x_\mu}+m_0\right]\psi(x), \quad (1)$$

for a spin-$\frac{1}{2}$ field ψ with mechanical mass m_0, where L_2 is not invariant under C or P transformation but is invariant under CP (or T) transformation. For simplicity we shall consider only CP-invariant interactions throughout this paper.

Since the free particle is interacting with its self-field, L_1 does not express the free part of the Lagrangian density for the dressed spin-$\frac{1}{2}$ field considered. When all interactions are renormalizable and invariant under both C and P transformations, as is well known, $(L_1-\delta m\bar{\psi}\psi)$ is the free part of the Lagrangian density for the dressed particle, where δm is the self-energy of the particle. In our more general case it will be shown in Sec. 2 that, in addition to the self-mass term, the self-interaction induces another term $(-a\bar{\psi}\gamma_\mu\gamma_5\cdot\partial\psi/\partial x_\mu)$, where $\gamma_5{}^2=1$ and a is a real constant. This term should be added to the free part of the Lagrangian density and consequently be subtracted from L_2, as the self-mass term is, to perform the renormalization consistently.

To discuss the magnitude of the coefficient a of the parity-nonconserving counter term, consider the Lagrangian density

$$-\bar{\psi}(x)\left[\gamma_\mu(1+a\gamma_5)\frac{\partial}{\partial x_\mu}+\mu\right]\psi(x) \quad (2)$$

or the Dirac equation

$$[i\gamma p(1+a\gamma_5)+\mu]\psi(p)=0 \quad (3)$$

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for the free dressed particle. From (3) we get the energy-momentum relation

$$p^2(1-a^2)+\mu^2=0$$

for the free particle, where the metric $p^2=\mathbf{p}^2-p_0^2$ is used. When $a^2>1$, this particle propagates in vacuum faster than the velocity of light. As far as we are concerned with covariant theory, this case can never happen. When $a^2=1$, μ^2 should be zero, and Eq. (3) reduces to the equation for the neutrinos with two components.[1] For any spin-$\frac{1}{2}$ particle with nonvanishing mass, a^2 should be less than unity. For the last case it will be shown in Sec. 2 that $a^2\leqslant\frac{1}{3}$ when L_2 does not contain the term $(\lambda\bar{\psi}\gamma_\mu\gamma_5\cdot\partial\psi/\partial x_\mu)$ as a primary interaction. When $1>a^2>\frac{1}{3}$, "ghost states" exist.

To perform the renormalization consistently, the parity-nonconserving counter term $(a\bar{\psi}\gamma_\mu\gamma_5\cdot\partial\psi/\partial x_\mu)$ must be introduced. This will be discussed in Sec. 3. The renormalization for the case in which the term $(\lambda\bar{\psi}\gamma_\mu\gamma_5\cdot\partial\psi/\partial x_\mu)$ is one of primary interactions will be discussed in Sec. 4.

This paper will consider only the spin-$\frac{1}{2}$ particle with nonvanishing mass, i.e., the case $a^2<1$. Now we may ask: Has the coefficient a any physical meaning? This question arises from the fact that the Γ matrix defined by

$$\Gamma_\mu=\gamma_\mu(1+a\gamma_5)/(1-a^2)^{1/2} \tag{4}$$

satisfies the usual commutation relation

$$\{\Gamma_\mu,\Gamma_\nu\}=\{\gamma_\mu,\gamma_\nu\}=2\delta_{\mu\nu},$$

so that Eq. (3) reduces to

$$(i\Gamma p+m)\psi(p)=0, \tag{5}$$

where $m=\mu(1-a^2)^{-1/2}$. Equation (5) shows that the coefficient a has no physical meaning for free particles. It should be noted that the term $(a\gamma_5)$ in Γ_μ comes from the self-interaction and, therefore, the definition of $\bar{\psi}$ is not $\psi^*\Gamma_0$ but $\psi^*\gamma_0$, where ψ^* is the Hermitian conjugate of ψ. When the former definition $\bar{\psi}=\psi^*\Gamma_0$ is misused, the Lagrangian density (2) is not a Hermite operator. To study whether or not the coefficient has any physical meaning when the spin-$\frac{1}{2}$ particle is interacting with other fields, we shall calculate the energy of a charged spin-$\frac{1}{2}$ particle in an external weak electromagnetic field. It will be shown in Sec. 5 that the coefficient has a physical meaning because of the definition of $\bar{\psi}$ mentioned above.

[1] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957); A. Salam, Nuovo Cimento **5**, 299 (1957); L. Landau, Nuclear Phys. **3**, 1271 (1957). To derive the equation for a two-component neutrino from Eq. (3), we shall use the representation

$$\boldsymbol{\gamma}=\begin{pmatrix}0 & -i\boldsymbol{\sigma}\\ i\boldsymbol{\sigma} & 0\end{pmatrix},\quad \gamma_0=\begin{pmatrix}0&1\\1&0\end{pmatrix}\quad\text{and}\quad \gamma_5=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

for γ matrix and the notation $\psi=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$, where σ_i is 2×2 Pauli spin matrix. Then Eq. (3) reduces to

$$(1+a)(\boldsymbol{\sigma}\mathbf{p}+p_0)\psi_1=\mu\psi_2,$$
$$(1-a)(-\boldsymbol{\sigma}\mathbf{p}+p_0)\psi_2=\mu\psi_1,$$

which are the desired equations when $a^2=1$.

In Sec. 6, a renormalizable example is considered to discuss the magnitude of the coefficient a. In the usual sense, weak interactions are unrenormalizable. Therefore, in Sec. 7 a cutoff Λ is introduced and the coefficients are calculated for the electron and the muon. Their coefficients are positive definite and nearly equal to each other, and $a(\Lambda)\lesssim10^{-2}$ when $\Lambda\lesssim300$ BeV.

From Eq. (5), one gets the equal-time anticommutation relation

$$\{\psi(x),\bar{\psi}(y)\}\,|_{x_0=y_0}=\Gamma_0\delta(\mathbf{x}-\mathbf{y}) \tag{6}$$

for the field ψ in interaction representation. It will be shown in Sec. 2 that the equal-time anticommutation relation for the field ψ_H in the Heisenberg representation is given by

$$\langle\{\psi_H(x),\bar{\psi}_H(y)\}\rangle_0\,|_{x_0=y_0}=\Gamma_0\left[1-\frac{a(a+\gamma_5)}{(1-a^2)}\right]\delta(\mathbf{x}-\mathbf{y}), \tag{7}$$

the right-hand side of which is different from that of (6) provided that $a\neq0$.

