(*d,p*) Reaction on the Titanium Isotopes*

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(Received 1 March 1963)

The (*d,p*) reaction on Ti^{46}, Ti^{47}, Ti^{48}, Ti^{49}, and Ti^{50} has been studied at an incident-deuteron energy of 21.4 MeV. Absolute differential cross sections and angular distributions for a number of transitions are reported. A number of previously unreported levels are found in Ti^{51}, Ti^{50}, Ti^{49}, Ti^{48}, and Ti^{47} while a few levels reported in the literature (e.g., the 600-keV level in Ti^{51}) do not appear to exist. The angular distributions are compared with distorted-wave Born approximation (DWBA) calculations, and the spectroscopic factor is obtained for a number of transitions. The results are compared with those from the (*d,t*) reaction. The present experiment indicates that seniority mixing occurs in the titanium isotopes.

I. INTRODUCTION

THE (*d,p*) reaction on the stable isotopes of titanium (Ti^{46}, Ti^{47}, Ti^{48}, Ti^{49}, and Ti^{50}) has been studied for deuterons incident at an energy of 21.4 MeV. The present experiment is a low-resolution experiment whose primary objective was to obtain absolute differential cross sections for the transitions to a few well-separated levels. Since the incident-deuteron energy is the same for all five isotopes, it is possible to compare the absolute cross sections for transitions from different isotopes and to make a comparison with the strength (or spectroscopic factor) obtained in a previous experiment from the (*d,t*) reaction on the titanium isotopes.[1] Furthermore, the availability of the angular distributions from the elastic deuteron scattering at 21.4 MeV permits a comparison with distorted-wave Born approximation (DWBA) calculations both for the case in which best-fit potentials to the individual isotopes are used and for the case in which an averaged potential with an appropriate correction for the nuclear radius is used. Few (*d,p*) experiments have been done at incident energies in the vicinity of 20 MeV, and it is not *a priori* clear that the l value of angular distributions at these energies can be identified unambiguously. The present experimental work can therefore also be used to investigate whether or not the angular distributions are characteristic of a given l value.

II. EXPERIMENTAL

The experiment was performed with the 60-in. scattering chamber[2] and the 21.4-MeV deuteron beam of the Argonne 60-in. cyclotron. The detection system used in most of the work consisted of an $E-(dE/dx)$ telescope with NaI(Tl) crystals. The arrangement has been described in detail in Ref. 1. The resolution width obtained with this system was usually somewhat better than 1%. In some instances spectra were obtained with a NaI(Tl) as the dE/dx detector and a Li-diffused silicon detector[3] as the E detector. The isotopic composition of the target foils has been determined by means of a mass spectrometer[4] and has been tabulated in Ref. 1. The spectra were recorded on a multichannel analyzer for a predetermined charge on the collector cup. The analyzer was then switched to the "subtract" mode and the contributions from isotopes other than the main isotope were subtracted. It was assumed in this correction that all targets other than the one under investigation were pure. The error resulting from this assumption is negligible compared with other errors. The target thickness was determined by means of the area/weight method. The uncertainty in target thickness is estimated to be about 5%.

The energy calibration was obtained from the range-energy relation as well as from the Q values of known reactions. The calibration is accurate to within 150 keV over the entire range. The absolute cross sections of the main peaks are reliable to approximately 5%. The energy spectra were obtained in 3° intervals from about 11° to 42°.

III. EXPERIMENTAL RESULTS

It is not clear *a priori* that angular distributions of (*d,p*) reactions on targets of intermediate mass at the incident energy used in this experiment can unambiguously determine the orbital angular momentum of the captured neutron. For the purpose of the present experiment it was considered adequate to establish that known transitions corresponding to the capture of f neutrons have a characteristic angular distribution and that this angular distribution is different from the ones obtained from transitions known to correspond to the capture of a p neutron. Angular distributions characteristic of $l=3$ transitions are well exemplified by the ground-state transitions of $Ca^{40}(d,p)Ca^{41}$ and $Ti^{48}(d,p)Ti^{49}$ and by the 160-keV transition in the $Ti^{46}(d,p)Ti^{47}$ reaction. The angular distributions for these three reactions are shown in Fig. 1. They show a primary peak near 24° and a secondary maximum near 55°. The

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] J. L. Yntema, Phys. Rev. **127**, 1659 (1962).
[2] J. L. Yntema and H. W. Ostrander, Nucl. Instr. Methods **16**, 69 (1962).
[3] Prepared by H. Mann, Electronics Division, Argonne National Laboratory.
[4] We are indebted to C. M. Stevens and his group for this analysis.

