Isospin Selection Rules for High-Energy Electron Scattering*

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The validity of Siegert's theorem and the isospin selection rule for electric and magnetic dipole transitions has been established for inelastic electron scattering, thus extending the well-known results obtained for real photons. Siegert's theorem is obtained with arbitrary electron wave functions for electric multipole transitions provided: $(k_0R)^2 \ll 1$, where k_0 is the energy transfer, and finite nuclear size effects are ignored. However, the latter assumption follows provided $(kR)^2 \ll 1$ (k is the momentum transfer) and this is valid only for small scattering angles ($\lesssim 1/ER$, E the primary energy). For light elements the isospin selection rule operates for $E1$ transitions in the forward cone only ($\lesssim 30°$). The $M1$ selection rule also follows with $(kR)^2 \ll 1$ and, therefore, operates in the same angular range. The angular distributions should exhibit an anomalous depression in the forward cone of half-angle about $1/ER$. Coulomb effects will then be decisive in determining the magnitude of the small-angle scattering. The same considerations are applicable to internal pair formation and internal conversion where the retardation assumption is valid in general under usual conditions.

I. INTRODUCTION

THE excitation of nuclear energy levels by the inelastic scattering of electrons from complex nuclei provides a convenient means of studying dynamic aspects of nuclear structure.[1] Electrons are particularly useful for this purpose since, in contrast to Coulomb excitation by nuclear particles, the interaction of electrons with the nucleus is purely electromagnetic even at high primary energies.

It is well known that the electroexcitation process differs from the excitation by γ-ray absorption through the presence of contributions from the longitudinal part of the virtual photons in the former. Nevertheless, it is possible to show that some features of the electroexcitation process are similar to properties exhibited by the γ-ray absorption. This comes about whenever it is possible to factor the relevant matrix elements for the electron process into a nuclear matrix element for the radiative transition and a purely electronic matrix element. This factorization, when it is valid, immediately permits one to apply the same selection rules to the two processes. It is to be emphasized that the conditions for the factorization[2] are rather restrictive as will be discussed further in the following.

In this paper we investigate the applicability in electroexcitation of the isotopic spin selection rule which operates for photon transitions.[3] It will be shown that this selection rule does indeed operate but under conditions somewhat more restrictive than those pertaining to γ-ray emission. As a result it will follow that for self-charge conjugate nuclei both $E1$ and $M1$ excitations (with $\Delta T=0$) should exhibit anomalous angular distributions and lower total cross sections than would be expected for normal transitions ($\Delta T \neq 0$ or $N \neq Z$).

For electric dipole transitions the validity of Siegert's theorem[4] must be established in order to obtain the isotopic spin selection rule. In the photon case Siegert's theorem follows if one can assume that $(k_0R)^2 \ll 1$ where k_0 is the energy transfer (in reciprocal length units) and R is the nuclear radius. For energy transfers of as much as 5–10 MeV, in the light nuclei we consider, this assumption is quite reasonable. In the electroexcitation process this assumption is necessary but not sufficient. A sufficient condition is obtained if $(kR)^2 \ll 1$ where k is the momentum transfer. This same condition (which includes the former one since $k > k_0$) is the basis for the idea of the equivalent spectrum.

For magnetic dipole transitions[5] the selection rule is not an absolute one in the sense that the γ-ray transition probability is small but not zero when the conditions for the $E1$ isospin selection rule are fulfilled. As a practical matter, Coulomb impurity and other effects[6] which break down the selection rule are such that the $E1$ and $M1$ forbidden transitions exhibit about the same inhibition factor, namely, 10^{-2} to 10^{-3}. The conditions for the $M1$ inhibition in electroexcitation are the same as for $E1$.

It will be recognized that the formalism used for the electroexcitation is completely equivalent to that employed in the calculation of internal pair formation.[7] The fact that here $k_0R > kR$ and that $(k_0R)^2 \ll 1$ for most practical cases implies that the isospin selection rule for $E1$ and $M1$ transitions apply to internal pair

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[1] W. C. Barber, in *Annual Review of Nuclear Science*, edited by E. Segrè (Annual Reviews Inc., Palo Alto, California, 1962), Vol. 12, p. 1. This review article gives references to earlier work.

[2] This is the basis for the factorization of cross sections into a photon cross section and an equivalent spectrum. See, for instance, Ref. 1.