2. RENORMALIZATION

We shall discuss the renormalization of a spin-$\frac{1}{2}$ field $(a^2<1)$ interacting with other fields by renormalizable interactions.[2] The Lagrangian density of the system is given by

$$\begin{aligned}
&L=L_0+L',\\
&L_0=-:\bar{\psi}_H(x)\left[\Gamma_\mu\frac{\partial}{\partial x_\mu}+m\right]\psi_H(x):,\\
&L'=:L_2:+\delta m:\bar{\psi}_H(x)\psi_H(x):\\
&\qquad+\frac{a}{(1-a^2)^{1/2}}:\bar{\psi}_H(x)\gamma_\mu\gamma_5\frac{\partial}{\partial x_\mu}\psi_H(x):,
\end{aligned} \tag{8}$$

where the symbol ψ_H is used for the field ψ in the Heisenberg representation, the notation $:X:$ means to take the normal product of the operators included in X, and L_2 is given by Eq. (1) in the Heisenberg representation. We shall tentatively assume that L_2 does not include the term $\lambda\bar{\psi}_H(x)\gamma_\mu\gamma_5(\partial/\partial x_\mu)\psi_H(x)$ as a primary interaction. Our proposal in Eq. (8) is that the parity-nonconserving term

$$\left(-\frac{a}{(1-a^2)^{1/2}}:\bar{\psi}_H(x)\gamma_\mu\gamma_5\frac{\partial}{\partial x_\mu}\psi_H(x):\right)$$

should be added to L_0 and subtracted from L', as is the self-mass term $(-\delta m:\bar{\psi}_H(x)\psi_H(x):)$, in order to perform the renormalization consistently.

We shall start with the definition of the modified propagator

$$\begin{aligned}
\langle T[\psi_H(x)\bar{\psi}_H(y)]\rangle_0&=-S_F'(x-y)\\
&=-\frac{1}{(2\pi)^4}\int d^4p\,e^{ip(x-y)}S_F'(p).
\end{aligned} \tag{9}$$

[2] The renormalizability of vector meson theories was considered, for example, by A. Salam, Phys. Rev. **127**, 331 (1962); T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962).

The improper self-energy part[3] Σ is related to S_F' by

$$S_F'(p)=S_F(p)+S_F(p)\Sigma(p)S_F(p)\,, \tag{10}$$

where $S_F(p)$ is given by

$$S_F(p)=-i\frac{i\Gamma p-m}{p^2+m^2-i\epsilon}\,. \tag{11}$$

One may express the improper self-energy part as[4]

$$\Sigma(p)=\int d^4(x-y)e^{-ip(x-y)}\langle T[O_H(x)O_H(y)]\rangle_0 -i\delta m-i\frac{a}{(1-a^2)^{1/2}}i\gamma p\gamma_5\,, \tag{12}$$

where δm is defined by

$$m=\frac{m_0}{(1-a^2)^{1/2}}+\delta m \tag{13}$$

and $O_H(x)$ by the equation of motion

$$\left(\Gamma_\mu\frac{\partial}{\partial x_\mu}+m\right)\psi_H(x)=O_H(x)\,. \tag{14}$$

Under the assumptions of Källén[5] and Lehmann[6] (except for the invariance under both C and P transformations), one obtains (Appendix 1)

$$\Sigma(p)=i\left\{-\delta m-\frac{a^2m}{1-a^2}+z\int_0^\infty dx^2[(m-x)\sigma_1'+\rho_2]\right\}-i^2\Gamma p\gamma_5\left\{\frac{a}{1-a^2}+z\int_0^\infty dx^2\rho_3'\right\}$$
$$+i(i\Gamma p+m)\left\{\frac{a^2}{1-a^2}+z\int_0^\infty dx^2\sigma_1'\right\}+i(i\Gamma p+m)z\int_0^\infty dx^2\frac{(i\Gamma p-x)\sigma_1'+\rho_2+i\Gamma p\gamma_5\rho_3'}{p^2+x^2-i\epsilon}(i\Gamma p+m)\,, \tag{15}$$

where z and the spectral functions σ_1', ρ_2, and ρ_3' are all real. Because of the CP invariance of L, the terms that are proportional to γ_5 cannot appear in expression (15). As will be shown in Appendix 1, two spectral functions σ_1' and ρ_3' may have the forms

$$\sigma_1'=\sigma_1+b\delta(x^2-m^2)\,,\qquad \rho_3'=\rho_3+c\delta(x^2-m^2)\,, \tag{16}$$

where σ_1 and ρ_3 no longer include the δ function. The remaining spectral function ρ_2 does not include the δ function.

The renormalization constant z_2 for the wave function $\psi_H(p)$ is determined by the conditions[3]

$$S_F'\propto z_2\,,\qquad [S_F'(p)S_F^{-1}(p)]\psi(p)=z_2\psi(p)\,, \tag{17}$$

where the brackets [ ] mean that S_F^{-1} should be replaced by S_F' before operating on $\psi(p)$. From Eqs. (10) and (15) and the condition (17), one obtains

$$\delta m=z_2\int_0^\infty dx^2[(m-x)\rho_1+\rho_2+am\rho_3]\,,\qquad z_2^{-1}=\int_0^\infty dx^2[\rho_1-a\rho_3]\,,\qquad \frac{a}{1-a^2}=-z_2\int_0^\infty dx^2\rho_3\,, \tag{18}$$

$$S_F'(p)=-iz_2\int_0^\infty dx^2\frac{(i\Gamma p-x)\rho_1+\rho_2+i\Gamma p\gamma_5\rho_3}{p^2+x^2-i\epsilon}\,, \tag{19}$$

where $\rho_1\equiv\delta(x^2-m^2)+\sigma_1$.

[3] F. J. Dyson, Phys. Rev. **75**, 486, 1736 (1949).

[4] K. Hiida and M. Sawamura, Progr. Theoret. Phys. (Kyoto) **14**, 167 (1955).

It is remarkable that all expressions in Eqs. (18) and (19) are independent of b and c appeared in Eq. (16). This independence comes from the nature of $\Sigma(p)$ which is independent of b and c. When Eq. (16) is substituted into the expression (15), the last term in (15) reduces to

$$-ibz(i\Gamma p+m)+iczi\Gamma p\gamma_5+iz(i\Gamma p+m)\times\int_0^\infty dx^2\frac{(i\Gamma p-x)\sigma_1+\rho_2+i\Gamma p\gamma_5\rho_3}{p^2+x^2-i\epsilon}(i\Gamma p+m)\,.$$

The first and the second terms in this expression are canceled by the corresponding terms which come from the third and the second terms in (15), respectively. Further, $\int_0^\infty dx^2(m-x)\sigma_1'=\int_0^\infty dx^2(m-x)\sigma_1$. Thus, it has been established that $\Sigma(p)$ is independent of b and c.