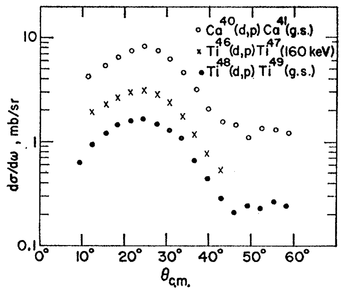

FIG. 1. Angular distributions of known $l=3$ transitions.

ground-state transition of $Ti^{50}(d,p)Ti^{51}$, the transition to the group of levels near 1.5 MeV in $Ti^{48}(d,p)Ti^{49}$, and the transition to the levels at 1.9 and 2.4 MeV in $Ca^{40}(d,p)Ca^{41}$ are known to correspond to the capture of p neutrons. The angular distributions are shown in Fig. 2. They show a primary maximum near 10° and a secondary maximum near 35°. This establishes that the angular distribution permits one to distinguish $l=1$ from $l=3$ transitions. The possibility that some transitions with other l values could cause confusion cannot be completely eliminated. In particular, the possibility of confusing $l=0$ and $l=3$ transitions has to be taken into consideration in the analysis.

A. $Ti^{50}(d,p)Ti^{51}$

The spectrum observed at 24° for the $Ti^{50}(d,p)Ti^{51}$ reaction is shown in Fig. 3. Previous (d,p) work[5] on Ti^{50} showed the ground-state transition, a consistent indication of a transition at 600±150 keV, and an excited group at 1.2 MeV which presumably was too wide to correspond to a single level. The present experiment shows the ground-state transition and additional peaks at 1.20, 1.45, 2.15, 3.00, and 4.1 MeV. The 1.2- and 1.45-MeV levels are not resolved. It is to be noted that the 600-keV level is not observed in the present experiment even though the resolution appears to be better than that in Ref. 5. The angular distributions of the ground-state group and the 1.2-MeV group are shown in Fig. 4. The curves are obtained from distorted-wave

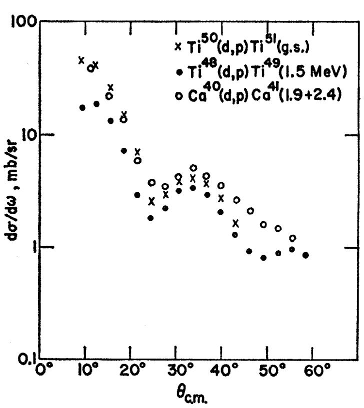

FIG. 2. Angular distribution of known $l=1$ transitions.

[5] G. F. Pieper, Phys. Rev. 88, 1299 (1952).

calculations made by Satchler.[6] Two sets of input data have been used in these calculations—one which employed the deuteron optical-model parameters adjusted to give the "best fit" to the angular distribution of the elastic deuteron scattering at 21.4 MeV for the particular isotope involved in the reaction, the other an "averaged" set of parameters which give a fair fit to the elastic deuteron-scattering data for all five titanium isotopes when the radius and mass have been appropriately adjusted. It is also assumed that the neutron is captured into a single-particle orbital with a binding energy equal to the separation energy $S=Q+2.23$ MeV. The curves have been adjusted to give a strength of 1 for the $\frac{3}{2}^-$ spin of Ti^{51} in the ground state and to correspond to the assumption that the spin of the 1.2-MeV state is $\frac{1}{2}^-$. No adjustment has been made for the difference in Q value between the ground state and the 1.2-MeV state. Such a correction would tend to raise

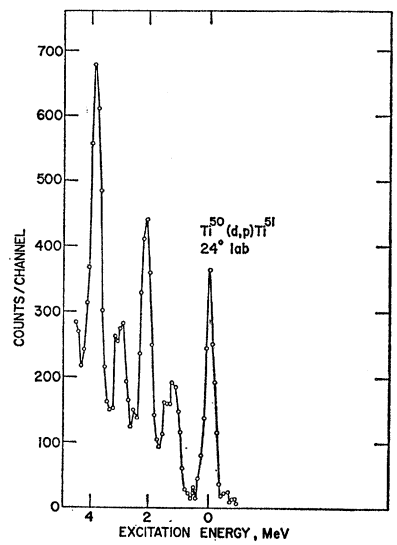

FIG. 3. Spectrum of the $Ti^{50}(d,p)Ti^{51}$ reaction at 24° lab.

the 1.2-MeV curve by a small amount. The curve shown is the one computed with the best-fit values from the deuteron elastic scattering on Ti^{50}. The strength in the case of the ground state is somewhat larger than 1. Use of the "average potential" parameters would change the value for the strength by about 10%.

