Electromagnetic Properties of the Neutrino

JEREMY BERNSTEIN* AND MALVIN RUDERMAN†
Department of Physics, New York University, New York, New York

AND

GERALD FEINBERG‡
Department of Physics, Columbia University, New York, New York
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In this note we make a detailed survey of the experimental information on the neutrino charge, charge radius, and magnetic moment. Both weak-interaction data and astrophysical results can be used to give precise limits to these quantities, independent of the supposition that the weak interactions are charge conserving.

I. INTRODUCTION

MOST physicists now accept the prospect that there are two neutrinos—ν_e and ν_μ—identical except for interaction (ν_e couples weakly with electrons and ν_μ with muons) and that these neutrinos have the simplest properties compatible with existing experimental evidence; i.e., zero mass, charge, electric, and magnetic dipole moments. However, the weak interactions have produced so many surprises that it is worthwhile, from time to time, to study the *experimental* limits that have been set on these quantities. In this note we present a systematic survey of the properties of the two neutrinos that can be inferred from experiment.

II. PROPERTIES

We begin by listing the properties of the neutrinos to be discussed: (a) mass, (b) helicity, (c) charge and electromagnetic moments. We do not have any new contributions to make with respect to (a) and (b), and most of the discussion that follows will be concerned with electromagnetics. However, the following summary may be helpful:

(a) Mass

(1) ν_e: The best experimental limit on m_{ν_e} appears to come from a measurement of the end point of the tritium β-decay spectrum.[1] With no assumptions about the specific form of the Fermi couplings, one has

$$m_{\nu_e} < 700 \text{ eV}. \tag{1}$$

If, however,[2] a strict V-A coupling is assumed—a coupling of the form $\gamma_\alpha(1+\lambda\gamma_5)$ with $\lambda=1$—then the tritium experiments give

$$m_{\nu_e} < 200 \text{ eV}, \tag{2}$$

and the experiments are consistent with $m_{\nu_e}=0$.

(2) ν_μ: The mass of the muon neutrino is the least well known of the parameters associated with either neutrino. The best measurements of it come from the energy-momentum balance in π decay. The experiment of Barkas *et al.*[3] gives[4]

$$m_{\nu_\mu} < 3.5 \text{ MeV}. \tag{3}$$

The reason for this uncertainty lies in the kinematic fact that the small neutrino mass is given as the difference between measured quantities of order 1. In the $\pi\rightarrow\mu+\nu$ decay, the accuracy with which the neutrino mass can be determined is given by

$$\Delta m_\nu \approx 100 \text{ MeV } (\Delta p/p)^{1/2}, \tag{4}$$

where Δp is the accuracy with which the muon momentum p is known. The use of the muon-decay spectrum to measure m_{ν_μ} is complicated by electromagnetic radiative corrections, and the limits set on m_{ν_μ} in this way are probably not as precise as those that come from π decay.

(b) Helicity

(1) ν_e: The helicity of the electron neutrino is measured indirectly by measuring the helicity of the other particles emitted along with it in a given reaction. The original measurement of the ν_e helicity, that of Goldhaber *et al.*,[5] gives

$$|\boldsymbol{\sigma}\cdot\mathbf{v}/v| \simeq -0.67 \pm 10\% \tag{5}$$

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† On leave from the Department of Physics, University of California, Berkeley, California.

‡ Alfred P. Sloan Foundation Fellow.

[1] D. R. Hamilton, W. P. Alford, and L. Gross, Phys. Rev. **92**, 1521 (1953); L. M. Langer and R. J. D. Moffat, *ibid.* **88**, 689 (1952). These measurements are discussed in some detail in a review article by C. S. Wu, in *Theoretical Physics in the Twentieth Century* (Interscience Publishers, Inc., New York, 1960).

[2] J. J. Sakurai, Phys. Rev. Letters **1**, 40 (1958).

[3] W. H. Barkas, W. Birnbaum, and F. M. Smith, Phys. Rev. **101**, 778 (1956). In this experiment the error is due both to the measurement of the muon momentum and to the uncertainty in the pion mass ($m_{\pi^+}=139.59\pm0.05$ MeV).

[4] Professor L. M. Lederman has pointed out to us that if the best recently measured value of the pion mass is used, the result of Barkas *et al.* (Ref. 3) is slightly improved to give $m_{\nu_\mu}<3$ MeV.