Among the many inequalities between the spectral functions ρ_i, one of the more useful ones is

$$\rho_1+a\rho_3\geqslant 0\,. \tag{20}$$

From the second and the third of Eqs. (18), one obtains another expression for z_2, namely,

$$z_2^{-1}=\frac{1-a^2}{1-3a^2}\int_0^\infty dx^2[\rho_1+a\rho_3]\,. \tag{21}$$

The renormalization constant z_2 should have the physical meanings of a probability. Use of the inequality (20) in Eq. (21) leads to the upper bound

$$a^2\leqslant\tfrac{1}{3}\,. \tag{22}$$

[5] G. Källén, Helv. Phys. Acta **25**, 416 (1952).

[6] H. Lehmann, Nuovo Cimento **11**, 342 (1954).

This is equivalent to

$$\left|z_2\int_0^\infty dx^2\rho_3\right|\leqslant\tfrac{1}{2}\sqrt{3}. \qquad (22')$$

The range $1>a^2>\frac{1}{3}$ or the equivalent $\infty>|z_2\int_0^\infty dx^2\rho_3|>\frac{1}{2}\sqrt{3}$ implies the existence of "ghost states," as in the Lee model.[7]

From Eqs. (17) and (18) one obtains

$$\langle\{\psi_H(x),\bar{\psi}_H(y)\}\rangle_0|_{x_0=y_0}=\Gamma_0\left[1-\frac{a(a+\gamma_5)}{1-a^2}\right]\delta(\mathbf{x}-\mathbf{y}) \qquad (7)$$

for the anticommutator at equal time $x_0=y_0$. Although the anticommutator in interaction representation (6) is unequal to that in the Heisenberg representation (7) (when $a\neq 0$), the proportionality of the field operators in the two representations

$$\psi_H(p)=z_2^{1/2}\psi(p),\quad \bar{\psi}_H(p)=z_2^{1/2}\bar{\psi}(p)$$

holds for $(i\Gamma p+m)=0$.

3. THE NECESSITY OF INTRODUCING THE PARITY-NONCONSERVING COUNTER TERM

In the preceding section, the concept of the renormalization of the parity-nonconserving term was introduced. In this section we shall show what happens when this concept is not introduced.

Since this concept is similar to that of mass renormalization, we may learn something about the former from the latter. Suppose the fictitious case in which the concept of mass renormalization has not been introduced. Then Eq. (12) is replaced by

$$\Sigma_1(p)=\int d^4(x-y)e^{-ip(x-y)}\times\langle T[O_{1H}(x)\bar{O}_{1H}(y)]\rangle_0-i\frac{a}{(1-a^2)^{1/2}}i\gamma p\gamma_5,$$

$$O_{1H}(x)=O_H(x)-\delta m\psi_H(x),$$

which leads to

$$\int_0^\infty dx^2[(m-x)\rho_1+\rho_2+am\rho_3]=0. \qquad (23)$$

Equation (23) means that

$$\Sigma_1(p)|_{(i\Gamma p+m)=0}=0.$$

But this is known to contradict the perturbation calculations. The contradiction shows the necessity of mass renormalization.

If we did not know that it was necessary to introduce the parity-nonconserving counter term, then we would write

$$\Sigma_2(p)=\int d^4(x-y)e^{-ip(x-y)}\langle T[O_{2H}(x)\bar{O}_{2H}(y)]\rangle_0-i\delta m,$$

$$O_{2H}(x)=O_H(x)-\frac{a}{(1-a^2)^{1/2}}\gamma_\mu\gamma_5\frac{\partial}{\partial x_\mu}\psi_H(x),$$

which leads to

$$\int_0^\infty dx^2\rho_3=0. \qquad (24)$$

Equation (24) was obtained by Sekine and discussed by Albright *et al.*[8] Contrary to the arguments given by the latter authors, however, the spectral function ρ_3 no longer contains the δ function $\delta(x^2-m^2)$. Again Eq. (24) means

$$\Sigma_2(p)|_{(i\gamma p+m)=0}=0,$$

which also contradicts the perturbation calculation in Sec. 6. This contradiction shows that it is also necessary to introduce the parity-nonconserving counter term.

4. PARITY-NONCONSERVING COUNTER TERM AS A PRIMARY INTERACTION

We have assumed that the parity-nonconserving term $\bar{\psi}_H(x)\gamma_\mu\gamma_5\cdot\partial\psi_H(x)/\partial x_\mu$ is not involved in L_2. The extension of this limitation is to assume that L includes the term

$$\lambda(1-a^2)^{-1/2}:\bar{\psi}_H(x)\gamma_\mu\gamma_5\frac{\partial}{\partial x_\mu}\psi_H(x): \qquad (25)$$

as a primary interaction. If the field ψ_H represents a charged field with spin $\frac{1}{2}$, the Lagrangian density L should also include the term

$$-ie\lambda(1-a^2)^{-1/2}:\bar{\psi}_H(x)\gamma_\mu\gamma_5A_{H\mu}(x)\psi_H(x): \qquad (26)$$

as a primary interaction because of the gauge invariance of the theory. Then λ should be very small. Although we do not like to take the parity-nonconserving electromagnetic interaction (26) as a primary interaction, for completeness we shall describe very briefly the results obtained by taking account of the term (25) as a primary interaction.

Both for charged and neutral fields, Eq. (12) is replaced by

$$\Sigma_3(p)=\int d^4(x-y)e^{-ip(x-y)}\langle T[O_{3H}(x)\bar{O}_{3H}(y)]\rangle_0-i\delta m-i\frac{a+\lambda}{(1-a^2)^{1/2}}i\gamma p\gamma_5,$$

where

$$O_{3H}(x)=O_H(x)+\frac{\lambda}{(1-a^2)^{1/2}}\gamma_\mu\gamma_5\frac{\partial}{\partial x_\mu}\psi_H(x).$$

[7] T. D. Lee, Phys. Rev. **95**, 1329 (1954).

[8] The author would like to thank Dr. K. Yamamoto for calling his attention (after Sec. 2 above was completed) to the papers of Sekine and of Albright *et al.* [K. Sekine, Nuovo Cimento **11**, 87 (1959); C. H. Albright, R. Haag, and S. B. Treiman, Nuovo Cimento **13**, 1282 (1959)].