It is clear that the angular distribution of the 1.2-MeV group is not fitted by the $l=1$ curve as well as the ground-state transition or the other angular distributions shown in Fig. 2. If one subtracts the $l=1$ curve from the data points on the assumption that the cross section at 9° is entirely due to the $l=1$ transition, the residual curve is fitted well by the $l=3$ distorted-wave

[6] I am indebted to Dr. Satchler for permission to use his results. He will discuss the optical-potential parameters in more detail in a later publication. It is sometimes found that a better fit to experiment is obtained if a lower cutoff is imposed on the radial integrals in stripping calculations. In the present work such a procedure usually gave a worse fit.

angular distribution. In subtractions from this and other angular distributions in this paper, we have used the experimental angular distributions shown in Fig. 1 for $l=3$ and those in Fig. 2 for $l=1$ rather than the results of the DWBA calculations. If one assumes that the transition proceeds to a $\frac{7}{2}^-$ state in Ti^{51}, the strength of the transition is approximately 0.08. This strength is somewhat larger than expected from the admixture of $2p_{3/2}$ obtained in the $Ti^{50}(d,t)Ti^{49}$ reaction. On the other hand, if one assumes that the transition proceeds to a $\frac{5}{2}^-$ state of Ti^{51}, this would appear to contain only a small fraction of the single-particle configuration.

The angular distributions of the other groups observed appear to contain admixtures of a number of l values and a detailed analysis requires a better energy resolution than is available at the cyclotron at this time.

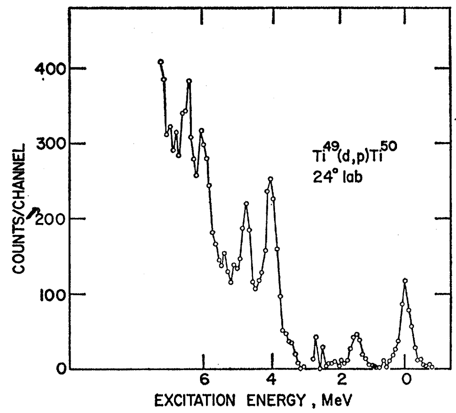

FIG. 5. Spectrum of the $Ti^{49}(d,p)Ti^{50}$ reaction at 24° lab.

B. $Ti^{49}(d,p)Ti^{50}$

The spectrum obtained at 24° for the $Ti^{49}(d,p)Ti^{50}$ is shown in Fig. 5. The ground-state transition, transitions

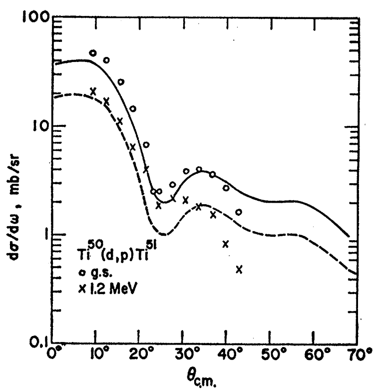

FIG. 4. Angular distributions of the $Ti^{50}(d,p)Ti^{51}$ ground-state transition and the 1.2-MeV group. The solid line was obtained when DWBA calculations for $l=1$ were adjusted to give a strength of 1 for a spin of $\frac{3}{2}^-$. The dashed curve is the same $l=1$ curve adjusted to give a strength of 1 for a spin of $\frac{1}{2}^-$.

to a level near 1.55 MeV, and a fairly complex level structure at excitation energies of 4.1 MeV and above are clearly indicated. There is also a group near 2.8 MeV. This latter group occurs at about the same energy as the ground-state transition of the $Ti^{48}(d,p)Ti^{49}$ reaction. The absolute value of the cross section for this latter transition has, therefore, a much greater uncertainty than the one to the ground state or the 1.56-MeV level.

The angular distributions of the transitions to the ground state and the 1.56-MeV level are shown in Fig. 6. The ground-state transition has a typical $l=3$ angular distribution. The solid curve is the one obtained from distorted-wave calculations and has been normalized to strength 1. This curve was again the one calculated for the "best fit" parameters and gives a strength of 5.5 for the transition, compared to a prediction of 8 based on the assumption of good seniority. The same result would be obtained for the "average" set. The transition to the 1.56-MeV state is shown to be fitted rather more poorly by the $l=1$ curve than was the $Ti^{50}(d,p)Ti^{51}$ ground-state transition. One can decompose this curve into a combination of $l=1$ and $l=3$. Since no 0^+ level is involved in this transition (the transition proceeds from a $\frac{7}{2}^-$ state to a 2^+ state), admixture of l values of the same parity is possible. The strength of the $l=3$ component can be estimated by making the assumption that all of the cross section at 9° arises from the $l=1$ component or, alternatively, that the cross section at 24° is entirely due to the $l=3$ component. This results in an $l=3$ cross section amounting to $35\pm5\%$ of the ground-state cross section. The angular distribution of the transition to the 2.8-MeV state is shown in Fig. 7. In previous work[5] a weak level was found at 3.0 MeV and at 2.80 MeV.[7] The 2.80-MeV transition was identified as an $l=0$ transition. If this level is the same as the level in Ti^{50} to which Sc^{50} decays, this would indicate that the ground-state parity of Sc^{50} is not the one expected on the basis of the shell model.