[5] M. Goldhaber, L. Grodzins, and A. Sunyar, Phys. Rev. **109**, 1015 (1958). A private communication from Dr. Goldhaber informs us that the inclusion of various nuclear corrections might bring the measured neutrino helicity up to -0.9, and that the experimental results are not incompatible with -1.

(measuring the circular polarization of the γ following electronic K capture).

A somewhat more precise number is obtainable by measuring the circular polarization of photons associated with bremsstrahlung of longitudinally polarized electrons emitted in nuclear β decay.[6] These measurements give a neutrino helicity consistent with -1 with an error of 5 to 10%.

(2) ν_μ: The best determination of the helicity of[7] ν_μ comes from a measurement of the electron-asymmetry associated with electrons emitted in the decay of polarized μ's from π decay. Bardon *et al.*[7] find that the muon helicity in π^- decay is $+0.9$, with an error of about 10%, and, hence, the helicity of $\bar{\nu}_\mu$ is determined to be $+0.9$ with the same error.

III. ELECTROMAGNETIC PROPERTIES

Electron-neutrino scattering by 1-photon exchange is described by the matrix element of the electromagnetic current in a one-neutrino state which can be written as

$$\langle \nu' | J_\alpha | \nu \rangle = \bar{\nu}[\gamma_\alpha F_1(q^2) + \gamma_\alpha \gamma_5 F_3(q^2) + \sigma_{\alpha\beta} q_\beta F_2(q^2) + \sigma_{\alpha\beta} q_\beta \gamma_5 F_4(q^2)]\nu. \quad (6)$$

A typical diagram contributing to the form factors in Eq. (6) is given in Fig. 1. In the figure, W represents the intermediate vector meson coupled to leptons. Under various assumptions about the weak couplings, Eq. (6) may be considerably simplified.

(1) If the theory is CP invariant, $F_4(q^2)=0$. This is a generalization of the well-known theorem that CP invariance is enough to rule out the existence of intrinsic electric-dipole moments for elementary particles.[8]

(2) If the neutrino wave functions, ν, have the property (the two-component theory)

$$\gamma_5 \nu = \nu, \quad \bar{\nu}\gamma_5 = -\bar{\nu}, \quad (7)$$

it then follows from Eq. (6) and the commutation relations of the γ that

$$\langle \nu' | J_\alpha | \nu \rangle = \bar{\nu}[\gamma_\alpha \{F_1(q^2) + F_3(q^2)\} + \sigma_{\alpha\beta} q_\beta \{F_2(q^2) + F_4(q^2)\}]\nu. \quad (8)$$

Furthermore, using Eq. (7) we see that the term proportional to $\sigma_{\alpha\beta}q_\beta$ vanishes, so that electromagnetic electron-neutrino scattering is described by one form factor, $F=F_1(q^2)+F_3(q^2)$, in the two-component theory. Moreover, it is generally supposed that $F(0)=0$, i.e., the neutrinos are neutral. We shall now begin a systematic discussion of the electromagnetic properties of the neutrino with some remarks on the experimental basis

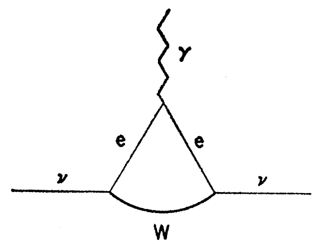

FIG. 1. One of the graphs which generates a charge radius for the electron's neutrino.

for supposing that the neutrino is actually neutral. As above, we separate the discussion into parts.

1. The Charge[9] of ν_e

A. Charge Conservation

If charge conservation is assumed in the decay

$$n \to p + e^- + \bar{\nu}_e,$$

it follows from the experiment of Zorn *et al.*[10] in which e_n and e_p+e are separately measured that

$$|e_{\bar{\nu}_e}| \leqslant 4\times 10^{-17} e. \quad (9)$$

(In what follows e will always stand for the electron charge.)

It is not completely understood why the electric charge, as measured by interaction with an electromagnetic field, should agree with the quantum numbers assigned by the charge-conservation law.[11] Indeed, it is possible to construct model theories in which this is not the case. Hence, we may ask for evidence about the neutrality of the neutrino which is not based on the use of charge conservation.

B. Elastic Scattering

In the electromagnetic interactions of the neutrino we shall suppose that charge is conserved and that such interactions (if they exist) can be computed using the conventional electrodynamics. It is not clear whether there is a consistent electrodynamic theory of a zero-mass charged fermion.[12] For purposes of the discussion that follows, we shall simply make use of the conventional formalism so long as it does not lead to obvious nonsense in the limit of zero mass.