[3] D. H. Wilkinson, in *Proceedings of the Rehovoth Conference on Nuclear Structure*, edited by H. J. Lipkin (North-Holland Publishing Company, Amsterdam, 1958), p. 175. G. Morpurgo, in *Nuclear Spectroscopy*, edited by G. Racah (Academic Press Inc., New York, 1962), p. 164. These discussions give references to both the experimental and theoretical work.

[4] See, for example, M. E. Rose, *Multipole Fields* (John Wiley & Sons, Inc., New York, 1955).

[5] G. Morpurgo, Phys. Rev. **110**, 721 (1958).

[6] G. Morpurgo, Nuovo Cimento **12**, 60 (1954); Phys. Rev. **114**, 1075 (1959).

[7] M. E. Rose, Phys. Rev. **76**, 678 (1949); **78**, 184 (1950).

formation with the same force as for the emission of the competing γ ray. Here the factorization of the matrix element is almost always a good approximation and, therefore, Coulomb effects and other effects which permit the transition to occur and which affect only the nuclear matrix element, do not produce any change in the pair formation coefficient, that is, the ratio of pairs to photons.

In principle, similar considerations apply to the internal conversion process. However, the internal conversion coefficients for dipole transitions, with the transition energies and values of Z for which the isospin selection rule would be expected to operate, are so small as to make the application of these considerations rather academic.

II. PROOF OF SIEGERT'S THEOREM FOR NONRADIATIVE TRANSITIONS

We will consider the interaction between the electron and the nucleus in the lowest order in the electromagnetic coupling constant, that is, we consider the exchange of one photon. The transition matrix element can then be written in the form[8]

$$V=\int[\mathbf{J}(\mathbf{x})\cdot\mathbf{I}\cdot\mathbf{j}(\mathbf{x}')-\rho_N(\mathbf{x})\rho_e(\mathbf{x}')]\times\frac{\exp(ik_0|\mathbf{x}-\mathbf{x}'|)}{|\mathbf{x}-\mathbf{x}'|}d\mathbf{x}d\mathbf{x}'. \quad (1)$$

In this expression, $(\mathbf{J}(\mathbf{x}),\ i\rho_N(\mathbf{x}))$ constitutes the nuclear transition four-current, and $(\mathbf{j}(\mathbf{x}'),\ i\rho_e(\mathbf{x}'))=-e(\psi_f{}^*(\mathbf{x}')\boldsymbol{\alpha}\psi_i(\mathbf{x}'),\ i\psi_f{}^*(\mathbf{x}')\psi_i(\mathbf{x}'))$ is the electron transition four-current, where ψ_i and ψ_f are initial- and final-state electron wave functions, respectively. The quantity k_0 which appears in the Green's function is the energy transfer to the nucleus, and $\mathbf{I}$ is the unit dyadic. We will ignore nuclear recoil.

The usual, and certainly the simplest, procedure at this juncture is to use plane waves for the electron and thereby obtain a nuclear matrix element of the form

$$\int J_\mu A_\mu d\mathbf{x},$$

where A_μ are the well-known Møller potentials. If one follows this procedure Siegert's theorem is obtained for electric multipoles if one assumes $(kR)^2\ll 1$, which implies $(k_0R)^2\ll 1$. In an effort to obtain a better understanding of this result and to remove the plane-wave restriction, we expand the scalar and dyadic Green's functions in Eq. (1) into multipole fields[4] and keep $\mathbf{j}$, ρ_e arbitrary for the moment. The required multipole expansions are

$$\frac{\exp(ik_0|\mathbf{x}-\mathbf{x}'|)}{|\mathbf{x}-\mathbf{x}'|}=4\pi ik_0\sum_{LM}j_L(k_0x_<)\times Y_L{}^{M*}(\hat{x}_<)h_L(k_0x_>)Y_L{}^M(\hat{x}_>), \quad (2)$$

and

$$\mathbf{I}\frac{\exp(ik_0|\mathbf{x}-\mathbf{x}'|)}{|\mathbf{x}-\mathbf{x}'|}=4\pi ik_0\sum_{LM\tau}\mathbf{A}_L{}^{M*}(\tau;\mathbf{x}_<)\mathbf{B}_L{}^M(\tau;\mathbf{x}_>). \quad (3)$$