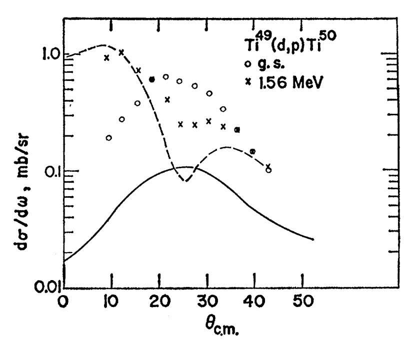

FIG. 6. Angular distributions of the $Ti^{49}(d,p)Ti^{50}$ ground-state transition and the transition to the 2^+ level at 1.56 MeV. The solid line was obtained when DWBA calculations for $l=3$ were adjusted to give a strength of 1 for the transition to the 0^+ state. The dashed line gives the shape of the $l=1$ angular distribution obtained from DWBA calculations and has been adjusted to fit the experimental points at smaller angles.

[7] N. I. Zaika and A. F. Nemets, Izv. Akad. Nauk. SSSR Ser. Fiz. **24**, 865 (1960).

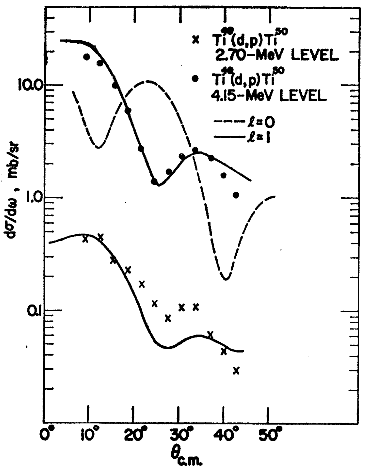

FIG. 7. Angular distribution of the $Ti^{49}(d,p)Ti^{50}$ transitions to the 2.78-MeV level and the 4.1-MeV group. The solid lines give the shape of the $l=1$ angular distribution obtained from DWBA calculations. The dashed line gives the $l=0$ curve obtained from DWBA calculations adjusted for a strength of 1 for the transition to a 4^- level.

Transitions to the 4^+ level and to the level at 3.3 MeV have been observed in the $V^{51}(n,d)Ti^{50}$ reaction.[8] Figure 7 shows the angular distribution (dashed curve) obtained by a distorted-wave calculation for $l=0$. It is clear that in the present experiment the admixture of $l=1$ and $l=3$ transitions (solid curve) is much to be preferred over the $l=0$ transition, especially when compared with the 1.56-MeV level whose parity is known. It should be emphasized that at this time we have not experimentally observed a well-isolated $l=0$ transition. However, since the shapes of the calculated $l=1$ and $l=3$ transitions agree so well with the experiment, one may expect a reasonably good fit for $l=0$ and $l=2$. Since the curve observed in the ground-state transition (Fig. 6) shows no indication whatever of the presence of a low-lying level with an $l=1$ angular distribution as suggested in Ref. 7, it appears that there is a discrepancy between the results of the two experiments. It should be noted that reasonable agreement is obtained for Ti^{47} and Ti^{48} as far as the shapes of the angular distributions of the lowest states are concerned. It is suggested that the discrepancy may have arisen from the low enrichment of the targets used in Ref. 7.

The cross section of the $l=3$ component of the transition to the 4^+ state is estimated to be $11\pm5\%$ of the ground-state cross section. The strength of the $l=1$ transition to the 2^+ level, which has to correspond to the capture of $p_{3/2}$ neutrons, is estimated from the distorted-wave analysis to be 0.2; the strength of the $l=1$ component to the 4^+ state is estimated at around 0.05. The angular distribution of the transition to the 4.1-MeV level is shown in Fig. 7 together with the $l=1$ angular distribution. It is not possible to estimate the strength of the $l=1$ transition since the spin of the state is not known. According to Hansen,[9] there are two levels near 4.1 MeV. It is certain that the level reported here at 2.80 MeV is the same as the level reported in Ref. 5 at 3.00 MeV. Experiments with better energy resolution show that the energy is 2.78 ± 0.05 MeV. There is a suggestion of a very weak transition to the 6^+ level at 3.25 MeV. In addition, strong transitions are observed to states near 4.85 MeV (probably double), 5.14, 5.32, 5.85, and 6.03 MeV. The level positions and spectroscopic factors are shown in Table I.

TABLE I. Levels observed in the titanium isotopes. The l values for the transitions are given if known. The spectroscopic factors are obtained from comparison with DWBA calculations. The validity of these values is discussed in the text.