In this spirit we may compute elastic $\nu-e$ scattering,

[6] See, for example, F. Boehm and A. N. Wapstra, Phys. Rev. **109**, 456 (1958), who found that electrons emitted from P^{32} have a longitudinal polarization of (-0.97 ± 0.06), which is consistent with an equal and opposite helicity for the associated antineutrino.

[7] M. Bardon, P. Franzini, and J. Lee, Phys. Rev. Letters **7**, 23 (1961).

[8] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957); L. Landau, Nucl. Phys. **3**, 127 (1957).

[9] We note that it follows from CP invariance or TCP invariance that if the neutrino had a charge, the antineutrino would have the opposite charge. This can be seen by considering the transformation properties of the matrix element $\langle \nu | J_\alpha | \nu \rangle$.

[10] J. C. Zorn, G. E. Chamberlin, and V. W. Hughes, Phys. Rev. **129**, 2566 (1963). In this experiment, e_n and e_p+e are measured. The experimental results are

$$e_n \leqslant (6.1\pm20)\times 10^{-18} e,$$
$$|e_p+e| \leqslant (-8.5\pm27)\times 10^{-18} e.$$

Charge conservation then implies the quoted limit for the neutrino charge.

[11] See, for example, G. Feinberg and M. Goldhaber, Proc. Nat. Acad. Sci. U.S.A. **45**, 1301 (1959) for a discussion of this point.

[12] T. D. Lee and M. Nauenberg (to be published).

assuming that the neutrino has a charge e_ν. The matrix element corresponding to Fig. 2 is

$$M=(4\pi/Q^2)e_\nu e\bar{e}_{p'}\gamma_\alpha e_p\bar{\nu}_{\nu'}\gamma_\alpha\tfrac{1}{2}(1+\gamma_5)\nu_\nu. \quad (10)$$

(We assume throughout that only left-handed neutrinos interact electromagnetically. This assumption does not seriously affect our conclusions.)

Using Eq. (4) we find that in the rest system of the electron

$$\frac{d\sigma(\nu)}{d\Omega}=\frac{1}{8}\frac{\alpha\alpha_\nu}{\nu^2}\frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}\frac{1}{(1+(2\nu/m)\sin^2\frac{1}{2}\theta)}. \quad (11)$$

We may compare this formula with the experimental results of Cowan and Reines.[13] In this experiment, an upper limit was set on the electromagnetic cross section of the neutrino by a search for recoil electrons (scattered by neutrinos emanating from a pile) with a minimum energy of 0.1 MeV. The neutrinos (actually anti-neutrinos) are distributed energetically according to a dimensionless normalized fission spectrum $n(\nu)$.

Hence, the quantity to be compared to experiment is

$$\sigma(\theta_{\min})=2\pi\int_{\nu_{\min}}^{\nu_{\max}}n(\nu)d\nu\int_{\theta_{\min}}^{\pi}\frac{d\sigma(\nu)}{d\theta}d\theta\sin\theta. \quad (12)$$

Here $0.1\text{ MeV}\leq\nu\leq0.5$ MeV, and θ is given in terms of the recoil-electron kinetic energy by the equation

$$\cos\theta=1-mT/\nu(\nu-T), \quad (13)$$

with

$$0.1\text{ MeV}\leq T\leq 2\nu^2/(m+2\nu), \quad (14)$$

where T is the recoil kinetic energy. To make an estimate of e_ν, we have replaced the neutrino energy spectrum by a mean value that we take as 0.4 MeV (the neutrinos with kinetic energy less than 0.21 MeV do not produce electrons energetic enough to be observable in the experiment). This average energy we denote $\bar{\nu}$. Thus,

$$\sigma(\theta_{\min})\simeq\frac{\pi}{2}\frac{\alpha\alpha_\nu}{\bar{\nu}^2}\left\{\frac{2}{1-x}-\left(1+\frac{2\bar{\nu}}{m}\right)\times\ln\left(\frac{1}{1-x}+\frac{\bar{\nu}}{m}\right)\right\}_{x=-1}^{x=x_{\max}}, \quad (15)$$

where $x=\cos\theta$. Putting in the numbers, we find in this way that

$$\alpha_\nu\leq10^{-20}. \quad (16)$$

In this calculation we have assumed that $F(q^2)\simeq F(0)=e_\nu$, where $F(q^2)$ is the electromagnetic form factor of the neutrino. The following rough argument indicates that this is an excellent approximation. A diagram like Fig. 1 will give rise to a mean-square radius of order

$$\langle r^2\rangle_{\text{av}}\simeq g^2(\hbar/m_Wc)^2\simeq Gm_n^2(\hbar/m_nc)^2\simeq10^{-32}\text{ cm}^2. \quad (17)$$

[13] C. L. Cowan, Jr., and Frederick Reines, Phys. Rev. **107**, 528 (1957).