Here $\tau=m,e,l$ refers to the three independent multipole fields: magnetic, electric, and longitudinal. The quantities $\mathbf{A}_L{}^M(\tau;\mathbf{x})$ can be expressed in terms of the irreducible tensors defined by Rose[4] as follows:

$$\mathbf{A}_L{}^M(m;\mathbf{x})=j_L(k_0x)\mathbf{T}_{LL}{}^M(\hat{x}), \quad (4a)$$

$$\mathbf{A}_L{}^M(e;\mathbf{x})=\left(\frac{L+1}{2L+1}\right)^{1/2}j_{L-1}(k_0x)\mathbf{T}_{LL-1}{}^M(\hat{x})-\left(\frac{L}{2L+1}\right)^{1/2}j_{L+1}(k_0x)\mathbf{T}_{LL+1}{}^M(\hat{x}), \quad (4b)$$

$$\mathbf{A}_L{}^M(l;\mathbf{x})=\left(\frac{L}{2L+1}\right)^{1/2}j_{L-1}(k_0x)\mathbf{T}_{LL-1}{}^M(\hat{x})+\left(\frac{L+1}{2L+1}\right)^{1/2}j_{L+1}(k_0x)\mathbf{T}_{LL+1}{}^M(\hat{x})=\frac{1}{k_0}\nabla(j_L(k_0x)Y_L{}^M(\hat{x})). \quad (4c)$$

The tensors $\mathbf{B}_L{}^M(\tau;\mathbf{x})$ are obtained from the corresponding $\mathbf{A}_L{}^M(\tau;\mathbf{x})$ by replacing the spherical Bessel function j_L, with a spherical Hankel function of the first kind, h_L. The expansions in Eqs. (2) and (3) are substituted into Eq. (1), and the nuclear transition in question is considered to select a pure electric multipole. The longitudinal term will also contribute in that case. This gives for the transition matrix element

$$\begin{aligned}V=4\pi ik_0\sum_{M\tau}\Bigg[\int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot\mathbf{A}_L{}^{M*}(\tau;\mathbf{x})\int_x^\infty d\mathbf{x}'\mathbf{j}(\mathbf{x}')\cdot\mathbf{B}_L{}^M(\tau;\mathbf{x}')+\int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot\mathbf{B}_L{}^M(\tau;\mathbf{x})\int_0^x d\mathbf{x}'\mathbf{j}(\mathbf{x}')\cdot\mathbf{A}_L{}^{M*}(\tau;\mathbf{x}')\Bigg]\\-4\pi ik_0\Bigg[\int_0^\infty d\mathbf{x}\rho_N(\mathbf{x})j_L(k_0x)Y_L{}^{M*}(\hat{x})\int_x^\infty d\mathbf{x}'\rho_e(\mathbf{x}')h_L(k_0x')Y_L{}^M(\hat{x}')\\-\int_0^\infty d\mathbf{x}\rho_N(\mathbf{x})h_L(k_0x)Y_L{}^M(\hat{x})\int_0^x d\mathbf{x}'\rho_e(\mathbf{x}')j_L(k_0x')Y_L{}^{M*}(\hat{x}')\Bigg]. \quad (5)\end{aligned}$$

[8] The units are such that $\hbar=c=$ rest mass of the electron $=1$. We have taken the matrix element to be the negative of the conventional one. Clearly, this makes no difference for our purposes.

The sum over τ includes e and l. This can be written as

$$V=V_0+V_c, \tag{6}$$

where

$$V_0=4\pi i k_0 \sum_{M\tau} \int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot \mathbf{A}_L{}^{M*}(\tau;\mathbf{x}) \int_0^\infty d\mathbf{x}'\mathbf{j}(\mathbf{x}')\cdot \mathbf{B}_L{}^M(\tau;\mathbf{x}') \\ -4\pi i k_0 \sum_M \int_0^\infty d\mathbf{x}\rho_N(\mathbf{x}) j_L(k_0 x) Y_L{}^{M*}(\hat{x}) \int_0^\infty d\mathbf{x}'\rho_e(\mathbf{x}') h_L(k_0 x') Y_L{}^M(\hat{x}'), \tag{7}$$