Nucleus	Energy (MeV)	l value	Spectroscopic factor
Ti^{51}	0	1	1.2
	1.20	1	1.2
	1.45	3	0.08
	2.15		
	3.00		
	4.10		
Ti^{50}	0	3	5.5
	1.56	1,3	0.2
	2.78	1,3	0.05
	4.1	1	
	4.85		
	5.14		
	5.32		
	5.85		
	6.03		
Ti^{49}	0	3	0.17
	1.38	1	0.3
	1.72	1	0.15 or 0.3
	2.5		
	4.41		
	4.66		
	4.98		
	5.31		
	5.57		
	5.80		
Ti^{48}	0.99	3	1.1
	2.30		
	2.43		
	3.16		
	3.25		
	3.40		
	4.35		
	4.68		
	4.83		
	5.56		
Ti^{47}	0.16	3	0.37
	1.56	1	0.39
	1.80	1	0.31–0.15
	2.15		
	2.58		
	2.81		
	3.71		
	3.93		
	4.35		
	4.43		
	4.68		
	5.06		
	5.31		
	5.56		

[8] K. Ilakovac, L. G. Kuo, M. Petravic, I. Slaus, P. Tomas, and G. R. Satchler, Phys. Rev. **128**, 2739 (1962).

[9] O. Hansen, Nucl. Phys. **28**, 140 (1961).

C. $Ti^{48}(d,p)Ti^{49}$

The spectrum of the $Ti^{48}(d,p)Ti^{49}$ is shown in Fig. 8. The levels at 1.38 and 1.76 MeV are obviously not resolved. The level structure above 2.5 MeV is quite complex and a detailed analysis of this region requires a much better energy resolution than was available in this experiment. The angular distribution of the ground-state transition is shown in Fig. 9. The two curves are the one obtained from the "best fit" optical-potential parameters (labeled B_3) and one obtained from the "average potential" (labeled B_{av}). It is clear that at small angles the shape of the B_3 distribution is in much the better agreement with the experimental result. The strength of the $l=3$ transition depends obviously on the choice of deuteron optical-model parameters. It is about 0.19 for B_{av} and 0.17 for B_3. The B_3 curve is drawn for a strength of 1, the B_{av} curve for a strength of 0.7.

The angular distribution of the group near 1.5 MeV is shown in Fig. 2. It is a typical $l=1$ curve. The main components are the transitions to the $\frac{3}{2}^-$ state at 1.38 MeV and the $\frac{1}{2}^-$ state at 1.72 MeV as shown by Rietjens, Bilaniuk, and Macfarlane.[10] Hansen[9] has found five levels between 1.373 and 1.758 MeV. However, only the two found by Rietjens *et al.* are excited in the (d,p) reaction. If one assumes the spin assignments[11] to be correct, the strength of the $p_{3/2}$ transition on the basis of the distorted-wave calculation is approximately 0.3 and the strength of the $p_{1/2}$ transition is also approximately 0.3. If one assumes that both of these transitions correspond to the capture of $p_{3/2}$ neutrons, the total strength would be 0.45. The group near 2.5 MeV shows an angular distribution which indicates the contributions of several levels with different l values. The $l=3$ component is estimated to be approximately 75±15% of the intensity of the ground-state transition. The strong group near 5 MeV contains contributions from states at 4.41, 4.66, 4.98, 5.31, 5.57, and 5.80 MeV.

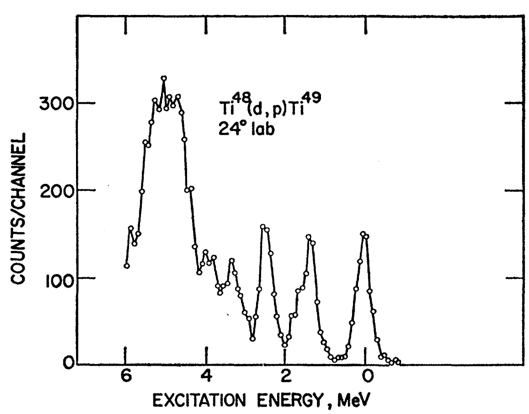

FIG. 8. Spectrum of the $Ti^{48}(d,p)Ti^{49}$ transition at 24° lab.

[10] L. H. T. Rietjens, O. M. Bilaniuk, and M. H. Macfarlane, Phys. Rev. **120**, 527 (1960).
[11] J. Vervier, Nucl. Phys. **26**, 10 (1961).

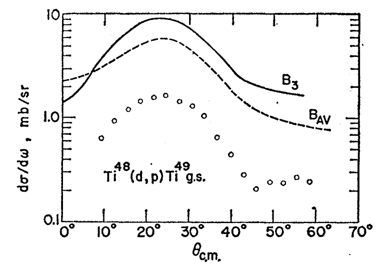

FIG. 9. Angular distribution of the ground-state transition of the $Ti^{48}(d,p)Ti^{49}$ reaction. The curves have been obtained from DWBA calculations for an $l=3$ transition. The curve B_3 was obtained with the "best fit" optical-potential parameters obtained from the elastic deuteron scattering at 21.4 MeV from Ti^{48} and has been adjusted for a strength of 1 and a transition to a state with spin $\frac{7}{2}^-$. The curve labeled B_{AV} was obtained from an "averaged set" of parameters which was obtained from the analysis of the elastic deuteron scattering from Ti^{46}, Ti^{47}, Ti^{48}, Ti^{49} and Ti^{50}. This curve has been adjusted to a strength of 0.7 for a transition to a state with spin $\frac{7}{2}^-$.

