the broadening to be expected from the dipolar interaction with the F^{19} moments. We have also observed the resonance in a NaF crystal at the same Larmor frequency.

In a separate experiment, we determined the sign of the F^{20} moment to be positive by comparing the depolarization produced by left and right hand circular polarization of H_1.[16] With $\omega/2\pi=4$ Mc/sec, the resonance frequency in 5015 Oe, we found $\alpha'=2.2\%$ ($\boldsymbol{\omega}\|\mathbf{H}_0$) and $\alpha'=1.2\%$ ($-\boldsymbol{\omega}\|\mathbf{H}_0$). α_0' was 2.8%. The slight depolarization for the case $\boldsymbol{\omega}\|\mathbf{H}_0$ is due to imperfect circular polarization of the rf field.

From the data represented in Figs. 5 and 6 we find the Larmor frequency to be 3999.2 ± 0.4 kc/sec in 5013.8 ± 0.3 Oe and 999.7 ± 0.4 kc/sec in 1253.4 ± 0.5 Oe. Without diamagnetic correction, the gyromagnetic ratio is, thus, $\gamma=+797.6\pm0.2$ cps/Oe and the nuclear g factor, $g=\mu/\hbar I$, is $g=+1.0463\pm0.0002$ nm/$\hbar$. Making the small correction for atomic diamagnetism, we obtain $g=+1.047\pm0.001$ nm/$\hbar$. Since $I=2$,[17] the magnetic moment is $\mu=+2.0926\pm0.0004$ nm (uncorrected) or $\mu=+2.094\pm0.002$ nm (corrected).

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Ground State of F^{20}†

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It is shown that in order to describe some properties of the F^{20} ground state, the $K=1$ and $K=2$ states of the Nilsson model must be strongly mixed. The primary ingredient in the description is, therefore, the rotational Coriolis term rather than the interaction between the odd neutron and odd proton.

INTRODUCTION

THE nucleus F^{20} lies in a region where a considerable amount of interpretation of nuclear properties has been carried out. This has been done either by means of a spherical shell model with residual two-body interactions or by means of the model of independent nucleons in a potential well of quadrupole deformation. The relationship between these interpretations has been demonstrated by means of the generating procedure,[1] and explicit calculations[2] showing their similarity have been done for nuclei of mass 18 and 19. The model with particles in a nonspherical potential well[3] is much easier to apply and generally gives the important features of the lowest states, so it is employed in the following treatment.

The model for odd-odd nuclei is that of a deformed core to which both the odd proton and the odd neutron are strongly coupled. There are then two states for the neutron-proton system, one with parallel projections of angular momentum on the nuclear-symmetry axis, the other with antiparallel projections. Rotational bands exist for each of these internal states and the energy separation of the bands is determined both by the rotational Hamiltonian and by the neutron-proton interaction. Gallagher and Moszkowski[4] studied the heavy odd-odd nuclei, and concluded that the lowest state appeared in the majority of cases to be consistent with the predictions of a neutron-proton force which preferred to align the intrinsic spins of the neutron and proton. While the rotational-energy differences are of secondary importance for the heavy nuclei, for light nuclei like F^{20}, the rotational energy ($\hbar^2/2\mathcal{I}$) has an order of magnitude of several hundred keV compared to tens of keV in the heavy nuclei. Therefore, the rotational terms are of greater importance, and in cases like F^{20} where the resultant states of the neutron-proton system can be mixed by the rotational Hamiltonian this mixing will be the dominant feature.

II. PROPERTIES OF THE F^{20} GROUND STATE

From the preceding paper[5] we know that the gyromagnetic ratio of F^{20} is $g=+1.046$. The angular momentum is $I=2^+$ (or 3^+) and the state decays[6] by an allowed beta transition to the first excited state ($I=2^+$) of Ne^{20} with a value of $\log ft=4.99$.

The Nilsson[3] picture for F^{20} is that of a core with prolate deformation, the odd-proton lying in level No. 6 with $k=\frac{1}{2}$ and the odd neutron in level No. 7 with $k=\frac{3}{2}$. The resultant neutron-proton states have projections of angular momentum $K=1$ and $K=2$ on the

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nuclear symmetry axis. The rule of Gallagher and Moszkowski[4] selects the $K=2$ state as lowest since the single-particle states both have spin projections $\Sigma=+\frac{1}{2}$ in the asymptotic limit. The g factor for the $K=2$ state is given simply by the expression derived by Bohr and Mottelson[7] for the state $I=K$,

$$(I+1)g=g_R+\langle K|\textstyle\sum_i \mu_z(i)|K\rangle. \quad (1)$$

Here $g_R\approx(Z/A)$ is the rotational g factor and μ_z is the usual single-particle magnetic-moment operator. The expectation value on the right of (1) has an upper bound of $\mu_p+\mu_n=0.88$, so that with $g_R=0.45$ the calculated g value for the state $I=K=2$ is less than $+0.45$. This number is less than half the experimental value, so that the ground state is evidently not the base of a $K=2$ band. Another argument against such an identification is that it would make the beta decay to Ne^{20} K forbidden if one wants to assign the $I=2^+$ state of Ne^{20} to the lowest $K=0$ rotational band. This does not agree with the observed value of $\log ft=4.99$.

