PHYSICAL REVIEW VOLUME 132, NUMBER 3 1 NOVEMBER 1963

Large-Angle Inelastic Scattering of 500-keV Electrons

J. W. Motz and R. C. Placious
National Bureau of Standards, Washington, D. C.
(Received 12 June 1963)

Experimental results are given for the energy spectra of electrons inelastically scattered at 100 deg, with 500-keV electrons incident on thin targets of gold and of aluminum. This large-angle inelastic scattering arises primarily from the two processes of bremsstrahlung and of atomic *K*-shell ionization with a small contribution from multiple-scattering effects. In the 300-keV energy region for the scattered electron, the sum of the theoretical cross sections evaluated with the Born calculations of Racah and of Weber, Deck, and Mullin for the respective two processes is approximately an order of magnitude smaller than the experimental values. This disagreement can be attributed to the breakdown in this energy region of both the high-energy approximation in the latter calculation and the Born approximation in both calculations.

1. INTRODUCTION

THERE are three important processes that contribute to the inelastic *single* scattering of electrons with initial energies in the region of 500 keV. First is electron-electron scattering for free electrons, second is electron-atom scattering with atomic excitation, and third is bremsstrahlung. For the first process of electron-electron scattering, accurate predictions of the cross section are given by the relativistic calculations of Møller.[1] On the other hand, for the remaining two processes, there is considerable uncertainty about the cross-section behavior because the available calculations are based on the Born approximation which breaks down in this energy region.[2] Such Born calculations for the cross-section differential in angle and energy of the scattered electron have been made by Ford and Mullin[3] and Weber, Deck, and Mullin[4] for the process of electron-atom scattering with the ionization of *K*-shell electrons, and by Racah[5] and McCormick, Keiffer, and Parzen[6] for the bremsstrahlung process.

Very few measurements have been reported for the electron spectrum produced by the latter two inelastic processes in this energy region. Experimental results obtained by Rampolla[7] do not show agreement with the cross-section behavior predicted by these Born calculations for 1.6-MeV electrons inelastically scattered at 120° by aluminum, nickel, and gold targets.

The purpose of the present investigation is to provide additional experimental cross-section data pertaining to the second and third processes specified above. In particular, experimental results are given for the dependence of the cross section on the energy of electrons scattered at 100 deg with 500-keV electrons incident on thin targets of gold and aluminum. At this large scattering angle of 100 deg, the cross section in the laboratory system for electron-electron single scattering is zero. In addition, if the targets are thin enough, multiple-scattering effects can be neglected. Therefore, for these experimental conditions, the energy spectrum of the scattered electrons gives information mostly about the two processes of electron scattering with atomic excitation and of bremsstrahlung.

2. EXPERIMENTAL DETAILS

The experimental arrangement for these measurements is shown in Fig. 1. A $\frac{1}{8}$-in.-diam beam of 500-keV electrons from the NBS constant potential accelerator is directed perpendicular to the target surface. The continuous energy distribution of the electrons scattered at 100 deg is determined by a sector-type magnetic analyzer with an energy resolution of approximately 10%, which was calibrated by the direct electron beam. The electrons are detected by a plastic scintillation spectrometer which is assumed to have a detection efficiency better than 98% for energies larger than 100

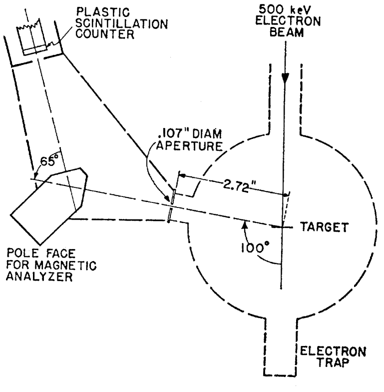

FIG. 1. Experimental arrangement for large-angle inelastic-scattering measurements. The target materials are aluminum and gold with thicknesses in the energy region from 10–20 $\mu g/cm^2$. The target chamber is insulated from the rest of the assembly and the total charge incident on the target in a given time is measured with a current integrator. The energy spectrum of the electrons scattered at 100 deg is measured with the magnetic analyzer.

---

[1] C. Møller, Ann. Physik **14**, 568 (1932).

[2] This breakdown for the bremsstrahlung process is described in Fig. 23, H. W. Koch and J. W. Motz, Rev. Mod. Phys. **31**, 920 (1959).

[3] G. W. Ford and C. J. Mullin, Phys. Rev. **110**, 520 (1958).

[4] T. A. Weber, R. T. Deck, and C. J. Mullin, Phys. Rev. **130**, 660 (1963).