and

$$V_c=4\pi i k_0 \sum_{M\tau} \left[\int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot \mathbf{B}_L{}^M(\tau;\mathbf{x}) \int_0^x d\mathbf{x}'\mathbf{j}(\mathbf{x}')\cdot \mathbf{A}_L{}^{M*}(\tau;\mathbf{x}') - \int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot \mathbf{A}_L{}^{M*}(\tau;\mathbf{x}) \int_0^x d\mathbf{x}'\mathbf{j}(\mathbf{x}')\cdot \mathbf{B}_L{}^M(\tau;\mathbf{x}')\right] \\ -4\pi i k_0 \sum_M \left[\int_0^\infty d\mathbf{x}\rho_N(\mathbf{x}) h_L(k_0 x) Y_L{}^M(\hat{x}) \int_0^x d\mathbf{x}'\rho_e(\mathbf{x}') j_L(k_0 x') Y_L{}^{M*}(\hat{x}') \right. \\ \left. - \int_0^\infty d\mathbf{x}\rho_N(\mathbf{x}) j_L(k_0 x) Y_L{}^{M*}(\hat{x}) \int_0^x d\mathbf{x}'\rho_e(\mathbf{x}') h_L(k_0 x') Y_L{}^M(\hat{x}')\right]. \tag{8}$$

The V_0 corresponds to a point nucleus, or better, no penetration of the electron inside the nucleus. The V_c are correction terms representing the penetration inside the nucleus.[9] In Eqs. (7) and (8) the electron wave functions appearing in the transition four-current, ψ_i and ψ_f, are to be taken as wave functions in the Coulomb field of a nucleus of finite size.[10] It is easily verified that for such wave functions all the integrals in Eqs. (7) and (8) converge, although this would not be the case had the wave functions been taken as those of an electron in the Coulomb field of a point nucleus. We first show that V_0 satisfies Siegert's theorem and then turn to the evaluation of V_c. Using the values of the spherical Bessel functions for small arguments

$$j_\lambda(y) \underset{y\to 0}{\longrightarrow} y^\lambda/(2\lambda+1)!!+O(y^{\lambda+2}), \tag{9}$$

and Eqs. (4b) and (4c), $\mathbf{A}_L{}^M(e;\mathbf{x})$ can be written in the limit $(k_0R)^2 \ll 1$, as

$$\mathbf{A}_L{}^M(e;\mathbf{x}) \cong \frac{1}{k_0}\left(\frac{L+1}{L}\right)^{1/2} \nabla(j_L(k_0x)Y_L{}^M(\hat{x})), \tag{10}$$

thus, in this approximation,

$$V_0=4\pi i k_0 \sum_M \int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot \nabla(j_L(k_0x)Y_L{}^{M*}(\hat{x})/k_0) \int_0^\infty d\mathbf{x}'\mathbf{j}(\mathbf{x}')\cdot \left[\left(\frac{L+1}{L}\right)^{1/2} \mathbf{B}_L{}^M(e;\mathbf{x}')+\mathbf{B}_L{}^M(l;\mathbf{x}')\right] \\ -4\pi i k_0 \sum_M \int_0^\infty d\mathbf{x}\rho_N(\mathbf{x}) j_L(k_0 x) Y_L{}^{M*}(\hat{x}) \int_0^\infty d\mathbf{x}'\rho_e(\mathbf{x}') h_L(k_0 x') Y_L{}^M(\hat{x}'). \tag{11}$$

We now perform an integration by parts in the first term, noting that the surface contribution vanishes since $\mathbf{J}(\mathbf{x})$ vanishes strongly at infinity. Moreover, the continuity equation for the nuclear transition current is

$$\nabla\cdot\mathbf{J}(\mathbf{x})=-ik_0\rho_N(\mathbf{x}), \tag{12}$$

so that

$$V_0=\int_0^\infty d\mathbf{x}\rho_N(\mathbf{x})\Lambda(\mathbf{x}), \tag{13a}$$

where[11]

$$\Lambda(\mathbf{x})=4\pi i k_0 \sum_M j_L(k_0x)Y_L{}^{M*}(\hat{x}) \\ \times \int_0^\infty d\mathbf{x}'\left[i\left(\frac{2L+1}{L}\right)^{1/2} h_{L-1}(k_0x')\mathbf{j}(\mathbf{x}')\cdot\mathbf{T}_{LL-1}{}^M(\hat{x}') \right. \\ \left. -\rho_e(\mathbf{x}')h_L(k_0x')Y_L{}^M(\hat{x}')\right]. \tag{13b}$$

This result establishes Siegert's theorem for the transition matrix element, $V=V_0+V_c$, provided V_c is negligible compared to V_0. In order to investigate the magnitude of V_c, we shall approximate the electron

[9] The matrix elements V_0 and V_c correspond to what is usually referred to in internal conversion theory as the static and dynamic effects of finite nuclear size, respectively, see, for example, M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958). Of course, in the present case we are eventually interested in a cross section rather than a branching ratio so that questions of nuclear dynamics are never completely eliminated.