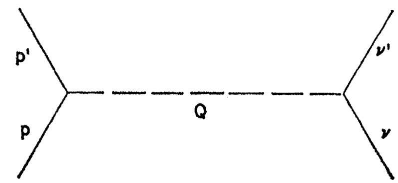

FIG. 2. A graph representing electron-neutrino scattering by photon exchange. Here p and p' are the initial and final momenta of the electron, while ν and ν' are the initial and final momentum of the neutrino. At the neutrino vertex, the matrix element of the current should be inserted.

The typical four-momentum transfers in the Cowan-Reines experiment are of the order of $m_e^2\simeq10^{21}$ cm^{-2}. Thus, the mean-square radius contribution to the scattering would be completely negligible. Below we discuss the experimental limits that have been set on the neutrino charge radius.

C. *Astrophysics*

The existence of a small electric charge, magnetic moment, or charge distribution for the neutrino would imply that neutrino-antineutrino pairs could be electromagnetically produced; a virtual photon could be converted into a neutrino-antineutrino pair. In any process in which electron-positron pairs can be made, neutrino pairs can also be produced but with two significant differences: (a) The charge on the neutrino, if any, is very small next to that of the electron, so that the probability of electromagnetically emitting neutrino pairs in any interaction is always very tiny; (b) the neutrino mass, if any, is probably much smaller than that of its associated lepton, so that the threshold for neutrino pair emission is very small and may be zero. Just because of their very weak interaction with matter, neutrino pair emission, if it exists, could play a very significant role in various stages of stellar evolution. The neutrinos, if produced at all, easily escape from the interior of a star without further interaction, while other forms of energy transmission (via photons or electrons) are limited by the slow diffusion from the interior to the surface. Indeed, even the very weak coupling between electrons and neutrino pairs which is suggested by various forms of the universal Fermi interaction may play a significant role in certain stages of stellar evolution.[14–17] So we shall exploit the known long life of our sun (at least 5×10^9 yr) to put an upper limit on its energy loss through neutrino pair emission and, hence, on the neutrino electric charge, moment, and charge radius.

We assume that the neutrino mass is not large com-

[14] B. Pontecorvo, Zh. Experim. i Teor. Fiz. **36**, 1615 (1959) [translation: Soviet Phys.—JETP **9**, 1148 (1959)].

[15] H. Y. Chiu and P. Morrison, Phys. Rev. Letters **5**, 573 (1960).

[16] V. I. Ritus, Zh. Experim. i Teor. Fiz. **41**, 1285 (1961) [translation: Soviet Phys.—JETP **14**, 915 (1962)].

[17] J. B. Adams, M. A. Ruderman, and C. H. Woo, Phys. Rev. **129**, 1382 (1963) contains further references.

pared to one keV; otherwise there is generally not enough energy to create them in stellar interiors where temperatures are typically 10^7–10^8°K.

Perturbation theory for the quantum electrodynamics of massless charged neutrinos is logarithmically divergent in that approximation in which photons have zero mass and infinite mean free path. But within a plasma, quantized transverse electromagnetic waves have the momentum and energy relation of particles with mass

$$\omega^2=\omega_P^2+k^2c^2, \tag{18}$$

where ω is the frequency, k the wave number of the "photon," and ω_P the plasma frequency

$$\omega_P^2=4\pi ne^2/m; \tag{19}$$

n, e, and m are the electron density, charge, and mass, respectively. At the core of the sun $n\sim10^{26}$/cc, so that the mass of a solar photon $\hbar\omega_P$ is approximately 400 eV, greater than twice the upper limit to the ν_e mass. (The mean free path of a photon in stellar matter is typically 1 g/cm^2, corresponding to a lifetime of more than 10^{-13} sec.; the imaginary part of the "photon" mass is then $\sim10^{-2}$ eV and negligible next to the real part.) A massive "photon" can then spontaneously decay into neutrino pairs. It can also have a certain amount of induced decay as a result of collisions with electrons, but as long as $\hbar\omega_P$ is not too small next to κT, these do not significantly change our results and shall be neglected.