D. $Ti^{47}(d,p)Ti^{48}$

The spectrum of the $Ti^{47}(d,p)Ti^{48}$ at 24° lab is shown in Fig. 10. The transition to the ground state is not observed. The transition to the 0.99-MeV state of Ti^{48} has a Q value of 8.40±0.05 MeV. The Q values reported in Refs. 5 and 7 are 8.14 MeV. Hansen[10] reports a Q value of 8.44 MeV. The transition to the ground state has an intensity of less than 3% of the one to the 2^+ state, if present at all. Transitions to states near 2.3, 3.3, and 4.3 MeV are observed. The only angular distribution which permits a unique l-value assignment, $l=3$, is the one to the 0.99-MeV state. From the distorted-wave calculation, the strength of the transition is 1.1. The group near 2.4 MeV contains contributions from both the 2.30- and 2.43-MeV levels. The group near 3.2 MeV contains contributions from the 3.16-, 3.25-, and 3.40-MeV levels with the major contribution from the 3.25-MeV level. There are indications for levels at 4.35, 4.68, 4.83, and 5.56 MeV.

E. $Ti^{46}(d,p)Ti^{47}$

The spectrum for the $Ti^{46}(d,p)Ti^{47}$ reaction is shown in Fig. 11. The resolution is not good enough to estimate

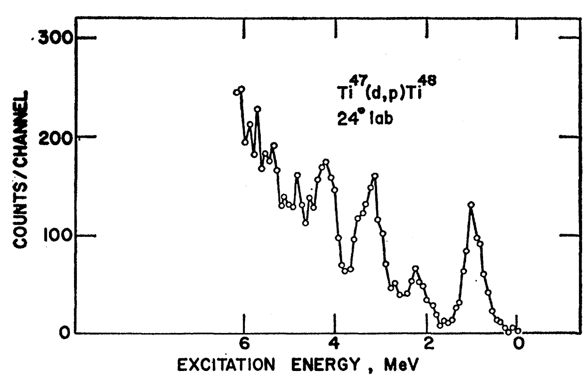

FIG. 10. Spectrum of the $Ti^{47}(d,p)Ti^{48}$ reaction at 24° lab.

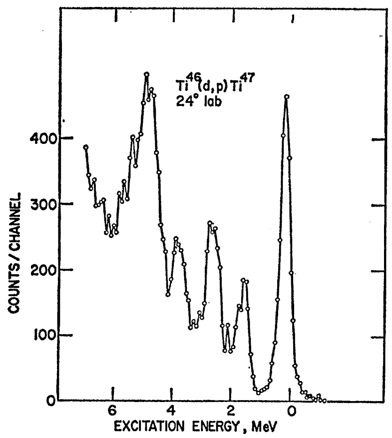

FIG. 11. Spectrum of the $Ti^{46}(d,p)Ti^{47}$ reaction at 24° lab.

is a $\frac{3}{2}^-$ level and the 1.80 level is a $\frac{1}{2}^-$ level and if one uses the intensity ratio from Ref. 10, then the strength for the 1.56-MeV level becomes 0.39 and for the 1.8-MeV level 0.31. It is clear that the 2.6-MeV group does not have an $l=1$ angular distribution. If we subtract from this group the $l=1$ contributions that might be expected from the transitions to the 2.58- and 2.81-MeV states, the remainder corresponds reasonably well to an $l=3$ angular distribution which at its maximum has a cross section of about 70±5% of the cross section to the 160-keV level of Ti^{47}.

The transition to the 2.15-MeV level is clearly present. The group near 3.8 MeV contains contributions from the 3.71- and 3.93-MeV levels. The group between 4 and 5 MeV contains contributions from the 4.35-, 4.43-, 4.68-, 5.06-, 5.31-, and 5.56-MeV levels.