Consequently, the third equation in (18), and Eqs. (21) and (22) are changed to

$$\frac{a+\lambda}{1-a^2}=-z_2\int_0^\infty dx^2\rho_3,$$

$$z_2^{-1}=\frac{1-a^2}{1-3a^2-2a\lambda}\int_0^\infty dx^2[\rho_1+a\rho_3],$$

$$\tfrac{1}{2}(1-3a^2)\geqslant a\lambda.$$

The anticommutator (7) at equal time is changed to

$$\langle\{\psi_H(x),\bar\psi_H(y)\}\rangle_0|_{x_0=y_0}=\Gamma_0\left[1-\frac{(a+\lambda)(a+\gamma_5)}{1-a^2}\right]\delta(\mathbf{x}-\mathbf{y}).$$

5. TWO-COMPONENT THEORY IN THE NONRELATIVISTIC REGION

As was shown by Eq. (5), and also will be shown in Appendix 2, the coefficient a has no physical meaning for free particles. To determine whether or not the coefficient a has any physical meaning when a spin-½ particle interacts with other fields, we shall study the equation of motion for a charged spin-½ particle in a weak external electromagnetic field.

The equation of motion (Appendix 2) is

$$H\psi=\frac{(1+a\gamma_5)}{(1-a^2)^{1/2}}E\psi, \tag{27}$$

where the Hamiltonian is expressed as

$$H=m\beta+e\phi\frac{(1+a\gamma_5)}{(1-a^2)^{1/2}}+\boldsymbol{\alpha}\frac{(1+a\gamma_5)}{(1-a^2)^{1/2}}(\mathbf{p}-e\mathbf{A}). \tag{28}$$

It should be noted that the form of Eq. (27) cannot be changed to the usual form

$$H'\psi=E\psi,$$

because $H'=(1-a\gamma_5)(1-a^2)^{-1/2}H$ is not a Hermitian operator and cannot be the Hamiltonian of the system.

To clarify the meaning of the coefficient a, let us divide Eq. (27) into two equations of motion, each of which concerns only one of the components ψ_1 or ψ_2 of the wave function

$$\psi=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}.$$

For this purpose we shall use the usual representation

$$\boldsymbol{\alpha}=\begin{pmatrix}0 & \boldsymbol{\sigma}\\ \boldsymbol{\sigma} & 0\end{pmatrix},\quad \beta=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \gamma_5=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}, \tag{29}$$

where $\boldsymbol{\sigma}$ is the 2×2 Pauli spin matrix. Substituting the representation (29) for the γ matrix into Eq. (27) yields the two equations

$$\begin{aligned}E_2\psi_1&=[e\phi-a\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\psi_1\\&\quad+[aE-ae\phi+\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\psi_2,\\E_1\psi_2&=[e\phi-a\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\psi_2\\&\quad+[aE-ae\phi+\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\psi_1,\end{aligned} \tag{30}$$

where $E_1=E+m(1-a^2)^{1/2}$ and $E_2=E-m(1-a^2)^{1/2}$.

Because of our assumption that the external electromagnetic field is very weak, we may express ψ_2 in terms of ψ_1 as

$$\psi_2=\sum_{n=0}^{\infty}\left(\frac{1}{E_1}[e\phi-a\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\right)^n\times\frac{1}{E_1}[aE-ae\phi+\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\psi_1, \tag{31}$$

which is obtained from the second of Eqs. (30). From Eqs. (30) and (31), the equation of motion for ψ_1 can be written

$$\left\{E_2-e\phi+a\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})-[aE-ae\phi+\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\times\sum_{n=0}^{\infty}\left(\frac{1}{E_1}[e\phi-a\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\right)^n\times\frac{1}{E_1}[aE-ae\phi+\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\right\}\psi_1=0. \tag{32}$$

Similarly, the equation of motion for ψ_2 is

$$\left\{E_1-e\phi+a\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})-[aE-ae\phi+\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\times\sum_{n=0}^{\infty}\left(\frac{1}{E_2}[e\phi-a\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\right)^n\times\frac{1}{E_2}[aE-ae\phi+\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})]\right\}\psi_2=0. \tag{33}$$

By the transformation

$$E\to-E,\quad \mathbf{p}\to-\mathbf{p},\quad\text{and}\quad e\to-e,$$

Eq. (33) reduces to Eq. (32). Therefore, *a particle and its antiparticle have the same value of a, including its sign.*

From Eq. (30) or Eqs. (32) and (33), the expressions

$$(E^2-m^2)\psi_i=0\quad(i=1,2) \tag{34}$$

are obtained for a free particle at rest. Since these equations are independent of a, we may take $a=0$ in

Eq. (30) or Eqs. (32) and (33) to obtain the equations of motion for the free particle at rest. Then

$$(E-m)\psi_1=0, \quad (E+m)\psi_2=0. \tag{35}$$

Therefore, the equations

$$(E+m)\psi_1=0, \quad (E-m)\psi_2=0$$

obtained from Eqs. (34) are singular at $a=0$. For a moving free particle, the expressions

$$(E^2-\mathbf{p}^2-m^2)\psi_i=0 \quad (i=1, 2) \tag{36}$$

are obtained from Eq. (30) or from Eqs. (32) and (33). Again these equations are independent of a; and therefore, *a has no physical meaning for free particles* (Appendix 2). In this section we shall consider only the positive-energy and negative-energy solutions for ψ_1 and ψ_2, *respectively*, that is, nonsingular solutions at $a=0$.

For a changed particle in a weak external electromagnetic field, we get from Eq. (32) that

$$\Big\{(E^2-m^2)(1-a^2)-E_1X-aEY-E_1Y\frac{aE}{E_1}-aEX\frac{aE}{E_1}$$
$$-E_1Y\frac{1}{E_1}Y-E_1Y\frac{1}{E_1}X\frac{aE}{E_1}-aEX\frac{1}{E_1}Y$$
$$-aEX\frac{1}{E_1}X\frac{aE}{E_1}+O\left(\frac{1}{E}\right)\Big\}\psi_1=0, \tag{37}$$

where $X=e\phi-a\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})$ and $Y=-ae\phi+\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})$. Because of our assumption that the external field is weak, we may neglect the terms

$$\mathbf{p}^2, \quad (e\phi)^2, \quad (e\mathbf{A})^2, \quad (e\phi)\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A}), \quad \text{etc.},$$

in comparison with m^2. In this approximation it may be shown that the left-hand side of Eq. (37) is proportional to $(1-a^2)$, which is not equal to zero. Taking the positive-energy solution for ψ_1 reduces Eq. (37) to

$$E\psi_1=\Big\{m+e\phi+\frac{1}{2m}\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})\cdot\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})$$
$$-\frac{a}{2m[1+(1-a^2)^{1/2}]}[e\phi\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})+\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})e\phi]$$
$$+\frac{a^2}{2m[1+(1-a^2)^{1/2}]^2}(e\phi)^2$$
$$+O\left(\frac{1}{m^2}\right)\Big\}\psi_1\equiv H'\psi_1. \tag{38}$$