[10] See, for example, M. E. Rose, *Relativistic Electron Theory* (John Wiley & Sons, Inc., New York, 1961), pp. 240–244.

[11] It is of interest to note that in Eq. (13b) the multipole appearing in the electronic matrix element is just the so-called "conventional gauge," Refs. 4 and 7.

wave functions by plane waves. We emphasize that our purpose is not one of precise evaluation of V_c, but rather the investigation of the conditions for $V_c \ll V_0$. It is expected that the conditions thereby established for the validity of the strong inequality will not be altered by the distortion of the wave functions from plane waves. This Born approximation for the correction term allows us to write for the electron transition current in V_c

$$\mathbf{j}(\mathbf{x}')=\mathbf{b}e^{i\mathbf{k}\cdot\mathbf{x}'}=4\pi\mathbf{b}\sum_{lm} i^l j_l(kx')Y_l^{m*}(\hat{x}')Y_l^m(\hat{k}), \tag{14a}$$

and the continuity equation for this current then gives

$$\rho_e(\mathbf{x}')=(ik_0)^{-1}\nabla\cdot\mathbf{j}(\mathbf{x}')=(\mathbf{k}\cdot\mathbf{b}/k_0)e^{i\mathbf{k}\cdot\mathbf{x}'}, \tag{14b}$$

where

$$\mathbf{b}=-e(u_f^*(\mathbf{p}')\boldsymbol{\alpha}u_i(\mathbf{p})). \tag{14c}$$

The integrations for V_c indicated in Eq. (8) are easily carried out by using the orthogonality relation for the spherical harmonics and the two radial integrals

$$\begin{aligned}\int_0^x x'^2dx'j_\lambda(kx')j_\lambda(k_0x')&=x^2(k^2-k_0^2)^{-1}[k_0j_\lambda(kx)j_{\lambda-1}(k_0x)\\&\qquad -kj_{\lambda-1}(kx)j_\lambda(k_0x)]\end{aligned} \tag{15a}$$

$$\begin{aligned}\int_0^x x'^2dx'j_\lambda(kx')h_\lambda(k_0x')&=x^2(k^2-k_0^2)^{-1}[k_0j_\lambda(kx)h_{\lambda-1}(k_0x)-kj_{\lambda-1}(kx)h_\lambda(k_0x)]\\&\qquad -i(k^2-k_0^2)^{-1}(k^\lambda/k_0^{\lambda+1}).\end{aligned} \tag{15b}$$

The result for V_c can then be written in the form

$$\begin{aligned}V_c=16\pi^2i^{L+1}(k^2-k_0^2)^{-1}\Bigg\{&\int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot\mathbf{T}_{LL+1}{}^M(\hat{x})\\&\times\mathbf{b}\cdot\mathbf{T}_{LL+1}{}^{M*}(\hat{k})\left[j_{L+1}(kx)-\left(\frac{k}{k_0}\right)^{L+1}j_{L+1}(k_0x)\right]\\&-\int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot\mathbf{T}_{LL-1}{}^M(\hat{x})\mathbf{b}\cdot\mathbf{T}_{LL-1}{}^{M*}(\hat{k})\\&\times\left[j_{L-1}(kx)-\left(\frac{k}{k_0}\right)^{L-1}j_{L-1}(k_0x)\right]\\&+\left(\frac{i\mathbf{k}\cdot\mathbf{b}}{k_0}\right)\int_0^\infty d\mathbf{x}\rho_N(\mathbf{x})Y_L{}^M(\hat{x})Y_L{}^{M*}(\hat{k})\\&\times\left[j_L(kx)-\left(\frac{k}{k_0}\right)^Lj_L(k_0x)\right]\Bigg\}.\end{aligned} \tag{16}$$