It has been shown[18] that to a very good approximation the quantum electrodynamics of massive photons (transverse plasmons) is exactly the same as conventional quantum electrodynamics, except for Eq. (18), i.e., it is identical to the theory for the transverse components in neutral vector-meson theory. If R_0 is the decay rate of a "photon" into neutrino pairs in its rest system, its decay rate when $k\neq0$ is $R_0\omega_P/\omega$ and the rate at which energy is converted into neutrino pairs is simply

$$\hbar\omega R_0\omega_P/\omega=\hbar R_0\omega_P,$$

independent of k. The total rate of neutrino emission per unit mass, $\mathcal{E}$, is then

$$\mathcal{E}=NR_0\omega_P\hbar/\rho, \tag{20}$$

with

$$N=2\int\frac{d^3k}{(2\pi)^3}\left[\exp\left(\frac{\hbar\omega}{\kappa T}\right)-1\right]^{-1} \tag{21}$$

and ρ the mass density. For the decay rate R_0 we have, for neutrinos of charge e_ν,

$$R_0=(e_\nu^2/\hbar c)\tfrac{1}{6}\omega_P. \tag{22}$$

[18] In a medium with a transverse dynamic dielectric constant $\epsilon^T(\omega,\mathbf{k})$ the usual normalization $[2\omega]^{-1/2}$ of the vector potential (which gives $E=\hbar\omega$) is replaced by $[\omega(2\mathcal{E}^T+\omega(\partial/\partial\omega)\epsilon^T)]^{-1/2}$ (Ref. 17). In a plasma with $\epsilon=1-(\omega_P^2/\omega^2)$ the bracket again becomes $[2\omega]^{-1/2}$.

In the special case of a star with $\kappa T\gg\hbar\omega_P$, Eq. (21) just gives the usual photon density, and such an expression is not an unreasonable estimate for N in the solar core,

$$N\sim\left(\frac{\kappa T}{\hbar c}\right)^3\frac{2}{\pi^2}\xi(3)\simeq0.244\left(\frac{\kappa T}{\hbar c}\right)^3. \tag{23}$$

From Eqs. (19), (20), (22), and (23) we have for the production of pairs, the loss per gram of stellar matter:

$$\mathcal{E}=0.04\left(\frac{e_\nu}{e}\right)^2\alpha\left(\frac{\kappa T}{\hbar c}\right)^3\frac{4\pi ne^2\hbar}{m\rho}. \tag{24}$$

For the solar core we take $T\sim1.5\times10^7$, $n\sim10^{26}$, $\rho\sim10^2$. Then

$$\mathcal{E}\sim(e_\nu/e)^2 10^{27}\ \text{ergs/g-sec}. \tag{25}$$

But the visible light radiated by the sun corresponds to an average energy production of about 1 erg/g-sec. The energy carried away by neutrinos cannot have been more than a factor of 10 greater than this without greatly shortening the life of the sun on the main sequence. For suppose the sun has been emitting 10 times as much energy in neutrinos as in photons over the past 5×10^9 yr. The source of such energy would be the conversion of H to He. From the known mass in the sun, we can estimate for how long the sun could have produced energy at a rate 10 times the visible rate. This turns out to be less than 10^9 yr. We can therefore conclude that the neutrino-energy loss cannot be too high, and that

$$(e_\nu/e)^2<10^{-26}, \tag{26}$$

or

$$e_\nu/e<10^{-13}. \tag{27}$$

This argument depends crucially on the assumption that ν or $\bar\nu$ absorption is negligible. If we did not know from other evidence that the interaction of neutrinos with matter was very weak, the neutrinos might be everywhere in thermal equilibrium and, thus, carry away an energy from the surface which, of necessity, would be about the same as that of electromagnetic radiation. With a weak coupling the ν, $\bar\nu$ are emitted directly from the hot core rather than the cooler surface, as is the case with stellar light.

The magnetic field of the sun is incapable of containing such a high neutrino flux even if the neutrinos possess a small charge e_ν of, say, $10^{-13}e$. If they were contained, the ν, $\bar\nu$ density would build up until $\gamma\rightarrow\nu+\bar\nu$ is balanced by $\nu+\bar\nu\rightarrow\gamma$. This will occur roughly when all neutrino states are filled up to a Fermi energy $E_F\sim\kappa T\sim1$ keV, corresponding to 10^{22} neutrinos/cc. These exert a pressure of 10^{13} dyn/cm^2 and would, therefore, require a magnetic field of 10^7 G to be contained.