It is evident that our new Hamiltonian H' is a Hermitian operator. Since H' was obtained without assuming $a^2\ll1$, Eq. (38) holds for $a^2<1$. In our Schrödinger picture, the strengths $\mathbf{E}$ and $\mathbf{H}$ of the electric and magnetic fields are related to ϕ and $\mathbf{A}$ by

$$\mathbf{E}=-\nabla\phi \quad \text{and} \quad \mathbf{H}=\nabla\times\mathbf{A}.$$

In terms of these field strengths, the equation of motion (38) can be rewritten as

$$\Big\{m+e\phi+\frac{(\mathbf{p}-e\mathbf{A})^2}{2m}-\frac{e}{2m}\boldsymbol{\sigma}\mathbf{H}$$
$$-\frac{a}{m[1+(1-a^2)^{1/2}]}e\phi\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})-\frac{iae}{2m[1+(1-a^2)^{1/2}]}\boldsymbol{\sigma}\mathbf{E}$$
$$+\frac{a^2}{2m[1+(1-a^2)^{1/2}]^2}(e\phi)^2+O\left(\frac{1}{m^2}\right)\Big\}\psi_1=E\psi_1. \tag{39}$$

Thus, it has been shown that *the coefficient a of the parity-nonconserving counter term has physical meaning*, at least for a charged particle with spin $\frac{1}{2}$.

The positive-energy solution for ψ_1 was taken to obtain Eqs. (38) and (39). If the negative-energy solution for ψ_1 is taken, it follows from Eq. (37) that the equation of motion for ψ_1 is

$$\Big\{-m+e\phi-\frac{(\mathbf{p}-e\mathbf{A})^2}{2m}+\frac{e}{2m}\boldsymbol{\sigma}\mathbf{H}$$
$$+\frac{a}{m[1-(1-a^2)^{1/2}]}e\phi\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})+\frac{iae}{2m[1-(1-a^2)^{1/2}]}\boldsymbol{\sigma}\mathbf{E}$$
$$-\frac{a^2}{2m[1-(1-a^2)^{1/2}]^2}(e\phi)^2+O\left(\frac{1}{m^2}\right)\Big\}\psi_1=E\psi_1, \tag{40}$$

instead of Eq. (39). Again this negative-energy solution is singular at $a=0$. In the same approximation, the negative-energy solution for ψ_2 is

$$\Big\{-m+e\phi-\frac{(\mathbf{p}-e\mathbf{A})^2}{2m}+\frac{e}{2m}\boldsymbol{\sigma}\mathbf{H}$$
$$+\frac{a}{m[1+(1-a^2)^{1/2}]}e\phi\boldsymbol{\sigma}(\mathbf{p}-e\mathbf{A})+\frac{iae}{2m[1+(1-a^2)^{1/2}]}\boldsymbol{\sigma}\mathbf{E}$$
$$-\frac{a^2}{2m[1+(1-a^2)^{1/2}]^2}(e\phi)^2+O\left(\frac{1}{m^2}\right)\Big\}\psi_2=E\psi_2. \tag{41}$$

It is remarkable that the coefficient of the parity-nonconcerving counter term is an observable.

6. A RENORMALIZABLE EXAMPLE

In this section we shall calculate the spectral functions ρ_i in the lowest order approximation of perturbation theory. As an example, we shall adopt here the following model which was considered by Sekine.[8] Two spin-$\frac{1}{2}$ fields ψ_H and χ_H interact through a spin-0 field ϕ_H

by the Lagrangian density, L_0 being given by Eq. (8) and L' by

$$L'=-:\bar{\chi}_H(x)\left[\Gamma_\mu'\frac{\partial}{\partial x_\mu}+m'\right]\chi_H(x):+:\varphi_H^*(x)[\Box-\mu^2]\varphi_H(x):+:\bar{\psi}_H(x)[g_1+g_2\gamma_5]\chi_H(x)\varphi_H^*(x):$$
$$+:\bar{\chi}_H(x)[g_1-g_2\gamma_5]\psi_H(x)\varphi_H(x):+\delta m:\bar{\psi}_H(x)\psi_H(x):+\delta m':\bar{\chi}_H(x)\chi_H(x):+\delta\mu^2:\varphi_H^2(x):$$
$$+\lambda_4:\varphi_H^4(x):+a(1-a^2)^{-1/2}:\bar{\psi}_H(x)\gamma_\mu\gamma_5\frac{\partial}{\partial x_\mu}\psi_H(x):+a'(1-a^2)^{-1/2}:\bar{\chi}_H(x)\gamma_\mu\gamma_5\frac{\partial}{\partial x_\mu}\chi_H(x):,\quad (42)$$

where the coupling constants g_1 and g_2 are real and

$$\Gamma_\mu'=\gamma_\mu\frac{(1+a'\gamma_5)}{(1-a'^2)^{1/2}}.$$

For stability of three fields, we shall assume the triangle relations

$$m<m'+\mu,\quad m'<m+\mu,\quad \mu<m+m'.\qquad (43)$$

From the Lagrangian density, the operator $O_H(x)$ defined by Eq. (14) is given by

$$O_H(x)=[g_1+g_2\gamma_5]\chi_H(x)\varphi_H^*(x)+\delta m\psi_H(x)+a(1-a^2)^{-1/2}\gamma_\mu\gamma_5\frac{\partial}{\partial x_\mu}\psi_H(x).\qquad (44)$$

From the definition of the spectral functions A_i given by the expression (A1), it follows that

$$\langle O_H(x)\bar{O}_H(y)\rangle_0=-\frac{1}{(2\pi)^3}\int d^4p\theta(p_0)e^{ip(x-y)}\times\int_0^\infty dx^2\delta(p^2+x^2)\times[(i\Gamma p-x)A_1+A_2+i\Gamma p\gamma_5A_3].\qquad (45)$$

On the other hand, in the lowest order of the coupling constants

$$\langle O_H(x)\bar{O}_H(y)\rangle_0=(g_1+g_2\gamma_5)\langle\chi(x)\bar{\chi}(y)\rangle_0\times(g_1-g_2\gamma_5)\langle\varphi^*(x)\varphi(y)\rangle_0.\qquad (46)$$