It is convenient for the sake of comparison to evaluate V_0 and $V=V_0+V_c$ in the Born approximation. This requires the same integrals as in Eq. (15) with the upper limit now taken infinite. The result for V_0 is

$$\begin{aligned}V_0=16\pi^2i^{L+1}(k^2-k_0^2)^{-1}&\\\times\Bigg\{&\int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot\mathbf{T}_{LL+1}{}^M(\hat{x})\mathbf{b}\cdot\mathbf{T}_{LL+1}{}^{M*}(\hat{k})\\&\times\left(\frac{k}{k_0}\right)^{L+1}j_{L+1}(k_0x)-\int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot\mathbf{T}_{LL-1}{}^M(\hat{x})\\&\times\mathbf{b}\cdot\mathbf{T}_{LL-1}{}^{M*}(\hat{k})\left(\frac{k}{k_0}\right)^{L-1}j_{L-1}(k_0x)+\left(\frac{i\mathbf{k}\cdot\mathbf{b}}{k_0}\right)\\&\times\int_0^\infty d\mathbf{x}\rho_N(\mathbf{x})Y_L{}^M(\hat{x})Y_L{}^{M*}(\hat{k})\left(\frac{k}{k_0}\right)^Lj_L(k_0x)\Bigg\},\end{aligned} \tag{17}$$

and adding these two results gives

$$\begin{aligned}V=16\pi^2i^{L+1}(k^2-k_0^2)^{-1}&\\\times\Bigg\{&\int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot\mathbf{T}_{LL+1}{}^M(\hat{x})\mathbf{b}\cdot\mathbf{T}_{LL+1}{}^{M*}(\hat{k})j_{L+1}(kx)\\&-\int_0^\infty d\mathbf{x}\mathbf{J}(\mathbf{x})\cdot\mathbf{T}_{LL-1}{}^M(\hat{x})\mathbf{b}\cdot\mathbf{T}_{LL-1}{}^{M*}(\hat{k})j_{L-1}(kx)\\&+\left(\frac{i\mathbf{k}\cdot\mathbf{b}}{k_0}\right)\int_0^\infty d\mathbf{x}\rho_N(\mathbf{x})Y_L{}^M(\mathbf{x})Y_L{}^{M*}(\hat{k})j_L(kx)\Bigg\}.\end{aligned} \tag{18}$$

This result is just the Møller potential result, and, in somewhat altered form, already appears in the literature.[12] Thus, V_0 is obtained from V by replacing $j_\lambda(kx)$ by $(k/k_0)^\lambda j_\lambda(k_0x)$. Clearly, the two are equal when the Bessel functions can be replaced by their leading terms. Hence, $V_c \ll V_0$ when $(kR)^2 \ll 1$. Another way of putting the matter is to refer to Eq. (16) from which it is clear that $V_c \ll V_0$ when

$$j_\lambda(kx)-(k/k_0)^\lambda j_\lambda(k_0x)\ll j_\lambda(kx)$$

or when $(kR)^2 \ll 1$. This is just the conclusion which is reached if we use the Møller potential from the beginning.

We are thereby driven back to the stringent condition $(kR)^2 \ll 1$ in order to establish Siegert's theorem. For incident electron energies, E, of 50 MeV or more, this inequality is fulfilled only for small angle scattering. For scattering through angles of about $1/ER$ (for which $kR\sim 1$) we must conclude that Siegert's theorem begins to break down. For example, for 100-MeV electrons scattered from O^{16}, the angle $1/ER$ is 37°. Below this angle the angular distribution in inelastic scattering should be expected to be anomalously small in those transitions which are forbidden for the radiative process.

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[12] See, for example, K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 (1956).