[5] G. Racah, Nuovo Cimento **11**, 476 (1934).

[6] P. T. McCormick, D. G. Keiffer, and G. Parzen, Phys. Rev. **103**, 29 (1956).

[7] D. S. Rampolla, dissertation, Department of Physics, University of Notre Dame, 1958 (unpublished).

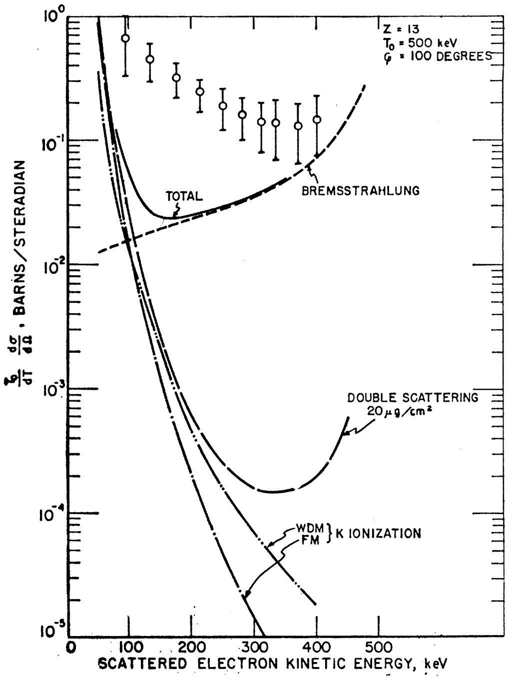

FIG. 2. Dependence of the quantity $(T_0/dT)(d\sigma/d\Omega)$, on the energy, T, of the scattered electron. The cross section, $dT(d\sigma/d\Omega)$ applies to the inelastic scattering of electrons with an initial kinetic energy T_0 of 500 keV from an aluminum target at a scattering angle φ of 100 deg. Experimental values are given by the open circles and the error limits include statistical and systematic errors. These values were obtained with target thicknesses in the range from 10–20 μg/cm². The cross-section curves for K ionization are evaluated from (Ref. 10) Eq. (17) of Ref. 3 (FM) and Eq. (25) of Ref. 4 (WDM). The cross-section curve for bremsstrahlung is evaluated from Eq. (1) of Ref. (6). The cross-section curve for double scattering is evaluated from Eq. (22) in Ref. 3 for a 20 μg/cm² target. The sum of these three curves (excluding FM) is given by the solid line.

keV. The total electron-charge incident on the target is measured with a current integrator. In order to minimize scattering effects, the transmitted electron beam enters an electron trap and the target chamber is lined with aluminum.

The targets consist of evaporated films of gold and aluminum with thicknesses in the region from 10 to 20 μg/cm². These films are evaporated on collodion backings approximately 5-μg/cm² thick. The small contribution (<10%) of the collodion backings is determined from separate measurements with collodion targets. For a given atomic number, measurements were made with different target thicknesses and within the experimental errors, the cross sections determined from the measurements were found to be independent of the target thickness.

3. RESULTS AND DISCUSSION

Experimental values of the cross section for the inelastic scattering of 500-keV electrons at 100 deg from aluminum and gold are given by the open circles in Figs. 2 and 3, respectively. The error limits represent maximum estimates given by the sum of the absolute values of all systematic errors which include uncertainties in the target thickness, the efficiency of the electron detector, and extraneous electron scattering effects. Compared to these systematic errors, the fractional standard deviation pertaining to the statistical counting errors is negligible.

Ford and Mullin[3] have pointed out the important competing processes that must be considered in evaluating large angle inelastic scattering in this energy region. These processes include bremsstrahlung, electron-Coulomb scattering with atomic excitation, and a double scattering process in which electron-electron scattering is followed by electron-Coulomb scattering without atomic excitation. For each of these processes it is necessary to calculate the dependence of the cross section on the scattered electron energy. A summary of the theoretical estimates for these processes is given in Figs. 2 and 3 and discussed below.

For the bremsstrahling process, Racah[5] and McCormick, Keiffer, and Parzen[6] have calculated the dependence of the cross section integrated over photon angle on the energy and angle of the scattered electron. Their calculations were made with the first Born approximation for an unscreened, point nucleus. The cross-section formula obtained from their calculations is given in Eq. (1) of Ref. 6. The behavior of the cross section predicted by this formula for the conditions of this experiment is shown by the dashed lines in Figs. 2 and 3.