The two expressions (45) and (46) leads to the equation in the momentum space,

$$\theta(p_0)[(i\Gamma p-[-p^2]^{1/2})A_1+A_2+i\Gamma p\gamma_5A_3]=\frac{1}{(2\pi)^3}\int d^4k\theta(p_0-k_0)\theta(k_0)\delta[(p-k)^2+m'^2]\delta[k^2+\mu^2]\times(g_1+g_2\gamma_5)[i\Gamma(p-k)-m'](g_1-g_2\gamma_5)\qquad (47)$$

for $p^2<0$. Performing the k integration in the above expression leads to

$$A_1=(g_1^2+g_2^2)\frac{x^2+m'^2-\mu^2}{2x^2}X(x),$$
$$A_2=\left\{(g_1^2+g_2^2)\frac{x^2+m'^2-\mu^2}{2x}-m'(g_1^2-g_2^2)\right\}X(x),$$
$$A_3=-g_1g_2\frac{x^2+m'^2-\mu^2}{x^2}X(x),\qquad (48)$$

$$X(x)=\frac{1}{(4\pi)^2}\theta[x-(m'+\mu)]\times\frac{[(x^2-m'^2-\mu^2)^2-4m'^2\mu^2]^{1/2}}{x^2}.$$

From Eqs. (48) and (A3), one gets

$$\sigma_1=\frac{1}{(x^2-m^2)^2}\left\{(g_1^2+g_2^2)(x^2+m^2)\times\frac{(x^2+m'^2-\mu^2)}{2x^2}+2mm'(g_1^2-g_2^2)\right\}X(x),$$
$$\rho_2=\frac{1}{(x+m)^2}\left\{(g_1^2+g_2^2)\frac{(x^2+m'^2-\mu^2)}{2x}-m'(g_1^2-g_2^2)\right\}X(x),\qquad (49)$$
$$\rho_3=g_1g_2\frac{(x^2+m'^2-\mu^2)}{x^2(x^2-m^2)}X(x).$$

It is evident that $\int_0^\infty dx^2\rho_3\neq0$.

At high energies the renormalized spectral functions in the lowest order of coupling constants behave as

$$\sigma_1\approx\frac{(g_1^2+g_2^2)}{32\pi^2}\frac{1}{x^2},$$
$$\rho_2\approx\frac{(g_1^2+g_2^2)}{32\pi^2}\frac{1}{x},\qquad (50)$$
$$\rho_3\approx\frac{g_1g_2}{16\pi^2}\frac{1}{x^2},$$

which show the logarithmic divergence of δm, z_2^{-1} and $\int_0^\infty dx^2\rho_3$. We now assume that the present theory in fact has the divergent nature indicated by the lowest order calculations, and further assume that the rough magnitudes of the exact spectral functions are given by Eq. (50). Then the conditions (22) and (22′) are satisfied when

$$\left|\frac{g_2}{g_1}\right|\gtrsim\sqrt{3}\ \text{or}\ \left|\frac{g_2}{g_1}\right|\lesssim\frac{1}{\sqrt{3}}.$$

7. ON THE ELECTRON AND THE MUON

In the usual sense, weak interactions are unrenormalizable, and our renormalization procedure described in Sec. 2 is not applicable to them. To estimate the rough magnitude of the coefficients a_e and a_μ for the electron and the muon in the lowest order of the weak coupling constant, therefore, we shall introduce a cutoff energy Λ.

When an intermediate vector boson is introduced, the Lagrangian density for weak interactions is usually assumed to be given by

$$L_{\text{weak}}=ig\bar{\psi}_{e,H}(x)\gamma_\alpha\tfrac{1}{2}(1+\gamma_5)\psi_{\nu e,H}(x)\varphi_{\alpha,H}(x)+ig\bar{\psi}_{\mu,H}(x)\gamma_\alpha\tfrac{1}{2}(1+\gamma_5)\psi_{\nu\mu,H}(x)\varphi_{\alpha,H}(x)+\text{Hermite conjugate}, \quad (51)$$

where ψ_e, ψ_μ, φ_α, $\psi_{\nu e}$, and $\psi_{\nu\mu}$ represent the electron, the muon, the boson, and the neutrinos associated with the electron and the muon, respectively. To calculate the coefficients a_e and a_μ to the order g^2, it is enough to use

$$O_{e,H}(x)=ig\gamma_\alpha\frac{1+\gamma_5}{2}\psi_{\nu e}(x)\varphi_\alpha(x)+\delta m_e\psi_e(x)+\frac{a_e}{(1-a_e^2)^{1/2}}\gamma_\alpha\gamma_5\frac{\partial}{\partial x_\alpha}\psi_e(x),$$

$$O_{\mu,H}(x)=ig\gamma_\alpha\frac{1+\gamma_5}{2}\psi_{\nu\mu}(x)\varphi_\alpha(x)+\delta m_\mu\psi_\mu(x)+\frac{a_\mu}{(1-a_\mu^2)^{1/2}}\gamma_\alpha\gamma_5\frac{\partial}{\partial x_\alpha}\psi_\mu(x), \quad (52)$$

for the electron and the muon, respectively. To calculate the spectral functions ρ_1 and ρ_2, it is necessary to add the terms concerned with electromagnetic interactions to the above expressions.

From Eqs. (45) and (52), one obtains

$$\theta(p_0)i\Gamma p\gamma_5 A_3=-\frac{ig^2}{(2\pi)^3}\int d^4k\theta(p_0-k_0)\theta(k_0)\delta[(p-k)^2]\delta[k^2+M_B^2]\times\gamma_\alpha\Gamma(p-k)\gamma_\beta\frac{\gamma_5}{2}\left[\delta_{\alpha\beta}+\frac{k_\alpha k_\beta}{M_B^2}\right] \quad (53)$$

instead of Eq. (47), where M_B is the boson mass. Performing the k integration in Eq. (53) and using the definition (A3) leads to

$$\rho_3=-\frac{g^2}{64\pi^2}\theta(x^2-M_B^2)\frac{(x^2-M_B^2)^2(x^2+2M_B^2)}{M_B^2(x^2-m^2)x^4}\leqslant 0, \quad (54)$$

where m is the mass of the electron or the muon. The electron and the muon have no strong interaction. Therefore, the renormalization constant z_2 for them would be very close to unity when a cutoff energy Λ is introduced. Since the coefficient a is proportional to the weak coupling constant g^2, it follows that $a(1-a^2)^{-1/2}\approx a\ll 1$. Taking these conditions into account, one gets both for the electron and the muon

$$a_e\approx a_\mu\approx-\int_{M_B^2}^{\Lambda^2}dx^2\rho_3\approx\frac{g^2}{(8\pi)^2}\frac{\Lambda^2}{M_B^2}>0, \quad (55)$$

where $\Lambda^2\gg M_B^2$ is assumed.