A universal neutrino degeneracy[19] (ν_e or ν_μ) which would suppress 1 keV ν, $\bar\nu$ production is also incompatible with observation.

[19] S. Weinberg, Phys. Rev. **128**, 1457 (1962).

A 1-keV ν is a 1-BeV ν as seen by a cosmic-ray proton of 10^{15}-eV energy, well below the highest energy primaries that have been seen. For such a neutrino the proton cross section is $\sim 10^{-38}$ cm^2, and the proton mean free path would be 10^{16} cm$=10^{-2}$ light years and much less for higher energy protons. Such local production (less than 10^{-2} the distance to the nearest star) and short life is inconsistent with any reasonable theory of cosmic-ray production and would require thousands of times more energy than can be accounted for by astronomical sources.[20]

Finally we note that the small charge e_ν is insufficient in itself to permit the ν or $\bar{\nu}$ to lose energy via inelastic coulomb scattering on electrons before leaving the sun. In any case, since they will be scattered but not absorbed by electrons or nuclei, at most this would mean that they could leave the sun with an energy corresponding to the surface temperature of 6000°K rather than the core temperature of 1.5×10^7. At worst this would reduce the bound on e_ν to $e_\nu<5\times10^{-12}e$.

For the scattering of keV neutrinos by electrons,

$$\frac{d\sigma}{d\Omega}\sim\frac{2\nu^2\alpha\alpha_\nu\cos^2\frac{1}{2}\theta}{[4\nu^2\sin^2\frac{1}{2}\theta+\omega_P{}^2/c^2]^2}, \tag{28}$$

where the shielding of the Coulomb field by the electron plasma is included. Therefore,

$$\sigma_{\nu e}<2\nu^2\alpha\alpha_\nu c^4/\omega_P{}^4, \tag{29}$$

with $\omega_P{}^2$ given by Eq. (19). For keV neutrinos and solar parameters,

$$\sigma_{\nu e}<(e_\nu/e)^2 10^{-28}\ \text{cm}^2; \tag{30}$$

or for our value (27),

$$\sigma_{\nu e}<10^{-44}\ \text{cm}^2,$$

a limit which is less than the part of the cross section which arises from the weak Fermi interaction and insufficient to result in any appreciable neutrino-energy loss.

2. The Charge of ν_μ

It is possible to obtain information about the charge of ν_μ by three methods, similar to three we have outlined for ν_e.

A. Charge Conservation

It is known from experiments on the energies of x rays emitted in the $3D_{5/2}-2P_{3/2}$ transition in μ-mesonic phosphorus, and from the muon $g-2$ experiment, that[21]

$$e_\mu/e=0.999993\pm0.000035. \tag{31}$$

Hence, assuming that charge is conserved in the decay $\mu^-\to e^-+\bar{\nu}_e+\nu_\mu$ and the result that $e_{\nu_e}\ll10^{-5}e$, we can conclude that

$$e_{\nu_\mu}<3\times10^{-5}e. \tag{32}$$

B. Scattering

Some information concerning experimental limits on electromagnetic interactions of ν_μ comes from the recent experiments on high-energy neutrino scattering.[22] A nonvanishing value for the matrix element of the current in a neutrino state would not effect the "charge-exchange" scattering of ν_μ, giving μ, which was actually observed. However, such a matrix element would contribute to processes like

(a) $\nu_\mu+\text{proton}\to\nu_\mu+\text{proton}$,

(b) $\nu_\mu+\text{proton}\to\nu_\mu+\text{proton}+\pi^0$.

According to the experimenters,[23] the first of these processes would have been observed if its cross section had been $\gtrsim10^{-37}$ cm^2, or about 10 times the observed charge-exchange cross section. This is because the only observable particle is the recoil proton, which usually does not leave the plate in the spark chamber. On the other hand, the second reaction will usually make a shower, which would make a visible track. No more than two events were seen which could be of this type, whereas some twenty events which are interpreted as

(c) $\nu_\mu+\text{nucleon}\to\mu+\text{nucleon}+\pi$,

were seen, corresponding to a cross section of about 10^{-38} for the latter events. It seems safe to conclude that the cross section for reaction (b) is less than 10^{-39}.

We can compare this to the cross section expected if the matrix element $\langle\nu|J_\alpha|\nu\rangle$ were nonzero. In this connection, it is useful to compare directly with experiments on production of pions by electrons.[24] In these experiments it is found that the cross section for production of pions by electrons of energy 400–700 MeV, and at momentum transfers of several hundred MeV/c, is about 10^{-30} cm^2. This may be compared with the upper limit of 10^{-39} for the corresponding neutrino process.