III. DISCUSSION OF THE ISOSPIN SELECTION RULES

We assume in the following that the scattering angle is sufficiently small as to allow the application of the condition

$$(kR)^2 \ll 1.$$

It follows *a fortiori* that $(k_0R)^2 \ll 1$. In this range of scattering angles ϑ,

$$k^2 \cong k_0^2 + (k_0/E)^2 + 2E^2(1-\cos\vartheta),$$

where $E \gg 1$ has also been assumed. We first discuss electric dipole transitions. Then with $L=1$ we obtain

$$V = \mathbf{a} \cdot \int_0^\infty \rho_N(\mathbf{x})\mathbf{x}d\mathbf{x}, \tag{19a}$$

where, with $M=\pm 1$ and 0,

$$a_M = \left(\frac{4\pi}{3}\right)^{1/2} ik_0^2 \int_0^\infty d\mathbf{x}' [i\sqrt{3}h_0(k_0x')\mathbf{j}(\mathbf{x}') \cdot \mathbf{T}_{10}{}^M(\hat{x}) - h_1(k_0x')\rho_e(\mathbf{x}')Y_1{}^M(\hat{x}')]. \tag{19b}$$

The integral in the vector $\mathbf{a}$ converges and other than this fact no further interest attaches to it.[13]

The form of Eq. (19) lends itself immediately to the derivation of the isospin selection rule as it appears in the literature.[14] Without repeating the well-known derivations we simply recall that the coordinates relative to the center of mass are introduced whereby the effective charges for neutron and proton appear. For this purpose it is convenient to write

$$\rho_N(\mathbf{x}) = e \sum_{l=1}^{Z} \int \delta(\mathbf{x}-\mathbf{x}_l)\Psi_f{}^*(\mathbf{x}_N)\Psi_i(\mathbf{x}_N)d\mathbf{x}_N,$$

where $\mathbf{x}_N$ is an abbreviation for the entire set of nucleon coordinates.

For $N=Z$ nuclei the effective charges of neutron and proton are, of course, $-e/2$ and $e/2$, respectively. The isoscalar part of V cancels and the isovector is an odd operator under the charge parity transformation which converts neutrons into protons and protons into neutrons. Thus, if the isospin is a good quantum number, the matrix element V vanishes between states of the same isotopic spin.

In real nuclei the matrix element is small but nonvanishing for the following reasons:

(1) Coulomb interactions and the neutron-proton mass difference make the isotopic spin partially nonconserved.

(2) The isospin selection rule is equivalent to a statement that the total momentum of the nucleus is zero in the rest system. Actually, the statement refers to bare nucleons. The total momentum is zero only when that of the π-meson field is taken into account.

(3) The nuclear recoil energy is not zero.

(4) Higher order terms in $(kx)^2$ are present.

While the first two effects are difficult to evaluate with any precision they contribute the major portion of the nonvanishing matrix element. The effect of higher order terms in the retardation expansion is easy to estimate. Considering this term alone, it would follow that at $\vartheta=0$ the matrix elements are smaller than those of the normal (nonforbidden on isospin grounds) transitions by a factor of order $(kR)^2$. Taking into account some small numerical factors one would conclude that the differential scattering cross section at $\vartheta=0$ in a light nucleus (O^{16} or N^{14} say) at $k_0=10$ ($\sim$5-MeV excitation) would be smaller than for normal transitions by a factor of about 10^{-6}. This is, no doubt, entirely unrealistic. Instead, one would obtain a far better estimate of the inhibition factor by using the empirical approach of estimating nuclear matrix elements from observed widths of γ transitions. As Eq. (19) shows, the matrix element is now factored into a nuclear matrix element, which is just that for γ rays, times an irrelevant constant coming from the electronic matrix element. Hence, under the circumstances that this result applies, the differential cross section for excitation through the isospin forbidden transitions is inhibited by the same factor (10^{-3} to 10^{-2} in most cases) as characterizes the γ transitions. In making this statement we use the almost trivial fact that the matrix element in the γ case is independent of the direction of the γ ray.

The consequence of all this is that the angular distribution of the inelastically scattered electrons for the transitions in question would be expected to exhibit a "hole" at small angles. As in the numerical example quoted at the end of the last section, the scattering cone in which this anomaly exists is difficult to observe but seems to be not inaccessible. Certainly, one alternative procedure would involve the scattering of low energy electrons (perhaps 10–20 MeV) in which case the anomalous cone opens up. In this way one may be able to use inelastic scattering to identify $\Delta T=0$ electric dipole transitions in $N=Z$ nuclei as a supplementary tool to the γ-ray emission studies. As far as known examples are concerned, one obviously finds fewer cases than in γ-ray emission because one is restricted to ground state transitions and stable nuclei. In Wilkinson's survey,[3] three $\Delta T=0$ ground-state transitions are listed: the 6.23-MeV level in N^{14}, and two levels at 7.12 and 9.58 MeV in O^{16}. Of these the N^{14} transition may actually be an $M1$ transition, but, as we discuss below, these transitions are similarly inhibited. The inelastic scattering could distinguish between $M1$ and $E1$ transitions if the large angle scattering is observed

[13] If we assume that $V_c \ll V_0$ for any realistic electron wave functions, Eq. (19) applies. Otherwise we are restricted to the Born approximation for which case $\mathbf{a}$ has been evaluated in the preceding section.