When the intermediate boson does not exist, the Lagrangian density (51) is replaced by

$$L_{\text{weak}}=\frac{G}{\sqrt{2}}\left[\bar{\psi}_{e,H}(x)\gamma_\alpha\frac{(1+\gamma_5)}{2}\psi_{\nu e,H}(x)\right]\times[\bar{\psi}_{\nu\mu,H}(x)\gamma_\alpha\tfrac{1}{2}(1+\gamma_5)\psi_{\mu,H}(x)]+\text{Hermite conjugate}, \quad (56)$$

from which

$$O_{e,H}(x)=\frac{G}{\sqrt{2}}\gamma_\alpha\tfrac{1}{2}(1+\gamma_5)\psi_{\nu e,H}(x)\times[\bar{\psi}_{\nu\mu,H}(x)\gamma_\alpha\tfrac{1}{2}(1+\gamma_5)\psi_{\mu,H}(x)]+\delta m_e\psi_{e,H}(x)+\frac{a_e}{(1-a_e^2)^{1/2}}\gamma_\alpha\gamma_5\frac{\partial}{\partial x_\alpha}\psi_{e,H}(x) \quad (57)$$

is obtained.

For the electron one gets

$$\theta(p_0)i\Gamma p\gamma_5 A_3\approx-\frac{G^2}{2}\frac{1}{(2\pi)^6}\int d^4kd^4q\theta(p_0-k_0)\theta(k_0-q_0)\theta(-q_0)\times\delta[(p-k)^2]\delta[(k-q)^2+m_\mu^2]\delta[q^2]\times\gamma_\alpha i\gamma(p-k)\gamma_\beta\tfrac{1}{2}(1+\gamma_5)S_p\{\tfrac{1}{2}(1+\gamma_5)\times[i\gamma(k-q)-m_\mu]\gamma_\beta\tfrac{1}{2}(1+\gamma_5)i\gamma q\gamma_\alpha\}. \quad (58)$$

Since this expression diverges, we shall introduce the cutoff factor $\theta(\Lambda+q_0)$ into the integrand. Long but straightforward calculations show that

$$\gamma_\alpha i\gamma(p-k)\gamma_\beta\tfrac{1}{2}(1+\gamma_5)\times S_p\{\tfrac{1}{2}(1+\gamma_5)[i\gamma(k-q)-m_\mu]\gamma_\beta\tfrac{1}{2}(1+\gamma_5)i\gamma q\gamma_\alpha\}\delta(q^2)=4[k^2-pk+pq-kq]i\gamma q(1+\gamma_5)\delta(q^2). \quad (59)$$

Substituting Eq. (59) into Eq. (58) and introducing the cutoff factor, one gets

$$\theta(p_0)i\Gamma p\gamma_5 A_3=-\frac{G^2}{(2\pi)^6}\int d^4kd^4q\theta(p_0-k_0)\theta(k_0-q_0)\theta(-q_0)\theta(\Lambda+q_0)\times\delta[(p-k)^2]\delta[(k-q)^2+m_\mu^2]\delta[q^2]\times[2pq-p^2-m_\mu^2]i\gamma q\gamma_5 \quad (60)$$

to the order G^2. This leads to

$$\rho_3 \approx -\frac{G^2}{24(2\pi)^4}\theta(x-m_\mu)\frac{\Lambda^3[2x+3\Lambda]}{x^2}, \quad (61)$$

where m_μ and m_e are neglected except for the factor $\theta(x-m_\mu)$. The expression (61) gives

$$a_e \approx \frac{G^2}{8(2\pi)^4}\Lambda^4\left(\ln\frac{\Lambda^2}{m_\mu^2}+\frac{4}{3}\right)>0. \quad (62)$$

The magnitude of the coupling constant is

$$G \approx \frac{\sqrt{2}g^2}{M_B^2} \approx \frac{4\times10^{-5}}{M_N^2},$$

where M_N is the nucleon mass. With this value of the coupling constant, the coefficient a_e of the electron is

$$a_e \approx \frac{10^{-5}}{\sqrt{2}(4\pi)^2}\left(\frac{\Lambda}{M_N}\right)^2 \quad (63)$$

when the intermediate boson exists, and

$$a_e \approx \frac{2\times10^{-10}}{(2\pi)^4}\left(\frac{\Lambda}{M_N}\right)^4\left[\ln\frac{\Lambda^2}{m_\mu^2}+\frac{4}{3}\right] \quad (64)$$

when the boson does not exist. On the assumption that $\Lambda \lesssim 300$ BeV, both expressions (63) and (64) give

$$0 < a_e \lesssim 10^{-2}.$$

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APPENDIX 1: DERIVATION OF EQ. (15)

Under the same assumptions (except for the invariance under C or P transformation) as Källén[5] and Lehmann[6] have made, one may obtain

$$\int d^4(x-y)e^{-ip(x-y)}\langle T[O_H(x)\bar{O}_H(y)]\rangle_0$$
$$= i\int_0^{\Lambda^2} dx^2 \frac{1}{p^2+x^2-i\epsilon} \times[(i\Gamma p-x)A_1+A_2+i\Gamma p\gamma_5 A_3], \quad (A1)$$

where the A_i are all real functions because of the CP invariance of L, and the cutoff Λ for massive intermediate states is introduced to avoid possible divergences.

To perform the renormalization, the expression (A1) will be expanded in powers of $(i\Gamma p+m)$ in such a manner that all terms appearing in the expanded expression are CP invariant. The result is

$$\int d^4(x-y)e^{-ip(x-y)}\langle T[O_H(x)\bar{O}_H(y)]\rangle_0$$
$$= -i\int_0^{\Lambda^2} dx^2\frac{(x+m)A_1-A_2}{(x^2-m^2)} - \Gamma p\gamma_5\int_0^{\Lambda^2} dx^2\frac{A_3}{(x^2-m^2)}$$
$$+i(i\Gamma p+m)\int_0^{\Lambda^2} dx^2\frac{(x+m)^2A_1-2mA_2}{(x^2-m^2)^2}$$
$$+i(i\Gamma p+m)\int_0^{\Lambda^2} dx^2\frac{1}{(p^2+x^2-i\epsilon)(x^2-m^2)^2}$$
$$\times\{(i\Gamma p-x)[(x+m)^2A_1-2mA_2]$$
$$+(x-m)^2A_2-(x^2-m^2)i\Gamma p\gamma_5 A_3\}(i\Gamma p+m). \quad (A2)$$

Introducing the new notations

$$z\sigma_1' = \frac{1}{(x^2-m^2)^2}[(x+m)^2A_1-2mA_2],$$
$$z\rho_2 = \frac{A_2}{(x+m)^2}, \quad (A3)$$
$$z\rho_3' = -\frac{A_3}{(x^2-m^2)}$$

and substituting Eq. (A2) into (12) leads to the expression (15). Although the spectral functions A_i do not include the δ function $\delta(x^2-m^2)$, the renormalized functions σ_1' and ρ_3' may include the δ function because of their definition (A3). This possibility was discussed in Sec. 2.