The process of electron-Coulomb scattering with

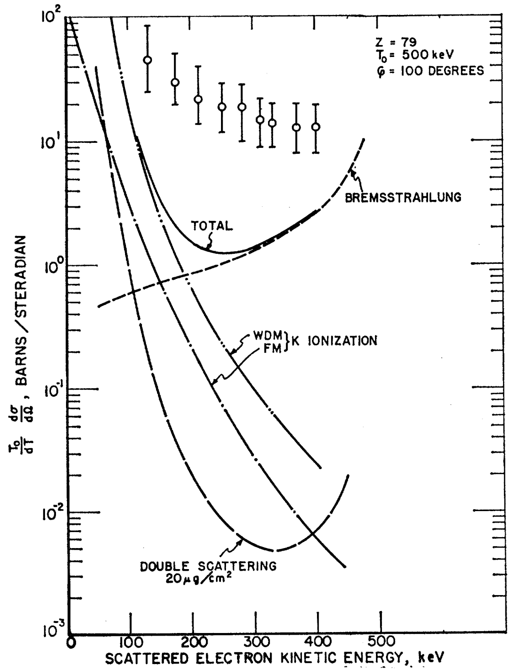

FIG. 3. Same as caption in Fig. 2 except that the aluminum is replaced by gold.

atomic excitation involves the different atomic shells. However, Ford and Mullin[3] have found that in this energy region only K-shell ionization is important for scattering angles larger than 90 deg, and the additional contribution of higher shell excitations is less than 5% of the K-shell ionization cross section. Few calculations of the K-ionization cross section are available. Born approximation formulas for the *total* K-ionization cross section which is not applicable to the present measurements were obtained by Perlman[8] and Burhop[9] with relativistic and nonrelativistic calculations, respectively. On the other hand, formulas for the K-ionization cross-section differential with respect to the energy and angle of one of the scattered electrons were obtained by Ford and Mullin[3] and Weber, Deck, and Mullin.[4] Ford and Mullin[3] used a nonrelativistic hydrogen-like wave function for the bound electron, and plane-wave functions for the incident and outgoing electrons. Weber, Deck, and Mullin recalculated the cross section in the first Born approximation by including first-order corrections of relativistic Coulomb effects on the bound electron and the low-energy outgoing electron and by neglecting the atomic binding energy. Furthermore, both of these latter calculations require that the initial electron kinetic energy is large compared to the electron rest energy. The cross-section formulas obtained from these calculations are given in[10] Eq. (17) of Ref. 3 and Eq. (25) of Ref. 4. The cross sections evaluated from these formulas are shown in Figs. 2 and 3.

In addition to the above two processes, Ford and Mullin[3] have estimated the cross section for double scattering (electron-electron scattering followed by electron-Coulomb scattering without atomic excitation) which depends on the target thickness. If the target is not thin enough, more detailed multiple-scattering calculations are necessary. The present measurements were carried out with target thicknesses in the range from 10 to 20 μg/cm^2, and the experimental cross sections in the energy region from 100 to 400 keV for the scattered electron were found to be independent of the target thickness within the limits of the experimental errors shown in Figs. 2 and 3. The Ford-Mullin estimates of the cross section for double scattering which were evaluated from Eq. (22) in Ref. 3 for 20 μg/cm^2 targets are shown in Figs. 2 and 3.

The total cross section estimated from the sum of the cross sections for the above three processes is shown by the solid lines in Figs. 2 and 3. This sum is obtained with the cross-section values given by the 20-μg/cm^2-target thickness for the double-scattering process and by the Weber-Deck-Mullin rather than the Ford-Mullin cross section for the K-ionization process.

The results in Figs. 2 and 3 show that in the energy region of 300 keV for the scattered electrons, the experimental values of the cross section for both gold and aluminum are approximately an order of magnitude larger than the estimated total cross section shown by the solid line. The reason for this disagreement can be attributed to the fact that the 500-keV energy for the initial electron imposes severe restrictions and both the high-energy approximation for the K-ionization calculations[4] and the Born approximation for both calculations[4,5] can be expected to break down.[2] Each of the two processes is dominant in a different region of the energy spectrum for the scattered electron; bremsstrahlung is important mostly above and K ionization mostly below 300 keV. Above 300 keV, the experimental cross sections tend to approach the theoretical values, and the agreement is better for the aluminum than for the gold. The results indicate that more accurate calculations should give much larger contributions to the cross section in the 300-keV region from both processes.

---

[8] H. S. Perlman, Proc. Phys. Soc. (London) **76**, 626 (1960).

[9] E. H. S. Burhop, Proc. Cambridge Phil. Soc. **36**, 43 (1940).

[10] According to a private communication from Dr. C. J. Mullin, this formula must be multiplied by a factor of 4 in order to apply to the two K-shell electrons.