To obtain information about the ν_μ charge from the experimental limit on pion production, we suppose that the ν_μ has a charge, but no other electromagnetic interaction, just as for the electron. Then the matrix element for pion production by neutrinos is proportional to the charge, and we find that the ratio of cross sections for production of pions by neutrinos or by electrons is given by

$$\sigma(\nu p\to\nu p\pi)/\sigma(ep\to ep\pi)=e_\nu{}^2/e^2. \tag{33}$$

[20] This argument can be used to show that if there exists a universal neutrino degeneracy, the Fermi level for both ν_e and ν_μ and their antineutrinos must be well below 100 eV.

[21] See G. Shapiro and L. M. Lederman, Phys. Rev. **125**, 1022 (1962), where the experimental references are also given.

[22] G. Danby, J. M. Gaillard, K. Goulianos, L. M. Lederman, N. Mistry, M. Schwartz, and J. Steinberger, Phys. Rev. Letters **9**, 36 (1962).

[23] See the discussion following the talk by M. Schwartz, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics CERN* (CERN, Geneva, 1962), p. 817.

[24] W. K. H. Panofsky and E. A. Allton, Phys. Rev. **110**, 1155 (1958).

From the experimental numbers quoted, we obtain

$$e_\nu^2/e^2 < 10^{-39}/10^{-30} = 10^{-9}, \tag{34}$$

or again

$$e_\nu < 3\times 10^{-5} e.$$

C. Astrophysical Evidence

If the mass of ν_μ is less than 1 keV, the arguments given in Sec. III. 1*C* apply for it as well, and we find that

$$e_{\nu_\mu} < 10^{-13} e. \tag{35}$$

This is, of course, a much more satisfactory limit, and in view of the difficulty in improving the experiments leading to the limits in 2*A*, *B*, it would be desirable to remove the mass restriction in 2*C*.

3. The Charge Radius of ν_e

In the previous section we entertained the possibility that ν_e might not be neutral. In this section we suppose that ν_e is neutral; i.e., $F(0)=0$, but we ask how big a charge radius for the neutrino is consistent with experiment. In the two-component theory, a neutral neutrino cannot interact with a real photon, so that the dominant electromagnetic interaction is proportional to the charge radius.

A. Electromagnetic Scattering

For a neutral neutrino we may write at small momentum transfers

$$F(q^2) \simeq -\tfrac{1}{6} e\langle r^2\rangle q^2. \tag{36}$$

In this approximation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\langle r^2\rangle^2 \nu^2}{18} \frac{\cos^2(\theta/2)}{[1+(2\nu/m)\sin^2(\theta/2)]^3}. \tag{37}$$

To compare with the Cowan-Reines experiment, we must once again integrate over a range of angles and energies determined by the conditions of the experiment. In this case

$$\sigma(\theta_{\min}) = m^2(\pi/18)\alpha^2\langle r^2\rangle^2 \times\left\{\frac{(\nu/m)x-\frac{1}{2}}{[1+(\nu/m)-(\nu/m)x]^2}\right\}_{x=-1}^{x_{\max}}. \tag{38}$$

As in the charge case, we replace the neutrino spectrum by an average energy $\bar{\nu}=0.4$ MeV. Putting in the numbers, we find from this experiment

$$\langle r^2\rangle \leq 1.6\times 10^{-29}\ \text{cm}^2. \tag{39}$$

B. Astrophysics

As we have seen, the photons in a star behave as if they have the effective mass ω_P. Such a photon has a finite amplitude for decay into a particle with a charge radius. This amplitude is obtained from Eq. (22) of 1*C* by the replacement

$$e_\nu \to \tfrac{1}{6} e(\omega_P/c)^2\langle r^2\rangle. \tag{40}$$

Using this, in the expression for R_0, we find a limit for the charge radius,

$$(\omega_P/c)^2\langle r^2\rangle < 6\times 10^{-13}; \tag{41}$$

or, putting in the solar parameters,

$$\langle r^2\rangle < 2\times 10^{-27}. \tag{42}$$

The astrophysical limit on the charge radius is not as good as the scattering limit, since in the scattering, the neutrinos are at a considerably higher energy.