Therefore, it appears that something more is needed, and since the $K=1$ and $K=2$ bands can be mixed by the Coriolis term of the rotational Hamiltonian it seems reasonable to investigate the effects of including this possibility.

III. BAND MIXING

The Hamiltonian which is to be diagonalized is

$$H=H(\text{Nils.})+H(\text{Rot.})+V(np). \quad (2)$$

Here

$$H(\text{Rot.})=A[I^2+\mathcal{J}^2-I_3{}^2-\mathcal{J}_3{}^2]-A[I_+\mathcal{J}_-+I_-\mathcal{J}_+], \quad (3)$$

where $\mathcal{J}$ refers to the angular momentum of the neutron-proton system, I is the total angular momentum, and $A=[\hbar^2/2\mathcal{I}]$ is the unit of rotational energy. Aside from the neutron-proton interaction, $V(np)$, this is just the problem of coupling between the symmetric-rotor and the extra-nucleon system. This latter problem is reviewed thoroughly in an article by Kerman,[8] so only the special features of the F^{20} problem are discussed in the following.

Since we consider two states of the neutron-proton system, eigenfunctions of $\mathcal{J}_3$ with eigenvalues of magnitude $K=1$ and $K=2$, there will be a 2×2 matrix to diagonalize for each value of I greater than one. The matrix elements are

$$\langle H\rangle_{KK}=\langle H(\text{Nils.})\rangle_{KK}+\langle V\rangle_{KK} +A[I(I+1)-2K^2+\langle\mathcal{J}^2\rangle_{KK}], \quad (4)$$

$$\langle H\rangle_{12}=-A(I(I+1)-2)^{1/2}\langle\mathcal{J}_-\rangle_{12},$$

where the matrix elements on the right are between neutron-proton functions, X_K, in the nuclear coordinate system. The important parameter which determines the mixing of states is the ratio of $(\langle H\rangle_{22}-\langle H\rangle_{11})$ to $2\langle H\rangle_{12}$. The matrix elements of $\mathcal{J}^2$ and $\mathcal{J}_-$ are functions of the deformation measured by the Nilsson parameter η. However, for the X_K of F^{20}, the difference $(\langle\mathcal{J}^2\rangle_{22}-\langle\mathcal{J}^2\rangle_{11})$ equals 3, independent of η, so that

$$\langle H\rangle_{22}-\langle H\rangle_{11}=\langle V\rangle_{22}-\langle V\rangle_{11}-3A.$$

The difference due to the neutron-proton interaction should have a magnitude of a few hundred keV and, according to the Gallagher-Moszkowski rule, should favor $K=2$. Therefore, $\Delta V\equiv\langle V\rangle_{22}-\langle V\rangle_{11}$ is about equal to $-A$, and since $\langle\mathcal{J}_-\rangle_{12}$ is about 2 to 3, the magnitude of $2\langle H\rangle_{12}$ is generally considerably bigger than the difference of diagonal elements. Thus, results are quite insensitive to the value of ΔV, and the calculations done with $\Delta V=0$ and $\Delta V=-A$ show little difference.

The ground state for the calculations is always $I=2$. The wave functions is given by

$$\psi=\alpha\psi(K=1)+(1-\alpha^2)^{1/2}\psi(K=2). \quad (5)$$

The g factor for the state $I=2$ is given by

$$g=g_R+\tfrac{1}{6}[\alpha^2\langle G_z\rangle_{11}+2(1-\alpha^2)\langle G_z\rangle_{22} +2\alpha(1-\alpha^2)^{1/2}\langle G_-\rangle_{12}], \quad (6)$$

where

$$G_z=\textstyle\sum_i\{(g_l(i)-g_R)l_z(i)+(g_s(i)-g_R)s_z(i)\}$$

and $G_-=G_x-iG_y$.

The results for the ground state are summarized in Table I as a function of the Nilsson parameter η. The results of Table I are for $\Delta V=0$ but the effect of using $\Delta V=-A$ is only to lower α by about 7%, so the change is negligible. Two points about the g values are worth noting. One point is that the contribution of g_R is very small, for if one evaluates the coefficient it is 0.09, 0.16, and 0.20 for $\eta=+2$, $+4$, and $+6$, respectively. Therefore, g_R contributes less than 0.1 to the total g value. The second point is that since the $\langle G_{zz}\rangle_{KK}$ values are negative, the term which raises g to $+1$, from its value of less than 0.5 for a pure $K=2$ band, is the last term of (6). This in turn depends on α which the band mixing calculation gives with the sign and magnitude desired for agreement with experiment.