a reliable upper limit for the transition to the ground state. The transition to the 160-keV level is obviously much stronger than the ground-state transition. Other groups are observed to states near 1.6, 2.6, 3.9, and 5.0 MeV. It is clear that the transition reported in Ref. 7 as the ground-state transition cannot be the ground-state transition but is the one to the 160-keV level. There is no indication of a level in the neighborhood of 600 keV. Such a level was reported by Rietjens *et al.*[10] but not by Hansen.[9] The group near 1.6 MeV has a shoulder at about 1.85 MeV. These states presumably are the strongly excited $l=1$ transitions reported at 1.56 and 1.80 MeV. The group near 2.6 MeV appears to be composed of at least three strong components. Rietjens *et al.*[11] report fairly strong transitions at 2.58 and 2.81 MeV. The group near 3.9 MeV shows indications of a level near 3.7 MeV. The angular distributions of the 160-keV, 1.6-, and 2.6-MeV groups are shown in Fig. 12. The comparison of the distorted-wave calculation with the experimentally obtained cross section gives a strength of 0.37 for this transition. A comparison of the angular distribution of the 1.6-MeV group with the ones shown in Fig. 2 shows that this group has a typical $l=1$ angular distribution. If both levels are assumed to have a spin of $\frac{3}{2}^-$, the resulting strength is 0.54. If one assumes that the 1.56-MeV level

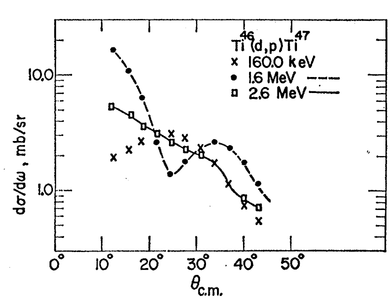

FIG. 12. Angular distribution of the $Ti^{46}(d,p)Ti^{47}$ transitions to the 160-keV level and the groups at 1.6 and 2.4 MeV. The curves have been drawn through the experimental points. At 39°, the 1.6-MeV point coincides with the 160-keV point. At 33° and 36°, the 2.6-MeV points coincide with the 160-keV points.

IV. DISCUSSION

It is clear that the $l=1$ and $l=3$ angular distributions are sufficiently different at an incident-deuteron energy of 21 MeV to permit unambiguous assignments of l values in the (d,p) reaction in a region where other l values do not occur. Furthermore, it appears that the intensity of $l=3$ transitions relative to that of $l=1$ transitions is higher at 21 MeV than at lower energies. The distorted-wave calculations seem to be in reasonable agreement with the experimentally observed angular distributions over the limited angular range used in this experiment. The strengths calculated from the "averaged potential" curves appear usually to be within 10% of the one computed with the curves computed with the "best fit" parameters of the deuteron optical-model potential. In the one case in which it seems reasonable to assume that the strength of the transition is known, i.e., the $Ti^{50}(d,p)Ti^{51}$ ground-state transition, the agreement between experiment and calculation appears rather satisfactory. There would, therefore, appear to be a reasonable probability that the same set of distorted-wave calculations can be used to estimate the $p_{3/2}$ and $p_{1/2}$ strength in those cases in which there are no $2p$-wave neutrons in the ground-state configuration of the target nucleus and in which the Q values of the transition is quite comparable to that of the ground-state transition of $Ti^{50}(d,p)Ti^{51}$. Such a situation exists in the transitions in the 1.6-MeV excitation region of both $Ti^{46}(d,p)Ti^{47}$ and $Ti^{48}(d,p)Ti^{49}$. The Q values in those cases are approximately 5.2 and 4.3 MeV, compared with a Q value of 4.1 MeV for the Ti^{50} case. From the results given in the previous section, it would then follow that the total $p_{3/2}$ strength in $Ti^{48}(d,p)Ti^{49}$ is about 0.5 if all of the $l=1$ transitions are assumed to correspond to the capture of $p_{3/2}$ neutrons and about $\frac{1}{3}$ if one assumes that the level with the highest Q value is the $p_{3/2}$ level and the other the $p_{1/2}$ level. This result is rather different from the conclusion of Ref. 10.

On the assumption that the states would have good seniority, Macfarlane and French[12] have predicted marked fluctuations in transition strength in the ground-state transitions near the end of the $f_{7/2}$ shell. The predicted strengths for the $Ti^{46}(d,p)Ti^{47}$, $Ti^{48}(d,p)Ti^{49}$, and $Ti^{49}(d,p)Ti^{50}$ are $\frac{1}{2}$, $\frac{1}{4}$, and 8, respectively. Since the ground-state spin of Ti^{47} is $\frac{5}{2}^-$, we assume that in the reaction on Ti^{46} the strength to be considered is that of the transition to the 160-keV state rather than to the ground state. The experimental results given above were 0.37, 0.19, and 5.5. The ratios of the strengths are seen to be in excellent agreement with the predictions of Macfarlane and French. The differences in absolute value might be considered to be rather minor if one considers the present state of the theory. Although the spin factors would lead one to expect fluctuations in cross section near the end of a shell, these are compensated to a large extent by the fluctuations in strength. However, such a compensation should not occur near the beginning of the shell. It should be pointed out that the predictions of Macfarlane and French are based on the assumption that the ground state of Ti^{49} and the 160-keV state of Ti^{47} have seniority 1. However, the (d,t) reactions on the titanium isotopes indicated strongly that this is not the case and, in particular, that the ratio of the strength of the $Ti^{49}(d,p)Ti^{50}$ reaction to that of the $Ti^{48}(d,p)Ti^{49}$ reaction should be considerably smaller than the 29:1 ratio obtained from the distorted-wave calculations.