4. Charge Radius of ν_μ

If the ν_μ has a charge form factor, then the matrix element for pion production of a fixed momentum transfer will be proportional to the charge form factor at that momentum transfer. When the neutrino charge vanishes, as seems likely from our previous considerations, it is not a bad approximation, at the momentum transfers involved in the ν_μ scattering experiments, to replace $F(q^2)$ by the charge radius term

$$F(q^2) \simeq -\tfrac{1}{6} eq^2\langle r^2\rangle. \tag{43}$$

The cross section for neutrino-pion production at a given energy and q^2 can then be expressed in terms of the corresponding electron cross section by the relation

$$\frac{d\sigma(\nu p\to\nu p\pi)}{d\sigma(ep\to ep\pi)} = \frac{q^4}{36}\langle r^2\rangle^2. \tag{44}$$

If we take the ratio to be $<10^{-9}$ for $q=500$ MeV, we obtain

$$\langle r^2\rangle \leq 10^{-30}\ \text{cm}^2. \tag{45}$$

If the ν_μ mass is less than 1 keV, it is possible to use the astrophysical evidence for it also, and obtain the same limit on the charge radius as for the electron

$$\langle r^2\rangle < 2\times 10^{-27}\ \text{cm}^2. \tag{46}$$

5. The Magnetic Moment of ν_e

A. Elastic Scattering

The Cowan-Reines experiment was actually analyzed[25] to set a limit on the magnetic moment of the electron's neutrino. The conclusion from the fact that the neutrino-electron scattering cross section is measured to be $<4\times 10^{-43}$ cm² is that

$$f < 1.4\times 10^{-9}. \tag{47}$$

(f stands for the magnetic moment in Bohr magnetons.)

[25] C. L. Cowan, Jr., and Frederick Reines, Phys. Rev. **107**, 528 (1957).

B. Astrophysics

We can get an estimate of f by letting

$$e_\nu \longrightarrow (\omega_P/c)f$$

in Eq. (22) of (1C). This is the proper replacement for the decay of a massive photon into a particle with a magnetic moment f. Using the results of 1C we then find

$$f_{\nu_e} < 10^{-10}. \tag{48}$$

6. The Magnetic Moment of ν_μ

A. Astrophysics

As above, if $m_{\nu_\mu} < 1$ keV, we use the approximate arguments of 5B to conclude

$$f_{\nu_\mu} < 10^{-10}. \tag{49}$$

B. Evidence from Pion Production

If the ν_μ had a magnetic moment, this would contribute to the production of pions in the experiments discussed. We have not made a detailed analysis of the expected cross section, but it appears that the limit to be extracted from the data is of the order

$$f_{\nu_\mu} < 10^{-8}. \tag{50}$$

IV. CONCLUSION

We have seen that the experimental evidence presently available has given no indication of electromagnetic interactions for neutrinos. The evidence is rather convincing that the electric charge and the magnetic moment of the neutrino are both zero, as we expect. We summarize the results in Table I.

The experiments are not yet sensitive enough to measure a neutrino charge form factor of the size that we would expect from the weak interactions, for example, via diagram (1). However, it is important to note that the vector meson theory of weak interactions has not yet yielded a unique prediction for this form factor, and it is perhaps unwarranted to make naive assumptions about the theoretically expected value. The anticipated development of better techniques for doing high-energy neutrino-scattering experiments should be very helpful in giving more information about the neutrino-charge form factor. If, as expected at present, the intrinsic weak amplitude for noncharge-exchange neutrino-baryon scattering is small, then the dominant contribution to such scattering will come from the neutrino charge form factor. It is, therefore, to be hoped that experiments to measure such scattering will be performed.

TABLE I. Summary of the known limits for the electromagnetic interactions of neutrinos.

Property	ν_e	ν_μ
Charge	$<4\times10^{-17}e$ from charge conservation	$<10^{-13}e$ from astrophysics, if $m_{\nu_\mu}<1$ keV
	$<10^{-13}e$ from astrophysics	$<3\times10^{-5}e$ from charge conservation
	$<3\times10^{-10}e$ from electron-neutrino scattering	$<3\times10^{-5}e$ from pion production by neutrinos
Magnetic moment (in Bohr magnetons)	$<10^{-10}$ from astrophysics	$<10^{-10}$ from astrophysics, if $m_{\nu_\mu}<1$ keV
	$<1.4\times10^{-9}$ from neutrino-electron scattering	$<10^{-8}$ from pion production by neutrinos
Charge radius (in cm)	$<4\times10^{-15}$ from electron-neutrino scattering	$<10^{-15}$ from pion production by neutrinos
	$<4\times10^{-14}$ from astrophysics	$<4\times10^{-14}$ from astrophysics, if m_{ν_μ} <1 keV

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