An added test of the wave function is found in calculating the β-decay matrix element to the lowest $I=2$ state of Ne^{20}. The Ne^{20} state is assumed to be part of the $K=0$ ground-state band, based on having a full Nilsson level No. 6. The decay therefore proceeds by

TABLE I. Results of the band-mixing calculation as a function of the Nilsson (Ref. 3) parameter η.

η	$\langle\mathcal{J}_-\rangle_{12}$	$\langle G_z\rangle_{11}$	$\langle G_z\rangle_{22}$	$\langle G_-\rangle_{12}$	$\alpha(I=2)$	$g(I=2)$
+2	2.60	−2.71	−0.62	4.99	0.60	0.95
+4	2.22	−3.36	−0.30	5.27	0.58	1.03
+6	1.97	−3.67	−0.17	5.35	0.57	1.05

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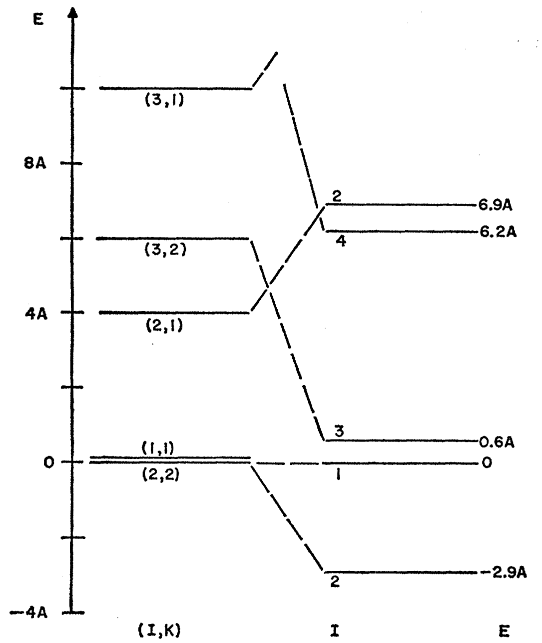

FIG. 1. The energy spectrum of F^{20} in units of $(\hbar^2/2I) \doteq 0.3$ MeV. Spectrum before band mixing is on the left, resultant is on the right. The neutron-proton interaction in this case is $\Delta V = -A$.

an allowed Gamow-Teller transition and using the β-decay constants chosen by Konopinski[9] the ft value is

$$ft = (4.30\times 10^3)|M_{\rm GT}|^{-2}. \quad (7)$$

The transition takes place only from the $K=1$ component of F^{20} to the $K=0$ state of Ne^{20} so that

$$|M_{\rm GT}|^2 = 2\alpha^2|\langle S_x - iS_y\rangle_{01}|^2.$$

The results in column two of Table II show that the calculation is compatible with the observed value of $\log ft = 4.99$.

The spectrum of energy levels from the two mixed bands is given in Fig. 1. The energy is in units of A which is about 0.3 MeV, so that the low excited states

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TABLE II. Transitions involving the F^{20} ground state as a function of the Nilsson parameter. Column two concerns the beta decay to Ne^{20}. The last two columns are $M1$ transition strengths in units of (nuclear magnetons)^2.

η	$\log ft(2^+ \to 2^+)$	$B_{M1}(1^+ \to 2^+)$	$B_{M1}(3^+ \to 2^+)$
2	4.78	15.8	2.44
4	5.06	19.5	2.57
6	5.31	21.0	2.53

lie at about 1 MeV. Experimentally there are four excited states of positive parity[10] between 0.65 and 1.06 MeV, and six more below 3 MeV. In order to carry out adequate calculation of the levels one should include the possibility of bands based on exciting either the odd neutron or the odd proton to their respective adjacent Nilsson levels. Such excitations presumably require only about two MeV as indicated by Paul's treatment[11] of F^{19}. The inclusion of such bands would also have some effect on the spacing of the low states of Fig. 1, but it may be that, as in F^{19}, a good first approximation is obtained by including only the two lowest bands. Values for the $M1$ transition strengths between low states of F^{20} are also given in Table II, and these are apparently strong $M1$ transitions.

IV. CONCLUSIONS

The gyromagnetic ratio and the beta transition probability of the ground state of F^{20} are adequately explained in terms of the mixing of a $K=1$ and a $K=2$ state. The ground state is not determined by the residual neutron-proton interaction, but rather by the Coriolis term which is dominant because the rotational energy is large in light nuclei. The bands are strongly mixed, and such an effect must be included whenever ΔK is unity.

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