[14] See G. Morpurgo, Ref. 3.

since the Coulomb matrix elements (arising from the longitudinal field) give an entirely different differential cross section.

If the scattering at small angles is too difficult to observe it may be worthwhile to consider total inelastic cross sections. While these are difficult to evaluate in view of the fact that one needs to assume a nuclear model to evaluate even the shape, as well as the magnitude of the angular distribution, one may consider energies large enough so that almost all of the scattering occurs in a narrow cone in the forward direction. For instance, for $E=200$ (100 MeV) 90% of the cross section is estimated to arise from scattering events with $\vartheta<30°$. This refers to a normal transition. If the transition is forbidden by isospin selection rules this 90% of the cross section would be largely wiped out and the total cross section would be anomalously small. A systematics of total cross sections would clearly be necessary to establish the identity of the forbidden transitions.

Turning now to the magnetic dipole transitions the situation is only slightly different in that the matrix element does not vanish even when the corrections (1)–(4) above are ignored. It is, however, anomalously small under these conditions due to an accidental cancellation.[5] To establish this we can easily obtain the magnetic multipole matrix elements. For $L=1$ this is

$$V=-i(6\pi)^{1/2}\sum_{\mu}\mu b_{\mu}\sum_{M}D_{M,-\mu}{}^{1}(\hat{k})\times\int_0^\infty \mathbf{J}(\mathbf{x})\cdot\mathbf{A}_1{}^M(m;\mathbf{x})d\mathbf{x}.$$

Here $D_{M,-\mu}{}^{1}(\hat{k})$ is an element of the rotation matrix with arguments given by the unit vector $\hat{k}$. If we write $\mathbf{J}$ as a sum of convective and spin currents with the neutrons contributing only to the latter, then in the usual way we obtain with $(kR)^2\ll1$ the result

$$\int\mathbf{J}\cdot\mathbf{A}_1{}^M d\mathbf{x}=\text{const}\int\Psi_f{}^*\boldsymbol{\xi}_M\cdot\Big[\sum_{l=1}^{Z}(\mathbf{L}+\mu_p\boldsymbol{\sigma})_l+\sum_{l=Z+1}^{A}\mu_n\boldsymbol{\sigma}_l\Big]\Psi_i d\mathbf{x}_N.$$

Here μ_p and μ_n are the proton and neutron magnetic moments in nuclear magnetons, and $\boldsymbol{\xi}_M$ are the spherical basis unit vectors.[4] In this form Morpurgo's analysis is directly applicable, and we find that for $\Delta T=0$ transitions in $N=Z$ nuclei

$$\int\mathbf{J}\cdot\mathbf{A}_1{}^M d\mathbf{x}=\tfrac{1}{2}(\mu_p+\mu_n-\tfrac{1}{2})\int\Psi_f{}^*\sum_{l=1}^{A}\boldsymbol{\sigma}_l\cdot\boldsymbol{\xi}_M\Psi_i d\mathbf{x}_N. \quad (20)$$

The factor in front of the integral on the right-hand side of Eq. (20) is 0.19. For a normal transition this factor would be replaced by $\frac{1}{2}(\mu_p-\mu_n)=2.35$. Therefore, the square of the matrix element is inhibited by the factor $(0.19/2.35)^2=0.7\times10^{-2}$ just as in the case of the $M1$ γ transition. Again this result applies in the forward directed cone where $(kR)^2\ll1$. Within the accuracy of our previous rough approximation, the peaking of the angular distribution of the $M1$ differential cross section[15] is the same as for $E1$. The angle below which a given fraction of the total cross section is included is then about as before. Consequently, all the remarks made in connection with the forbidden $E1$ transitions apply to $M1$ transitions as well.

[15] I. N. Sneddon and B. F. Touschek, Proc. Roy. Soc. (London) **A193**, 344 (1948).