APPENDIX 2. QUANTIZATION OF THE FREE FIELD

The Lagrangian density for a free spin-½ field is

$$L_0 = -:\bar{\psi}(x)\left[\Gamma_\mu\frac{\partial}{\partial x_\mu}+m\right]\psi(x):, \quad (A4)$$

where the definition of $\bar{\psi}$ is not $\psi^*\Gamma_0$ but $\psi^*\gamma_0$, and ψ^* is the Hermitian conjugate of ψ. Because of this definition of $\bar{\psi}$, the quantization of the free field is a little different from that in the usual case and the Schrödinger equation for the free particle has the unusual form (27).

When expanded into positive-frequency and negative-frequency parts, the expression for $\psi(x)$ is

$$\psi(x) = \frac{1}{(2\pi)^{3/2}}\int d\mathbf{p}\left(\frac{m}{E_p}\right)^{1/2}$$
$$\times\sum_{\mu=1,2}\{\exp(i\mathbf{px}-iE_px_0)a_\mu(\mathbf{p})U^\mu(\mathbf{p})$$
$$+\exp(-i\mathbf{px}+iE_px_0)b_\mu^*(\mathbf{p})V^\mu(\mathbf{p})\}, \quad (A5)$$

where $E_p=(\mathbf{p}^2+m^2)^{1/2}$, $\sum_\mu$ means the summation over all possible spin states, and the spinors U and V satisfy

$$(i\mathbf{\Gamma p}-\Gamma_0 E_p+m)U^\mu(\mathbf{p})=0\,,\qquad (i\mathbf{\Gamma p}-\Gamma_0 E_p-m)V^\mu(\mathbf{p})=0\,. \tag{A6}$$

As in the usual case, a_μ and b_μ will be quantized as

$$\{a_\mu(\mathbf{p}),a_\nu^*(\mathbf{p}')\}=\{b_\mu(\mathbf{p}),b_\nu^*(\mathbf{p}')\}=\delta_{\mu\nu}\delta(\mathbf{p}-\mathbf{p}')\,, \tag{A7}$$

and the other anticommutators are zero. Further we shall take the conditions for the orthonormality of the spinors U, V and their adjoint spinors $\bar{U}$, $\bar{V}$ in the form

$$\bar{U}^\mu(\mathbf{p})U^\nu(\mathbf{p})=-\bar{V}^\mu(\mathbf{p})V^\nu(\mathbf{p})=\delta_{\mu\nu}\,. \tag{A8}$$

From Eqs. (A6) and (A8) one may obtain a number of relations for quadratic form in the spinors, the most important of which are

$$\begin{aligned}
&U^{\mu*}(\mathbf{p})\frac{(1+a\gamma_5)}{(1-a^2)^{1/2}}U^\nu(\mathbf{p})=V^{\mu*}(\mathbf{p})\frac{(1+a\gamma_5)}{(1-a^2)^{1/2}}V^\nu(\mathbf{p})=\frac{E_p}{m}\delta_{\mu\nu}\,,\\
&U^{\mu*}(\mathbf{p})\frac{(1+a\gamma_5)}{(1-a^2)^{1/2}}V^\nu(-\mathbf{p})=V^{\mu*}(\mathbf{p})\frac{(1+a\gamma_5)}{(1-a^2)^{1/2}}U^\nu(-\mathbf{p})=0\,,\\
&\sum_{\mu=1,2}U_\alpha{}^\mu(\mathbf{p})\bar{U}_\beta{}^\mu(\mathbf{p})=-\frac{(i\mathbf{\Gamma p}-\Gamma_0E_p-m)_{\alpha\beta}}{2m}\,,\\
&\sum_{\mu=1,2}V_\alpha{}^\mu(\mathbf{p})\bar{V}_\beta{}^\mu(\mathbf{p})=-\frac{(i\mathbf{\Gamma p}-\Gamma_0E_p+m)_{\alpha\beta}}{2m}\,.
\end{aligned} \tag{A9}$$

Using the usual canonical formalism, we may express the dynamical variables of the free field in terms of operators a_μ and b_μ. The results are exactly the same as those of the usual case in which L_0 is invariant under both C and P transformations. For example, the Hamiltonian of the system is given by

$$H=i\int d\mathbf{x}:\psi^*(x)\frac{(1+a\gamma_5)}{(1-a^2)^{1/2}}\frac{\partial}{\partial x_0}\psi(x):\,, \tag{A10}$$

which has the same form as Eq. (27). By use of Eqs. (A5), (A7), and (A9), the Hamiltonian can be expressed in terms of a_μ and b_μ in the form

$$H=\sum_\mu\int d\mathbf{p}E_p\{a_\mu^*(\mathbf{p})a_\mu(\mathbf{p})+b_\mu^*(\mathbf{p})b_\mu(\mathbf{p})\}\,.$$

From Eqs. (A5)–(A9) one obtains

$$\{\psi_\alpha(x),\bar{\psi}_\beta(y)\}=-\frac{1}{(2\pi)^3}\int d^4p e^{ip(x-y)}[\theta(p_0)-\theta(-p_0)]\times(i\Gamma p-m)_{\alpha\beta}\delta(p^2+m^2)$$

for the anticommutation relation,

$$\{\psi_\alpha(x),\bar{\psi}_\beta(y)\}_{x_0=y_0}=(\Gamma_0)_{\alpha\beta}\delta(\mathbf{x}-\mathbf{y}) \tag{6}$$

for the equal-time anticommutation relation, and

$$\langle T[\psi_\alpha(x)\bar{\psi}_\beta(y)]\rangle_0\equiv-S_{F\alpha\beta}(x-y)=\frac{i}{(2\pi)^4}\int d^4p e^{ip(x-y)}\frac{(i\Gamma p-m)_{\alpha\beta}}{p^2+m^2-i\epsilon} \tag{11}$$

for the propagator of the field.