This ratio of 29:1 for the strength of the $Ti^{49}(d,p)Ti^{50}$ relative to that of the $Ti^{48}(d,p)Ti^{49}$ reaction should be compared with the ratio of about 17.3:1 obtained by comparing the (d,t) reactions on Ti^{50} and Ti^{49} with distorted-wave calculations in which the neutron was assumed to be picked up from an orbital with a binding energy which was taken to be the same in both nuclei, instead of being adjusted for the difference in Q value as in the (d,p) calculations. A ratio of about 19.5:1 was also obtained by Lawson and Zeidman[13] who used a model which approximates the residual nucleon-nucleon interaction. The calculated strength of 0.19 for the $Ti^{48}(d,p)Ti^{49}$ ground-state transition is smaller than the value of 0.31 obtained from the (d,t) reaction on Ti^{49} or the value of 0.25 calculated by Lawson and Zeidman. The strength of 5.5 in $Ti^{49}(d,p)Ti^{50}$ is considerably larger than the 4.80 obtained theoretically or the 4.20 found experimentally for Ti^{50} $(d,t)Ti^{49}$. The uncertainties involved in the calculation of strength from distorted-wave calculations have been discussed in some detail by Satchler and it is quite clear that differences of $\pm 25\%$ are readily obtained. Therefore, the differences between the theoretically expected strength and the one obtained from the distorted-wave calculations for a given transition are not necessarily significant. On the other hand, the discrepancies between *ratios* of transition strengths would be expected to be considerably smaller. However, there is a rather large discrepancy between the 29:1 ratio obtained from the (d,p) reaction with the assumption that the neutron binding energy is given by $B=Q+2.3$ MeV and the 17.3:1 ratio obtained from the (d,t) results with the assumption that the neutron binding energy is a constant. If the binding energy associated with the neutron orbit is equated to the separation energy $B=|Q|+6$ MeV in the distorted-wave calculations for (d,t) reactions, then the ratio of the corrected cross sections for the (d,t) reactions on Ti^{50} and Ti^{49} becomes approximately 27:1. Unfortunately, one then cannot obtain a reasonable internal consistency in the titanium (d,t) data nor obtain a reasonable value for the strength. It would, therefore, appear that this comparison indicates that the Q-value dependence of the strength used in the (d,p) distorted-wave analysis is too strong. Since the values of the strength of the $p_{3/2}$ and $p_{1/2}$ transitions in $Ti^{46}(d,p)Ti^{47}$ and $Ti^{48}(d,p)Ti^{49}$ were obtained for approximately the same Q value as the ground-state transition of the $Ti^{50}(d,p)Ti^{51}$, for which reasonable agreement was shown to exist, these values should be somewhat more reliable.

The admixture of $l=3$ transitions in $Ti^{49}(d,p)Ti^{50}$ reactions to the 2^+ and 4^+ states are in rather good agreement with the predictions of Lawson's model, if one assumes that they correspond to the capture of $f_{7/2}$ neutrons. The $l=3$ component of the transition to the 2^+ state is approximately an order of magnitude larger than would be expected for $f_{7/2}$ neutron capture if the seniority of the Ti^{49} ground state were 1. It would appear improbable that this $l=3$ transition corresponds to $f_{5/2}$ neutron capture since in $Ca^{48}(d,p)Ca^{49}$ this transition proceeds to states with an excitation energy considerably higher than 1.56 MeV. The sums of these three $l=3$ transition strengths is 8 ± 1 compared with the value of 8 expected for the $f_{7/2}$ strength. Although there is some uncertainty about the intensity of the $l=3$ component to the 2.5-MeV state in $Ti^{48}(d,p)Ti^{49}$, it seems that the intensity of the $Ti^{46}(d,p)Ti^{47}$ $l=3$ transition to the 2.5-MeV group can be estimated reasonably well. This intensity is observed for transitions to states at about the same energy as the $l=3$ transitions found in $Ti^{48}(d,t)Ti^{47}$ and the ratio of the cross sections is also quite comparable. It would, therefore, seem likely that this $l=3$ component proceeds to $\frac{7}{2}^-$ states in Ti^{47}.

ACKNOWLEDGMENTS

I wish to thank G. R. Satchler and M. H. Macfarlane for helpful discussions and W. Ramler and the cyclotron group for their cooperation. I am indebted to W. J. O'Neill for technical assistance and to J. Van Zytveld for his help in the numerical computations.

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